

ELEX 3525 - Answers to Lecture 2 Exercises

$$Z_0 \approx \frac{120}{\sqrt{\epsilon_r}} \ln\left(\frac{2S}{D}\right)$$

Exercise 1: What is the characteristic impedance of UTP made from 24-gauge wire with polyethylene insulation ($\epsilon_r = 2.2$) of 0.25mm thickness?

$$\begin{aligned} 24 \text{ ga} &= 0.5 \text{ mm dia (D)} \\ &= 2 \times \text{radius of wire} + 2 \times \text{thickness of ins.} \\ &= 2 \times 0.25 + 2 \times 0.25 \\ &= 0.5 + 0.5 = 1 \end{aligned}$$

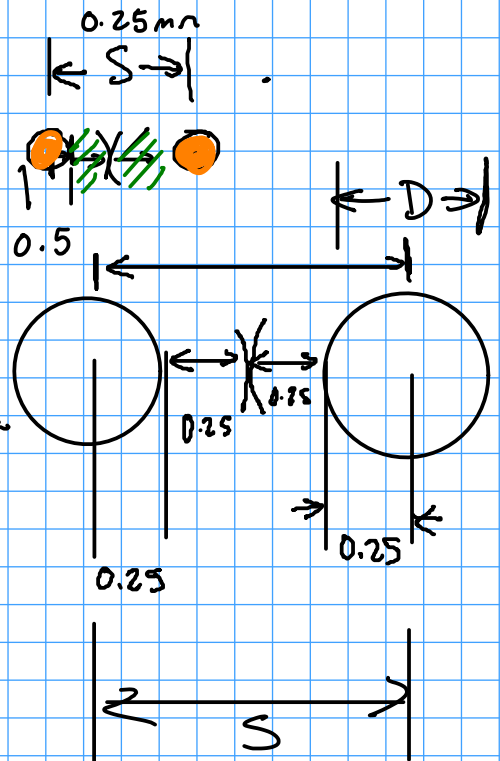
$$S = 4 \times 0.25 = 1 \text{ mm}$$

$$D = 0.5 \text{ mm}$$

$$Z_0 = \frac{120}{\sqrt{2.2}} \times \ln\left(\frac{2 \cdot 1}{0.5}\right) \approx \frac{120}{1.4} \ln(4)$$

$$\approx 100 \cdot 2 = 200$$

$$= 112 \, \Omega$$



$$Z_0 \approx \frac{138}{\sqrt{\epsilon_r}} \log_{10}\left(\frac{D}{d}\right)$$

Exercise 2: What is the characteristic impedance of a coax cable with a 0.8mm diameter center conductor, 3.5mm

diameter shield and foamed polyethylene between them that has a dielectric constant of 1.5?

$$\begin{aligned} d &= 0.8 \\ D &= 3.5 \\ \epsilon_r &= 1.5 \end{aligned}$$

$$Z_0 = \frac{130}{\sqrt{1.5}} \log_{10}\left(\frac{3.5}{0.8}\right) \approx 100 \log_{10}(4)$$

$$\approx 100 \cdot 0.6 = 60$$

$$\approx 72$$

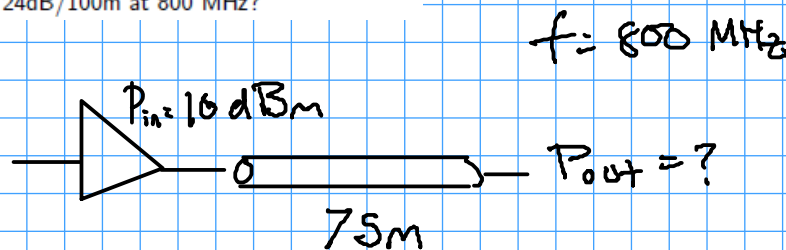
$$\begin{aligned} \log_{10}(2) &= 0.3 \\ \log_{10}(4) &= 0.6 \end{aligned}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

Exercise 3: What is the characteristic impedance of a lossless cable with an inductance of 94 nH per foot and capacitance of 17pF/ft?

$$Z_0 = \sqrt{\frac{94 \times 10^{-9}}{17 \times 10^{-12}}} \approx \sqrt{\frac{100 \times 10^{-9}}{20 \times 10^{-12}}} \\ \approx \sqrt{5 \times 10^3} = 74 \Omega$$

Exercise 4: An 800 MHz signal is output from a CATV amplifier at a power level of 10dBm. What power level would you expect at the other end of a 75m run of co-ax whose loss is specified as 24dB/100m at 800 MHz?



