Design of High-Performance Filter Banks for Image Coding

Di Xu   Michael D. Adams

Dept. of Elec. and Comp. Engineering
University of Victoria, Canada

IEEE Symposium on Signal Processing and Information Technology, 2006

Outline

Outline

Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)
- good frequency selectivity
- certain vanishing moment

Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)
- good frequency selectivity
- certain vanishing moment

Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)

$$G_{\mathrm{SBC}} = \prod_{k=0}^{L-1} \left(\frac{\alpha_k}{A_k B_k}\right)^{\alpha_k}, \quad \text{where}$$

$$A_k = \sum_{m \in \mathbb{Z}} h'_{\mathrm{h}k}(m) \sum_{n \in \mathbb{Z}} h'_{\mathrm{v}k}(n) \sum_{p \in \mathbb{Z}} h'_{\mathrm{h}k}(p) \sum_{q \in \mathbb{Z}} h'_{\mathrm{v}k}(q) r(m-p, n-q),$$

$$B_k = \alpha_k \sum_{m \in \mathbb{Z}} g'^2_{\mathrm{h}k}(m) \sum_{n \in \mathbb{Z}} g'^2_{\mathrm{v}k}(n), \qquad \alpha_k \text{ is sampling factor,}$$

$$r(x,y) = \begin{cases} \rho^{|x|+|y|} & \text{for separable model} \\ \rho^{\sqrt{x^2+y^2}} & \text{for isotropic model,} \end{cases}$$

- good frequency selectivity
- certain vanishing moment

Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)
- good frequency selectivity
- certain vanishing moment

Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)
- good frequency selectivity
- certain vanishing moment

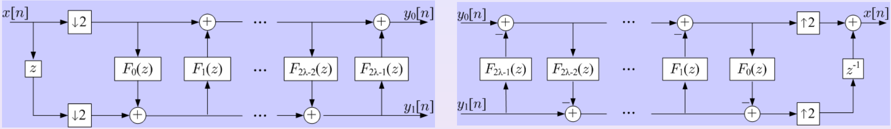
Desirable Characteristics

- perfect reconstruction
- linear phase
- high coding gain (two models)
- good frequency selectivity
- certain vanishing moment

Problems?
Difficult to obtain all properties
Difficult to obtain a good tradeoff

Outline

Lifting Scheme



(a) analysis side (b) synthesis side

Figure: The lifting realization of a 1-D two-channel filter bank.

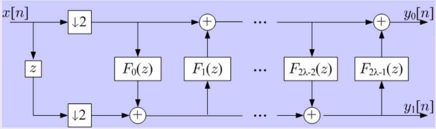
- perfect reconstruction
- linear phase

⇒ easily imposed by lifting realization

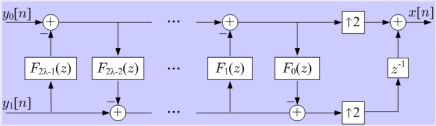
- high coding gain
- good frequency selectivity
- certain vanishing moment

⇒ properties left to design

Lifting Scheme



(a) analysis side



(b) synthesis side

Figure: The lifting realization of a 1-D two-channel filter bank.

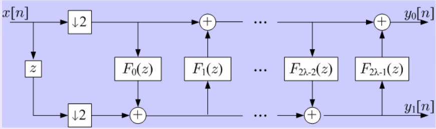
- perfect reconstruction
- linear phase

$\Rightarrow$ easily imposed by lifting realization

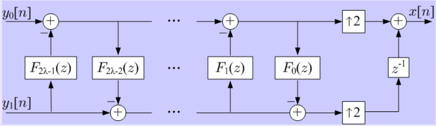
- high coding gain
- good frequency selectivity
- certain vanishing moment

$\Rightarrow$ properties left to design

Lifting Scheme



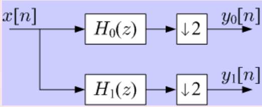
(a) analysis side



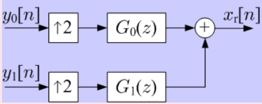
(b) synthesis side

Figure: The lifting realization of a 1-D two-channel filter bank.

