Assignment 3

Due December 3, 2018. Submit your assignment using the appropriate Assignment folder on the course web site. Assignments submitted after the solutions are made available will be given a mark of zero. **Show how you obtained your answers.**

Question 1

A student in class suggested using repetition for error detection or correction.

1. What is the minimum distance of a (Nk, k) code where N is the number of repetitions of k data bits?

   Hint: Find the answer for $k = 1$ and $N = 1, 2, ...,$ then for $k = 2$, then generalize.

2. What value of N would guarantee detecting 1 error per codeword? What would be the code rate?

3. What value of N would guarantee correcting 3 or fewer errors per codeword?

Question 2

In a lecture it was suggested that the parity bits of an FEC code could be computed as modulo-2 sums of certain data bits in the codeword.

A simple method is to arrange the data bits in a rectangular matrix and compute one modulo-2 sum for each column and one for each row. These parity bits are transmitted along with the data bits. At the receiver the parity bits are re-computed and any discrepancies point to an error in a particular row and column.

For example, for 9 data bits we would use a 4×4 matrix where the last column and row contain the modulo-2 sums of the respective rows and columns:

0	1	1	0
1	0	1	0
1	1	1	1
0	0	1	1

If you receive the following bits:

0	0	1	1	1
0	1	0	1	0
1	1	1	1	0
0	1	0	0	1
0	1	0	1	0

(a) Is there an error?

(b) Which bit is in error?

(c) How many errors can this type of code correct?

(d) What is the code rate when the number of data bits is $k = m^2$?

(e) What is the code rate where there are $k = 64$ data bits?

Question 3

Impulse noise with a period of about 16.6 ms and duration of 40 μs interferes with a communication system using FEC and block interleaving. The data rate is 100 kb/s and the FEC codeword size is 256 bits.

(a) What are the minimum block interleaver width and depth that ensure at most one bit per FEC codeword is affected by the noise?

(b) What delay is added due to interleaving? Assume the block must be fully filled at the transmitter before transmission begins and fully filled at the receiver before any of the bits are made available.

Question 4

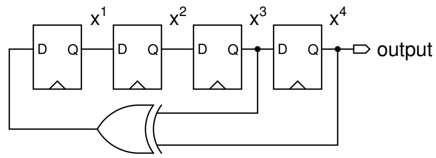
A LFSR will generate a PRBS if the feedback taps correpond to a primitive polynomial[1] with coefficients in GF(2). These taps can be used in two ways: using external or internal feedback.

For $n = 4$ a primitive polynomial is $x^4 + x^3 + 1$ and we can implement LFSRs for this polynomial in the following ways:

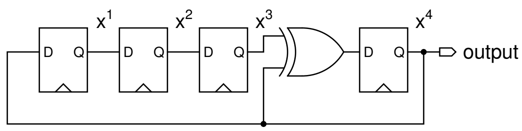
External Feedback. As shown in the lecture notes, the next bit fed into the shift register is the sum, in GF(2), of the shift register bits selected by the

[1] Similar to a prime number, a primitive polynomial is one with no factors other than itself and 1.

feedback polynomial. The bits are numbered with the oldest (rightmost, if shifting right) bit as x^n:



Internal Feedback. This operates similar to polynomial division: as we shift we subtract the feedback polynomial only if the rightmost bit (if shifting right) of the shift register is 1:



(a) Compute the sequence of $2^n - 1$ (binary) values of the shift registers for both structures. Assume the initial state is all-1's.

(b) Consider the sequence of output (rightmost) bits as the PRBS. Does each sequence have the required number of 1's and 0's for a ML-PRBS?

(c) Does each sequence meet the *period* requirement for an ML-PRBS?

(d) How are the two PRBS sequences related?