Lecture 10 - Error Detection and Correction

Exercise 1: Compute the modulo-4 checksum, C, of a frame with byte values 3, 1, and 2. What values would be transmitted in the packet? What would be the value of the sum at the receiver if there were no errors? Determine the sum if the received frame was: 3, 1, 1, C? 3, 1, 2, 0, C? 1, 2, 3, C?

3+1+2=6   6mod4=2

would transmit 3,1,2,2

at receiver the sum would be: 3+1+2+2=8, and 8mod4=0

if received 3+1+1+2, sum is 7; 7mod4=3 so errror detected

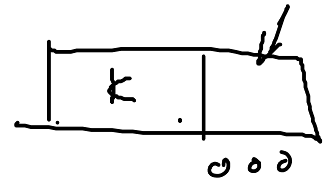
if received 3+1+2+0+2, sum is still 8; 8mod4=0 so appended zeros are not detected

if received 1+2+3+2, sum is also still 8; 8mod4=0 so transposition is not detected

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

$1+0+1 = 2 \qquad 2 \bmod 2 = \underline{\underline{0}}$

Exercise 3: How many possible code words are there for an (n, k) code? How many possible parity bit patterns are possible for each code word?



– 2^k possible

– 1 possible set of parity bits for each k-bit data portion.

$$\begin{array}{r} 0111 \\ 1011 \\ \hline 1+1+0+0 = 2 \end{array}$$

Exercise 4: What is the Hamming distance between the codewords 11100 and 11011? What is the minimum distance of a code with the four codewords 0111, 1011, 1101, 1110?

$$\begin{array}{r} 11100 \\ 11011 \\ \hline 1+1+1 = 3 \end{array}$$

$D_{min} = 2$

	0111	1011	1101	1110
0111	0	2	2	2
1011	2	0	2	2
1101	2	2	0	2
1110	2	2	2	0

Exercise 5

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What is the code rate of a code with 4 codewords each of which
is 4 bits long? {\em Hint: If a code has $2^k$ codewords, what
is $k$?}.
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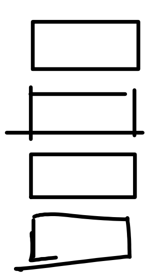

$k = 2$

$n = 4$

$$\frac{k}{n} = \frac{2}{4} = \frac{1}{2}$$

"rate $\frac{1}{2}$" code.

Exercise 6: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the received decide was sent? What bit was most likely in error? Is it possible that the other codeword was sent?

	101	010
110	2	1

most likely in error

receiver should choose

yes, 101 could have been sent but channel would have caused 2 errors.

Exercise 7: What is the minimum distance for the code in the previous exercise? How many errors can be detected if you use this code? How many can be corrected? What are n, k, and the code rate (k/n)?

010
101

$d_{min} = 1+1+1 = 3$

guaranteed to detect $d_{min} - 1 = 2$ errors

" " correct $\left\lfloor \frac{d_{min}-1}{2} \right\rfloor = \left\lfloor \frac{3-1}{2} \right\rfloor = 1$ error

100	010	101
	100	100
	1+1+0 = 2	0+0+1 = 1

receive

$n = 3$

$k = 1$

$\frac{k}{n} = \frac{1}{3}$

8

Exercise 9: What are the units of Energy? Power? Bit Period? How can we compute the energy transmitted during one bit period from the transmit power and bit duration?

Energy Joules

Power Joule / s = W

Bit period : s

J = J/s s

E = P · T

$E_b = P \cdot T_b$

$E_b = S \cdot T_b$

Exercise 10: A system needs to operate at an error rate of 10^{-3}. Without FEC it is necessary to transmit at S= 1W at a rate of 1 Mb/s. When a rate-1/2 code is used together with a data rate of 2 Mb/s the power required to achieve the target BER decreases to 500mW. What is the channel bit rate in each case?

What is the information rate in each case? What is E_b in each case? What is the coding gain?

		w/o coding	w/ coding
channel bit rate		1×10^6	2×10^6
data (information) rate		1×10^6	$2 \times 10^6 \cdot \frac{1}{2} = 1 \times 10^6$ (rate)
E_b	S	1 W	$\frac{1}{2}$ W
	T_b	10^{-6}	10^{-6} $\left(\frac{1}{f_b}\right)$
	E_b	1 μJ/bit	0.5 μJ/bit

coding → $\frac{E_b\ (w/o)}{E_b\ (w/)} = \frac{1}{0.5} = 2$ (3dB)

Exercise 11: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?

$k = 1$

$n = 2$

$K = 7$

$\text{rate} = \frac{k}{n} = \frac{1}{2}$

Exercise 12: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?

	1	2	3
A →	A	A	
B →		B	B

in

$\text{rate } \frac{k}{n} = \frac{3}{4}$

out

Exercise 13: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 9 and 10 of the interleaved sequence have errors, where would the errors appear in the de-interleaved sequence? If the receiver could correct up to one error per 4-bit word, would it be able to correct all the errors without interleaving? With interleaving?

(see lecture notes)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1, 2, 3, 4, 5, 6, 7, ~~8~~, ~~9~~, ~~10~~, 11, 12, 13, 14, 15, 16

1, 5, 9, 13, 2, 6, 10, ~~14~~, ~~3~~, ~~7~~, 11, 15, 4, 8, 12, 16

1	2	~~3~~	4
5	6	~~7~~	8
9	10	11	12
13	~~14~~	15	16

$\rightarrow$ 1, 2, ~~3~~, 4, 5, 6, ~~7~~, 8, 9, 10, 11, 12, 13, ~~14~~, 15, 16

= GF(256)

Exercise 14: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?

byte interleaving.