Solutions to Assignment 1

Question 1

(a) Since we have no other information about the source other than the pixel probabilities, we treat each pixel as a message.

The 0-value pixel has a probability of $1/2$ and so the sum of the probabilities of the others must be $1 - 1/2 = 1/2$. Since each of the other 63 values is equally likely, the probability of each of these values must be $1/2/63 = 1/126$.

The entropy of a source is defined as:

$$H = \sum_i (-\log_2(P_i) \times P_i) \quad \text{bits/message}$$

$$= -\log_2\left(\frac{1}{2}\right) \cdot \frac{1}{2} + 63 \times \left(-\log_2\left(\frac{1}{126}\right) \times \frac{1}{126}\right)$$

$$\approx 0.5 + 3.5 = 4 \quad \text{bits/pixel}$$

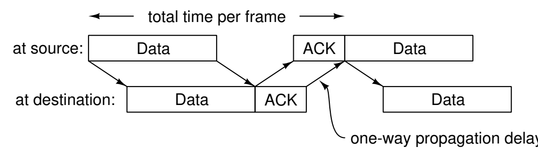
(b) Each frame of $1920 \times 1020 = 1\,958\,400$ pixels scanned every 16 ms (62.5 Hz) results in 122.4 Mpixels/s and an information rate of $122.4 \times 4 \approx 488$ Mbits/s.

Question 2

(a) Since the throughput for each frame-plus-acknowledgement pair is the same, we can compute the throughput based on the throughput of sending one frame and receiving its acknowledgement.

The payload of a frame is 1150 bytes. The total time required to transmit the frame, the 100 bytes of overhead and receive the 50-byte acknowlegement is $(1150 + 100 + 50)$ bytes $\times$ 8 bits/byte $\div$ 1×10^9 bits/second $= 10.4$ µs. The throughput is thus 1150/10.4 µs $= 110.6 \times 10^6$ bytes/s $= 885$ Mbps.

(b) The one-way free-space propagation delay over 300 m is: 300 m $\div$ 3×10^8 m/s $= 1$ µs. The two-way delay is thus 2 µs. If there is a delay between each frame equal to this, the time required to transmit one frame increases by 2 µs as shown below (not to scale):



The total time required to transmit each frame now increases to $10.4 + 2 = 12.4$ µs and the new throughput is 1150/12.4 µs $\approx 92.7 \times 10^6$ bytes/s $= 741$ Mbps.

Question 3

This is an example of many possible answers for this question.

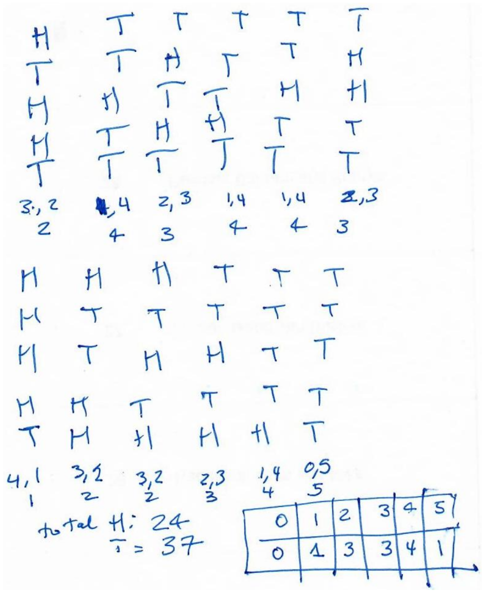
(a) The simplified Chinese character for the color blue (according to `https://translate.google.com`) is: 蓝.

(b) According to `https://www.unicode.org/cgi-bin/GetUnihanData.pl` the Unicode code point for this character is: `U+84DD`.

