Solutions to Assignment 2

Question 1

(a) The 16550 has eight registers accessible using address lines A0-A2.

However, when the Divisor Latch Address Bit (DLAB, bit 7 in register 3 "Line Control Register") is set to 1 then registers 0 and 1 are the LS and MS bytes of a 16-bit clock divider ratio.

Two of the other eight registers (0=THR and 2=FCR) are write-only and selected when doing a write and two are read-only (0=RBR and 2=IIR) and selected when doing a read.

Thus as shown in Table 1 there are actually 12 registers (6 read/write, 2 read-only, 2 write-only and 2 enabled by the DLAB bit).

Depending on how the answer is phrased, it could be 8 (directly addressable registers), 10 (indirectly addressable registers) or 12 (registers that can store different values).

(b) According to section 8.5 and Table 4, to obtain a baud rate of 19200 bps with a clock frequency of 1.8432 MHz the divider ratio would be $1843200/(16 \times 19200) = 6$. Thus DLM (the MS byte of the divisor) would be set to 0 and DLL (LS byte) would be set to 6.

The table in the datasheet shows the divider values required for a few of the possible bit rates. Although the highest rate in the table is 56000, the maximum baud rate would correspond to a clock divider ratio of 16 (DLM=0, DLL=1) and a bit rate of 115.2 kb/s. For this reason you will often see this value quoted as the maximum speed of many serial interfaces.

(c) According to Section 8.6.3 and Table 1, bit 3 of the Line Status Register (register 5) register indicates a framing error. It is set to 1 when a framing error is detected.

(d) Section 8.6.3 says that the Break Indicator bit is set when the serial input line stays high (logic 0) for more than the duration of a character (including start, data, parity and stop bits).

(e) According to the Wikipedia entry, a UART can detect a false start bit by sampling multiple times after a rising edge of the serial input signal. If the start bit is too short it is declared a "false" start bit. The definition of "too short" is implementation dependent. The 16550 samples 16 times per bit so this allows it to detect short start bits.

Question 2

(a) The -6 dB bandwidth is measured between frequencies at which the magnitude of the amplitude response is 0.5. From the graph this bandwidth is 5.38 MHz.

(b) The value of α is the ratio of the total bandwidth (measured between frequencies where the response is 0) to the 6dB bandwidth, minus one. From the graph this is $(6/5.38) - 1 = 0.115$ or 11.5%.

(c) The 3 dB bandwidth is measured between frequencies where the the response is $\frac{1}{\sqrt{2}}$. The equation for the raised-cosine amplitude response is $\frac{1}{2}(1 + \cos(\frac{\pi\Delta_f}{2d}))$ where Δ_f is the offset from the start of the raised-cosine response and $2d$ is the (frequency) span of the raised-cosine portion as shown in the graph. Thus:

$$\frac{1}{2}\left(1 + \cos\left(\frac{\pi\Delta_f}{2d}\right)\right) = \frac{1}{\sqrt{2}}$$

from which we can solve for Δ_f:

$$\Delta_f = \frac{2d}{\pi}\cos^{-1}\left(\sqrt{2} - 1\right)$$

and find $\Delta_f \approx 0.728d$. From the graph, the 3 dB bandwidth is $5.38 - 2d + 2\Delta_f$. Since $d = 0.31$ the 3 dB bandwidth is $5.38 - 2 \times 0.31 + 2 \times 0.728 \times 0.31 =$ and since d is given as 0.31 MHz, the 3 dB bandwidth is $\approx$ 5.21 MHz.

Question 3

(a) NEXT (near-end crosstalk) is the crosstalk from a transmitter to signals received at the same location as the transmitter. Thus NEXT would be measured at the side of the line with the test signal generator.

(b) Using the equation with $f = 100$, the loss should be

$$27.1 - 16.8\log_{10}(100/100) = 27.1 \text{ dB}$$

An attenuation (loss) of 27.1 dB corresponds to a voltage ratio of $10^{(-27.1/20)} \approx 0.044$ and the received voltage should be less than $0.044 \times 1 = 44$ mV to comply with the spec.

(c) We can repeat the above calculation with $f = 1$ and obtain a minimum loss of 60.7 dB and a maximum received voltage of 0.923 mV.

Question 4

(a) The formula for the RMS voltage is:

$$\sqrt{\sum_i p_i v_i^2}$$

For four equally-likely values $p_i = 0.25$. Let the scaled voltage corresponding to +1V be v. Then the scaled RMS voltage is:

$$\sqrt{0.25\left((+3v)^2 + v^2 + (-v)^2 + (-3v)^2\right)} = \sqrt{5}\,v$$

Setting this equal to 0.224 V, we find $v = (0.224/\sqrt{5}) \approx 100$ mV.

(b) The question asks for the noise power (the variance, σ^2) that results in a probability of error of 10^{-6} assuming that an error happens when the noise voltage exceeds half of the difference between two adjacent voltage levels which is $\frac{1}{2}2v = v$.

This is the probability $P(x > t) = 10^{-6}$ where t is the normalized threshold. The normal CDF, $P(x < t)$ is what we can usually compute but since the distribution is symmetrical around the mean (zero here), $P(x > t) = P(x < -t)$.

