Introduction to Digital Communication

Exercise 1: Give an example of a communication system. If you can, identify the source, transmitter, channel, receiver and destination.

	tx	Channel	rx
TV		→ free space	
cell phone		→ F/S	
Bluetooth		→ F/S	
RC		→ F/S	

email

Exercise 2: Give an example of a communication network. If you can, identify different the type(s) of channels used.

Ethernet – TX –□– RX

Internet –
- copper
- fibre optic
- free space

P.S.T.N. – twisted pair

Exercise 3: Speech is intelligible if you restrict the sounds to frequencies below about 4 kHz. What is the minimum sampling rate that should be used to sample speech ~~so that it will be intelligible~~?

max = 4 kHz

A signal-to-noise power ratio of about 48 dB is considered "toll quality" (the SNR conventional telephone networks provide). How many bits of quantization are required to obtain a quantization SNR equivalent to "toll quality" speech?

What if the signal was a video signal with a 5 MHz bandwidth and required a quantization SNR of 40 dB?

What are the resulting bit rates in the two examples above ?

$f_s \geq 2 \cdot$ max freq.

$\geq 2 \cdot 4\,kHz = 8\,kHz$

$SNR_{(quant.)} \approx 6 \cdot B$ (dB) $= 48$ dB.

$B = \frac{48}{6} = 8$ bits

in telephony systems: $8000 \frac{samples}{s} \cdot 8$ bits/sample $= 64k$ bits/s

for 5 MHz bandwidth & 40 dB quant. SNR

$f_s = 10$ MHz

$B = 7$ bits

overall bit rate = 70 Mb/s.

Exercise 4: Write the sequence of bits that would be transmitted if the 16-bit value 525 was transmitted with the bytes in little-endian order and the bits lsb-first. Write the sequence of bits that would be transmitted in "network order" and the bits msb-first.

$525_{10} = 20d_{16}$

02	0d

0d 02

0000 1101 0000 0010

bit: 1011 0000 0100 0000 → time

02 0d

0000 0010 0000 1101 → time

Exercise 5: How many bits would be required to uniquely identify 100,000 different characters? (Hint: $2^{16} = 65536$).

$2^{17} = 2 \times 65535 > 100,000$

so 17 bits is enough.

Exercise 6: The Chinese character for "Rice" (the grain) is □□ with Unicode value (code point) U+7C73. What is the UTF-8 encoding for this character?

7C73 → 0111 1100 0111 0011 (7 C 7 3; z | y | x)

米

Table 3-6. UTF-8 Bit Distribution

Scalar Value	First Byte	Second Byte	Third Byte	Fourth Byte
00000000 0xxxxxxx	0xxxxxxx			
00000yyy yyxxxxxx	110yyyyy	10xxxxxx		
zzzzyyyy yyxxxxxx	1110zzzz	10yyyyyy	10xxxxxx	
000uuuuu zzzzyyyy yyxxxxxx	11110uuu	10uuzzzz	10yyyyyy	10xxxxxx

1110 0111 ① E 7

10 11 0001 ② B 1

1011 0011 ③ B 3

U8 → E5 BC 84

→ 1110 0101 (z) 1011 1100 (y) 1000 0100 (x)

Unicode → 0101 1111 00 000100

5 F 0 4

Exercise 7: Convert the decimal number 525 to a 16-bit (two-byte) binary number. How would you write this in hexadecimal notation?

Find the ASCII codes for the *characters* '525'. Write out the bits of the sequence that would be transmitted assuming each character is encoded in UTF-8. *Hint: the UTF-8 character code for a digit is 0x30 plus the value of the digit.*

Which of these two sequences of bits is the text format and which is the binary format? How many more bits would need to be stored or transmitted for the text format?