Canonical Form:

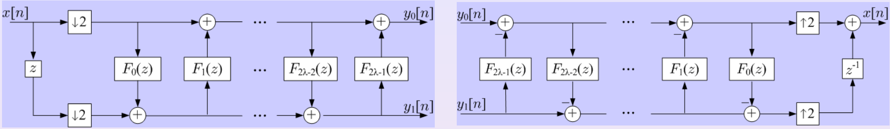


(a) analysis side



(b) synthesis side

Lifting Scheme



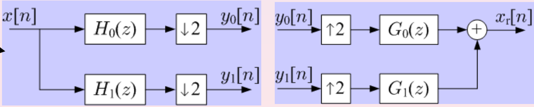
(a) analysis side (b) synthesis side

Figure: The lifting realization of a 1-D two-channel filter bank.

$$H_0(z) = H_{0,0}(z^2) + zH_{0,1}(z^2) \quad \text{and} \quad H_1(z) = H_{1,0}(z^2) + zH_{1,1}(z^2),$$

where $$\begin{bmatrix} H_{0,0}(z) & H_{0,1}(z) \\ H_{1,0}(z) & H_{1,1}(z) \end{bmatrix} = \prod_{k=0}^{\lambda-1} \left(\begin{bmatrix} 1 & F_{2k+1}(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ F_{2k}(z) & 1 \end{bmatrix} \right)$$

Canonical Form:



(a) analysis side (b) synthesis side

Constrained Optimization

Objective function:

$$G(\boldsymbol{x}) = \begin{cases} G_{\text{sep}}(\boldsymbol{x}) & \text{separable only} \\ G_{\text{iso}}(\boldsymbol{x}) & \text{isotropic only} \\ \min\{G_{\text{sep}}(\boldsymbol{x}), G_{\text{iso}}(\boldsymbol{x})\} & \text{joint.} \end{cases}$$

Stopband energy: $b_k(\boldsymbol{x}) \triangleq \int_{S_k} |\hat{h}_k(\omega, \boldsymbol{x})|^2 d\omega, \quad k \in \{0, 1\}.$

Moment functions: $c_k(\boldsymbol{x}) \triangleq \|\boldsymbol{m}_k(\boldsymbol{x})\|, \quad k \in \{1, 2, \ldots, n\}.$

Abstract Optimization Problem

$$\begin{aligned} \text{maximize} \quad & G(\boldsymbol{x}) \\ \text{subject to:} \quad & b_k(\boldsymbol{x}) \leq \varepsilon_k, \ k \in \{0, 1\} \ \text{and} \\ & c_k(\boldsymbol{x}) \leq \gamma_k, \ k \in \{1, 2, \ldots, n\}. \end{aligned}$$

Details for Different Objective Functions

Optimization Scheme

Highly Nonlinear → Iterative SOCP Algorithm

1. reduce order at an operating point $\boldsymbol{x}$ by using Taylor series approximation ⇒ functions of $\boldsymbol{\delta}$
2. impose an additional constraint, s.t. δ is small
3. solve the second-order cone programming (SOCP) problem
4. update operating point $\boldsymbol{x} = \boldsymbol{x} + \delta$
5. go to step 1, unless algorithm converges

maximize $\quad \nabla^T G(\boldsymbol{x})\boldsymbol{\delta}$

subject to:

$$\|\boldsymbol{Q}_k^{1/2}(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{q}_k(\boldsymbol{x})\| \le \varepsilon_k - b_k(\boldsymbol{x}) + \boldsymbol{q}_k^T(\boldsymbol{x})\boldsymbol{q}_k(\boldsymbol{x}),\ k \in \{0, 1\},$$

$$\|\nabla^T \boldsymbol{m}_k(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{m}_k(\boldsymbol{x})\| \le \gamma_k,\quad k \in \{1, 2, \ldots, n\}, \text{ and}$$

$$\|\boldsymbol{\delta}\| \le \beta,$$

where

$$\boldsymbol{Q}_k(\boldsymbol{x}) = \int_{S_k} \nabla_{\boldsymbol{x}} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega,$$

$$\boldsymbol{q}_k(\boldsymbol{x}) = \boldsymbol{Q}_k^{-1/2}(\boldsymbol{x}) \int_{S_k} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega.$$

Optimization Scheme

Highly Nonlinear → Iterative SOCP Algorithm

1. reduce order at an operating point $\boldsymbol{x}$ by using Taylor series approximation ⇒ functions of $\boldsymbol{\delta}$
2. impose an additional constraint, s.t. $\boldsymbol{\delta}$ is small
3. solve the second-order cone programming (SOCP) problem
4. update operating point $\boldsymbol{x} = \boldsymbol{x} + \boldsymbol{\delta}$
5. go to step 1, unless algorithm converges

maximize $\quad \nabla^T G(\boldsymbol{x})\boldsymbol{\delta}$

subject to:

$$\|\boldsymbol{Q}_k^{1/2}(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{q}_k(\boldsymbol{x})\| \leq \varepsilon_k - b_k(\boldsymbol{x}) + \boldsymbol{q}_k^T(\boldsymbol{x})\boldsymbol{q}_k(\boldsymbol{x}),\ k \in \{0, 1\},$$

$$\|\nabla^T \boldsymbol{m}_k(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{m}_k(\boldsymbol{x})\| \leq \gamma_k,\quad k \in \{1, 2, \ldots, n\},\ \text{and}$$

$$\|\boldsymbol{\delta}\| \leq \beta,$$

where

$$\boldsymbol{Q}_k(\boldsymbol{x}) = \int_{S_k} \nabla_{\boldsymbol{x}} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega,$$

$$\boldsymbol{q}_k(\boldsymbol{x}) = \boldsymbol{Q}_k^{-1/2}(\boldsymbol{x}) \int_{S_k} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega.$$

Optimization Scheme

Highly Nonlinear → Iterative SOCP Algorithm

1. reduce order at an operating point $\boldsymbol{x}$ by using Taylor series approximation ⇒ functions of $\boldsymbol{\delta}$
2. impose an additional constraint, s.t. $\boldsymbol{\delta}$ is small
3. solve the second-order cone programming (SOCP) problem
4. update operating point $\boldsymbol{x} = \boldsymbol{x} + \boldsymbol{\delta}$
5. go to step 1, unless algorithm converges

maximize $\nabla^T G(\boldsymbol{x})\boldsymbol{\delta}$

subject to:

$$\|\boldsymbol{Q}_k^{1/2}(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{q}_k(\boldsymbol{x})\| \leq \varepsilon_k - b_k(\boldsymbol{x}) + \boldsymbol{q}_k^T(\boldsymbol{x})\boldsymbol{q}_k(\boldsymbol{x}),\ k \in \{0, 1\},$$

$$\|\nabla^T \boldsymbol{m}_k(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{m}_k(\boldsymbol{x})\| \leq \gamma_k,\ k \in \{1, 2, \ldots, n\},\ \text{and}$$

$$\|\boldsymbol{\delta}\| \leq \beta,$$

where

$$\boldsymbol{Q}_k(\boldsymbol{x}) = \int_{S_k} \nabla_{\boldsymbol{x}} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega,$$

$$\boldsymbol{q}_k(\boldsymbol{x}) = \boldsymbol{Q}_k^{-1/2}(\boldsymbol{x}) \int_{S_k} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega.$$

Optimization Scheme

Highly Nonlinear → Iterative SOCP Algorithm

1. reduce order at an operating point $\boldsymbol{x}$ by using Taylor series approximation ⇒ functions of $\boldsymbol{\delta}$
2. impose an additional constraint, s.t. $\boldsymbol{\delta}$ is small
3. solve the second-order cone programming (SOCP) problem
4. update operating point $\boldsymbol{x} = \boldsymbol{x} + \boldsymbol{\delta}$
5. go to step 1, unless algorithm converges

maximize $\quad \nabla^T G(\boldsymbol{x})\boldsymbol{\delta}$

subject to:

$$\|\boldsymbol{Q}_k^{1/2}(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{q}_k(\boldsymbol{x})\| \leq \varepsilon_k - b_k(\boldsymbol{x}) + \boldsymbol{q}_k^T(\boldsymbol{x})\boldsymbol{q}_k(\boldsymbol{x}), \; k \in \{0,1\},$$

$$\|\nabla^T \boldsymbol{m}_k(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{m}_k(\boldsymbol{x})\| \leq \gamma_k, \; k \in \{1,2,\ldots,n\}, \text{ and}$$

$$\|\boldsymbol{\delta}\| \leq \beta,$$

where

$$\boldsymbol{Q}_k(\boldsymbol{x}) = \int_{S_k} \nabla_{\boldsymbol{x}} \hat{h}_k(\omega,\boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega,\boldsymbol{x}) d\omega,$$

$$\boldsymbol{q}_k(\boldsymbol{x}) = \boldsymbol{Q}_k^{-1/2}(\boldsymbol{x}) \int_{S_k} \hat{h}_k(\omega,\boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega,\boldsymbol{x}) d\omega.$$

