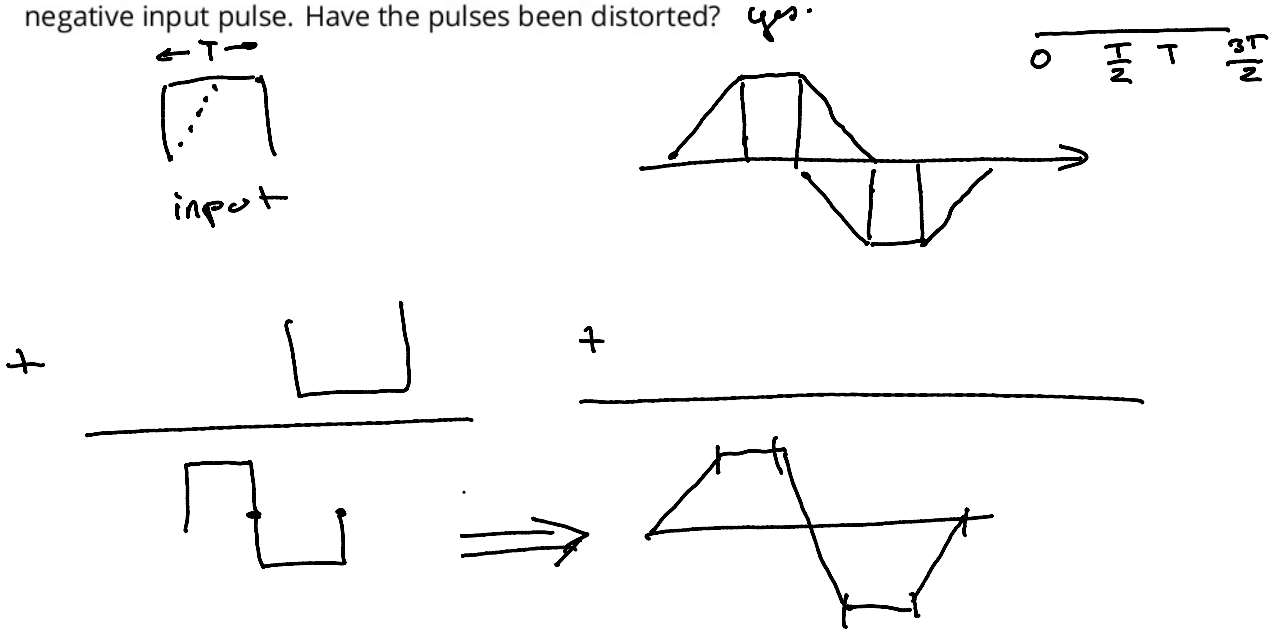
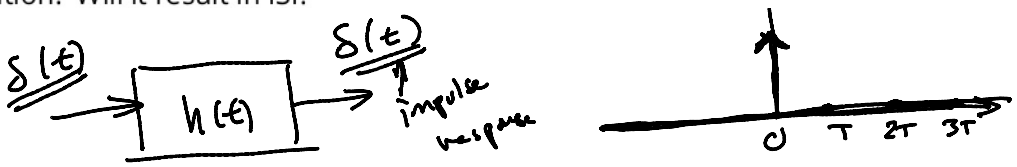
Data Transmission over Bandlimited Channels

Exercise 1: Draw a square pulse of duration T. Draw the pulse after it has passed through a linear low-pass channel that results in rise and fall times of $T/2$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse. Have the pulses been distorted? yes.



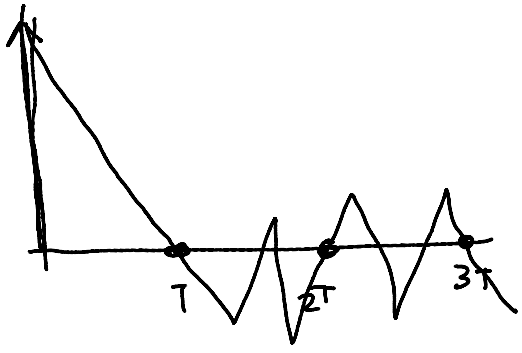
Exercise 2: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?



Yes it is zero at multiples of T.

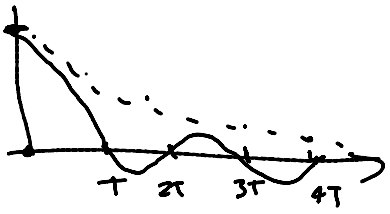
No ISI.

