## Solutions to Midterm Exam

## Question 1

(a) $\log _{2}(4)=2$ bits can represent four symbols. Since the duration of each symbol is $1 \mu \mathrm{~s}$, the bit rate is $\frac{2}{10^{-6}}=2 \mathrm{MHz}$.
(b) The symbol rate is one symbol every microsecond or 1 MHz .
(c) The information rate can be computed as the product of entropy (bits per symbol) times the symbol rate (symbols per second). The entropy can be computed as $H=\sum_{i}\left(-\log _{2}\left(P_{i}\right) \times P_{i}\right)=$ $-0.1 \log _{2}(0.1)-02 . \log _{2}(0.2)-0.3 \log _{2}(0.3)-$ $0.4 \log _{2}(0.4) \approx 1.846$ bits/symbol. At a symbol rate of 1 MHz this is about $1.85 \mathrm{Mbits} / \mathrm{s}$.
(d) The baud rate is defined by the minimum time between signal level transitions. In this case it is $0.5 \mu \mathrm{~s}$ so the baud rate is 2 MHz .

## Question 2

There were two RS-232 waveforms:

and

(a) The bit values from MS to LS can be read from right to left from the diagram: $10010111=0 \times 97$ and $11100011=0 x E 3$.
(b) The bit rate is the inverse of the bit durations which are approximately 9600 bps and 19,200 bps.
(c) The bit following the start bit and the 8 data bits is high ( 0 ) so it cannot be a stop bit and must be a parity bit. In both cases the number of 1's is odd (5, including the parity bit) so odd parity was used.

## Question 3

We can solve the Friis path loss equation:

$$
P_{R}=P_{T} G_{T} G_{R}\left(\frac{\lambda}{4 \pi d}\right)^{2}
$$

for the distance as a function of the other variables:

$$
d=\frac{\lambda}{4 \pi}\left(\frac{P_{R}}{P_{T} G_{T} G_{R}}\right)^{-\frac{1}{2}}
$$

which are all given: $\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{900 \times 10^{6}}=\frac{1}{3} \mathrm{~m}, P_{R}=$ $1 \mu \mathrm{~W}, P_{T}=1 \mathrm{~W}$ or $10 \mathrm{~W}, G_{T}=G_{R}=10^{10 / 10}=10$, $P_{R}=1 \times 10^{-6}$. The two answers are: 265 m (for 1 W ) and 839 m (for 10 W ).

This question was marked out of 4 as noted on the question although the summary at the front shows it being worth 3 marks. Thus the total number of marks on the exam was 34 , not 33 .

## Question 4

The generator polynomial is $x^{3}+x+1$ (1011) which is of order 3 so the CRC has 3 bits. The messages, including the CRC were 011001 or 011101 . Dividing the messages by the generator polynomial we get remainders:

$$
\begin{gathered}
1011 \mid 011001 \\
--- \\
1100 \\
1011
\end{gathered}
$$

```
----
    1111
    1011
    ----
    100
```

In this case the remainder is not zero so there must have been an error, and:

```
1011/011101
        1110
        1 0 1 1
        ----
        1011
        1011
        ----
```

and in this case there is no remainder so there is not likely to have been an error.

## Question 5

An FEC code uses the following four codewords:

```
000110
```

011011
101100
110001
(a) $k=\log _{2}(4)=2$ is the number of data bits, $n=6$ is the number of bits per codeword and $k / n=$ $2 / 6=1 / 3$ is the code rate.
(b) The bit differences between the different codewords are shown below:

|  | 000110 | 011011 | 101100 | 110001 |
| :--- | :--- | :--- | :--- | :--- |
| 000110 | 000000 | 011101 | 101010 | 110111 |
| 011011 | 011101 | 000000 | 110111 | 101010 |
| 101100 | 101010 | 110111 | 000000 | 011101 |
| 110001 | 110111 | 101010 | 011101 | 000000 |

and the corresponding Hamming distances are:

|  | 000110 | 011011 | 101100 | 110001 |
| :---: | :---: | :---: | :---: | :---: |
| 000110 | 0 | 4 | 3 | 5 |
| 011011 | 4 | 0 | 5 | 3 |
| 101100 | 3 | 5 | 0 | 4 |
| 110001 | 5 | 3 | 4 | 0 |

so the minimum distance of the code is $d_{\min }=3$.
(c) Since $d_{\text {min }}=3$ this code is guaranteed to detect $d_{\text {min }}-1=3-1=2$ errors.
(d) Since $d_{\text {min }}=3$ this code is guaranteed to correct $\left\lfloor\frac{d_{\text {min }}-1}{2}\right\rfloor=\left\lfloor\frac{3-1}{2}\right\rfloor=1$ error.
(e) Yes, if the codeword received is not a valid codeword (one in the code), the channel must have introduced an error.
(f) The codeword most likely to have been transmitted is the codeword with the minimum distance to the received codeword. The bit differences and Hamming distances are shown below:

|  | 000110 | 011011 | 101100 | 110001 |
| :---: | :---: | :---: | :---: | :---: |
| 111100 | 111010 | 100111 | 010000 | 001101 |
| $d$ | 4 | 4 | 1 | 3 |
| 100110 | 100000 | 111101 | 001010 | 010111 |
| $d$ | 1 | 5 | 2 | 4 |

and the decoder will choose the codeword at the smallest distance from the received codeword ( 101100 for the first example and 000110 for the second).
(g) It is more likely that we receive codewords with fewer errors. However, in the general case where each error is independent, any number of errors is possible. Since the channel could have introduced any number of errors it is possible that any other codeword could have been transmitted.

## Question 6

(a) Stop and Wait ARQ would be best suited for a communication system operating over a short distance and requiring low-cost hardware because the short distance implies low delay between transmission and ACK. Stop and Wait ARQ also has the lowest implementation cost so it would be most suitable for this application.
(b) If the output data is on TxD (transmit data) the device is a DTE (terminal), if the output is on RxD it is a DCE (communications equipment modem). A "null modem" (or "crossover cable") would allow these two devices to communicate.
(c) The maximum allowed cable length for a 10BaseT Ethernet link is 100 m . If the dielectric
constant of the cable is $\varepsilon_{r}=2.2$ the velocity factor is $V F=\frac{1}{\sqrt{2.2}}=0.66$ and the propagation delay is $\frac{100}{0.66 \times 3 \times 10^{8}}=0.5 \mu \mathrm{~s}$. Due to the error in the question, an answer with $\varepsilon_{r}$ and $V F$ exchanged would also be marked correct.
(d) The minimum channel bandwidth required to transmit a NRZ signal without ISI is half of the symbol rate $(0.5 \mathrm{MHz}$ for $1 \mathrm{Mb} / \mathrm{s}$ and 1 MHz for $2 \mathrm{Mb} / \mathrm{s}$ ). A channel with the minimum bandwidth would have a "brick-wall" frequency response $(\alpha=0)$. This is constant from a frequency of 0 (DC) to half of the symbol rate and zero above that:

(e) A transformer is often used to implement a balun ${ }^{1}$ for use on twisted-pair links (e.g. Ethernet over twisted pair).
(f) An audio amplifier amplifies baseband (rather than passband) signals so its distortion is more like to be specified and measured using THD.

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[^0]:    ${ }^{1}$ Refer to the "Transformers" section in Lecture 6 and the corresponding learning objective.

