ELEX 3525 : Data Communications
2019 Fall Term

Assignment 2

Due Tuesday, November 12, 2019. Submit your assignment using the appropriate Assignment folder on the course web site. Assignments submitted after the solutions are made available will be given a mark of zero. **Show how you obtained your answers.**

Question 1

The expectation operator $E[x]$ denotes the average (mean) of x. It is a linear operator so we can expand the expression for the average power[1] of a sum of two voltages a and b as:

$$E[(a+b)^2] = E[a^2] + 2E[ab] + E[b^2]\,.$$

Evaluate the left- and right-hand sides of the above equality for the following cases:

(a) When a and b are constant (DC) voltages with $a = 2$ V and $b = 3$ V.

(b) When a is a constant 2 V and b is a random signal that is equally likely to have values +1 V and −1 V.

(c) When a and b are independent random signals, each equally likely to have values +1 V and −1 V.

Question 2

A noise signal has a constant (uniform) probability density function over the range ±100 mV. In other words, the probability of any voltage between −100 mV and +100 mV is the same.

(a) Draw the probability density function.

(b) Knowing that the integral of this function (area under the curve) has to be equal to 1, what is the height of the curve?

(c) What are the units of the probability density function? *Hint: the area is a probability and has no units.*

(d) What is the probability that the noise falls between 30 mV and 50 mV?

[1]Normalized to an impedance of 1 ohm.

Question 3

A system transmits data as 8 data bits plus one parity bit per word – a total of 9 bits per word. Noise causes random bit errors.

(a) How many different error patterns are possible? *Hint: assume a '1' indicates a bit is in error and a '0' indicates a bit is not in error. How many patterns of 0's and 1's are possible? Remember to include the possibility of errors in the parity bit.*

(b) How many of these patterns contain zero errors?

(c) How many contain exactly one error?

(d) How many contain exactly two errors?

Assume the probability of any data bit being received in error is $P_e = 1\%$. If errors happen independently, the probability of a *specific* error pattern that contains n bit errors is $P = P_e^n \cdot (1-P_e)^{(9-n)}$. For example, the probability of the error pattern with errors in the first and last bits (1 0000 0001) is $(0.01)^2(0.99)^7 = 0.932 \times 10^{-4}$.

The probability of receiving any one of various error patterns is the sum of the probabilities of each of the patterns. For example,the probability of receiving the error pattern 1 0000 0001 or the error pattern 1 1000 0000 is $\approx 1.86 \times 10^{-4}$.

What are the probabilities of receiving:

(e) A word with zero errors.

(f) A word with exactly one error.

(g) A word with exactly two errors.

Question 4

A remote radio unit is connected to a power and control unit using 200 m of 14/2 cable. This cable con-

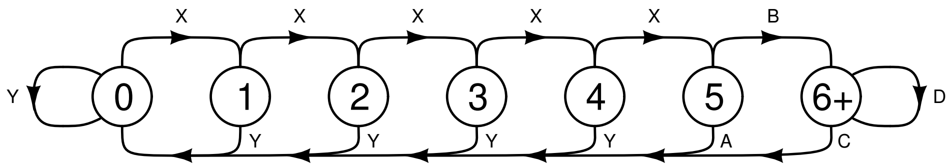

Figure 1: State transition diagram for an HDLC framing detector.

tains a 14-gauge ground wire and two 14-gauge conductors. One of these conductors is used to supply power at 48 VAC with the ground wire used as the return.[2] The other conductor is used for a unidirectional RS-232 data circuit and shares the same ground return as the power circuit.

(a) If the power cable is supplying 480 W, what is the *peak* voltage offset of the ground point at the remote end (relative to ground at the control unit)?

(b) Will the data circuit operate properly if the RS-232 link uses voltages of ±5 V ? If not, why not?

Hint: 14-gauge copper wire has a resistance of about 5 ohms per km.

Question 5

The diagram in Figure 1 is a state transition diagram for an HDLC framing detector. States are labelled with the number of consecutive '1' bits received.

(a) What events are indicated by the transitions labelled X, Y, A, B, C and D (e.g. start of frame, framing error, etc.)?

(b) What should the receiver do with the bit received that causes the state transition labelled A?

Question 6

Framing for a "T1" link uses a frame-synchronization bit inserted every 192 bits for a total of 193 bits per frame. The receiver detects loss of frame synchronization when it does not see the expected value in the framing bit position.

Since the frame contains arbitrary data there is a chance that the receiver will fail to notice the loss of synchronization until a certain number of frames have been observed.

Assuming the data is random and that 0's and 1's in the data are equally likely:

(a) A receiver starts looking for frame sync at an arbitrary position within the frame. What is the probability that loss of frame sync will not be detected when looking at this first bit?

(b) What is the probability that loss of synchronization will not be detected for 8 successive frames? *Hint: the probability of N independent events is the product of their individual probabilities.*

(c) For 40 successive frames?

(d) "T1" links operate at 1.544 Mb/s. How long does it take to transmit 40 frames?

[2] This is a low-voltage AC circuit and the designer was trying to reduce costs.