Exercise 3: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines.



Exercise 4: Why does the sinc() function have periodic zero-crossings? yes at multiples of T
Why does the amplitude decay?

$$h(t) = \text{sinc}(\frac{t}{T}) = \frac{\sin(\pi t/T)}{\pi t/T}$$

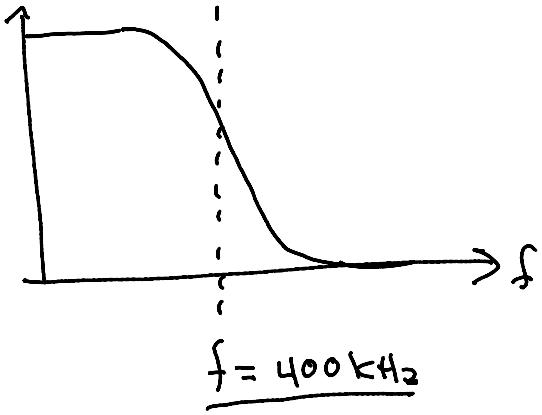


at $t = T$ $\sin(\pi T/T) = 0$

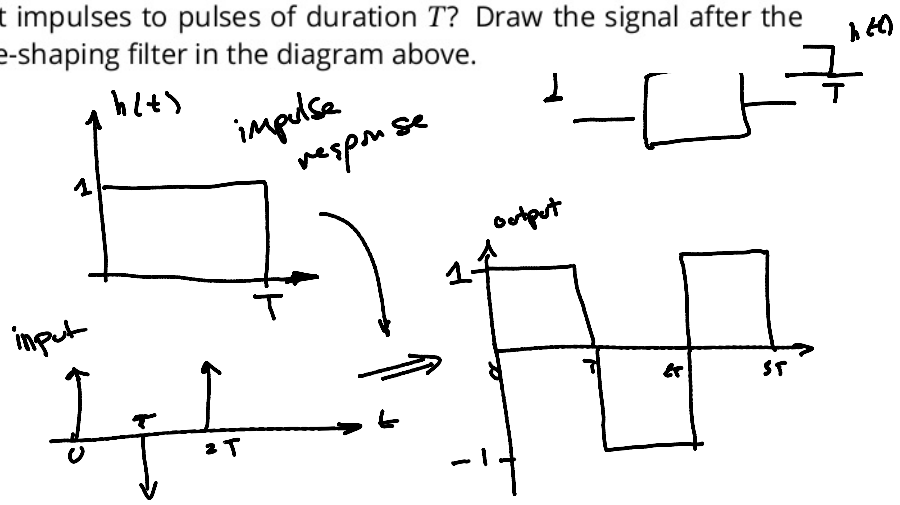
$t = 2T$ $\sin(2\pi) = 0$

$\vdots$

Exercise 5: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses. – f_{symbol}.

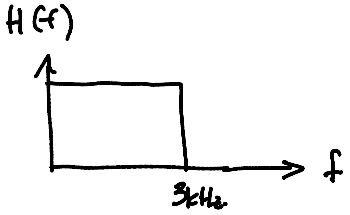


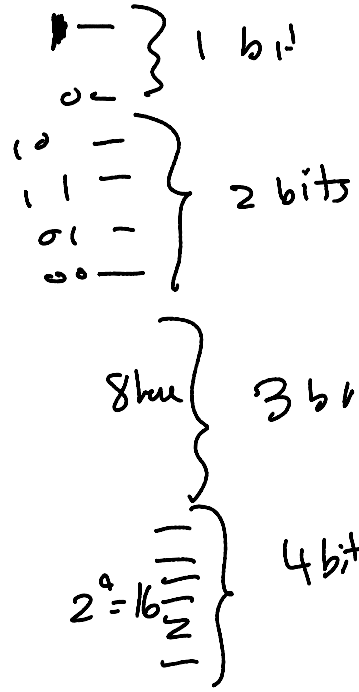
Exercise 6: Draw the impulse response of a filter than converts input impulses to pulses of duration T? Draw the signal after the pulse-shaping filter in the diagram above.



brick wall

Exercise 7: A channel has a 3 kHz bandwidth and meets the Nyquist non-ISI conditions with ~~[illegible]~~ How many levels are required to transmit 24 kb/s over this channel using multi-level signalling?





what is f_{symbol} for no ISI?

