Introduction to Digital Communication

Exercise 1: Give examples of communication systems. Identify the source, channel and sink for each one.

"landline" – person, telephone cable, person

broadcast TV – video camera?, free space, person

Exercise 2: Give some examples of communication networks and identify different types of channels that each typically uses.

"Internet" – WiFi, Ethernet, optical fiber

Exercise 3: Speech is intelligible if you restrict the sounds to frequencies below about 4 kHz. What is the minimum sampling rate that should be used to sample speech so that it will be intelligible?

A signal-to-noise power ratio of about 48 dB is considered "toll quality" (the SNR conventional telephone networks provide). How many bits of quantization are required to obtain a quantization SNR equivalent to "toll quality" speech?

What if the signal was a video signal with a 5 MHz bandwidth and required a quantization SNR of 40 dB?

sample at $> 2 \times 4$ kHz $= 8$ kHz

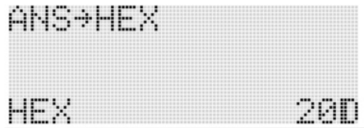
48 dB $= 6B$ $\quad B = \frac{48}{6} = 8$ bits/sample

5 MHz → sample at > 10 MHz

40 dB ⇒ sample with $\frac{40}{6} \approx 6$ bits/sample

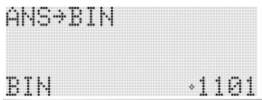
Exercise 4: (1) Write the sequence of bits that would be transmitted if the 16-bit value 525 was transmitted with ~~the bytes~~ (2) in little-endian order and the bits lsb-first. Write the sequence of bits that would be transmitted in “network order” and the bits msb-first.

525 =



= 0x020d

02 ← MS Byte; 0d ← L.S. Byte



0D → 0000 1101

02 → 0000 0010

m.s. bit first →

0000 0010 0000 1101 ← network order

(0000 0010 = MS Byte; 0000 1101 = L.S. Byte)

l.s. bit first →

1011 0000 (L.S. Byte)

l.s. bit first →

0100 0000 (M.S. Byte)

Exercise 5: How many bits would be required to uniquely identify 100,000 different characters? (Hint: $2^{16} = 65536$).

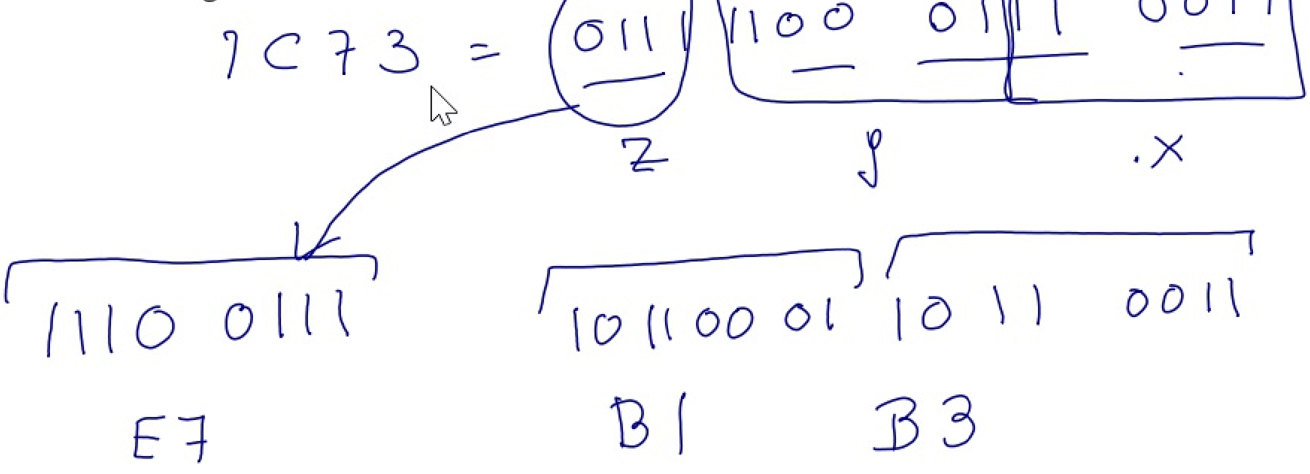
00,000 different characters? (Hint: $2^{16} = 65536$).

