ELEX 3525 : Data Communications
2020 Fall Term

Noise

This chapter describes noise and how to compute noise voltage probabilities for Gaussian noise.
After this chapter you should be able to: compute SNR and compute the probability that a Gaussian source will lie within a certain range.

Noise and SNR

Noise is a unpredictable ("random") signal that is added to the desired signal. Noise can be added by the channel or by the receiver.

Some sources of noise include the thermal noise that is present in any resistor at temperatures above 0 K, "shot" noise generated by semiconductor devices, electrical equipment such as motors and some lights, lightning and the sun.

Noise is the phenomenon that ultimately limits the performance of any communication system. Noise may cause errors in digital communication system or degrade the quality of an analog signal.

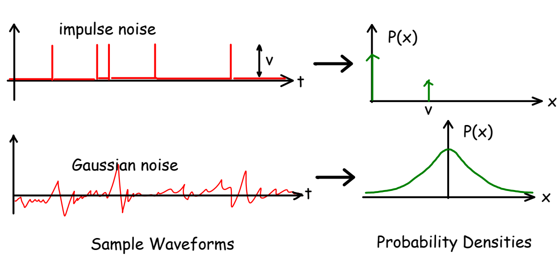
One important metric is the signal-to-noise ratio (SNR) which is the ratio of signal power to noise power.

Exercise 1: A sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

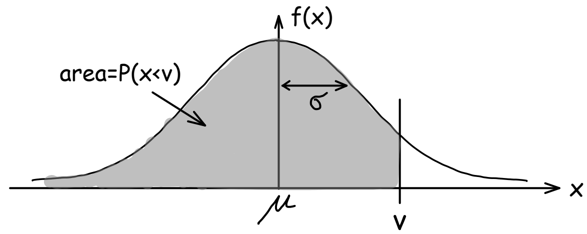
Nearby machinery is adding noise onto the line. The voltage of this noise signal is measured with an RMS voltmeter as 100mVrms. The characteristic impedance of the line is 600Ω and it is terminated with that impedance. Why was an RMS voltmeter used? What is the signal power? What is the noise power? What is the SNR?

Gaussian Probability Distribution

In addition to the distribution of the noise power in frequency, we also need to know how the voltage is distributed. For example, impulse noise has only two voltage levels (zero and the peak value of the impulse).



Signals that result from the sum of many small independent events have a probability distribution known as a Gaussian distribution. In communication systems this usually happens due to the sum of of voltages produced by the actions of very many individual photons, electrons or molecules.



The Gaussian distribution is the familiar "bell curve" or "normal" distribution. The probability is a maximum at the average value and drops off to smaller probabilities at larger or smaller voltages.

The Gaussian distribution is determined by two values: the mean (μ) and the variance (σ^2).

It's often useful to know the probability that the voltage of a Gaussian noise signal will exceed a certain voltage. If the noise signal x has a DC (mean) value μ and an RMS *AC* (zero-mean) voltage σ then the probability that the noise voltage is less than v is given by the Gaussian (Normal) cumulative distribution function (CDF). This is the area under the Gaussian distribution curve to the left of (less than) the value v.

Exercise 2: Would you use AC or DC coupling to measure: (a) σ, (b) μ, and (c) the RMS power? Would you measure the average or RMS power in each case? What is the RMS power of the signal x if it has a mean (DC) value of $\mu = 2$ V and $\sigma = 3$ V?

The plot in Figure 1 shows the shape of the Gaussian density function and also gives the cumulative probabilities along a second x-axis.

To find the probability that the voltage is greater than v we can use the fact that the sum of all probabilities is 1. Thus $P(x > v) = 1 - P(x \leq v)$.

To compute $P(v)$ we first compute a normalized value, t by subtracting the mean, μ, and dividing by

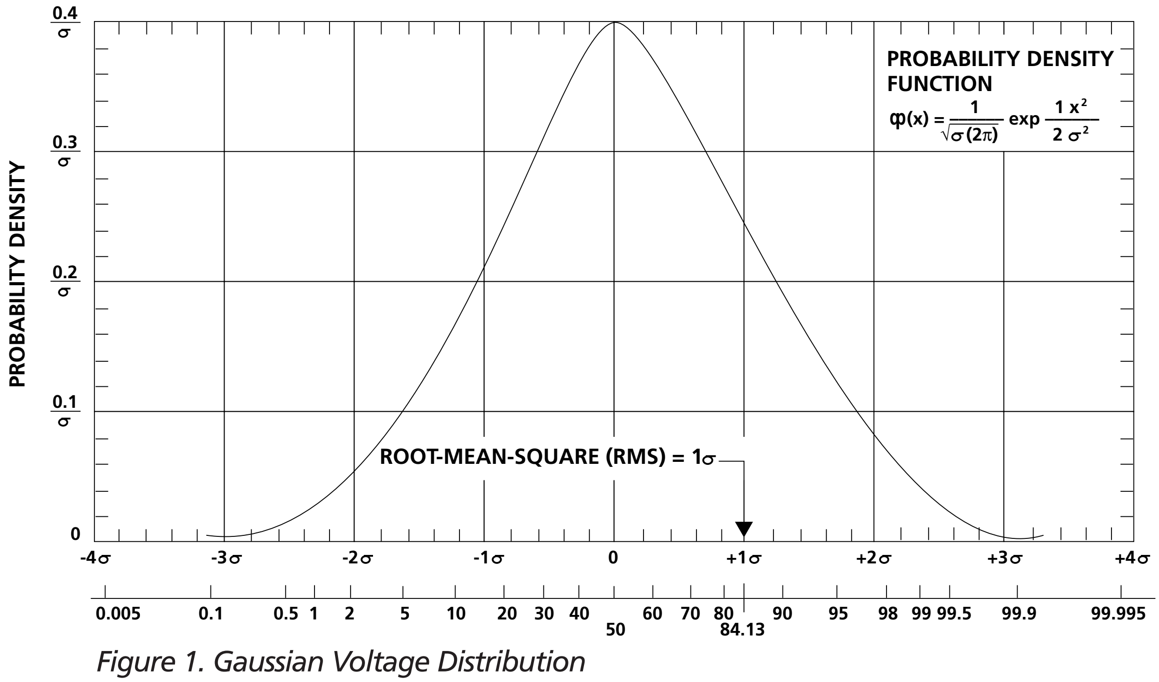


Figure 1: Gaussian density function and values of the cumulative distribution.

the standard deviation, σ, of the distribution:

$$t = \frac{v - \mu}{\sigma}$$

Exercise 3: What are the units of t?

Exercise 4: The output of a noise source has a Gaussian (normally) distributed output voltage. The (rms) output power is 20mW and the output impedance is 100Ω. What fraction of the time does the output voltage exceed 300mV? Hint: the variance (σ^2) of a signal is the same as the square of its RMS voltage.

Some calculators will compute this ($P()$ function) or you can use the figure above.