$f_{symbol} = 6$ kHz

24 kbps @ 6k symbols/sec.

24000 bits/second · 6000 symbols/second

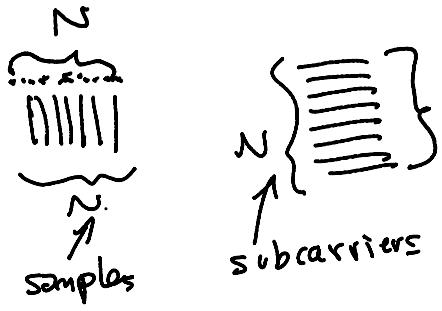
bits/symbol = ? $\frac{24000}{6000} = 4$ bits/symbol

need $2^4 = 16$ levels for 4 bits/symbol.

Exercise 8: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

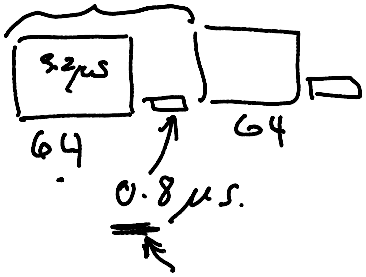
$f_s = 20 \times 10^6$ samples/s

$N = 64$

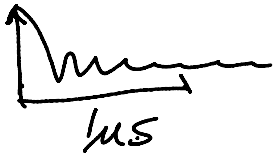


$\frac{64 \text{ samples}}{20 \times 10^6 \text{ samples/s}} = 3.2\mu s$.

$? = 3.2 + 0.8 = 4\mu s$



$0.8\mu s \cdot 20 \times 10^6 \frac{\text{samples}}{\text{sec}} = 16$ samples



Exercise 9: What is capacity of a binary channel with a BER of $\frac{1}{8}$ (assuming the same BER for 0's and 1's)? *Hint:* $log_2(\frac{7}{8}) \approx -0.2$.

$\log_2 \frac{1}{8}$

$= \log_2 2^{-3}$

$= -3$

$$C = 1 - (-p \log_2 p - (1-p)\log_2(1-p))$$

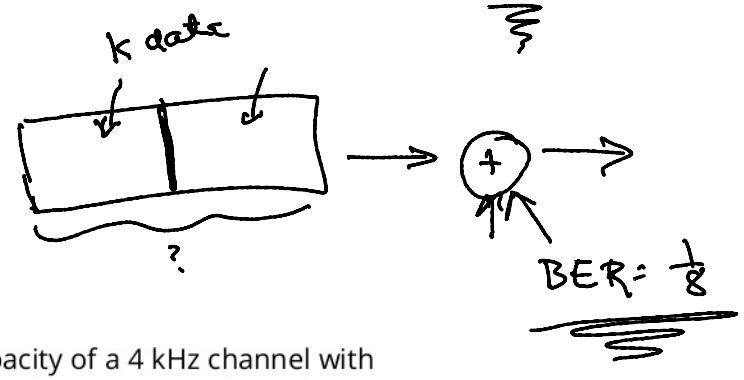
$$C = 1 - \left(\left(-\frac{1}{8}\right)(-3) - \left(1-\frac{1}{8}\right)(-0.2)\right)$$

$$= 1 - \left(\frac{3}{8} + \left(\frac{7}{8}\right)(0.2)\right)$$

$$= 1 - \left(\frac{3}{8} + \frac{1.4}{8}\right)$$

$$= 1 - \left(\frac{4.4}{8}\right) = \frac{3.6}{8}$$ bits per message

information bits per channel bit



Exercise 10: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 4000 \cdot \log_2\left(1 + 10^{\frac{30}{10}}\right)$$

$$\approx 4000 \cdot 10 \approx 40 \text{ kb/s}$$

Exercise 11: Can we use compression to transmit information faster than the (Shannon) capacity of a channel? To transmit data faster than capacity? Explain.

NO — compression doesn't change information.

YES - using compression.

Exercise 12: What do the Nyquist no-ISI criteria and the Shannon Capacity Theorem limit? What channel parameters determine these limits?

Nyquist - symbol rate for no ISI.
- depends on channel impulse response.

Shannon - information rate
- depends on channel:
  AWGN: bandwidth & SNR
  BSC: error rate.