$$24 \text{ dB} / 100 \text{ m} \quad \text{Loss} = 24 \text{ dB} / 100 \text{ m} \cdot 75 \text{ m} \\ = 18 \text{ dB}$$

$$P_{\text{out}} = P_{\text{in}} - \text{loss} \\ = 10 \text{ dBm} - 18 \text{ dB} \\ = -8 \text{ dBm}$$

$$VF = \frac{1}{\sqrt{\epsilon_r}}$$

Exercise 5: What is the velocity factor for a cable with polyethylene insulation ($\epsilon_r = 2.2$)? How long would it take for a signal to propagate 100m? For a cable with air dielectric?

for $\epsilon_r = 2.2$

$$VF = \frac{1}{\sqrt{2.2}} = 0.674$$

$$v = \underbrace{3 \times 10^8}_{c} \cdot 0.66 = 2 \times 10^8$$

$$t = \frac{d}{v} = \frac{100}{2 \times 10^8} = 50 \times 10^{-8} = 500 \times 10^{-9} = 500 \text{ ns}$$

for air $VF = 1$

$$t = \frac{100}{3 \times 10^8} = 330 \text{ ns}$$

$$125 \mu\text{m} = 125 \times 10^{-6} \text{ m}$$

$$\frac{125 \times 10^{-6}}{1 \times 10^3} \text{ mm} \approx \frac{1}{8}$$

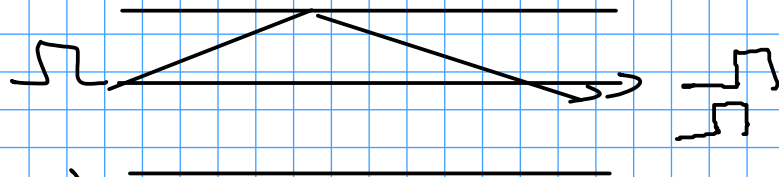
Exercise 6: If the optical signal wavelength is 1330nm what is the frequency?

$$c = \lambda f$$

$$f = \frac{c}{\lambda} = \frac{2 \times 10^8}{1.3 \times 10^{-6}} \text{ (assumed)}$$

$$= 1.5 \times 10^{14} = 150 \times 10^{12}$$

$$= 150 \text{ THz}$$



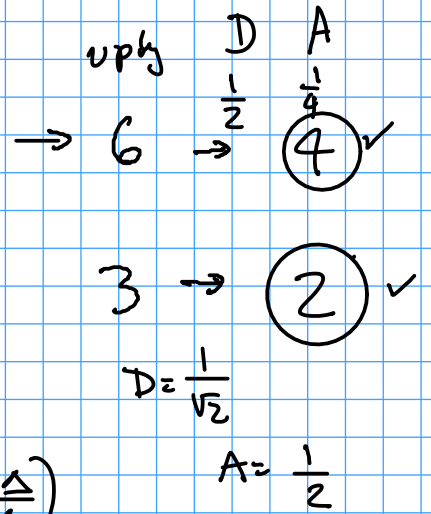
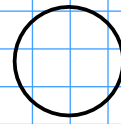
Exercise 7: How much does a cable's resistance increase when the gauge size increases by 6? By 3? Hint: a wire's resistance is proportional to its cross-sectional area.

inversely

gauge ↑ 6
diameter ↓ × 1/2

↑ +3
↓ 1/√2

resistance ≈ gauge



$$D = \left(\frac{1}{2}\right)^{\left(\frac{\Delta}{6}\right)}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

Where P_R and P_T are the received and transmitted powers, G_T and G_R are the gains of the transmit and receive antennas, λ is the wavelength and d is the distance from transmitter to receiver.

Exercise 8: A point-to-point link uses a transmit power of 1 Watt, transmit and receive antennas with gains of 20dB and operates at 3 GHz. How much power is received by the receiver?

$$P_T = 1W$$

$$G_T = 20dB$$

$$G_R = 20dB$$

$$f = 3GHz$$

$$P_T = 10 \log \left(\frac{1000}{1mW} \right) = 30dBm$$

$$c = \lambda f$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 10^{-1} \text{ m} \quad \text{assume } d = 100m$$

$$\begin{aligned} \text{in dB(m): } P_R &= P_T + G_T + G_R + 20 \log \left(\frac{\lambda}{4\pi d} \right) \\ &= 30 + 20 + 20 + 20 \log \left(\frac{0.1}{4\pi \cdot 100} \right) \\ &= 30 + 20 + 20 - 82 \\ &= -12 dBm \end{aligned}$$