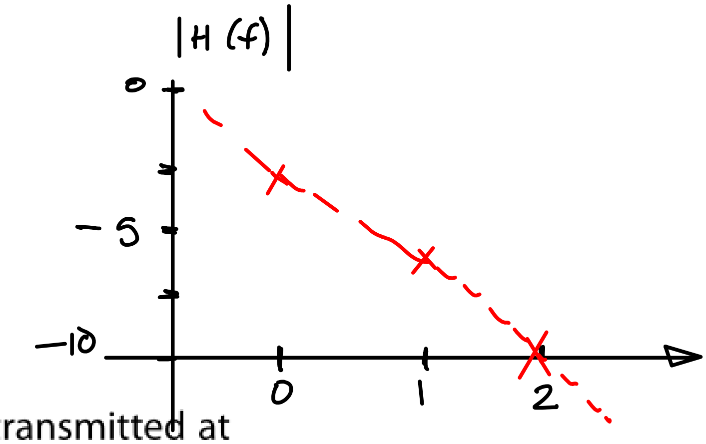
Lecture 4 - Channel Characteristics and Impairments

Exercise 1: A 10 dBm signal is applied to one end of a 50 ohm co-ax cable at frequencies of 1, 10 and 100 MHz. At the other end you measure voltages of 7, 4 and 0 dBm respectively. Plot the amplitude of the transfer function of the channel formed by this cable. Show dB on the vertical axis and log of frequency on the horizontal axis.

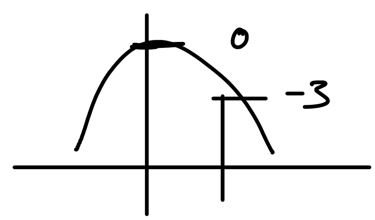
$$H(f) = \frac{V_{OUT}}{V_{in}}$$

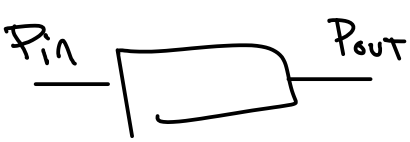
f	$\log(f)$	V_{OUT}	V_{IN}	$\lvert H(f)\rvert$ (dB)
1	0	7	10	−3
10	1	4	10	−6
100	2	0	10	−10

$$H(f)\,dB = V_{out\,(dB)} - V_{in\,(dB)}$$



Exercise 2: How much *power* would a signal transmitted at the edge of the 3 dB bandwidth passband have compared to the the power it would have if transmitted at the frequency with the lowest loss? What would be the ratio of the *voltages*? What if the bandwidth was defined as the 6 dB bandwidth?





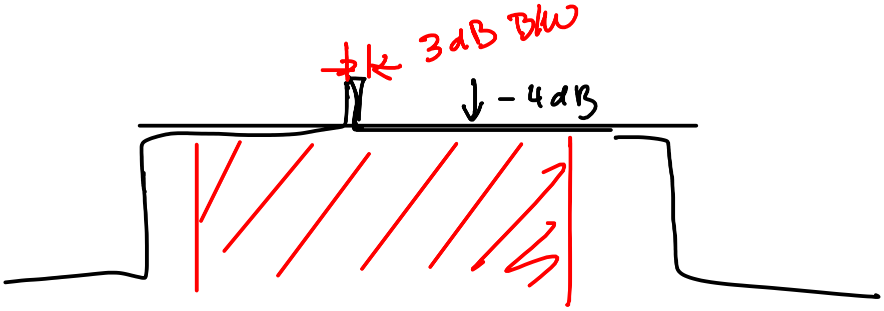
$$\frac{P_{OUT}}{P_{in}} = -3\,dB \qquad 10^{\frac{-3}{10}} \approx \frac{1}{2}$$

$$20\log\left(\frac{V_{OUT}}{V_{IN}}\right) = -3 \qquad \left(10^{\frac{-3}{10}}\right)^{\frac{1}{2}}$$

$$\log\left(\frac{V_{OUT}}{V_{in}}\right) = \frac{-3}{20}$$

$$\frac{V_{OUT}}{V_{in}} = 10^{\frac{-3}{20}} = \frac{1}{\sqrt{2}}$$

$$\text{Power ratio} = 10^{\frac{-6}{10}} = \left(10^{\frac{-3}{10}}\right)^2 \approx \frac{1}{4} \qquad V.\ \text{ratio} = 10^{\frac{-6}{20}} \approx \frac{1}{2}$$



Exercise 3: If t is a time, which happens first, t or $t-\delta$? If $f(t)$ and $f(t-\delta)$ are the input and output of a channel with delay, which is the input and which the output? What is the delay?

