Solutions to Assignment 2

Corrected answer to Question 2 (refractive index $n = \frac{1}{\sqrt{\varepsilon_r}}$).

Question 1

According to the Friis equation the received power is:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

For each of the scenarios in this question we select the frequency band that would result in the highest received signal power. However, since the transmit power (P_T) and distance ($(\frac{1}{4\pi d})^2$) are the same at both frequencies, we can base the decision on the product $G_T G_R \lambda^2$.

At 2.4 GHz the wavelength is $\lambda = \frac{c}{f} = \frac{3\times10^8}{2.4\times10^9} = 0.125$ m. and at 5 GHz the wavelength is $\lambda = \frac{3\times10^8}{5\times10^9} = 0.06$ m.

(a) The path loss (gain) [1] defined as $(\frac{\lambda}{4\pi d})^2$ is higher (lower gain) at 5 GHz by a factor of $(\frac{5}{2.4})^2 = 4.3 = 6.4$ dB (the received signal is 6.4 dB lower at 5 GHz).

(b) For an effective areas of $A_e = 0.5$ m^2 the transmit and receive antenna gains at 2.4 GHz are $G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi\times0.5}{0.125^2} = 402 = 26.0$ dB each. At 5 GHz the antenna gains are $G = \frac{4\pi0.5}{0.06^2} = 1745 = 32.4$ dB each. *At 5 GHz each antenna has 6.4 dB higher gain.*

Because the received signal power is affected by both the transmit and receive antenna gains, the signal power at 5 GHz will be 6 dB higher if both antennas are directional. Thus for antennas with equal *effective areas* we would choose the 5 GHz band.

(c) (i) If one antenna was omni-directional (0 dB gain) the difference in antenna gains would be 6.4 dB and the received power would be the same in both cases. There would not be any advantage to either frequency.

(ii) If both antennas were omni-directional the antenna gains would be the same at both frequencies and the received power would be 6.4 dB lower at 5 GHz. In this case we would choose the 2.4 GHz band.

Question 2

Using the datasheet for Corning SMF-28 single-mode fiber:

(a) The refractive index, $n = \frac{1}{\sqrt{\varepsilon_r}}$ is specified as 1.4676 at 1310 nm and 1.4682 at 1550 nm corresponding to velocities of propagation of $v = \frac{c}{n} \approx 2.04 \times 10^8$ m/s.

(b) At a 1550 nm wavelength the maximum attenuation is specified as ≤ 0.18 dB/km. The attenuation for a 20 km link would be ≤ 3.6 dB.

(c) For a signal with a bandwidth of 10 GHz transmitted at a center wavelength of 1530 nm.

(i) The optical signal's frequency is $f = \frac{c}{\lambda} = \frac{3\times10^8}{1530\times10^{-9}} = 196$ THz and the signal's bandwidth is $\frac{10\times10^9}{196\times10^{12}} \approx 51.0 \times 10^{-6}$ times the carrier frequency.

(ii) The wavelength span is $51.0\times10^{-6}\times1530 = 78.0 \times 10^{-3}$ nm.

(iii) The maximum (chromatic) dispersion is specified as 18 ps/(nm·km) at 1550 nm. Assuming the same value[2] for 1530 nm, a 20 km link with a 7.8 nm wavelength span would have a maximum dispersion of $18 \times 78 \times 10^{-3} \times 20 = 28$ ps.

(iv) For a symbol rate of 1 GHz the symbol duration is 1 ns and the ratio of dispersion to symbol duration is $28/1000 = 2.8$ %.

[1] This number much less than 1 so is typically called the path loss but strictly speaking it's a "path gain." When specifying it in dB we simply change the sign and call it a path loss.

[2] You could also interpolate the value at 1530 from the values at 1550 and 1625 nm.

Question 3

If delay (τ) increases linearly from 5.000 000 s to 5.000 100 s as the frequency (f) increases from 1 to 1.1 kHz, the delay at 1.05 kHz would be 5.000 050.

The group delay is defined as the change (or slope) of the phase shift, given by $-2\pi f\tau$, versus angular frequency (ω).

There are various ways to find a solution. As an approximation we could assume a constant group delay (constant slope) between the two frequencies. The difference in phase is $-2\pi(f_2\tau_2 - f_1\tau_1)$ and the difference in (angular) frequency is $2\pi(f_2 - f_1)$. The value of the group delay is:

$$\frac{-(1100 \times 5.000\,100 - 1000 \times 5.000\,000)}{1100 - 1000} = -5.001\,\text{s}$$

We could also write an equation for phase versus frequency:

$$\Delta\Theta(f) = -2\pi f\tau(f) = -2\pi f(5 + 1 \times 10^{-6}(f - 1000))$$

and the derivative with respect to (angular) frequency:

$$\frac{d\Delta\Theta(f)}{df} = -5 - 1 \times 10^{-6}(2f - 1000)$$

which at f=1.05 kHz is also -5.001 s.

Question 4

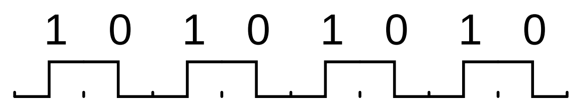
A normal distribution has tails that extend infinitely in both directions away from the mean. This would imply a non-zero probability of having a negative number of lighting strikes per month. This is clearly not possible. This situation usually arises from fitting measured data to a poorly-chosen model.

(a) Assuming the model is correct, the probability of fewer than 20 lightning strikes in one month is $P(t = \frac{\nu-\mu}{\sigma}) = P(\frac{20-10}{6}) = P(1.67) = 0.95254$ The probability of more than 20 lightning strings is thus about 5%.

(b) The probability there are fewer than -2 lightning strikes is zero as discussed above.

Question 5

If a low-to-high transition encodes a '1' bit, the waveform that would be used to transmit an 8-bit-long 10 Mb/s Manchester-encoded dotting sequence (alternating 1's and 0's) beginning with a 1 would be:



The frequency of the Manchester-encoded waveform is half of the bit rate or 5 MHz.