ELEX 3525 : Data Communications
2019 Fall Term

Solutions to Assignment 2

Question 1

The mean power of the sum of two signals, a and b, can be re-written as:

$$E[(a+b)^2] = E[a^2] + 2E[ab] + E[b^2]\,.$$

The expected value (mean) of a random variable x that takes on values x_i with probabilities P_i is:

$$E[x] = \sum_i P_i x_i\,.$$

In this question a and b can be constants ($P_i = 1$) or can take on two equally-likely values ($P_i = 1/2$). The expression within the expectation operator can thus take on one, two or four possible values.

(a) When a and b are constant (DC) voltages with $a = 2$ V and $b = 3$ V there is only one possible outcome.

The left-hand side (LHS) expression has the value $(a+b)^2 = (2+3)^2 = 5^2 = 25$ and the right-hand side (RHS) expression has the value $2^2 + 2(2\times 3) + 3^2 = 4 + 12 + 9 = 25$.

(b) When a is a constant 2 V and b is a random signal that is equally likely to have values +1 V and −1 V, then there are two possible outcomes: $a = 2, b = -1$ and $a = 2, b = +1$, each with probability $1/2$.

The LHS expression has the value $1/2(2-1)^2 + 1/2(2+1)^2 = 1/2 + 9/2 = 5$.

The RHS expression has the value $2^2 + 2\big(1/2(2\times -1) + 1/2(2\times 1)\big) + \big(1/2(1^2) + 1/2(-1)^2\big) = 4 + 2(-1+1) + (1/2 + 1/2) = 4 + 0 + 1 = 5$.

(c) When a and b are both independent random signals, each equally likely to have values +1 V and −1 V then there are four possible combinations of a and b, each with equal probability $P_i = 1/4$: $(+1,+1)$, $(+1,-1)$, $(-1,+1)$, and $(-1,-1)$.

The LHS expression is:

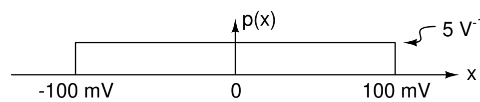
$$1/4(1+1)^2 + 1/4(1-1)^2 + 1/4(-1+1)^2 + 1/4(-1-1)^2 = 1 + 0 + 0 + 1 = 2\,.$$

The RHS expression is:

$$\begin{aligned} & 1/4(+1)^2 + 1/4(+1)^2 + 1/4(-1)^2 + 1/4(-1)^2 \\ & + 2\cdot 1/4\big((1\cdot 1) + (1\cdot -1) + (-1\cdot 1) + (-1\cdot -1)\big) \\ & + 1/4(+1)^2 + 1/4(-1)^2 + 1/4(+1)^2 + 1/4(-1)^2 \\ & = 1 + 0 + 1 = 2\,. \end{aligned}$$

Question 2

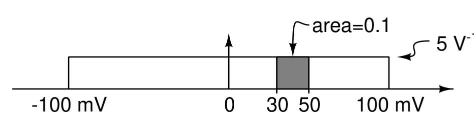
(a) Since the probability density function is constant, the graph has a constant value between −100 and +100 mV:



(b) The integral of this function (area under the curve) must be one. The area is also $0.2h = 1$ where h is the height of the curve. Then $h = 1/0.2 = 5$.

(c) If the units of probability are unitless and the x-axis has units of volts, this probability density must have units of 1/volts (V^{-1}).

(d) The probability is the integral of the curve between 30 mV and 50 mV and is $5\times(0.05-0.03) = 0.1$ (10%):



Question 3

(a) There are $2^9 = 512$ possible combinations of 0's and 1's in 9 bits and thus 512 possible error patterns, one of which is the no-error pattern.

(b) One of these patterns (0 0000 0000) contains zero errors.

(c) Nine of these patterns contain exactly one error (1 0000 0000 through 0 0000 0001).