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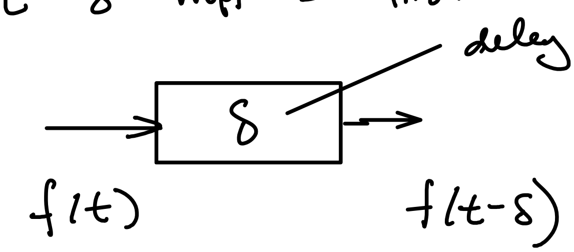
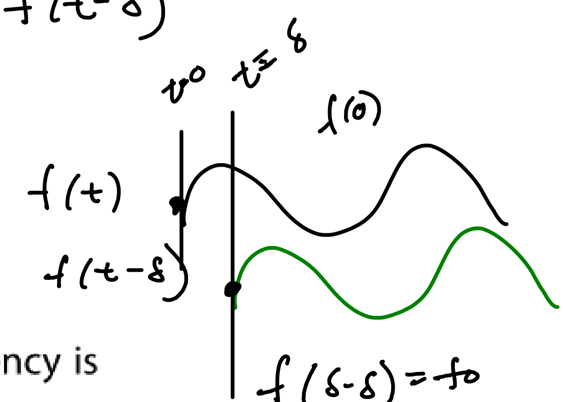
$t - \delta$ happens first.

$f(t)$
$f(t-\delta)$





Exercise 4: If the phase changes by $\frac{\pi}{2}$ and the frequency is 1 kHz, what is the delay? If the phase at the input is $\frac{\pi}{4}$, what is the phase at the output?

$$\Delta\theta = -2\pi f \tau$$

$f = 1\,\text{kHz}$ $\quad \Delta\theta = \frac{\pi}{2}$ $\quad \tau = \frac{\Delta\theta}{-2\pi f} = \frac{\frac{\pi}{2}}{-2\pi\, 1000} - 0.25\ \text{ms}$

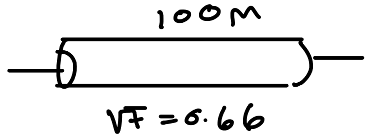
$\tau = -0.25\ \text{ms.}$ $\quad f = 1\,\text{kHz}$ $\quad \Delta\theta = -2\pi\, 1000 \cdot (0.25 \times 10^{-3}) = \frac{\pi}{2}$

$$\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

$VF = \frac{v}{c}$

$v = VF \cdot c$

Exercise 5: A 100m transmission line has a velocity factor of 0.66. Plot the phase response of the cable over the frequency range 0 to 6 MHz.

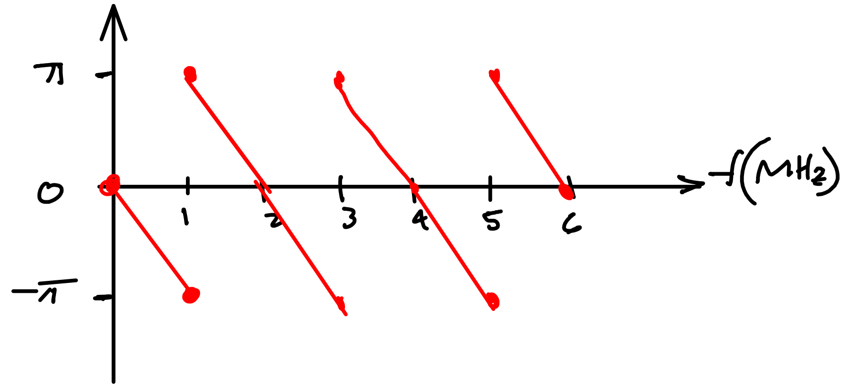


$v = 2 \times 10^8$ m/s

$\tau = \frac{d}{v} = \frac{100}{2\times10^8} = 50 \times 10^{-8}$

$\tau = 0.5 \times 10^{-6}$

$\Delta\theta = -2\pi f \tau$



f	$\Delta\theta$
1×10^6	$-2\pi \cdot 1\times10^6 \cdot 0.5 \times 10^{-6} = -\pi$

Exercise 6: A telephone line is being used to transmit symbols at a rate of 300 symbols/second using frequencies between 800 and 1200 Hz. If the group delay variation must be less than 10% of the symbol period, what is the maximum allowable variation in group delay over this frequency range?

