Introduction to Digital Communication

Exercise 1: Give an example of a communication system. If you can, identify the source, transmitter, channel, receiver and destination.

transmitter : } cell phone
receiver :

source : } person
sink :

channel : free space.

Exercise 2: Give an example of a communication network. If you can, identify different the type(s) of channels used.

- internet
- WIFI , ethernet cable, fiber-optic cable

Exercise 3: Speech is intelligible if you restrict the sounds to frequencies below about 4 kHz. What is the minimum sampling rate that should be used to sample speech so that it will be intelligible?

A signal-to-noise power ratio of about 48 dB is considered "toll quality" (the SNR conventional telephone networks provide). How many bits of quantization are required to obtain a quantization SNR equivalent to "toll quality" speech?

What if the signal was a video signal with a 5 MHz bandwidth and required a quantization SNR of 40 dB?

What are the resulting bit rates in the two examples above ?

audio

$f_{sample} = 2 \times f_{max} = 2 \times 4\text{ kHz} = 8\text{ kHz}$

$48\text{ dB} = 6B \rightarrow B = \frac{48}{6} = 8\text{ bits}$

$f_{bit} = 8 \frac{\text{bits}}{\text{sample}} \times 8{,}000 \frac{\text{samples}}{\text{second}} = 64{,}000 \frac{\text{bits}}{\text{second}}$

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video

$f_{max} = 5\text{ MHz}$

$f_{sample} = 2 \times 5\text{ MHz} = 10\text{ MHz}$ (Msamples/s)

$SNR(\text{quant}) \geq 40\text{ dB}$

$40 = 6B \quad B = 6.\frac{2}{3}$ (use 7 bits)

$f_{bit} = 10\text{ MHz} \times 7\text{ bits} = 70\text{ Mb/s}$

Exercise 4: Write the sequence of bits that would be transmitted if the 16-bit value 525 was transmitted with the bytes in little-endian order and the bits lsb-first. Write the sequence of bits that would be transmitted in “network order” and the bits msb-first.

$525_{10} = 20D_{16}$ $\quad 0x020D = 0000\ 0010\ 0000\ 1101$

(msbit, lsbit; MS Byte, LS Byte)

$= 0\times2^{15} + 0\times2^{14} + \ldots + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

1011 0000 0100 0000 (Little endian, lsb first)

0000 0010 0000 1101 (big endian, msb first).

Exercise 5: How many bits would be required to uniquely identify 100,000 different characters? (Hint: $2^{16} = 65536$).

17 bits

Exercise 6: The Chinese character for “Rice” (the grain) is ▯ with Unicode value (code point) U+7C73. What is the UTF-8 encoding for this character?

米 7C73

① convert to binary

7 C 7 3

0111 1100 0111 0011

② which row of table?

Scalar Value	First Byte	Second Byte	Third Byte	Fourth Byte
00000000 0xxxxxxx	0xxxxxxx			
00000yyy yyxxxxxx	110yyyyy	10xxxxxx		
zzzzyyyy yyxxxxxx	1110zzzz	10yyyyyy	10xxxxxx	
000uuuuu zzzzyyyy yyxxxxxx	11110uuu	10uuzzzz	10yyyyyy	10xxxxxx

③ z = 0111
y = 110001
x = 110011

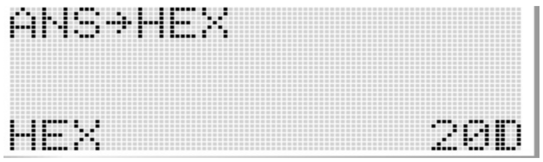
④ add prefixes:

1110 0111 = E7
10 11 0001 = B1
10 11 0011 = B3

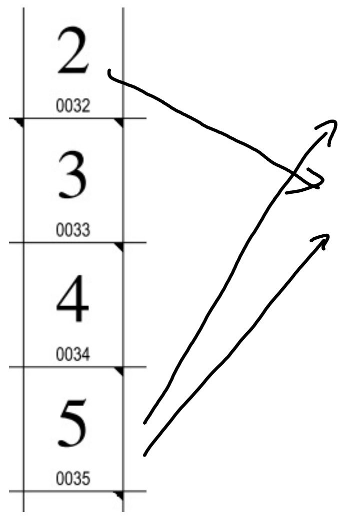
Exercise 7: Convert the decimal number 525 to a 16-bit (two-byte) binary number. How would you write this in hexadecimal notation?

