Lecture 3 - Channel Characteristics and Impairments

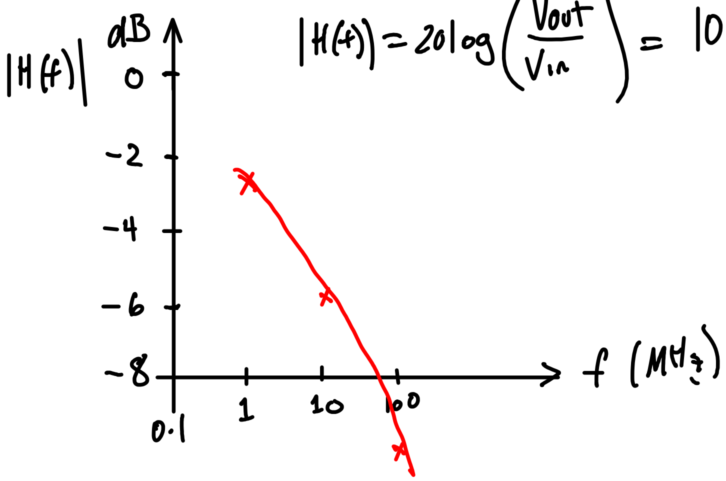
Exercise 1: A 10 dBm signal is applied to one end of a 50 ohm co-ax cable at frequencies of 1, 10 and 100 MHz. At the other end you measure voltages of 7, 4 and 0 dBm respectively. Plot the amplitude of the transfer function of the channel formed by this cable. Show dB on the vertical axis and log of frequency on the horizontal axis.

$$\frac{P_{out}}{P_{in}}(\omega) \equiv P_{out} - P_{in} \text{ (dBm)}$$

10 dBm

f	P_{out}	
1	7	−3
10	4	−6
100	0	−10

$$|H(f)| = 20\log\left(\frac{V_{out}}{V_{in}}\right) = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$



$$-3\text{dB} = \frac{P_{out}}{P_{in}} = \frac{1}{2}$$

$$\left(\frac{V_{out}}{V_{in}}\right) = \left(\frac{1}{\sqrt{2}}\right)^2$$

Exercise 2: How much *power* would a signal transmitted at the edge of the 3 dB bandwidth passband have compared to the the power it would have if transmitted at the frequency with the lowest loss? What would be the ratio of the *voltages*? What if the bandwidth was defined as the 6 dB bandwidth?

(1) $-3\text{dB} = 10\log\left(\frac{P_{out}}{P_{in}}\right); \quad \frac{P_{OUT}}{P_{IN}} = \frac{1}{2}$

(2) $= 20\log\left(\frac{V_{OUT}}{V_{in}}\right); \quad \frac{V_{OUT}}{V_{in}} = \frac{1}{\sqrt{2}}$

(3) $-6\text{dB} = \quad \frac{P_{OUT}}{P_{in}} = \frac{1}{4}$

$= \quad \frac{V_{OUT}}{V_{in}} = \frac{1}{2}$

Exercise 3: A 100m transmission line has a velocity factor of 0.66. Plot the phase response of the cable over the frequency range 0 to 6 MHz.

$VF = 0.66$

$c = 3 \times 10^8 \ m/s$

$v = VF \cdot c = 2 \times 10^8 m/s$

$d = 100m$

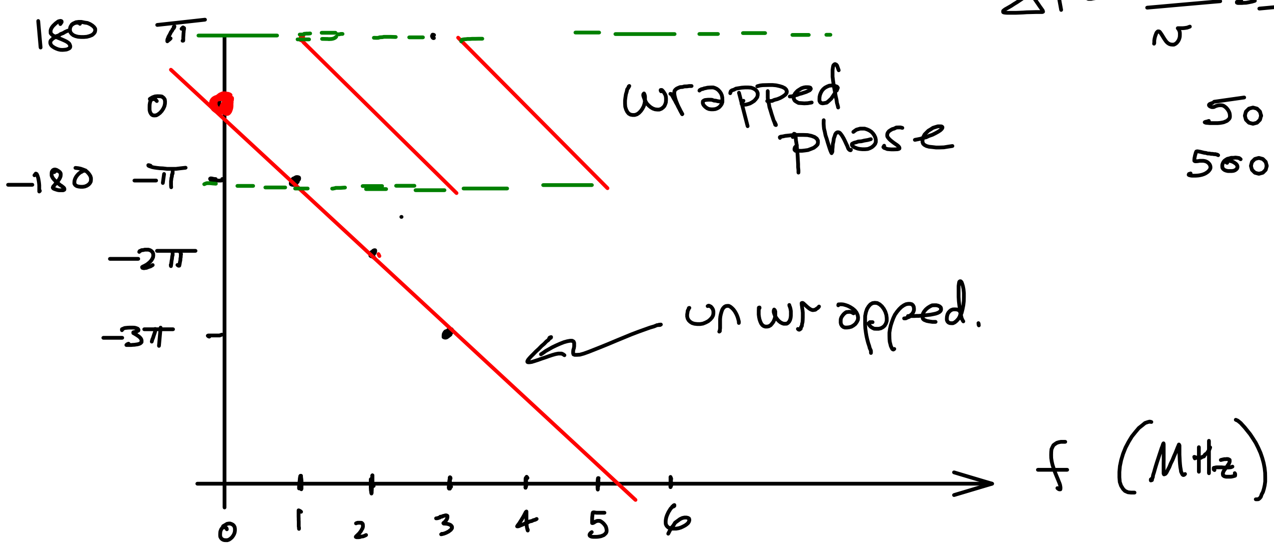
$$\Delta T = \frac{d}{v} = \frac{100}{2 \times 10^8} = 50 \times 10^{-8} = 500 \times 10^{-9} \ \left(\tfrac{1}{2} \mu s\right)$$