$f_{sym} = 300$

$T_{sym} = \frac{1}{300} \approx 3.3$ ms.

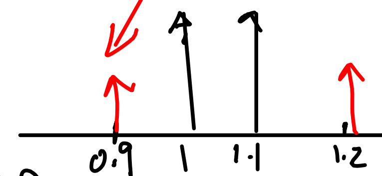
group delay < 10% of T_{sym} = 0.1×3.3 ms $\doteq \underline{\underline{333 \mu s.}}$

Exercise 7: The input to a ~~non-ideal~~ non-linear amplifier is the sum of two sine waves at frequencies of 1 and 1.1 MHz. What are the frequencies of the harmonics of these frequencies? What are the frequencies of the (positive) third-order IMD products?

$f_1 = 1$

$f_2 = 1.1$

3rd order IM products



harmonics:

fundamental	2nd	3rd	4th	...
1	2	3	4	
1.1	2.2	3.3	4.4	

3rd-order IMD: $2f_1 - f_2 = 2 - 1.1 = 0.9$

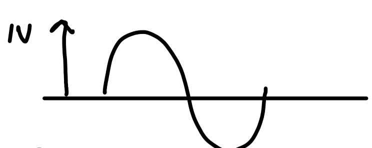
$2f_2 - f_1 = 2.2 - 1 = 1.2$

Exercise 8: A sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

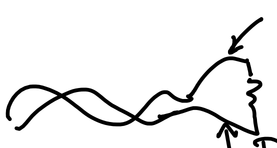
Nearby machinery is adding noise onto the line. The voltage of this noise signal is measured with an RMS voltmeter as 100mVrms. The characteristic impedance of the line is 600Ω and it is terminated with that impedance. Why was an RMS voltmeter used? What is the signal power? What is the noise power? What is the SNR?

Why RMS? → noise is not sinusoidal so only true RMS VM will give accurate power measurement.

$V_{peak} = 1\,V$



$V_{RMS} = \frac{1}{\sqrt{2}} A = \frac{1}{\sqrt{2}}$



$P_{sig} = \frac{V}{R} \cdot V = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{600} = \frac{1}{1200}$

$\frac{S}{N} = \frac{\frac{1}{2}}{.01} = 50 \approx 17\,dB$

$P_{noise} = \frac{V^2}{R} = \frac{(0.1)^2}{600} = \frac{.01}{600}$

Exercise 9: Would you use AC or DC coupling to measure: (a) σ, (b) μ, and (c) the RMS power? ~~Would you measure the average or RMS power in each case~~? What is the RMS power of the signal x if it has a mean (DC) value of $\mu = 2$ V and $\sigma = 3$ V?

$\mu = 2V$

$\sigma = 3V$

(a) σ = rms voltage AC

(b) μ = average voltage DC

(c) rms power. $= 2^2 + 3^2 = 4+9 = 13$

if $R = 1\,\Omega$ ("normalized" power)

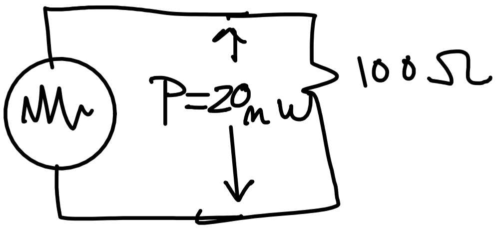
Exercise 10: What are the units of t?

$$t = \frac{v - \mu}{\sigma}$$

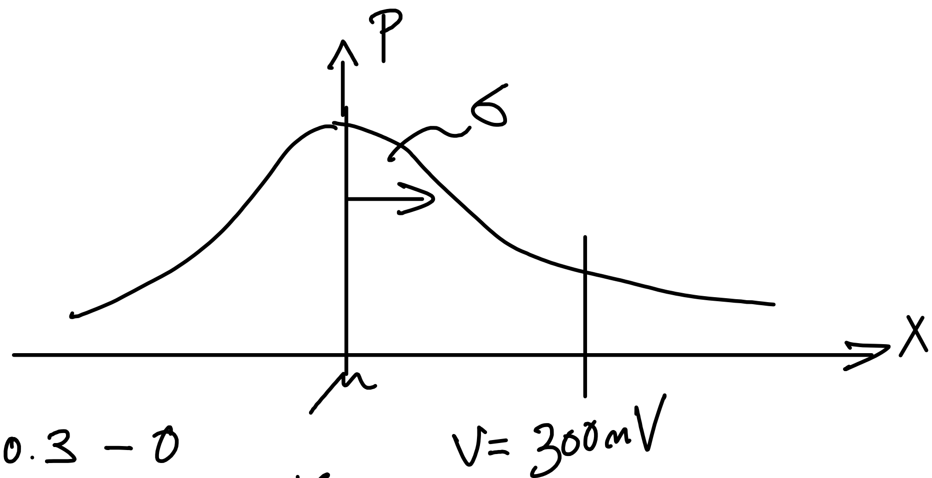
unit less.

