Solutions to Final Exam

December 21: Added a note to Question 4 that the answer is only valid for Gaussian noise.

Question 1

There were two version of this question: for LATIN SMALL LETTER G WITH CARON (ǧ), **U+01E7** and for LATIN CAPITAL LETTER WYNN (Ƿ), **U+01F7**.

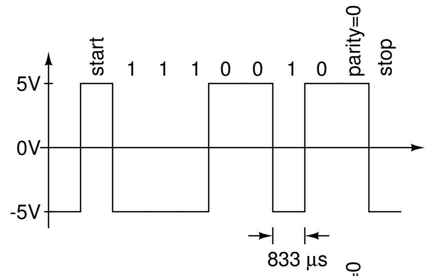
(a) These values have 9 non-zero bits and Table 3-6 from the Unicode standard shows they must be encoded using the second row of the table and thus require two (2) bytes.

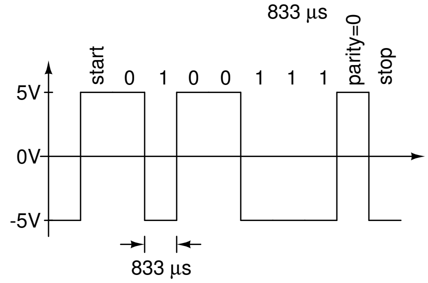
(b) The values of these bytes can be computed as:

- The binary representation of **0x01E7** is **0000 0001 1110 0111** from which y=**0001 11**, x=**10 0111**, and the two bytes are: **110 0001 11, 10 10 0111** which are **C7**, and **A7** in hex.
- The binary representation of **0x01F7** is **0000 0001 1111 0111** from which y=**0001 11**, x=**11 0111**, and the two bytes are: **110 0001 11, 10 11 0111** which are **C7**, and **B7** in hex.

Question 2

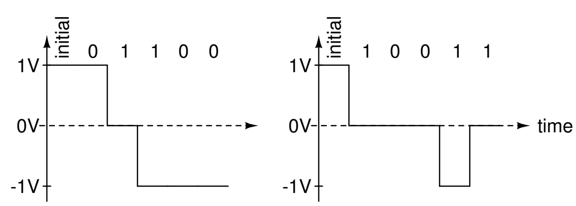
The waveforms that would be used to transmit the byte values 0x27 (binary 010 0111) and 0x72 (binary 111 0010) over an RS-232 asynchronous serial interface at a rate of 1200 bps with 7 bits per character, even parity and one stop bit are shown below. The parity bit is 0 in both cases because the data already has an even number of ones (4). The bit duration is the inverse of the bit rate ($\frac{1}{1200} = 833.3\,\mu$s). The minimum voltage at the transmitter is ±5 V.





Question 3

The waveforms that would be used to transmit the bit sequence 0, 1, 1, 0, 0 and 1, 0, 0, 1, 1 using an MLT-3 line code assuming an initial voltage level of +1 V are shown below. There are three equally-spaced levels around zero (thus zero and ±1 V) and each '1' bit causes the level to change to the next level in the sequence $\{+1, 0, -1, 0\}$.



Question 4

The probability distribution of the noise must be known to answer this question. The following answer assumes the noise distribution is Gaussian.

The noise mean is $\mu = 0$ V and the two versions of the question specified standard deviations of $\sigma = 0.044$ or $\sigma = 0.088$ V with thresholds of $x = -0.1$ and $x = -0.2$ V respectively. Both give a normalized threshold of:

$$t = \frac{-0.1 - 0}{0.044} = \frac{-0.2 - 0}{0.088} \approx -2.27\,.$$

Using a calculator the probability of error, P_e, is $P(x < \frac{-0.1}{0.044}) \approx 1.15 \times 10^{-2}$. Or using the graph in the Lecture 4, $P_e \approx 1\%$.

Question 5

A binary symmetric channel (BSC) with an error rate of p has a capacity of:

$$C = 1 - (-p \log_2 p - (1-p)\log_2(1-p))\,.$$

For $p = 1 \times 10^{-3}$ and using the approximations that $\log_2(p) \approx -10$ and $\log_2(1-p) \approx 0.001$, C evaluates to:

$$\begin{aligned} C &\approx 1 - (-0.001(-10) - (0.999)0.001) \\ &= 1 - 0.01 + 0.001 \\ &= 0.991\,. \end{aligned}$$

bits per channel use.

If the channel is used 1000 times per second then the maximum information rate is $1000C = 991$ bps.

Question 6

(a) For an FEC code that uses only two codewords the minimum distance is the only distance and is the distance between the two codewords. There were two version of the question: one using codewords 1100 and 0011 and the other using the codewords 1001 and 0110. In both cases the Hamming distance is $d_{min} = 4$.

(b) The code is guaranteed to detect $d_{min} - 1 = 4 - 1 = 3$ errors.

(c) The code is guaranteed to correct

$$\lfloor \frac{d_{min} - 1}{2} \rfloor = \lfloor \frac{4-1}{2} \rfloor = \lfloor 1.5 \rfloor = 1$$

errors.

Question 7

The first 14 bytes of an Ethernet frame are the 6 bytes of the destination address, the 6 bytes of the source address and the two byte of the length/type field in that order.

There were two versions of this question for frames containing the bytes:

`08 00 27 da fa 8f` | `00 1d 7e 2f b5 9b` | `08 00`

and

`00 1d 7e 2f b5 9b` | `08 00 27 da fa 8f` | `08 00`

where the values in the three boxes indicate the destination address, source address and the Ethertype (Ethernet payload type) fields respectively.

Question 8

(a) For a communication system using differential voltage signalling with the voltage levels V_a and V_b the common-mode voltage is the average voltage ($(V_a + V_b)/2$) and the (absolute) differential voltage is the (absolute value of the) voltage difference ($|V_a - V_b|$).

There were two versions of the question, one with $V_a = -5$ and $V_b = +10$ and another with $V_a = -10$ and $V_b = +5$.

(i) The common-mode voltages are $(-5 + 10)/2 = 2.5$ V and $(-10 + 5)/2 = -2.5$ V.

(ii) The (absolute value of the) differential voltages are $|-5 - 10| = 15$ V and $|-10 - 5 = 15$ V.

(b) A ML PRBS generated using a circuit that contains m flip-flops generates a sequence where the longest run of 1's has a length m. There were two versions of the question for $m = 12$ and $m = 11$ which are also the lengths of the longest continuous sequence of ones in the generated bit sequence.

(c) Of the three types of ARQ covered in the course, stop-and-wait ARQ over a link that has a long delay relative to the packet duration will have a low throughput. However, Go-Back-N ARQ provides a high throughput when the error rate is low and is simpler than Selective-Repeat ARQ so it would be the best answer to this question.