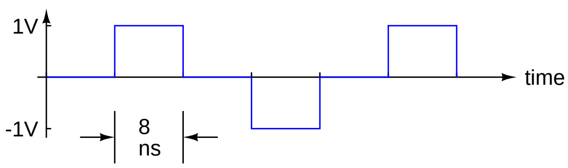
Solutions to Assignment 3

Corrected bit period in question 1 to $\frac{1}{125\times10^6} = 8$ *ns.*

Question 1

The 4B5B Idle symbol is '11111'. When encoded using MLT-3 this results in a transition every bit interval (every $\frac{1}{125\times10^6} = 8$ ns). The voltage levels are 0 and ±1 V. The waveform is thus:



Question 2

A channel excess bandwidth parameter (α) of 0.33 means that the total (maximum) bandwidth of the channel is $1+\alpha = 1.33$ times the minimum required for no ISI. The gain of the channel is zero ($-\infty$ dB) at a frequency of 60 MHz and so this is the total channel bandwidth. The minimum bandwidth for no ISI is $\frac{60}{1.33} = 45$ MHz. The minimum bandwidth is half of the symbol rate for which there would be no ISI so this symbol rate is $2 \times 45 = 90$ MHz.

Question 3

If a bit rate of 2 Mb/s is transmitted over a BSC there are 2 million channel uses per second. If the channel has a capacity of 1 Mb/s then the capacity is $\frac{1\times10^6}{2\times10^6} = 0.5$ bits per channel usage. The equation for the capacity (C) vs BER (p) for the BSC is:

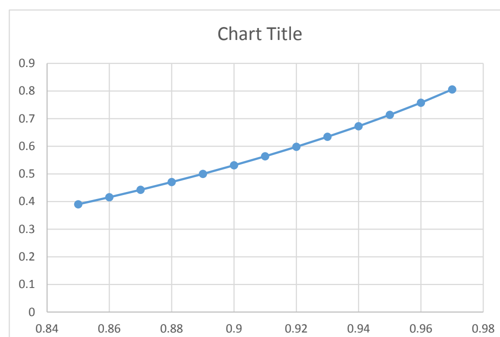
$$C = 1 - (-p\log_2 p - (1-p)\log_2(1-p))$$

There is no closed-form solution for p but we can find a numerical solution. You can re-write the equation as

$$1 - (-p\log_2 p - (1-p)\log_2(1-p)) - 0.5 = 0$$

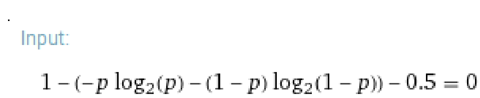
and solve for the roots of the equation. From the shape of the curve we know there will be two solutions: one between 0 and 0.5 and one between 0.5 and 1). There are many ways to find the roots:

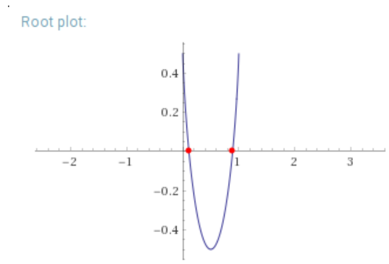
- Use a calculator. For example, on the Sharp EL-W516XBSL enter the equation in the display as a function of x and use Math->Solver to find the roots.
- Use a spreadsheet. You can enter the value of p in one cell, the equation in another and either iterate one value manually, compute the equation for a range of values or use the "solver" feature. For example, we can plot capacity vs p for $p = 0.85$ to 0.97 to find one of the roots:



- Use a root-finding function in a numerical analysis program such as `fzero` from Matlab or Octave.
- Use a web site such as Wolfram Alpha which provides an on-line front-end to Mathematica's numerical analysis features.

Screen captures from the last method are as follows:





Solutions:

$$\begin{pmatrix} p = 0.110028 \\ p = 0.889972 \end{pmatrix}$$

Thus the capacity is 1 Mb/s when the BER is 0.11 or 0.89.

Question 4

To find the remainder we can write the polynomial $x^6 + 1$ as 1000001 and $x^2 + 1$ as 101 and do long polynomial division using the arithmetic rules for coefficients from GF(2):

```
      10101
   --------
101|1000001
    101
    ---
     010
     ---
      100
      101
      ---
       010
       ---
        101
        101
        ---
         00
```

So the remainder is 0.
Note that this is not the CRC.

Question 5

For a 100 Mb/s Ethernet switch with 8 ports where packets are continuously being received on each port:

(a) if the destinations of the packets are equally divided among the 8 ports, then a total of 100 Mb/s ($8 \times \frac{1}{8} \times 100$ Mb/s) will be output on each port. The total throughput will thus be 800 Mb/s.

(b) if the destinations of the packets are all the same then only that one port will have any traffic flowing out of it and the throughput will be 100 Mb/s.