Find the ASCII codes for the *characters* '525'. Write out the bits of the sequence that would be transmitted assuming each character is encoded in UTF-8. *Hint: the UTF-8 character code for a digit is 0x30 plus the value of the digit.*

Which of these two sequences of bits is the text format and which is the binary format? How many more bits would need to be stored or transmitted for the text format?



$525_{10} = 0x20D$ (2 bytes) — binary



5 = 0x35
2 = 0x32
5 = 0x35

0x35 = 0011 0101 0x32 = 0011 0010 0x35 = 0011 0101

text format (3 bytes)

Exercise 8: We observe a source that outputs letters. Out of 10,000 letters 1200 were 'E'. What would be a reasonable estimate of the probability of the letter 'E'?

$$P = \frac{1200}{10000} \approx 12\%$$

$$I = -\log_2(P) \quad \text{e.g. } P = \frac{1}{256}$$

$$I = -\log_2\left(\frac{1}{256}\right) = -\left(-\log_2(256)\right) = \log_2 256 = 8$$

Exercise 9: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$$P_3 = 1 - (P_0 + P_1 + P_2) = 1 - (0.125 + 0.125 + 0.25)$$
$$= 1 - 0.5 = 0.5$$

$P_0 = 0.125 \quad I_0 = -\log_2(0.125) = -\log_2\left(\frac{1}{8}\right) = 3$

$P_1 = 0.125 \quad I_1 = 3$

$P_2 = 0.25 \quad I_2 = 2$

$P_3 = 0.5 \quad I_3 = 1$

$$H = \sum (-\log_2 P_i) \times P_i =$$
$$= 3 \cdot 0.125 + 3 \times 0.125 + 2 \times 0.25 + 1 \times 0.5$$
$$= \frac{3}{8} + \frac{3}{8} + \frac{1}{2} + \frac{1}{2} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4} \text{ bits/message}$$

$2^{10} =$

$1 = 2^0$
2
4
8
16
32
64
128
256
512
1Ki $1024 = 2^{10}$

$1024 \cdot 1024 = 2^{10} \cdot 2^{10}$

$1Mi = 2^{20}$

Exercise 10: How long will it take to transfer 1 M(i)Byte at a rate of 10 kb/s?

1 MByte = 2^{20} bytes

$$\frac{2^{20} \text{ (bytes)} \times 8 \text{ (bits/byte)}}{10{,}000 \ \frac{\text{bits}}{\text{second}}} = \approx 840 \text{ second}$$

$$\frac{2^{20} \times 8}{10000} = 838.8608$$

Exercise 11: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, the information rate and the baud rate?

4 symbols $\Rightarrow \log_2 4 = 2$ bits/symbol

bit rate: $2 \text{ bits} / 1\,\mu s \Rightarrow \dfrac{2 \text{ bits}}{1\times10^{-6}\,s} = \dfrac{2\times10^{6} \text{ bits}}{1\,s}$

symbol rate: 1 symbol / 1 μs $\Rightarrow$ 1 MHz

	P_i	$-\log_2 P_i$
P_0 =	0.4	1.3
P_1 =	0.3	1.7
P_2 =	0.2	2.3
P_3 =	0.1	3.3

$$H = \sum_{i=0}^{3} \left(-\log_2 P_i\right)\cdot P_i$$

$$= \left(-\log_2 0.4\right)\cdot 0.4 + \left(-\log_2 0.3\right)0.3 + \left(-\log_2 0.2\right)0.2 + \left(-\log_2 0.1\right)0.1$$

$$= 0.52 + 0.51 + 0.46 + 0.33 = 1.03 + 0.79 = 1.82$$

≈ 1.82 bits/message.

