ASSIGNMENT 3 SOLUTIONS

Q.1

$-3dB$ is a voltage ratio of $10^{\left(-\frac{3}{20}\right)} = 0.707$

$-6dB$ " " $10^{\left(\frac{-6}{20}\right)} = 0.5$

equation of line $H(f) = 1 - \frac{f\ (kHz)}{100}$

$$f = 100\left(1 - H(f)\right)$$

$$f(-3dB) = 100\left(1 - 0.707\right) \simeq \underline{\underline{29\ kHz}}$$

$$f(-6dB) = 100\left(1 - 0.5\right) = \underline{\underline{50\ kHz}}$$

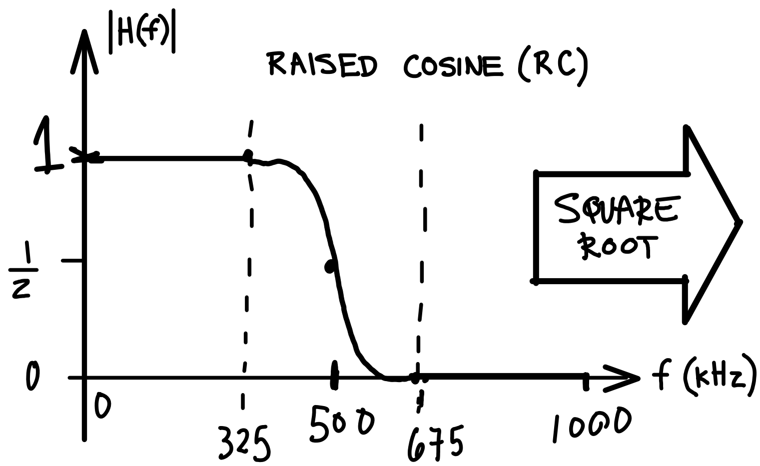
Q.2

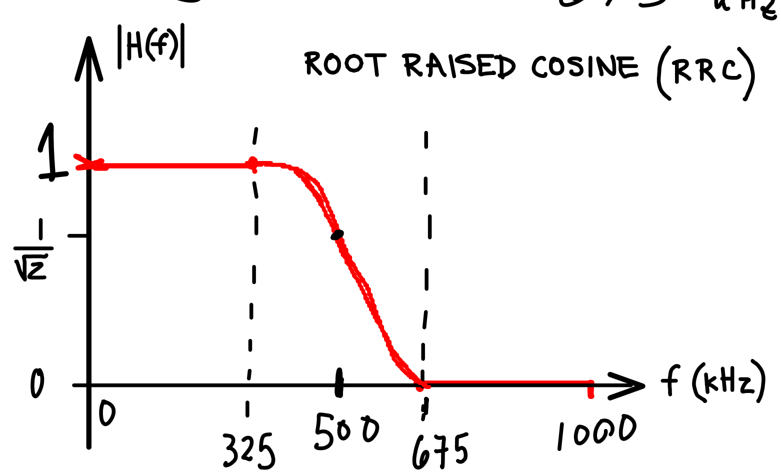
(a) for a 1MHz symbol rate the RC response must be symmetrical about $\frac{f_{sym}}{2} = 500\ kHz$.

the RC response extends over $\frac{f_{sym}}{2}(1 \pm \alpha)$

for $\alpha = 0.35$: $500(1 \pm 0.35)\ kHz = 500 \cdot 0.65 = 325\ kHz$

to $500 \cdot 1.35 = 675\ kHz$





at $f = 500\ kHz$

$H(f) = \frac{1}{2}$

(for symmetry)

at $f = 500\ kHz$

$\underline{\underline{H(f) = \frac{1}{\sqrt{2}}}}$

(b) The transmit filter does not include pulse-shaping portion of the RRC response that converts impulses to pulses (has $\frac{\sin(x)}{x}$ frequency response)

Q.3

(a) at 2kHz the phase shift is $\frac{\pi}{2}$

$$\theta = 2\pi f t$$

$$t = \frac{\theta}{2\pi f} = \frac{\frac{\pi}{2}(\pm n \cdot 2\pi)}{2\pi \cdot 2\times10^3}$$

minimum delay is for $n=0$ $\quad t = \frac{1}{8\times10^3} = 0.125\times10^{-3}$

$= 125\mu s.$

next longest possible delay is for $n=1$:

$$t = \frac{\frac{5\pi}{2}}{2\pi \cdot 2\times10^3} = \frac{5}{8}\times10^{-3} = 625\ \mu s$$

(b) A linear system can only change the amplitude and phase of individual frequency components.

