# Solutions to Midterm Exam

## Question 1

The entropy of a source is computed as:

$$H = \sum_i \left(-\log_2(P_i) \times P_i\right) \quad \text{bits/message}$$

In this question a message is a run.

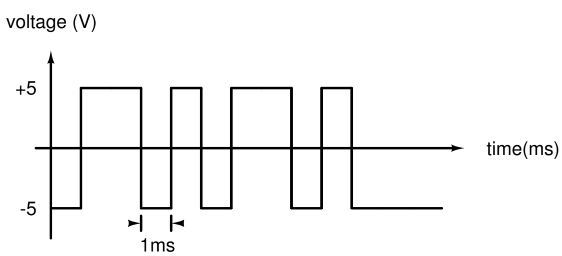
There were two versions of the question. The computation of the entropy for each is shown below:

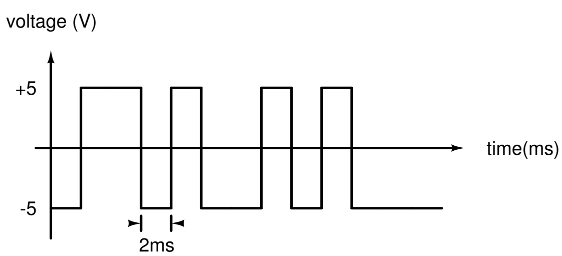
| P | log2(P) | Plog2(P) |
|---|---|---|
| 0.40 | 1.32 | 0.53 |
| 0.30 | 1.74 | 0.52 |
| 0.15 | 2.74 | 0.41 |
| 0.15 | 2.74 | 0.41 |
| | | 1.87 |

| P | log2(P) | Plog2(P) |
|---|---|---|
| 0.4 | 1.32 | 0.53 |
| 0.3 | 1.74 | 0.52 |
| 0.2 | 2.32 | 0.46 |
| 0.1 | 3.32 | 0.33 |
| | | 1.85 |

## Question 2

There were two waveforms:




(a) The bit rate is the inverse of the bit period. For a bit period of 2 ms the bit rate is 500 Hz and for a bit period of 1 ms the bit rate is 1 kHz. For an asynchronous serial interface there is a maximum of one transition per bit so the baud rate is the same as the bit rate.

(b) For the first waveform the bits transmitted in order from LS to MS bit are 0101 1010 (high is 0 and low is 1). In order from MS to LS bit the value is 0101 1010, hex 0x5A.

For the second waveform the bits transmitted in order from LS to MS bit are 0101 0010 (high is 0 and low is 1). In order from MS to LS bit the value is 0100 1010, hex 0x4A.

(c) Assuming UTF-8 encoding, code points less than 128 are encoded with one byte so the code points are 0x5A and 0x4A respectively. The corresponding Unicode characters (which are the same as ASCII in this case) are found in the "C0 Controls and Basic Latin" Unicode table as 'Z' and 'J' respectively.

## Question 3

The characterisitic impedance of twisted pair with dielectric insulation is approximately:

$$Z_0 \approx \frac{120}{\sqrt{\varepsilon_r}} \ln\left(\frac{2S}{D}\right) \quad \Omega$$

where $D$ is the wire diameter and $S$ is the separation (distance between centers).

The diameter of $n$ AWG wire is approximately:

$$d \approx 8 \times 2^{-\frac{n}{6}} \text{ mm}$$

There were two versions of this question, one with 14-gauge wire and 8 cm spacing and the second with 12-gauge wire and 10 cm spacing. For air, $\varepsilon_r = 1$.

For the first version, the wire diameter is $D = 8 \times 2^{-14/6} \approx 1.6$ mm, $S = 80$ mm and $Z_0 = \approx 120 \ln\left(\frac{2\cdot 80}{1.6}\right)$ ..

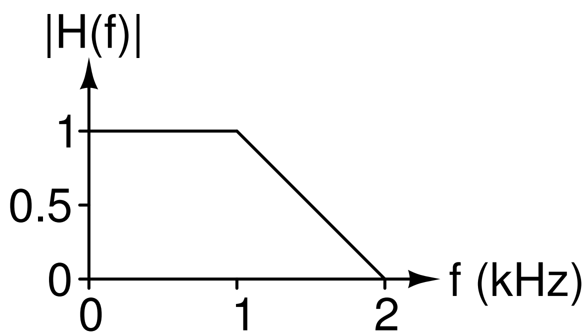
For the second version, the wire diameter is $D = 8 \times 2^{-12/6} = 2$ mm, $S = 100$ mm and $Z_0 = \approx 120 \ln\left(\frac{2\cdot 100}{2}\right)$.

In both cases $Z_0 \approx 553\Omega$.

Due to the poorly-worded question, I also accepted answers where a dielectric constant of 2.2 was assumed which would give $Z_0 \approx 553/\sqrt{2.2} = 372\ \Omega$.

## Question 4

For the following frequency response:




(a) This is a low-pass channel.

(b) In the rolloff region between 1 and 2 kHz $|H(f)| = 2 - f$ or $f = 2 - |H(f)|$.

The $-6$ dB bandwidth is the frequency range over which the response of the filter is within 6 dB of the maximum gain. $-6\text{ dB} = 10^{-6/20} \approx 0.5$. The frequency at which the gain is $-6$ dB is $2 - 0.5 = 1.5$ kHz and this is the bandwidth.

Similarly, $-3\text{ dB} = 10^{-3/20} = 0.707$ and the -3 dB bandwidth is $2 - 0.707 \approx 1.293$ kHz.

(c) At 1.5 kHz $|H(f)| = 0.5$ and this is the ratio of the output to input voltages. So if the input was 10 mV the output would be 5 mV and if the input was 100 mV the output would be 50 mV.