in bits/second

≈ 1.82 bits/~~msg~~ $\cdot\ 10^6$ ~~msg~~/s $= 1.82$ Mbps

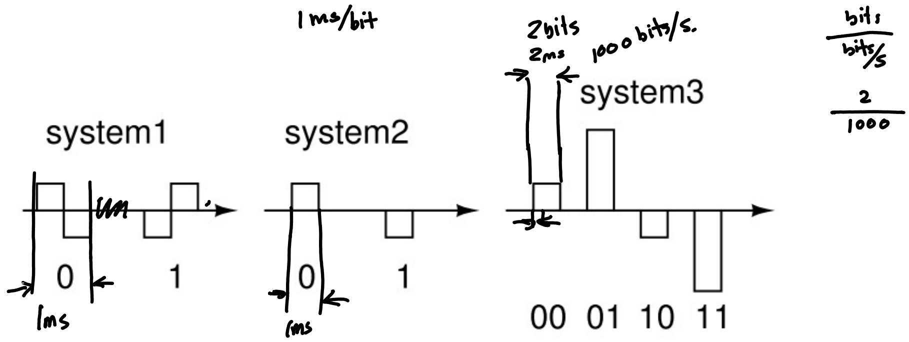
if baud rate = symbol rate

1 MHz

if baud rate = $\dfrac{1}{\text{shortest duration}}$

$= \dfrac{1}{0.5\,\mu s} = 2$ MHz

Exercise 12: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). Assuming each system transmits at 1000 bits per second, what are the baud rates in each case? How many different symbols are used by each system? What are the symbol rates? Assuming each symbol is equally likely, what are the information rates?



2 2 4 symbols

$\frac{1 \text{ symbol}}{1 \times 10^{-3} \text{ s}}$ = 1000 symbols (Hz)

same 1 kHz

$\frac{1}{.002}$ = 500 Hz

2 ms

smallest time difference = 0.5 ms =1 ms

baud rate = 2 kHz 1 kHz 500 Hz

Exercise 13: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

10^6 frames · 100 bits

$FER = \frac{56}{10^6} = 56 \times 10^{-6}$

56 frames w/ errors

40 have 1 bit errors = 40 bit errors

15 ha 2 bit " = 30 bit errors

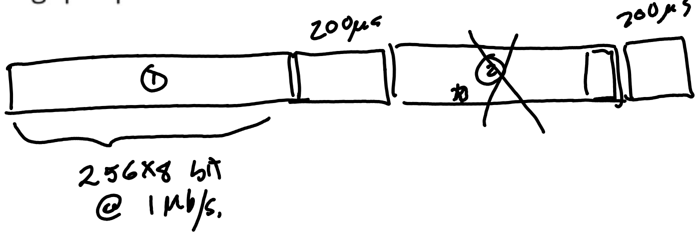
1 3 " " 3 bit errors

total 73 bit errors.

