Solutions to Quiz 1

Question 1

A signal has frequency components from 0 to 15 kHz. What is the minimum sampling rate that should be used?

The minimum sampling rate should be twice the highest frequency (or bandwidth for a bandpass signal). In this case it's $f_s \geq 2 \times 15 = 30$ kHz.

Question 2

After quantizing a signal, the quantization SNR was found to be 60 dB. Approximately how many bits were used to quantize each sample?

The quantization SNR in dB is approximately given by $6B$ where B is the number of bits. Thus $B = 60/6 = 10$ bits.

Question 3

The value 0x1F7A (hexadecimal notation) is transmitted in little-endian byte order with the most-significant bit (msb) first. What are the first four bits transmitted?

If the bytes are transmitted in little-endian byte order the least-significant byte, 0x7, is transmitted first. In binary this is `0111 1010`. If these bits are transmitted msb first, the first four bits transmitted are `0111`.

Question 4

The Unicode code point for the CYRILLIC CAPITAL LETTER ZHE (Ж) is U+0416 (0x0416 in hexadecimal notation).

1. *How many bytes would be transmitted if this character were encoded using UTF-8?*

   In binary the code point is `0000 0100 0001 0110`. Since the most significant five bits are zero it can be UTF-8 encoded using the second row of the table with two (2) bytes.

2. *What are the values of the 'x', 'y' and 'z' bit fields defined in Table 3-6 "UTF-8 Bit Distribution" of the Unicode standard? Omit any fields that do not apply.*

   Referencing the field definitions, there are no z bits, $y =$`100 00`, and $x =$`01~0110`.

3. *What are the values of the bytes of the UTF-8 encoding in hexadecimal notation?*

   Referencing the encoding rules, the first byte is `110 10000` or 0xD0 and the second byte is `10 01 0110` or 0x96.

Question 5

A communication system transmits three different messages with probabilities 0.25, 0.5 and 0.25.

1. *How many bits of information are transmitted by each message?*

   The number of bits of information in each message is given by $-\log_2(P)$. For the messages with $P = 0.25 = 1/4$, $I = -\log_2(1/4) = 2$ bits For those with $P = 0.5 = 1/2$, $I = -\log_2(1/2) = 1$ bits.

2. *What is the entropy of this system in units of bits per message?*

   $H = 0.25 \times 2 + 0.5 \times 1 + 0.25 \times 2 = 1.5$ bits/message.

3. *If the system transmits 10,000 messages per second, what is the information rate in bits per second?*

   The information rate is the product of the entropy and the message rate: $1.5 \times 10^4 = 15000$ bits/second