00000

$2^{16} = 65535$ 2 bytes.

$2^{16} \cdot 2^{1} = 2^{17} = 128\text{K}$

Exercise 6: The Chinese character for "Rice" (the grain) is "□" with Unicode value (code point) U+7C73. What is the UTF-8 encoding for this character?



Exercise 7: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$P_0 = 0.125 = \frac{1}{8}$

$P_1 = $ " "

$P_2 = 0.25 = \frac{1}{4}$

$P_3 = 1 - \left(\frac{1}{8}+\frac{1}{8}+\frac{1}{4}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

	$-\log_2(P_i)$		P_i
$I_0 = -\log_2(P_0) =$	3	$\cdot$	$\frac{1}{8}$
$I_1 =$	3	$\cdot$	$\frac{1}{8}$
$I_2 =$	2	$\cdot$	$\frac{1}{4}$
$I_3 =$	1	$\cdot$	$\frac{1}{2}$

$$\sum_i -\log_2(P_i) \cdot P_i = 3\cdot\frac{1}{8} + 3\cdot\frac{1}{8} + 2\cdot\frac{1}{4} + 1\cdot\frac{1}{2}$$

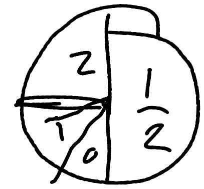
$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{2} = 1.75 \text{ bits/message.}$$

Exercise 8: We observe a source that outputs letters. Out of 10,000 letters 1200 were 'E'. What would be a reasonable estimate of the probability of the letter 'E'?

$$\frac{1200}{10{,}000} \approx 12\%$$

Exercise 9: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$\frac{1}{8}, \frac{1}{8}, \frac{1}{4}$



$$P_3 = 1 - \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right) = \frac{1}{2}$$

for M_0, $I_0 = -\log_2 P_0 = -\log_2\left(\frac{1}{8}\right) = 3$ bits

M_1 " " $= 3$

M_2 $I_2 =$ $= \log_2\left(\frac{1}{4}\right) = 2$ bits

M_3 $I_3 =$ $= \log_2\left(\frac{1}{2}\right) = 1$ bit.

$$H = \sum_i I_i P_i = 3 \cdot \frac{1}{8} + 3 \frac{1}{8} + 2 \frac{1}{4} + 1 \cdot \frac{1}{2}$$

$$= \frac{6}{8} + \frac{1}{2} + \frac{1}{2} = 1.75 \text{ bits/message}$$

information rate $= 1.75 \frac{\text{bits}}{\text{msg}} \cdot 100 \frac{\text{msg}}{\text{sec}}$

175 bits/second

if $P_i = \frac{1}{4}$ $I_i = 2$ bits

$H = 2$ bits/msg

Exercise 10: How long will it take to transfer 1 MByte at a rate of 10 kb/s?

$1 \times 10^6 \rightarrow$ bytes $= 8 \times 10^6$ bits

1 MByte ~~$2^{20} \rightarrow$~~

10 kbit/s

$$\frac{8 \times 10^6}{10^4} = 800$$

Exercise 11: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, the information rate and the baud rate?

bit rate: $\frac{2 \text{ bits}}{1 \times 10^{-6} \text{ s}} = \frac{2}{1} \times 10^{6} \frac{\text{bits}}{\text{s}}$

symbol rate: $\frac{1 \text{ symbol}}{1 \times 10^{-6} \text{ s}} = 1 \times 10^{6} \frac{\text{symbols}}{\text{second}}$

$\log_a b = \frac{\log b}{\log a}$

information rate:

$P_0 = 0.4 \quad I_0 = 1.3$

$P_1 = 0.3 \quad I_1 = 1.7$

$P_2 = 0.2 \quad I_2 = 2.3$

$P_3 = 0.1 \quad I_3 = 3.3$

entropy $H = 0.4 \cdot 1.3 + 0.3 \cdot 1.7 + 0.2 \cdot 2.3 + 0.1 \cdot 3.3$

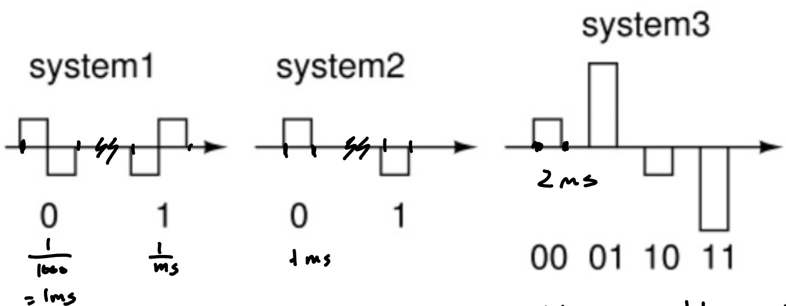
≈ 1.8 bits/message

information rate $= 1.8 \text{ bits/msg} \cdot \frac{1 \text{ msg}}{1 \times 10^{-6}}$

$= 1.8$ Mbits/s

baud rate $= \frac{1}{0.5 \, \mu s} = 2 \times 10^{6}$ baud $= 2$ MHz

Exercise 12: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). Assuming each system transmits at 1000 bits per second, what are the baud rates in each case? How many different symbols are used by each system? What are the symbol rates? Assuming each symbol is equally likely, what are the information rates?



$2 \frac{\text{bits}}{\text{symbol}}$ $1000 \frac{\text{bits}}{\text{s}}$: $\frac{2 \text{ b/sym}}{1000 \text{ b/s}}$ s/symbol

2 ms/symbol

bit rate = $\frac{2 \text{ bits}}{2 \text{ ms}}$ = 1000 b/s.

baudrates: $\frac{1}{0.5\text{ms}}$ = 2KHz $\frac{1}{1\text{ms}}$ = 1KHz $\frac{1}{2\text{ms}}$ = 500 Hz

symbols: 2 2 4

symbol rates: 1 kHz 1KHz 500 Hz.

information rates: $\frac{1 \text{ bit}}{\text{ms}}$ = $\frac{1 \text{ bits}}{\text{ms}}$ = $\frac{2 \text{ bits}}{2\text{ms}}$ = all 1000 b/s

Exercise 13: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

$$FER = \frac{\#\text{ frames w/ error}}{\#\text{ frames}} = \frac{56}{1 \times 10^6} = 56 \times 10^{-6} \approx 6 \times 10^{-5}$$

$$BER = \frac{}{\#\text{ bits}} = \frac{40 \times 1 + 15 \times 2 + 3 \times 1}{100 \times 10^6} = \frac{73}{10^8} =$$

$$= 73 \times 10^{-8} = 7.3 \times 10^1 \times 10^{-8}$$

$$= 7.3 \times 10^{1-8} = 7.3 \times 10^{-7}$$

Exercise 14: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?

$$\frac{\text{useful data}}{\text{time}} = \frac{\quad}{1\,s.}$$

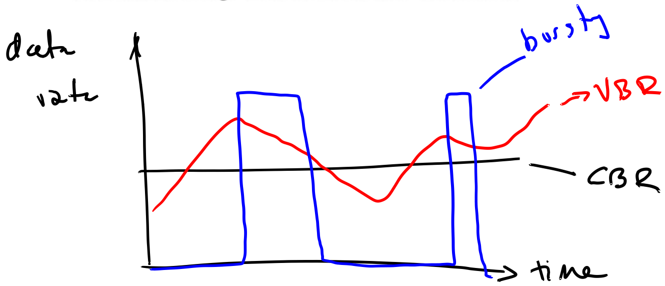
frame + guard time $= \frac{256 \times 8\ b}{1 \times 10^6\ b/s} = 2048 \times 10^{-6}\ s. + 200\mu s. = 2.2\ ms$

$\#\ \text{frames} = \frac{1}{2.2 \times 10^{-3}} = 445$ frames

per user $= \frac{445}{4} = 111$ frames. $\times$ 200 bytes $\times$ 8 bit/byte

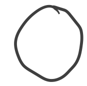
$$\simeq \frac{177\ kb}{1\,s} = 177\ kb/s.$$

Exercise 15: Plot some sample data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)



Exercise 16: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, “texting”, downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?

36,000 km.



30×10^6 m

3×10^8 m/s.

100ms.

	tolerate error	tolerate delay
phone call	yes	10's of ms ok.
texting	somewhat	yes.
download	No	yes.
stream video	No	somewhat.

Exercise 17: Highlight or underline each term where it is defined in these lecture notes.