Exercise 11: The output of a noise source has a Gaussian (normally) distributed output voltage. The (rms) output power is 20mW ~~and the output~~ driving an impedance ~~is~~ of 100Ω. What fraction of the time does the output voltage exceed 300mV? Hint: the variance (σ^2) of a signal is the same as the square of its RMS voltage.

and $V_{avg} = 0$



$$P = \frac{\sigma^2}{R} = \frac{(V)^2}{100} = 0.020$$

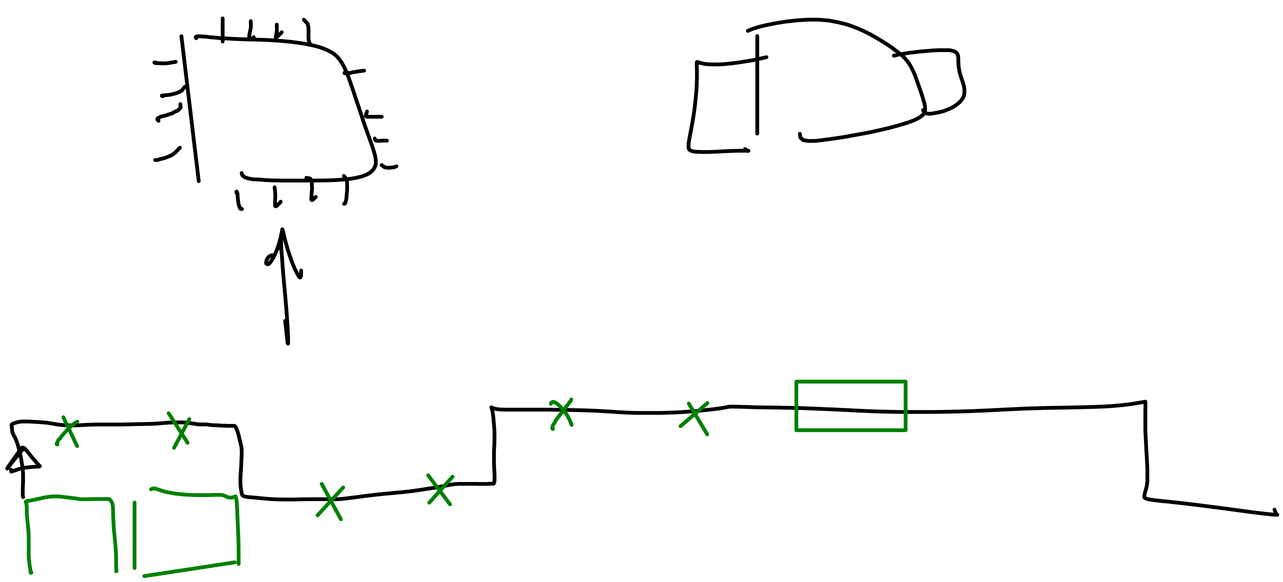


$$V = \sqrt{.02 \cdot 100}$$

$$\sigma = \sqrt{2}$$

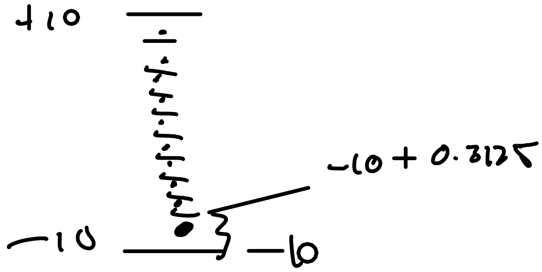
$$t = \frac{v - \mu}{\sigma} = \frac{0.3 - 0}{\sqrt{2}} = 0.212$$

$$\text{prob}\,(x < 0.3) = P(t)$$



Midterm: next Monday @ $8^{30} \to 10^{20}$

October 17



egg 3

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$\rightarrow$ $10 \times 10^6 \times 8$ — payload

$\rightarrow$ time $\leftarrow$ $\leftarrow$ overhead