# Solutions to Mid-Term Exam

## Question 1

(a) If we treat each street state as a message, the entropy of the source is:

$$H = \sum_i (-\log_2(P_i) \times P_i) \quad \text{bits/message}$$

where $P_i$ is 0.5, 0.25, 0.04, 0.1 for $i = 0, \ldots, 3$. These values don't add up to 1 either a message is not given or one or more of the probabilities are incorrect.

If we simply apply the formula to the values given the entropy can be calculated as:

$$\begin{aligned} &-(0.5\log(0.5) + 0.25\log(0.25) + \\ &\quad 0.04\log(0.04) + 0.1\log(0.1))\,/\log(2) \\ &\qquad = 1.25 \text{ bits/message} \end{aligned}$$

The calculation and another possible answer where the second probability has been increased to 45% so the probabilities add up to 1 is shown below:

| Probability | | | Probability | |
|---|---|---|---|---|
| 0.50 | 0.50 | | 0.50 | 0.50 |
| 0.25 | 0.50 | | 0.45 | 0.52 |
| 0.04 | 0.19 | | 0.04 | 0.19 |
| 0.01 | 0.07 | | 0.01 | 0.07 |
| Entropy= | 1.25 | | Entropy= | 1.27 |

(b) If the best possible compression method is used the required data rate will be the same as the information rate. To transmit the traffic state information for 1000 streets in 60 seconds will require a data rate of $\frac{1000\times1.25}{60} \approx 21$ b/s.

## Question 2

The Cyrillic small letter 'ZHE' (Ж) has the Unicode code point U+0436.

The code point in binary is 0000 0100 0011 0110. Since the most-significant 5 bits are zero this value should be encoded using two bytes (second row of the UTF-8 encoding table).

The MS 5 bits are yyyyy=10000 and the LS 6 bits are xxxxxx=110110.

From the table the first byte is 110 10000 (0xd0) and the second byte is 10 110110 (0xb6).

## Question 2

The equation for the characteristic impedance of a coaxial cable is

$$Z_0 \approx \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right)$$

We are given $Z_0 = 50$, $d$ as 24 gauge which is 0.5 mm. During the exam $\varepsilon_r$ was given as 2.2. We can solve for $D$:

$$D = \exp\left(\frac{Z_0\sqrt{\varepsilon_r}}{60}\right) d \approx 1.7 \text{ mm}$$

## Question 4

(a) The -10 dB bandwidth is where the amplitude response is 10 dB below the maximum. In this case it is 4.5 kHz.

(b) The stopband attenuation in this case is 20 dB.

(c) At a frequency of 4.5 kHz the attenuation is 10 dB. The output voltage will thus be $V_{out} = V_{in} \times 10^{10/20} = 0.32\ \text{mV}_{rms}$.

(d) The attenuation in the transition between passband and stopband increases by 20 dB from 4 to 5 kHz or 20 dB/kHz. A frequency higher than 4 kHz is 8 kHz (a doubling of frequency) so the attenuation would increase to $20 \times 4 = 80$ dB and this is the rolloff in dB per octave.

## Question 5

A signal will be undistorted only if all of its frequency components are attenuated and delayed equally. The channel must thus have a constant amplitude response and a linear phase response over the signal's bandwidth.

In the example shown the amplitude response is constant from 0 to 2 kHz but the phase response is only linear over the range 0 to 1 kHz. Thus the maximum bandwidth of a baseband signal that will be undistorted is 1 kHz.