ELEX 3525 Lecture 9 Notes

Exercise 1: Compute the modulo-8 (3-bit) checksum of a frame with values 4, 1, and 3. Would an error be detected if the received frame was 4, 1, 3, 0? How about if the received frame was 1, 4, 3?

100
+001
101
+011
000

100001011

100001011000

$4+1+3 = 8 \bmod 8 = 0$

$4+1+3+0 = 8 \bmod 8 = 0$

$1+4+3 = 8 \bmod 8 = 0$

100001011000

checksum.

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

modulo2 sum = remainder after dividing sum by 2.

$1+0+1 = 2$ $\quad \frac{2}{2} = 1$ remainder 0

$\therefore$ modulo-2 sum = 0

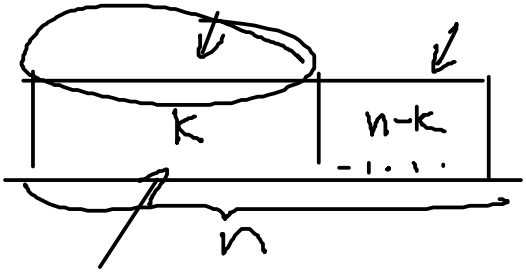
if the # of 1's is even, the sum is even & modulo-2 sum is zero (0).

in c : $n \,\%\, 2 \equiv n \,\&\, 1$

$1 \oplus 1 \oplus 0 \oplus 1 \oplus 1$

even parity:
1's is even
odd parity
1's is odd.

Exercise 3: How many different code words (different blocks) does an (n, k) code have? How many different patterns of $n - k$ parity bits are there?



2^k patterns (codewords)

2^{n-1}

2^n

2^k

2^{n-k} possible patterns of parity bits.

01101

$k = 5$

$2^5 = 32$

$n = 8$

$n - k = 3$

00000 101
00001 110
00010 111

Exercise 4: What is the Hamming distance between the code-words 11100 and 11011?

$$\begin{array}{r} 1\,1\,1\,0\,0 \\ 1\,1\,0\,1\,1 \\ \hline 0+0+1+1+1 = 3 \end{array} \leftarrow \text{Hamming distance}$$

Protocol	network	3
Layers	MAC	2
	PHY	1

Exercise 5: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the received de-

valid codewords { 101, 010

min distance = 3

$n = 3$ block size

$k = 1$ data bit

$\log_2$ (# of valid codewords)

$n - k = 2$ parity bits

receive: 110

distance from: 101

$$\begin{array}{r} 110 \\ \hline \sum 0+1+1 = 2 \end{array}$$

distance from: 010

$$\begin{array}{r} 110 \\ \hline \sum 1+0+0 = 1 \end{array}$$

this valid codeword has minimum distance from the received codeword so we guess/select that this was the one transmitted.

Exercise 6: Assume errors on a channel cause a frame error rate of 50%. When a rate-1/2 FEC code is used the frame error rate drops to 2%. Compute the throughputs with and without coding relative to the uncoded and error-free channel. What other advantages might the use of FEC provide?

$\sum w_1 \ominus w_2$

4×3

$C(n, k)$

$C(4, 2) = 6$

rate-$\frac{1}{2}$ $\quad \frac{k}{n} = \frac{1}{2}$

$k = 1N$	$n-k = 1N$

$n = 2N$

FER = 50% — half of the frames have errors

with FEC, FER = 2%

ignoring retransmissions

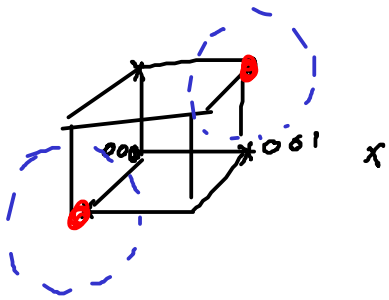
- use FEC: 2% FER & rate-$\frac{1}{2}$ $= 0.98 \times \frac{1}{2} = 0.49$
- no FEC: 50% FER & rate = 1 $= 0.50 \times 1 = 0.5$
- no errors: 0% FER & rate = 1 $= 1.0 \times 1 = 1$

including 1st retransmission:

$$\frac{0.49}{1+0.02} \approx \text{avg} \leftarrow \text{total time}$$

$$\frac{0.5}{1+0.5} \approx 0.3$$

010 → 100
101



$$\frac{1}{1+0.02+0.000\ldots}$$

$$\frac{1}{1+0-5+0.25}$$

Exercise 7: A system without coding needs to transmit at 1W to transmit 1 Mb/s at an error rate of 10^{-3}. When a rate-1/2 code is used the power to transmit the necessary 2Mb/s of data data and parity bits decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is E_b? What is the coding gain?

energy = power x time

J = W · seconds, where seconds = $\frac{1}{f_b}$

	w/ coding	w/o coding
Ch. bit rate	2×10^6	1×10^6
Inf. bit rate	1×10^6	1×10^6
E_b	$0.5 \cdot \frac{1}{1 \times 10^6} = \frac{1}{2} \times 10^{-6}$ J	$1 \times \frac{1}{10^6} = 10^{-6}$ J

Coding Gain. = 2 (3dB)

Exercise 8: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?

k = 1

n = 2

K = 7 (6 SR outputs + current input)

rate (R) = $\frac{1}{2}$

Exercise 9: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?