(d) The number of patterns that contain two errors is the number of combinations of 9 bit positions taken two at a time (drawing two items from a collection of nine without replacement). The expression for this is $C(n, k)$ or $\binom{n}{k}$ where $n = 9$ and $k = 2$ and is equal to:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{9!}{2!7!} = \frac{9 \times 8}{2} = 36 .$$

(e) The probability of receiving a word with zero errors is the probability of receiving the error pattern with $n = 0$ errors which is:

$$P = 1 \times P_e^0 \cdot (1 - P_e)^9 = 1 \times 1 \cdot (0.99)^9 \approx 0.913 .$$

(f) The probability of receiving a word with one error is the sum of the probabilities of receiving the nine possible error patterns with $n = 1$ errors which is:

$$P = 9 \times P_e^1 \cdot (1 - P_e)^8 = 9 \times 0.01 \cdot (0.99)^8 \approx 0.083 .$$

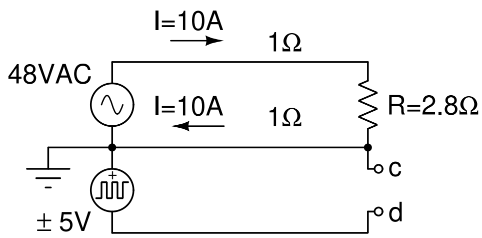
(g) The probability of receiving a word with two errors is the sum of the probabilities of receiving the 36 error patterns with $n = 2$ errors which is:

$$P = 36 \times P_e^2 \cdot (1 - P_e)^{(9-2)} = 36 \times 10^{-4} \cdot (0.99)^7 \approx 0.0034 .$$

From these values we can also calculate that the probability of 3 or more errors (i.e. not 0, 1 or 2 errors) is $1 - (0.913 + 0.033 + 0.003) \approx 4 \times 10^{-4}$.

Question 4

The following schematic shows the connection between the remote radio unit and the power and control unit:



The 200 m of 14-gauge cable has a resistance of 200 m × 5 mΩ/m = 1 ohm per conductor.

(a) To supply 480 W the RMS current on the power conductor must be $I = P/V = 480/48 = 10$ A. The RMS voltage between ground and point **c** in the schematic will be $V = IR = 10 \cdot 1 = 10$ V. The peak voltage will be $\sqrt{2}$ times larger or ≈ 14.1 V

(b) The data signal seen at the far end will be the voltage difference between **c** and **d**. Assuming no voltage drop in the data circuit, this voltage difference will be the sum of the ±5 V data signal and the 14.1 V noise voltage on the shared return conductor. Since the peak noise voltage is more than the noise margin of the data circuit (5 V), errors will be introduced and the data circuit will not operate properly.

Question 5

(a) **X**: This transition happens when the number of consecutive 1's increases by 1. Thus it indicates the reception of a '1' bit.

Y: This transition happens when the number of consecutive 1's is reset to 0. Thus it indicates the reception of a '0' bit.

A: This transition happens when the number of consecutive 1's changes from 5 to 0. Thus it indicates the reception of a '0' bit after 5 '1' bits. This is a "stuffed" bit.

B: This transition happens when the number of consecutive 1's goes from 5 to 6. Thus it indicates the reception of the final '1' bit of a flag sequence.

C: This transition happens when the number of consecutive 1's goes from 6 to 0, indicating the reception of a '0' bit after 6 or more consecutive '1' bits. This is either the end of the flag sequence or an error condition (if more than 6 consecutive '1' bits).

D: This transition happens when more than 6 consecutive 1's are received. This indicates an error (sometimes treated as an "abort" condition which causes the frame to be discarded).

(b) The receiver should ignore the "stuffed" received bit that causes the state transition labelled A.

Question 6

(a) If a receiver starts looking for the frame sync bit at an arbitrary position within the frame and both bit values are equally likely, the probability that it will find the expected bit value and thus not detect the loss of frame sync is $1/2$.

(b) The receiver will check the frame sync bit every 193 bits. The receiver will not detect frame sync for 8 consecutive frames if the bits being tested correspond to the expected frame sync bit value in all eight frames. The probability of this is $(1/2)^8 = 2^{-8} \approx 4 \times 10^{-3}$.

(c) The probability of not detecting sync loss for 40 consecutive frames is $2^{-40} \approx 1 \times 10^{-12}$.

(d) At 1.544 Mb/s it takes $\frac{40 \text{ frames} \times 193 \text{ bits/frame}}{1.544 \times 10^6 \text{ bits/second}} =$ 5 ms to transmit 40 frames.