∴ a 2 kHz signal will not be distorted in the sense that the output will look like a sine wave of the same frequency.

However, the frequency response is flat between 1 & 2 kHz ∴ the two components will see different delays and the waveform will be distorted (the shape will be different).

(c) group delay is slope of phase response:

from	to	slope
0	1 kHz	$\pi/2$ / kHz
1	3 kHz	0
3	5 kHz	$\pi/2$ / kHz

maximum group delay $= \frac{\pi}{2}$ radians $\div$ $2\pi \times 1000$ radians/sec.

$= \frac{1}{4000}$ seconds $= 250\,\mu s$.

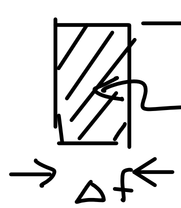
(this is also the variability in group delay in this example)

Q.4

total power $= 3\,\mu W/Hz \times 2\,MHz + 2\times(1\,\mu W/Hz \times 2\,MHz)$

$= 6\,W + 2\times 2\,W = 10\,W$

$\frac{1}{2}\%$ of total power $= 10 \times 0.005 = 5\times10^{-2}\,W$



— $1\,\mu W/Hz$

area $= 5\times10^{-2}\,W = \Delta f \cdot 1\,\mu W/Hz$

$$\Delta f = \frac{5\times10^{-2}\,W}{1\times10^{-6}\,W/Hz} = 50\,kHz$$

99% bandwidth $= 6\,MHz - (2\times 50\,kHz) = 5.9\,MHz$.

Q.5

$10 \pm 0.5\,MHz = 9.5$ and $10.5\,MHz$

3rd-order products are at $\pm n f_1 \pm m f_2$ where $\begin{cases} n+m=3 \\ n>0 \\ m>0 \end{cases}$

values of n, m are $(1,2), (2,1)$

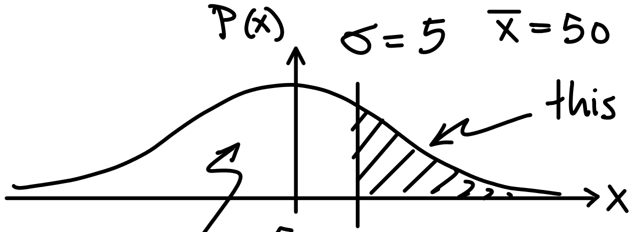
3rd order IMD frequencies are $\pm 9.5 \pm 2\times 10.5 = \pm 9.5 \pm 21$

and $\pm 2\times 9.5 \pm 10.5 = \pm 19 \pm 10.5$

$$\left.\begin{aligned} 9.5+21 &= \boxed{30.5} \\ 9.5-21 &= -11.5 \\ -9.5+21 &= 11.5 \\ -9.5-21 &= -30.5 \end{aligned}\right\} MHz \qquad \left.\begin{aligned} 19+10.5 &= 29.5 \\ 19-10.5 &= 8.5 \\ -19+10.5 &= -8.5 \\ -19-10.5 &= -29.5 \end{aligned}\right\} MHz$$

There are no components withing 1 MHz of 10 MHz. (between 9 and 11 MHz).

Q. 6

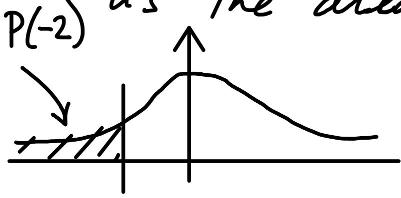


this area is $1 - P(t) = 1 - P(2) \approx 1 - 0.97 = 0.03$

CDF is this area $= 0.97$

$$t = \frac{60-50}{5} = \frac{10}{5} = 2 \qquad P(2) \approx \frac{1}{1+e^{-1.7\cdot 2}} \approx 0.97$$

by symmetry we can also compute it as the area to the left of -2 i.e. $P(-2)$:



$$P(-2) \approx \frac{1}{1+e^{-1.7(-2)}} = 0.03$$

(actual value is 0.023)

Q.7

$\sigma^2 = 0.5\ V^2$ (volts^2)