$$\Delta\theta = -2\pi f \Delta T$$
$$= -2\pi f \cdot 0.5 \times 10^{-6}$$
$$= -\pi f$$

$y = mx$

$\Delta\theta = \underline{\underline{-2\pi\Delta T}} \ f$

(f in MHz)



what if (e.g.) $\Delta T = \frac{1}{f}$

$$\Delta\theta = -2\pi f \Delta T$$
$$= \cdot 2\pi f \cdot \frac{1}{f}$$
$$= 2\pi$$

Exercise 4: A telephone line is being used to transmit symbols at a rate of 300 symbols/second. If the group delay must be less than 10% of the symbol period, what is the maximum allowable group delay?

$f_{symbol} = 300 \ Hz$

$T_{symbol} = \frac{1}{300} = 3.3ms$

group delay $<$ 10% of T_{symbol}

$< 0.1 \times$

$< \underline{330 \mu s.}$

Exercise 5: The input to a non-ideal amplifier is the sum of two sine waves at frequencies of 1 and 1.2 MHz. What are the frequencies of the even harmonics of these frequencies? Of the odd harmonics? What are the frequencies of the third-order IMD products?

$$\text{even harmonics} = (2, 4, 6, \ldots) \times \begin{cases} 1 = 2, 4, 6, \ldots \\ 1.2 = 2.4, \ldots \end{cases}$$

$$\text{odd harmonics} = (3, 5, 7, \ldots) \times \begin{cases} 1 = 3, 5, \ldots \\ 1.2 = 3.6, 6, 8.4, \ldots \end{cases}$$

7=4+3

9=5+4

for 3rd order products $m+n = 3$

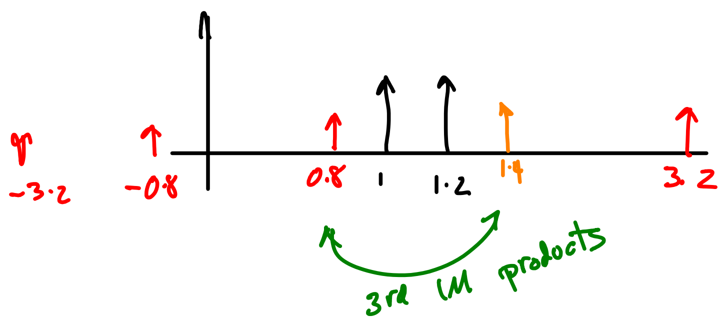
possible values of m, n = 2, 1 or 1, 2

$$f_{IM} = \pm m f_1 \pm n f_2 = \pm \underset{m}{2}(1) \pm \underset{n}{1}(1.2)$$

$m=2$ $n=1$

2	+1.2	3.2
2	−1.2	0.8
−2	+1.2	−0.8
−2	−1.2	−3.2

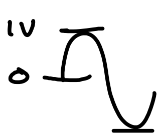
$\pm 2 \times 1$ $\pm 1 \times 1.2$



Exercise 6: A sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

Nearby machinery is adding noise onto the line. The voltage of this noise signal is measured with an RMS voltmeter as 100mVrms. The characteristic impedance of the line is 600Ω and it is terminated with that impedance. Why was an RMS voltmeter used? What is the signal power? What is the noise power? What is the SNR?

Why RMS voltmeter?
Because signal is not a sine wave & ordinary meters only measure the rms voltage for a sine wave.



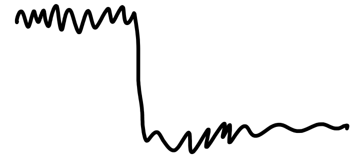
$V_{rms} = \frac{A}{\sqrt{2}} = 0.707$ signal

$V_{rms} = 100\text{ mV}$ noise

$$P_{signal} = \frac{V^2}{R} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{600}$$

$$P_{noise} = \frac{V^2}{R} = \frac{(0.1)^2}{600}$$

$$SNR = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{(0.1)^2} = \frac{\frac{1}{2}}{.01} = 50$$



$$10 \log (50) = 17\,dB$$

Exercise 7: What are the units of N assuming the units above?

$N = kTB$

Watts.

