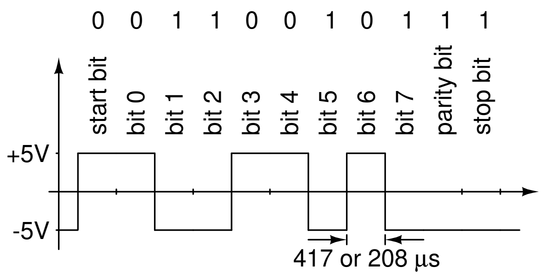
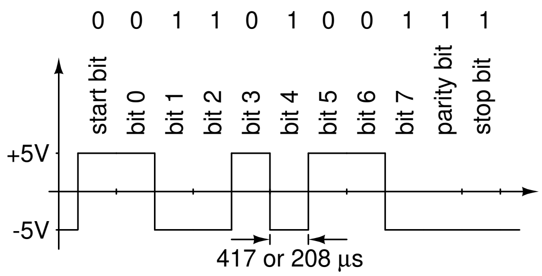
Solutions to Midterm Exam

Question 1

The waveform that would be used by a typical ansynchronous serial ("RS-232") interface to transmit the 8-bit values 0xA6 (`1010 0110` and 0x96 (`1001 0110`) using 8 bits per character odd parity with are shown below. The bits are transmitted in order from LS to MS bit with a logical '1' transmitted as a low voltage. The minimum voltage levels at the transmitter are ±5 V. The bit durations are the inverse of the bit rate, 417 µs for 2400 bps and 208 µs for 4800 bps.





Question 2

If the probability of a space is $25\% = \frac{1}{4}$ then the sum of the other message probabilities must be $1 - \frac{1}{4} = \frac{3}{4}$ and since all other message have the same probability, the probability of each is $\frac{3}{4}/6 = \frac{1}{8}$.

The entropy of a message source is defined as:

$$H = \sum_i (-\log_2(P_i) \times P_i) \quad \text{bits/message}$$

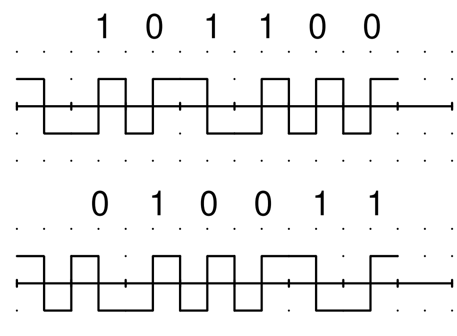
Where $P_i = \frac{1}{4}$ for spaces and $\frac{1}{8}$ for the digits. The entropy is thus:

$$H = -\log_2(\frac{1}{4}) \times \frac{1}{4} + 6 \times (-\log_2(\frac{1}{8}) \times \frac{1}{8})$$

$$= \frac{2}{4} + \frac{6 \times 3}{8} = \frac{22}{8} = 2.75 \text{ bits/message}$$

Question 3

The following waveform shows differential Manchester line code used to transmit the bits shown. The coding convention described in the lecture notes is that ones are encoded as a different waveform than the previous one and zeros as the same waveform.



Question 4

A channel adds zero-mean Gaussian noise with a variance of $\sigma^2 = 63\ \text{mV}^2 = 0.063\ \text{V}^2$, $\sigma = \sqrt{0.063} = 0.25$ V or $\sigma^2 = 28\ \text{mV}^2 = 0.028\ \text{V}^2$, $\sigma = \sqrt{0.028} = 0.167$ V to a signal. The receiver makes errors whenever the level of the noise exceeds $v = +0.6$ V or $v = +0.4$ V. We can compute the normalized threshold as $t = (v - \mu)/\sigma$ which is $t = 0.6/0.25 = 2.4$ or $t = 0.4/0.167 = 2.4$.

In both cases the probability that the voltage is less than the normalized threshold is $P(2.4)$ which can be obtained using a calculator or from the diagram in Lecture 4 as approximately 99.2%.

The error rate is the probability that the signal is *greater* than the threshold is $1 - P(2.4) = 1 - 0.992 \approx 0.8\%$.