Solutions to Assignment 3

Question 1

(a) For $k = 4 \times 4 = 16$ data bits, there will be four rows and four columns and thus $n - k = 4 + 4 = 8$ parity bits[1] and $n = 16 + 8 = 24$. The code rate will be $k/n = 16/24 = 2/3$.

(b) Any error in a row or column, including the parity row or column, will cause that row or column to have the opposite parity and thus errors in parity bits will be detected.

(c) If there are three errors in one row (but not two or four) then the parity check for that one row will fail and the column parity checks for those columns will also fail. The three errors can be corrected by inverting all of bits in that row that also fail their column parity checks.

Three errors in one column can be corrected in a similar way.

Note that errors that are not restricted to one row or column (e.g. along a diagonal) do not uniquely constrain the error locations and these error patterns cannot be corrected.

Question 2

(a) Since the generator polynomial is of order 1 (two bits), the length of the CRC is 1 bit.

(b) The remainder of dividing the message by the generator polynomial is 0 as can be found by long division as shown in Figure 1.

(c) With a generator polynomial of order 1 there is one bit in the partial remainder at each step of the division (or two if you include the leading zero).

(d) If the two bits being considered in a step of the long division are **ab**, then:

```
    -------
11 |1010110
    11
    --
     11
     11
     --
      00
      --
       01
       --
        11
        11
        --
         00
         --
          0
```

Figure 1: Long division for Question 2(b).

- if **a** is 1 then the next partial remainder is **a'** $= \mathbf{b} \oplus 1 =$ NOT **b**,
- if **a** is 0 then the next partial remainder is **a'** $=$ **b**.

(e) If we write out the truth table:

		a	
	a′	0	1
b	0	0	1
	1	1	0

we can recognize this operation as **a'** $=$ **a** $\oplus$ **b** (or **a'** $=$ **a** XOR **b**).

(f) At each step of the division each partial remainder is computed as the XOR of the previous partial remainder and the next bit. The final partial remainder is thus the XOR of all of the bits.

This is *even* parity because the number of '1' bits, including the parity bit, is an even number.

[1] It's also possible to have an additional parity bit for the parity row and column in which case the code rate is $16/(16+9) = 16/25 = 0.64$.

Question 3

(a) A channel with a low-pass flat frequency response (also called a "brick wall" response) is an example of a channel that meets the Nyquist no-ISI criteria.

The maximum symbol rate (in Hz) that can be transmitted without ISI over a channel with a -6 dB bandwidth of B is $2B$.

The -6 dB bandwidth of this channel is 1 MHz. Therefore over this channel we can transmit at a symbol rate of $2B = 2$ MHz without ISI.

(b) Since any number of bits can be transmitted per symbol, there is no maximum data rate that can be transmitted over this channel without ISI.

(c) For an AWGN channel, the maximum information rate that can be transmitted with an arbitrarily low error rate is given by the channel capacity of $C = B \log_2(1 + S/N)$. In this case $B =$ 1 MHz and $S/N = 2$ (3 dB) so $C = 10^6 \log_2(1 + 2) \approx 1.6$ Mb/s.

Although the Shannon capacity theorem does not relate capacity to error rate, in practice exceeding the channel capacity results in a very high error rate (much higher than 10^{-15} as mentioned in the question). Thus one upper limit that we can specify with the information given is a maximum information rate of ≈ 1.6 Mb/s.