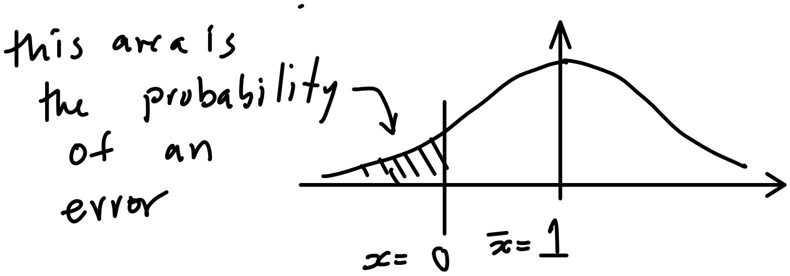
signal is $\pm 1V$

signal power is $\frac{V^2}{R} = \frac{1}{R}$ $\left(\frac{V^2}{\Omega}\right)$

noise power is $\frac{0.5}{R}$ $\left(\frac{V^2}{\Omega}\right)$

$SNR = \frac{1}{0.5} = 2 = \underline{3dB}$

An error will be made if the noise level greater than +1 V when signal is −1 V or if noise is less than −1 V when signal is +1 V.



for signal = +1V: $P(error) = P\left(\frac{0-1}{\sqrt{0.5}}\right)$

$= P(-\sqrt{2}) \simeq 0.08$

variance $(\sigma^2) = 0.5$

standard deviation $(\sigma) = \sqrt{0.5}$

$$P(-\sqrt{2}) \simeq \frac{1}{1 + e^{1.7 \cdot \sqrt{2}}} = 0.08$$

by symmetry both probabilities of error (for ±1) are equal so the bit error rate is ≃ 8%.

Q.8

- This is an "eye diagram"
- The channel does not meet the no-ISI criteria because there is ISI (the "eye" is not "open" at any point).
- This does not imply anything about the error rate. Many approaches can be used to recover the data (equalization, sequence estimation, etc.)

Q.9

- 3 pins (ground, Tx Data & Rx Data) are sufficient to transmit & receive data.
- V.28 section 4 says the generator (transmitter) voltage shall not exceed ±15V (RS-232 says up to ±25V) so the voltages are too high & the signal is not compliant.
- the bit rate is $\frac{1}{\text{bit duration}} = \frac{1}{1\,ms} = 1\,kHz$ (kb/s)
- the bits transmitted are:

```
0   1 0 1 1 0 1 0 1 1
↑   ↑           ↑ └─┘
start LS        MS  stop
bit  bit        bit bits/idle
```

NOTE: if 8 data bits were transmitted then the value is 0xAD (not an ASCII character)

in msb first order: 0 1 0 1 1 0 1 → 2 D

from ASCII table 0x2D is '–' (minus sign or dash)

Q. 10

bandwidth = 1 MHz

minimum sampling rate (real samples) = 2 MHz

20% higher: $2\ MHz \times 1.2 = \underline{\underline{2.4\ MHz}}$

guard time duration = $10\mu s$ = $10\% \times$ block duration

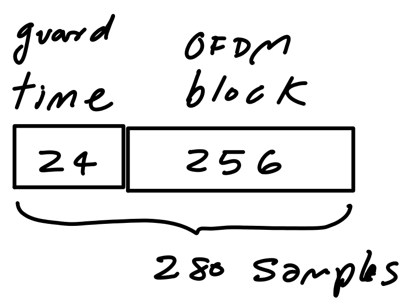
$\therefore$ block length $\geq \dfrac{10\mu s}{0.1} = 100\mu s = \underline{\underline{1 \times 10^{-4}\ s}}$

$100\mu s \times 2.4 \times 10^{6}$ samples/second = $\underline{\underline{240 \text{ samples}}}$

next largest power of 2 is 256

$\therefore$ block size should be $\underline{\underline{N = 256}}$

(guard time $\simeq 10\mu s \times 2.4 \times 10^{6} = 24$ samples)



Q. 11

bandwidth $B = 6\ MHz$

$SNR = 16\ dB = 10^{\left(\frac{16}{10}\right)} = 40$

Shanon capacity $C = B \cdot \log_2\left(1 + \frac{S}{N}\right)$

$= 6 \times 10^{6} \cdot \log_2(1 + 40)$

$= 6 \times 10^{6} \cdot \dfrac{\log_{10}(41)}{\log_{10}(2)} = 6 \times 10^{6} \dfrac{1.6}{0.3}$

$\underline{\underline{\simeq 32\ Mb/s}}$

(b) for a capacity of $C = 20 Mb/s$

$B = 6 MHz$

$$\log_2(1 + S/N) = \frac{20}{6}$$

$$\frac{S}{N} = 2^{\frac{20}{6}} - 1 = 10^{\log 2 \cdot \frac{20}{6}} - 1 \approx 9 \approx 10 \text{ dB}$$