$525_{10} = 20D_{16} = 0x20d$

binary needs **16 bits**

[5][2][5] = 0x35, 0x32, 0x35

text

text needs 3 X 8 = **24 bits**

Exercise 8: We observe a source that outputs letters. Out of 10,000 letters 1200 were 'E'. What would be a reasonable estimate of the probability of the letter 'E'?

fraction of events with that outcome.

$$\frac{1200}{10,000} = 12\%$$

Exercise 9: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$\log_2 \frac{1}{2} =$

$2^{-1} = \frac{1}{2} = 0.5$

$\log_2 2^x = x \qquad \log_2 2^{-1} = -1$

$I = -\log_2 \frac{1}{2} = -(-1) = 1$

$P_0 = 0.125 = \frac{1}{8} \rightarrow -\log_2 \frac{1}{8} = -\log_2 2^{-3} = 3$ bit

$P_1 = 0.125 = \frac{1}{8} \rightarrow = 3$ bit

$P_2 = 0.25 = \frac{1}{4} \rightarrow = 2$ bit

$P_3 = 1 - (0.25 + 0.125 + 0.125) = 1 - \frac{1}{2} = \frac{1}{2} \rightarrow 1$ bit

Entropy

$H = 3 \cdot \frac{1}{8} + 3 \frac{1}{8} + 2 \frac{1}{4} \cdot 1 \cdot \frac{1}{2}$

$= \frac{6}{8} + 1 = 1.75$ bits/message

Information Rate.

1.75 bits/msg · 100 msg/s = 175 bits/second

$P_0 = P_1 = P_2 = P_3 = \frac{1}{4}$ $(-\log_2 P_i)$

$I = 2$ bits/msg.

$H = 2$ bits/msg.

200 bits/second

B= bytes
b= bits.

Exercise 10: How long will it take to transfer 1 MByte at a rate of 10 kb/s?

1 MByte at 10 kb/S.

1 MBytes = 8×10^6 = 8 Mbits

$\frac{8 \times 10^6}{}$ ↑ 8 Mbits of data comm.

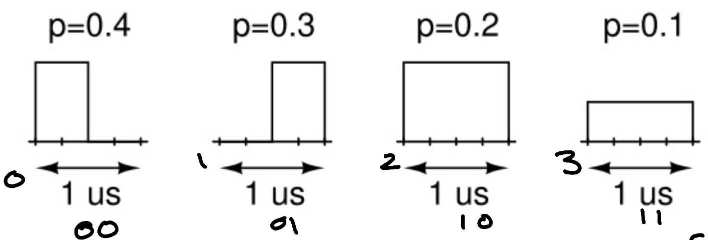
$\frac{8 \times 2^{20}}{}$ ↑ 8M bits of RAM

$\frac{8 \times 10^6}{10 \times 10^3} = 8 \times 10^2$

b / $\frac{b}{s}$ → s

800 seconds

Exercise 11: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, the information rate and the baud rate?



✓ symbol rate $= \frac{1}{1\mu s} = 10^6 =$ 1 M symbol/s

$I_0 = -\log_2(0.4)$

$\vdots$

$I_3 = -\log_2(0.1)$

$H = 0.4(-\log_2(0.4)) + 0.3(-\log_2 0.3) + 0.2(-\log_2 0.2) + 0.1(-\log_2 0.1)$

$= 1.82$

✓ information rate $= H \cdot 1 \times 10^6$ ← bits/second of information.

$= \underline{\underline{1.82 \times 10^6}}$

with 2 bits can select 1 of $2^2 = 4$ symbols.

∴ transmitting 2 bits of data per symbol

data rate is $\underline{\underline{2 \text{ Mb/s}}}$.

Exercise 12: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). Assuming each system transmits at 1000 bits per second, what are the baud rates in each case? How many different symbols are used by each system? What are the symbol rates? Assuming each symbol is equally likely, what are the information rates?

Exercise 13: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

Exercise 14: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?

Exercise 15: Plot some sample data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)

Exercise 16: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, “texting”, downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?

Exercise 17: Highlight or underline each term where it is defined in these lecture notes.