# Lecture 9 - Error Detection and Correction

3 + 1 + 2 = 6

6 modulo 4 = 2

$$4\overline{)6}\,^{1}$$
2

**Exercise 1**: Compute the modulo-4 checksum, *C*, of a frame with byte values 3, 1, and 2. What values would be transmitted in the packet? What would be the value of the checksum at the receiver if there were no errors? Determine the checksum if the received frame was: 3, 1, 1, *C*? 3, 1, 2, 0, *C*? 1, 2, 3, *C*?

C = checksum is −2
or 2

3, 1, 2, 2 ← c

at receiver if no errors, sum = 8 mod 4 = 0
(or 4 mod 4 = 0)

3, 1, 1, 2 : 3 + 1 + 1 + 2 = 7 7 mod 4 = 3 ⇒ not zero error!

3, 1, 2, 0, 2 : 3 + 1 + 2 + 0 + 2 = 8 8 mod 4 = 0 ⇒ no errors

1, 2, 3, 2 : 1 + 2 + 3 + 2 = 8 8 mod 4 = 0 ⇒ no errors.

$$2\overline{)1}\,^{0}$$
1

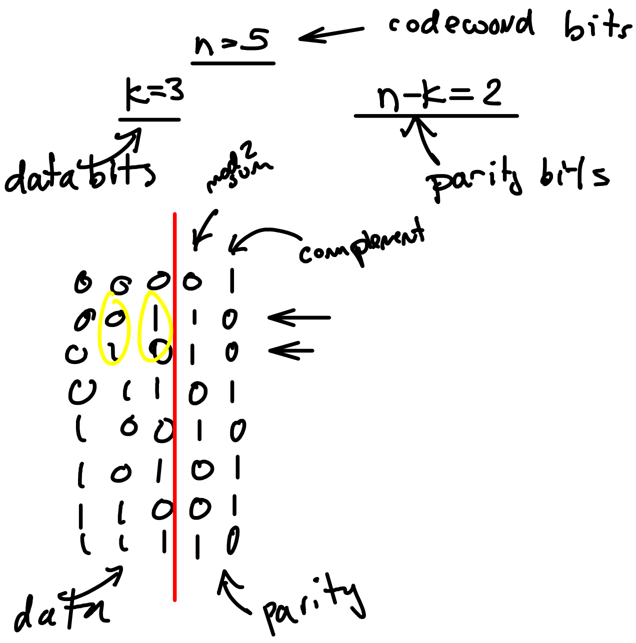
**Exercise 2**: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

modulo 2 sum is remainder after dividing by 2
can be 0 even # of 1's (sum is even #)
or 1 odd # of 1's (sum is odd #)

e.g. 1, 0, 1 = 2 mod 2 = 0

**Exercise 3**: How many possible code words are there for an $(n, k)$ code? How many possible parity bit patterns are possible for each code word?

- possible $2^k$ valid codewords
- only one parity pattern is possible for each codeword.



0 0 0 | 0 1
0 0 1 | 1 0
0 1 0 | 1 0
0 1 1 | 0 1
1 0 0 | 1 0
1 0 1 | 0 1
1 1 0 | 0 1
1 1 1 | 1 0

0 1 0 1 1

**Exercise 4**: What is the Hamming distance between the codewords 11100 and 11011? What is the minimum distance of a code with the four codewords 0111, 1011, 1101, 1110?

$$\begin{array}{c} 1\,1\,1\,0\,0 \\ 1\,1\,0\,1\,1 \\ \hline 0+0+1+1+1 \end{array} = 3 \text{ is Hamming distance}$$

0111
1011
1101
1110

1100 2
1010 2   0110 2
1001 2   0101 2   0011 2

minimum distance of this code $(D_{min}) = 2$.

**Exercise 5:** A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the received decide was sent? What bit was most likely in error? Is it possible that the other codeword was sent?

$k=1$ $n=3$

valid codewords $\left\{ \begin{array}{l} 101 \\ 010 \end{array} \right.$

receiver receives 110

vs 101 → 2

vs 010 → 1

receiver "guesses" that 010 was transmitted because it has smallest Hamming distance from all valid codewords.

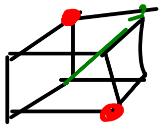
**Exercise 6:** What is the minimum distance for the code in the previous exercise? How many errors can be detected if you use this code? How many can be corrected? What are $n$, $k$, and the code rate $(k/n)$?

k n-k

101

010

$1+1+1=3$



for this code $d_{min} = 3$

# errors guaranteed to detect $= d-1 = 2$

# " " " correct $= \left\lfloor \frac{d-1}{2} \right\rfloor = \frac{3-1}{2} = 1$

$n = 3$ bits /codework

$k = \log_2(\text{\# codewords}) = \log_2 2 = 1$

$n-k = 3-1 = 2$ parity

rate $= \frac{k}{n} = \frac{1}{3}$

$\left\lfloor \frac{4-1}{2} \right\rfloor = \lfloor 1.5 \rfloor$

$= 1$

**Exercise 7**: Assume 1000-bit frames are being transmitted at 1 Mb/s. What is the throughput if there are no errors? Now assume errors introduced by the channel cause a frame error rate of 90%. What is the throughput?

Now assume we use a rate-1/2 FEC code. How many bits must be transmitted in each frame? What is the throughput if there are no errors? Now assume that with FEC coding the receiver corrects most of the errors and the frame error rate drops to 1%. What is the new throughput?

Now assume the FEC can only correct 10% of the frames, what is throughput?

When is it worthwhile to use FEC? What other advantage might the use of FEC provide?

In all cases above ignore the effect of retransmissions.

1000 frames second

x 1000 bits/frame

$= 10^6$ bits/second.

w/ 90 FER

throughput = 10% · 1Mb/s =

= 100 kb/s

rate $\frac{1}{2}$: $\frac{k}{n} = \frac{1}{2}$ $\frac{k=1000}{n=2000}$

each frame has 1000 data bits / 2000 bits per codeword.

at $\frac{1\text{Mb/s}}{2000 \text{ bits/frame}}$ = 500 frames/s

throughput w/ no errors = 500 kb/s (500 x 1000 bits)

FER = 1% 0.99 X 500 frames/s = 495 frames/s

throughput = 495 x 1000 = 495 kb/s

~~**Exercise 8**: What are the units of Energy? Power? Bit Period? How can we compute the energy transmitted during one bit period from the transmit power and bit duration?~~

~~**Exercise 9**: A system needs to operate at an error rate of $10^{-3}$. Without FEC it is necessary to transmit at 1W at a rate of 1 Mb/s. When a rate-1/2 code is used together with a data rate of 2 Mb/s the power required to yachieve the target BER decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is $E_b$ in each case? What is the coding gain?~~

skipped

**Exercise 10**: Assuming one bit at a time is input into the encoder in the diagram above, what are $k$, $n$, $K$ and the code rate?

$k = 1$ bit in at a time

$n = 2$ bits out at a time

rate $= \frac{k}{n} = \frac{1}{2}$

$K = 7$

**Exercise 11**: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?

**Exercise 12**: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 9 and 10 of the interleaved sequence have errors, where would the errors appear in the deinterleaved sequence? If the receiver could correct up to one error per 4-bit word, would it be able to correct all the errors without interleaving? With interleaving?

skipped

**Exercise 13**: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?

**Exercise 14**: How many different patterns of $n - k$ parity bits are there? Assuming all parity bit sequences are equally likely, what is the probability that a randomly-chosen code word has the same parity bits as another codeword?