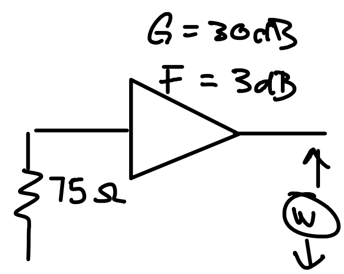
k — J/K

T — K

B — Hz — $\frac{1}{s}$

$$\frac{J}{K} \cdot K \cdot \frac{1}{s} = \frac{J}{s} = \underline{\text{Watts.}}$$

Exercise 8: A line amplifier for a cable TV system amplifies the range of frequencies from 54-1002 MHz. The amplifier has a gain of 30 dB and a noise figure of 3 dB. If we connect a 75Ω resistor (the input impedance of the amplifier) to the input how much power will we measure at the output of the amplifier?



$N = kTBF$ (assuming gain = 0dB = 1)

$$= -174 + 10 \log (B) + \underbrace{10 \log (F)} + \underline{\text{gain}}$$

$B = 1002 - 54$

$\approx 1000\ MHz$

$\approx 10^9\ Hz$

$$= -174 + 10 \log (10^9) + 3 + 30$$

$$\approx -174 + 90 + 3 + 30 \equiv 1.4 \times 10^{-23} \times 290 \times 10^9 \times 2 \times 1000$$

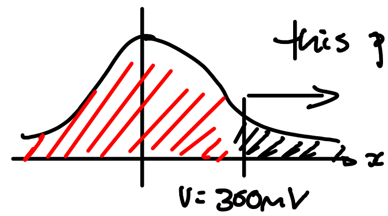
$$=$$

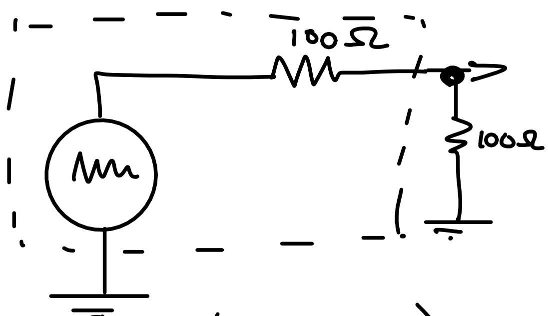
$$= \begin{array}{r} -174 \\ 123 \\ \hline -51\ dBm \end{array}$$

Exercise 9: What are the units of t?

$$t : \frac{\text{volts} - \text{volts}}{\text{volts}} \quad ; \quad \text{unitless ratio}$$

Exercise 10: The output of a noise source has a Gaussian (normally) distributed output voltage. The (rms) output power is 20mW and the output impedance is 100Ω. What fraction of the time does the output voltage exceed 300mV? Hint: the variance (σ^2) of a signal is the same as the square of its RMS voltage.





$P = 20$ mW rms

$V = 300$mV

$$t = \frac{v - \mu}{\sigma}$$

assume: - DC (average, mean) output is zero $\quad \mu = 0$

& - load is same as Z_{out}

$$P = \frac{V^2}{R} \qquad V_{rms} = \sqrt{R \cdot P}$$

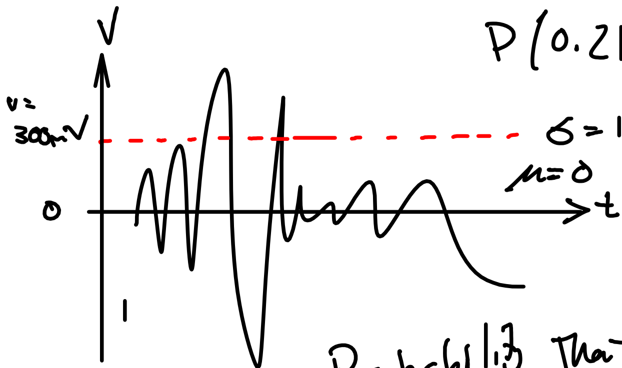
$$V_{rms} = \sqrt{100 \cdot 0.020} = \sqrt{2} \approx 1.41$$

$$\boxed{\begin{array}{l} \text{by definition } V_{rms} \equiv \sigma \\ \text{" \quad " DC voltage} \equiv \mu \end{array}}$$

$$t = \frac{v-\mu}{\sigma} = \frac{0.300 - 0}{1.41} = 0.21$$

$$P(x < 0.3V) = \mathbb{P}(0.21)$$

Gaussian CDF



$$P(0.21) =$$

P(0.3÷√2)= 0.583998

$$\frac{1}{1+e^{-1.7\times 0.21}} = 0.588314025$$

Probability that noise voltage < 300mV = 0.58

$$v = 300\,mV \xRightarrow{\text{normalized}} t = 0.21$$

Prob. that noise voltage > 300mV

$$= 1 - 0.58 = 0.42$$

$$\mu = V_{avg} = \frac{x_1 + x_2 + x_3}{3} \qquad \sigma = V_{rms} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2}{3}}$$

(Lab 5 Notes)

X = "My name, 12345 \n\r"

'\n'

X = char [] = { 'M', 'y', . . . – '\r', 0 };

- output all the characters

output a character →
- output start bit
- output all the data bits
- output stop bit

output N samples

0x35

```
  0 0 X 1 0 1 0 1
& 0 0 1 0 0 0 0 0
-----------------
  0 0 X 0 0 0 0 0
```

& ← bitwise AND

&& ← logical AND