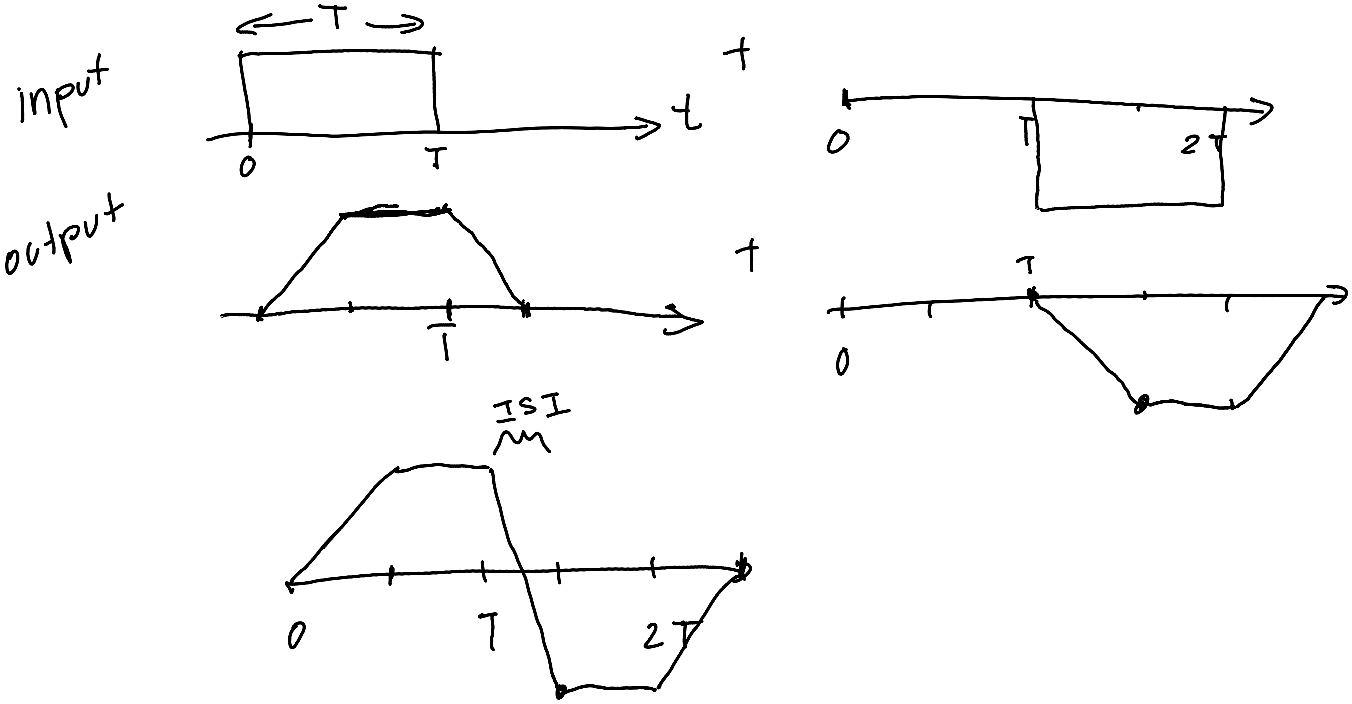
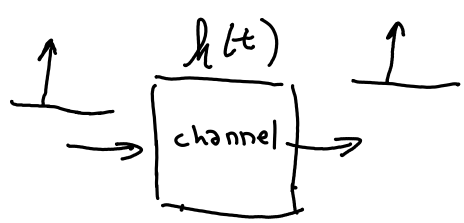
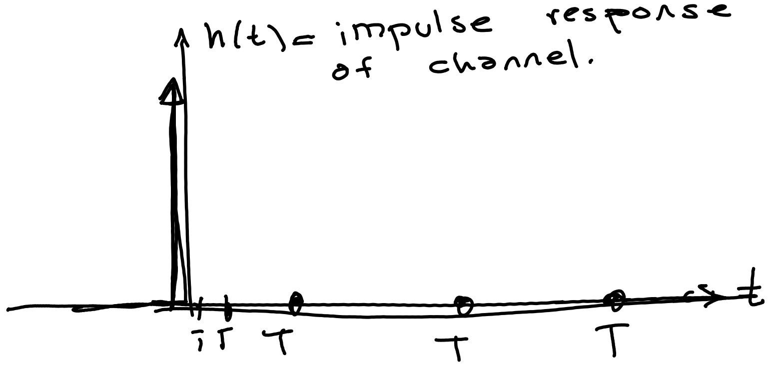
# Data Transmission over Bandlimited Channels

**Exercise 1**: Draw a square pulse of duration $T$. Draw the pulse after it has passed through a linear low-pass channel that results in rise and fall times of $T/2$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse. Have the pulses been distorted?