Optimization Scheme

Highly Nonlinear → Iterative SOCP Algorithm

1. reduce order at an operating point $\boldsymbol{x}$ by using Taylor series approximation ⇒ functions of $\boldsymbol{\delta}$
2. impose an additional constraint, s.t. $\boldsymbol{\delta}$ is small
3. solve the second-order cone programming (SOCP) problem
4. update operating point $\boldsymbol{x} = \boldsymbol{x} + \boldsymbol{\delta}$
5. go to step 1, unless algorithm converges

maximize $\nabla^T G(\boldsymbol{x})\boldsymbol{\delta}$

subject to:

$$\|\boldsymbol{Q}_k^{1/2}(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{q}_k(\boldsymbol{x})\| \le \varepsilon_k - b_k(\boldsymbol{x}) + \boldsymbol{q}_k^T(\boldsymbol{x})\boldsymbol{q}_k(\boldsymbol{x}), \; k \in \{0, 1\},$$

$$\|\nabla^T \boldsymbol{m}_k(\boldsymbol{x})\boldsymbol{\delta} + \boldsymbol{m}_k(\boldsymbol{x})\| \le \gamma_k, \; k \in \{1, 2, \ldots, n\}, \text{ and}$$

$$\|\boldsymbol{\delta}\| \le \beta,$$

where

$$\boldsymbol{Q}_k(\boldsymbol{x}) = \int_{S_k} \nabla_{\boldsymbol{x}} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega,$$

$$\boldsymbol{q}_k(\boldsymbol{x}) = \boldsymbol{Q}_k^{-1/2}(\boldsymbol{x}) \int_{S_k} \hat{h}_k(\omega, \boldsymbol{x}) \nabla_{\boldsymbol{x}}^T \hat{h}_k(\omega, \boldsymbol{x}) d\omega.$$

Outline

Choice of Objective Function

$$? \quad G(\boldsymbol{x}) = \begin{cases} G_{\text{sep}}(\boldsymbol{x}) & \text{separable only} \\ G_{\text{iso}}(\boldsymbol{x}) & \text{isotropic only} \\ \min\{G_{\text{sep}}(\boldsymbol{x}), G_{\text{iso}}(\boldsymbol{x})\} & \text{joint.} \end{cases}$$

▸ Test Environment

Table: Statistical results over all 26 test images and 5 bit rates

Transform	Mean (%)	Median (%)	Outperform (%)
9/7-sep	-0.0049	-0.0001	46.15
9/7-iso/jnt	0.1488	0.1070	87.69
6/14-sep	-0.5848	-0.5112	20.77
6/14-iso/jnt	0.0331	0.0279	61.54

Conclusion: better jointly optimizing both of the $G_{\text{sep}}(\boldsymbol{x})$ and $G_{\text{iso}}(\boldsymbol{x})$

Choice of Objective Function

$$G(\boldsymbol{x}) = \begin{cases} G_{\text{sep}}(\boldsymbol{x}) & \text{separable only} \\ G_{\text{iso}}(\boldsymbol{x}) & \text{isotropic only} \\ \min\{G_{\text{sep}}(\boldsymbol{x}), G_{\text{iso}}(\boldsymbol{x})\} & \text{joint.} \end{cases}$$

Test Environment

Table: Statistical results over all 26 test images and 5 bit rates

Transform	Mean (%)	Median (%)	Outperform (%)
9/7-sep	-0.0049	-0.0001	46.15
9/7-iso/jnt	0.1488	0.1070	87.69
6/14-sep	-0.5848	-0.5112	20.77
6/14-iso/jnt	0.0331	0.0279	61.54

Conclusion: better jointly optimizing both of the $G_{\text{sep}}(\boldsymbol{x})$ and $G_{\text{iso}}(\boldsymbol{x})$

Design Examples

Table: Characteristics of various filter banks

Transform	$\{L_k\}$	G_{sep}	G_{iso}	b_0	b_1	Van. Mom.
9/7-J	{2,2,2,2}	14.973	12.178	0.063	0.035	4, 9.560
9/7	{2,2,2,2}	14.933	12.181	0.057	0.035	2, 0.004
9/11	{4,2,2}	14.928	12.112	0.111	0.043	2, 0.276
13/11	{4,2,2,2}	15.041	12.206	0.030	0.027	2, 0.068
17/11	{2,2,4,4}	15.117	12.218	0.031	0.028	2, 0.337
13/15	{6,2,2}	14.641	12.074	0.094	0.035	2, 0.169

Table: Statistical results over all 26 test images and 5 bit rates

(compared with 9/7-J from JPEG-2000 standard)

Transform	Mean (%)	Median (%)	Outperform (%)
9/7	0.149	0.107	87.69
9/11	0.537	0.055	59.23
13/11	0.186	0.087	74.62
17/11	0.580	0.242	77.69
13/15	0.594	0.163	68.46

Design Examples (Cont'd)