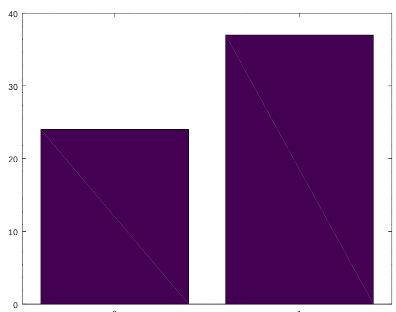
(c) The UTF-8 encoding is derived from the code point value as follows: the sequence `84 DD` has a binary representation `1000 0100 1101 1101` from which we can extract the bits: z=`1000`, y=`0100 11`, and x=`01 1101`, and derive the UTF-8 encoding in three bytes as: `1110 1000`, `10 010011`, and `10 011101` which is `0xE8`, `0x93`, `0x9D`.

Question 4

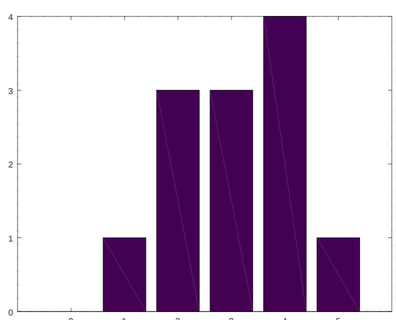
The results of flipping a coin 60 times is shown below (I wrote H and T instead of 0 and 1):



(a) The histogram for a random variable whose values are the 0 (H) and 1 (T) values above is:



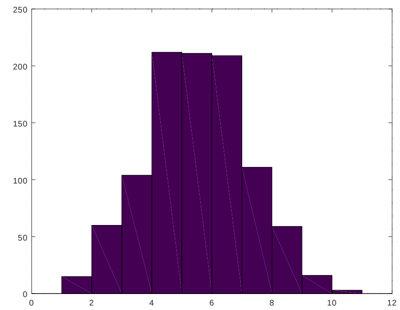
(b) The histogram for a random variable that is the sum of every five successive flips is:



(c) If we used a very large number of coin flips and summed a large number of 0/1 values the Central Limit Theorem says we would expect to observe a distribution similar to a Gaussian distribution.

To demonstrate this, the following Matlab code plots the histogram of the sums of 1000 columns each with 12 values:

```
x=rand(12,1000)>0.5;
hist(sum(x))
```



Question 5

(a) The quantization noise power is about $6B$ dB[1] less than the signal power so the signal-to-noise ratio is $6B$ dB. For $B = 24$ bits the quantization SNR would be $6 \times 24 = 144$ dB.

(b) For signal voltage of 1 Vrms, the signal power would be 1 W (into 1 Ω), the quantization noise voltage would be $1/10^{144/20} \approx 63$ nV. This is a very small voltage and 144 dB SNR is unlikely to be achieved in practice.

(c) Since the sampling rate must be twice the bandwidth in order to accurately reconstruct the signal, we could reconstructed frequencies from 0 (DC) up to $192/2 = 96$ kHz.

(d) The maximum frequency that humans can hear varies between individuals and decreases with age. The maximum is approximately 20 kHz, which is much less than the maximum frequency that this sound system is able to reproduce.

Question 6

(a) Since there are four symbols, each encodes $\log_2(4) = 2$ bits and since all are equally likely this is also the entropy (in bits/symbol).

[1] Note that 6B is the SNR in dB, not the ratio of the signal and noise powers.

(b) To compute an average rate we need to use the harmonic mean. An example would be computing the average speed for a trip driven at different speeds.

The mean symbol duration is $(2+3+4+5)/4 = 3.5$ ms and the mean symbol rate as $1/3.5\text{ ms} \approx 286$ Hz.

Note that computing the average of the three symbol rates $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})/4 \approx (0.5+0.33+0.25+0.2)/4$ kHz $= 320$ Hz gives the wrong answer.

(c) Since each symbol represents two bits of data, the average data rate is 2 bits/symbol $\times$ 286 symbols/s $=$ 571 bps.

(d) Since all symbols are equally likely (probability $1/4$), the number of information bits per symbol is also 2 ($4 \times 1/4 \times -\log_2 1/4 = 2$) and the information rate is the same as the data rate, 571 bps.

(e) The minimum time between level transitions would be 1 ms so the baud rate, according to the definition in the question, would be 1 kHz. If "baud rate" were defined as the symbol rate, the baud rate would the average symbol rate given above.