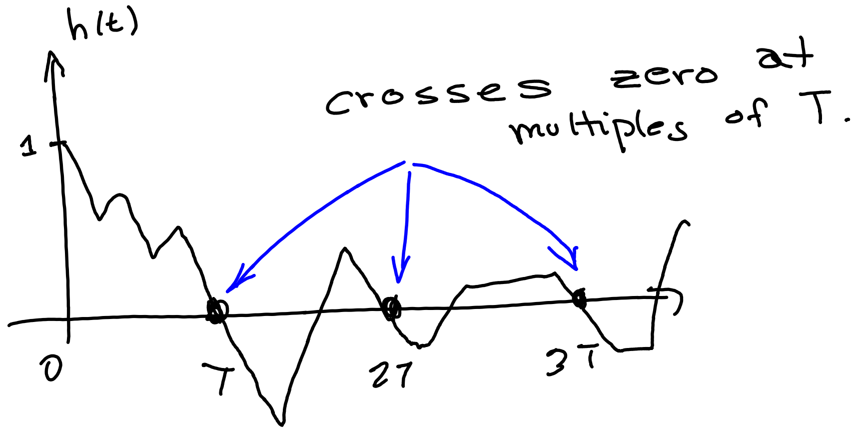

**Exercise 2**: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?




$$h(t) = \begin{cases} \text{impulse} & t = 0 \\ 0 & \text{elsewhere} \end{cases}$$

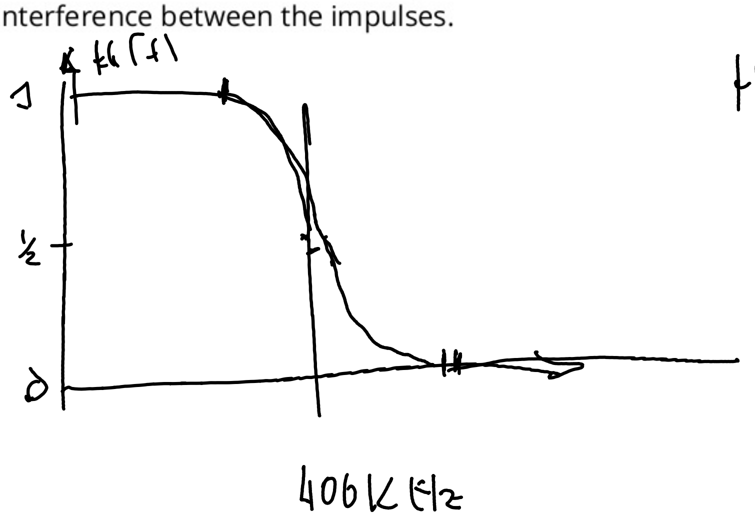
**Exercise 3**: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines.



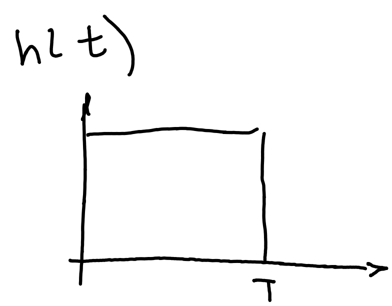

**Exercise 4**: Why does the sinc() function have periodic zero-crossings? Why does the amplitude decay?

- yes, because sin() is periodic.
- yes, because denominator ($\pi t/T$) increases with t

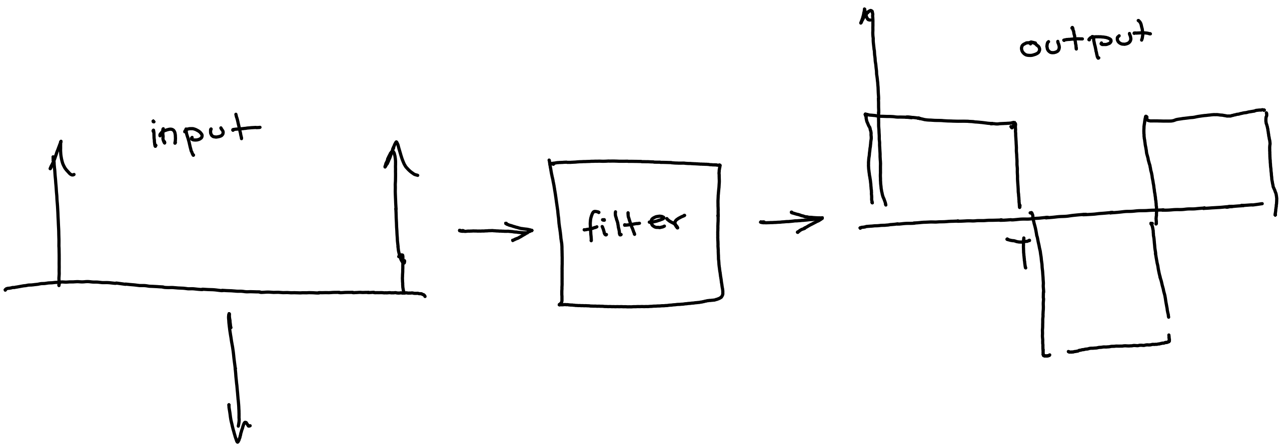
**Exercise 5**: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses.