Table: Specific results for three representative images

Image (model)	CR	PSNR (dB) 9/7-J	9/7	9/11	13/11	17/11	13/15
gold (sep)	8	36.75	36.88	37.34	36.85	37.17	**37.39**
	16	33.75	33.84	**34.00**	33.76	33.91	33.95
	32	31.23	31.27	**31.35**	31.24	31.32	31.27
	64	29.16	29.15	29.24	29.17	29.17	**29.25**
	128	27.32	27.32	**27.39**	27.35	27.37	27.34
target (—)	8	41.46	41.59	42.92	42.12	42.81	**43.11**
	16	33.54	33.55	33.47	33.83	**34.00**	33.45
	32	27.07	27.19	26.65	27.84	**27.88**	26.84
	64	22.70	22.84	22.35	23.11	**23.19**	22.42
	128	19.16	19.14	18.86	19.35	**19.46**	18.94
sar2 (iso)	8	30.32	30.35	30.30	30.33	30.33	**30.37**
	16	26.61	26.62	26.59	**26.62**	26.60	26.57
	32	24.69	**24.70**	24.65	24.69	24.69	24.70
	64	23.55	**23.55**	23.52	23.54	23.53	23.54
	128	22.73	22.73	22.70	**22.74**	22.74	22.67

Subjective Image Quality (compression ratio: 32)

(a) original image

(b) 9/7-J from JPEG 2000

(c) 9/7 design

Outline

Summary

- Designed filter banks with:
  - perfect reconstruction
  - linear phase
  - high coding gain
  - good frequency selectivity
  - certain prescribed moment properties
- Outperformed the well-known 9/7-J filter bank (from the JPEG-2000 standard)
- Proposed 9/7 design has same computational complexity as 9/7-J filter bank

Questions ?

Design Parameter Selection

- Frequency Response $L2$-norm Error: for a stopband width of $\frac{3\pi}{8}$, tolerances within $[0.02, 0.14]$ is highly effective
- Moment Constraint: the norm of the vector of zeroth dual and primal moments is less than $2 \cdot 10^{-5}$
- finding multiple solutions from many different initial points; effective when consider lifting-filter coefficients within $[-2, 2]$

Impulse Responses of the Lifting Filters

```
9/7-J from JPEG 2000:

-1.58613434   -1.58613434
-0.0529801185 -0.0529801185
 0.882911076   0.882911076
 0.443506852   0.443506852


proposed 9/7 design:

-1.49341357   -1.49341357
-0.0630113314 -0.0630113314
 0.794482374   0.794482374
 0.469650396   0.469650396
```

Constrained Optimization

separable only or isotropic only case

$$\begin{aligned} \text{maximize} \quad & G_{\text{sep}}(\boldsymbol{x}) \text{ or } G_{\text{iso}}(\boldsymbol{x}) \\ \text{subject to:} \quad & b_k(\boldsymbol{x}) \le \varepsilon_k, \; k \in \{0,1\} \text{ and} \\ & c_k(\boldsymbol{x}) \le \gamma_k, \; k \in \{1,2,\ldots,n\}. \end{aligned}$$

joint case

$$\begin{aligned} \text{maximize} \quad & t \\ \text{subject to:} \quad & G_{\text{sep}}(\boldsymbol{x}) \ge t, \\ & G_{\text{iso}}(\boldsymbol{x}) \ge t, \\ & b_k(\boldsymbol{x}) \le \varepsilon_k, \; k \in \{0,1\}, \text{ and} \\ & c_k(\boldsymbol{x}) \le \gamma_k, \; k \in \{1,2,\ldots,n\}. \end{aligned}$$

Return

Choice of Objective Function

- Test data: all of the 26 reasonably-sized continuous-tone grayscale images from the JPEG-2000 test set

Table: Characteristics of a subset of the test images

Image	Size, Precision	Model	Description
gold	720×576, 8	separable	houses and countryside
target	512×512, 8	—	patterns and textures
sar2	800×800, 12	isotropic	synthetic aperture radar

- Codecs: EZW, SPIHT, and MIC

Return

References

- Y. Chen, M. D. Adams, and W.-S. Lu
  Design of Optimal Quincunx Filter Banks for Image Coding.
  Proc. of IEEE International Symposium on Circuits and Systems, May 2006.
- J. Katto and Y. Yasuda
  Performance evaluation of subband coding and optimization of its filter coefficients.
  Proc. of SPIE Visual Communications and Image Processing, vol 1605, pp 95–106, Nov. 1991.
- M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret
  Applications of second-order cone programming.
  Linear Algebra and its Applications, vol 248, pp 193–228, Nov. 1998.