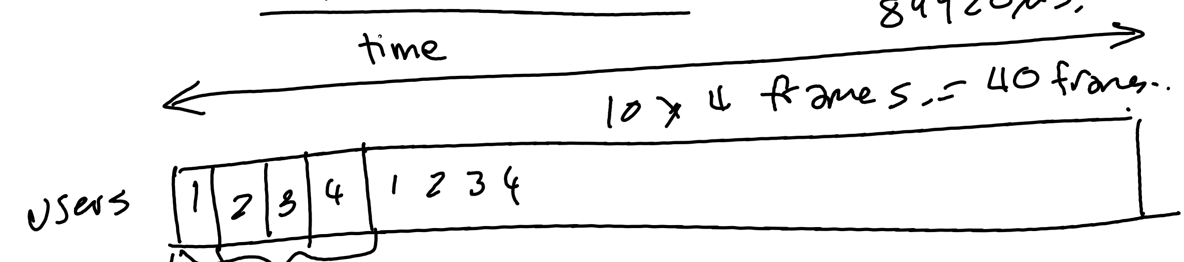
$\frac{73}{100 \times 10^6}$

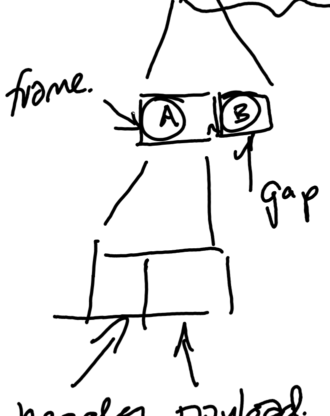
$BER = 73 \times 10^{-8}$

Exercise 14: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?



$$\frac{\#\text{ received useful bits}}{\text{time}}$$





total time
$= 40 \times 2.248$
$= 89920\ \mu s$

2.248 ms,
$= 2048\ \mu s$
$+ 200\ \mu s.$

$$= \frac{256\text{ bytes} \times 8\text{ bits/byte}}{1 \times 10^{6}}\ (A) + 200 \times 10^{-6}\ (B)$$

bps

$$\left[\ \frac{\text{bits}}{\frac{\text{bits}}{s}} = s.\ \right]$$

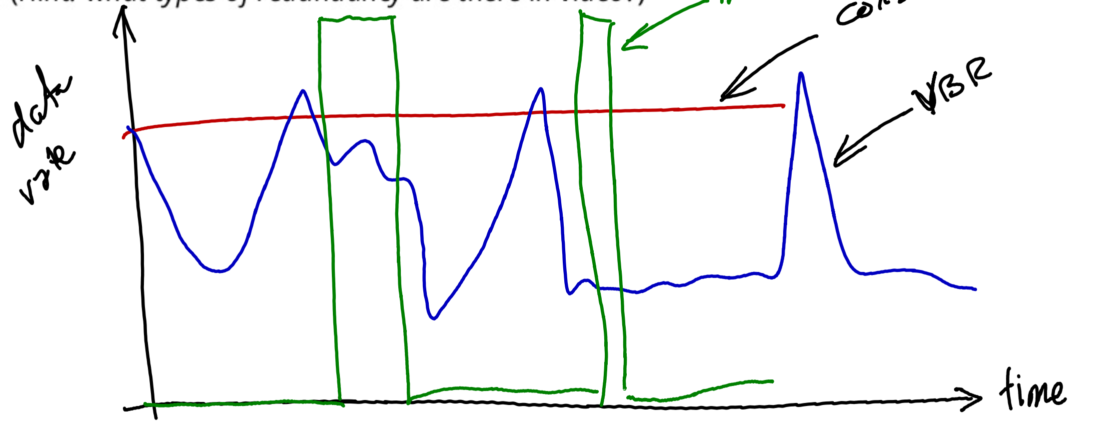
each user gets 10 frames
each frame has 200 bytes x 8 bits/byte
= 1600 bits of data.

total of 16000 bits in 89920 μs.

minus 10% of frames (or bits) are discarded due to errors ∴
16000 − 1600 = 14400 bits are received by one user.

$$\text{Throughput} = \frac{14400}{89920 \times 10^{-6}} = 160\ \text{kb/s}.$$

Exercise 15: Plot some ~~sample~~ examples of data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)



Exercise 16: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, "texting", downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?

tolerance to:

	errors	delay
phone call	H	L
"texting"	L	H
downloading	L	H
video (1-way)	H	H
video (2-way)	H	L

Exercise 17: Highlight or underline each term where it is defined in these lecture notes.