$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$




**Exercise 6**: Draw the impulse response of a filter than converts input impulses to pulses of duration $T$? Draw the signal after the pulse-shaping filter in the diagram above.

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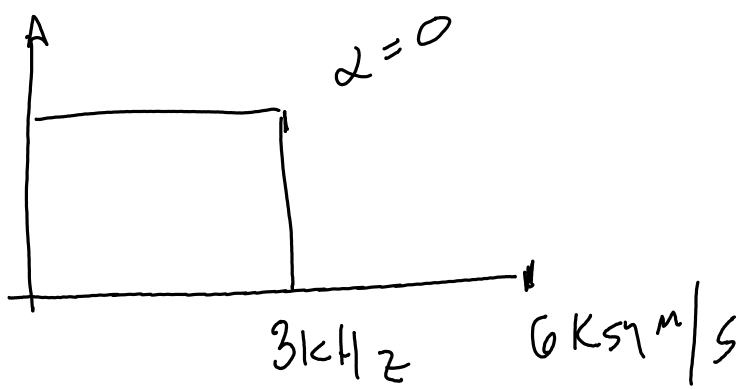
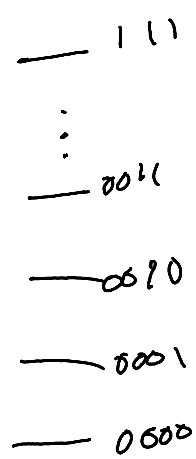
**Exercise 7**: A channel has a 3 kHz bandwidth and meets the Nyquist non-ISI conditions with $\alpha = \cancel{1}$. How many levels are required to transmit 24 kb/s over this channel using multi-level signalling?




$$24000 \text{ bits/s} \div 4 \frac{\text{bits}}{\text{symbol}} = 6000 \text{ symbols/s}$$

$$\text{bits/symbol} = \frac{24{,}000}{6000} = 4 \text{ bits/symbol}$$

need $2^b = 2^4 = 16$ levels

to transmit 4 bits/symbol.

**Exercise 8**: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$$f_s = 20MHz \Rightarrow T_s = \frac{1}{20\times10^6} = 50ns/sample$$

$$\text{Duration} = \underbrace{N \cdot T_s}_{\text{symbol}} + 0.8\mu s = 64 \cdot 0.05 + 0.8$$
$$= 3.2 + 0.8 = 4\mu s$$

$$\text{guard time (in samples)} = \frac{0.8}{0.05} = 16 \text{ samples.}$$

**Exercise 9**: What is capacity of a binary channel with a BER of $\frac{1}{8}$ (assuming the same BER for 0's and 1's)? *Hint:* $log_2(\frac{7}{8}) \approx -0.2$.

$$\log_2(p) = \log_2\left(\frac{1}{8}\right) = -3$$
$$\log_2(1-p) = \log_2\left(\frac{7}{8}\right) \approx -0.2$$

$$C = 1 - (-p\log_2 p - (1-p)\log_2(1-p))$$
$$\approx 1 - \left(-\frac{1}{8}\times -3 - \frac{7}{8}\times -0.2\right) = 1 - \left(\frac{3}{8} + \frac{1.4}{8}\right) = 1 - \frac{4.4}{8}$$
$$= \frac{3.6}{8}$$ (about ½ bit of information per transmitted bit)

**Exercise 10**: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

$$\frac{S}{N} = 30dB = 10^{\frac{30}{10}} = 1000$$

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$
$$= 4000 \cdot \log_2(1 + 1000)$$
$$\approx 40 kb/s$$

**Exercise 11**: Can we use compression to transmit information faster than the (Shannon) capacity of a channel? To transmit data faster than capacity? Explain.

No, compression (lossless) does not change the probability of a message & thus does not change the information rate. It cannot be used to transmit information faster than capacity.

No, compression can reduce the number of bits (data) to be transmitted, but the information rate is unchanged.

**Exercise 12**: What do the Nyquist no-ISI criteria and the Shannon Capacity Theorem limit? What channel parameters determine these limits?

Nyquist: limits symbol rate possible without ISI based on impulse response (or transfer function).

Shannon Capacity: limits information rate (with arbitrarily low error rate) based on channel probabilities:

- BER for BSC
- bandwidth & SNR for AWGN channel