We can solve for t in various ways:

(a) From tables. Unfortunately, most published tables of the normal cumulative distribution function (CDF) only show values up to $t = 3$ or 4

(b) From a graph. Similarly, the graph on page 6 of Lecture 4 only extends to probabilities of $P(t = -4) \approx 5 \times 10^5$. But this tells us that t must be more negative than -4.

(c) Using a calculator. Evaluating a few values with the Sharp FX-516 calculator finds $P(-4.7) \approx 1 \times 10^{-6}$ although it can't compute probabilities less than this.

(d) Using the Matlab (or octave). The function `norminv(1E-6)` returns -4.7534.

(e) Using Excel (or LibreOffice Calc) function. The function `NORMINV(1E-6,0,1)` also returns -4.7534.

(f) Using the logistic approximation. The formula can be solved to find $t \approx -\ln(10^6)/1.7 = -8$ which is a poor approximation (although $P(-8)$ is within 1% of $P(-4.7)$).

(g) On the web. On the Wolfram Alpha web site entering "normal distribution," and selecting "formula," and "probability" and entering `1e-6` shows that $P(-4.753) = 1 \times 10^{-6}$.

The normalized threshold ($-t = -4.753$) is related to the threshold voltage ($v = 0.1$ V) and noise mean ($\mu = 0$ V) and standard deviation ($\sigma\ V_{rms}$) by:

$$t = \frac{(v - \mu)}{\sigma}$$

from which we can solve for $\sigma = \frac{v}{t} = \frac{0.1}{4.753} = 21.0\ \text{mV}_{rms}$. The (normalized) noise power is $\sigma^2 \approx 443\ \mu\text{V}^2_{rms}$.

For the two voltage levels nearest zero errors happen if the noise is larger or smaller than v. Thus $2P(x < t) = 10^{-6}$, $t = 4.89$ and $\sigma^2 \approx 418\mu\text{V}^2_{rms}$.

The overall average error rate is the average of these two approximations. In this case the error rate is $1.5P(x < t) = 10^{-6}$, $t = 4.67$ and $\sigma^2 \approx 459\mu V^2_{rms}$.

The true noise power would be computed as σ^2/R but the resistance at the point where the voltage was measured, R, was not stated in the question.

(c) When using 4 different symbols we transmit $\log_2(4) = 2$ bits per symbol. For a bit rate of 1 Mb/s the symbol rate is 500 kHz.

The Nyquist no-ISI criteria is satisfied by a channel with a minimum bandwidth of half of the symbol rate or 250 kHz.

(d) The signal power is the square of the RMS power or 224^2 mV^2. The noise power is $\sigma^2 \approx 2$ mV^2. The SNR is thus about 113 (20.5 dB).

If the SNR of a band-limited AWGN channel is known then the Shannon capacity theorem can be used to compute the minimum SNR required for an arbitrarily-low error rate at an information rate C:

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

From which we can find

$$B = \frac{C}{\log_2(1 + S/N)} = \frac{10^6}{\log_2(114)} = 146 \text{ kHz}$$

Question 5

Since the total power is 100%, then 1% of the power is not included in the 99% power bandwidth. Since the Gaussian distribution is symmetrical, half (0.5%) of this power is above the bandwidth and half below. From Figure 1 in Lecture 4 a probability of 0.5% corresponds to a value of $t \approx 2.6$. Thus the frequency range extends over $\pm t\sigma = 2.8 \times 10 = 26$ kHz around the center frequency and the bandwidth is ≈ 52 kHz.

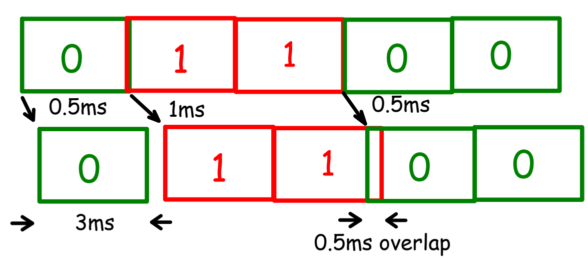
Question 6

(a) Note that the phase of a signal increases with time. A delayed signal (the signal at an earlier time) thus has a smaller phase. The phase shift through a channel (the phase of the delayed output minus the phase of the input) is therefore negative. Thus while the slope of the phase versus frequency curve is negative, the delay itself is positive.

The group delay is the (negative of) the slope of the phase versus frequency curve or the rate of change of phase with frequency. Since the phase response is linear between the measured frequencies, the slope, and thus the group delay is constant.

Between 1 and 1.5 kHz the phase shift expressed in cycles is $90/360 = 0.25$ cycles so the group delay is $0.25/500 = 500$ μs. Between 1.5 and 2 kHz the phase shift is $180/360 = 0.5$ cycles and thus the group delay is $0.5/500 = 1$ ms. The same calculation could be done using units of radians and radians/second.

(b) The symbols will be delayed relative to each other by different amounts due to the different group delays. The 1's (higher frequency symbols) will be delayed more than the 0's. As shown below, at the receiver the two one's will be delayed in time relative to the zeros and so the third zero will suffer ISI from the second '1'.



The duration of the ISI will be equal to the difference in group delays or 500 μs. Since the period is 3 ms, the fraction corrupted is $0.5/3 = 1/6$.

This is an idealized model. Switching between the two frequencies causes discontinuities in the signal which results in additional frequeny components and a more complex ISI calculation.