Examining the Economic Optimality of Automatic Generation Control

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***Abstract*—The automatic generation control (AGC) system is temporally situated between economic dispatch and synchronous generator dynamics, and its primary role is to regulate frequency within and tie-line flows between control areas. Given appropriate choice of participation factors (feed-forward controller gains that govern the disaggregation of the area-level power requirement to individual generators), the AGC can be engineered to nudge system dynamics toward a steady-state operating point corresponding to economic optimality. This paper establishes necessary and sufficient conditions under which a widely accepted choice of participation factors guarantees the alignment of steady-state synchronous generator outputs with a global minimum of a prototypical economic dispatch problem. In so doing, it resolves several ambiguities and formalizes technical assumptions governing the role of the standard AGC architecture in the context of economic dispatch and steady-state operation. Numerical case studies tailored to a modified version of the New England 39-bus 10-machine test system validate the theoretical results.**

***Index Terms*—Automatic generation control, economic dispatch, loss penalty factors, participation factors.**

I. Introduction

The automatic generation control (AGC) system is an integral component of the power system control architecture. The goal of AGC is to regulate system frequency and inter-area tie-line schedules to their steady-state nominal values [1]. An important consideration is how AGC can attain frequency and tie-line regulation in an economically optimal fashion. However, research pertinent to the standard industry-employed AGC system, described in, e.g. [2], has mainly focused on improving power quality and dynamic performance, paying scant attention to economic optimality in steady state. A survey of such efforts is available in [3]. In recent years, AGC economic optimality has received significant attention given the realization that greater frequency regulation effort would be required in the grid with increased penetration of variable renewable resources [4], [5]. Academic efforts have sought to *redesign* the AGC system so that it provably steers generator outputs to an optimizer of the companion economic dispatch problem [6]–[10]. In contrast, we consider a different problem and tackle the following fundamental research question:

> *Under what conditions is the standard AGC system employed in practice [2],* without any retrofits or modifications*, economically optimal?*

Surprisingly, the above question has not received due regard in previous work. Before detailing our contributions, we provide a comprehensive literature review of recent efforts that focus primarily on either *retrofitting* or *significantly modifying* the current implementation of AGC so that it guarantees economic optimality. We remark that these efforts do not address the more fundamental question of when the prevailing and standard AGC system utilized in practice aligns the system dynamics with economic optimality.

Related work. Academic efforts seeking to *redesign* the AGC system so it provably steers generator outputs to an optimizer of the companion economic dispatch problem in steady state have been proposed in [6]–[15]. Particularly, primal-dual gradient methods are proposed in [6]–[10] to solve optimization problems that jointly consider generator scheduling and frequency regulation. Newton's method is leveraged to design fast-converging distributed AGC systems in [11]–[13]. Consensus-based algorithms are used to steer generator outputs to the minimizers of economic dispatch problems in [14] and [15]. A distributed averaging proportional integral control for frequency regulation that realizes economic optimality in steady state is introduced in [16]. Furthermore, online feedback controllers that converge to the solution of pertinent cost minimization problems are proposed in [17]–[20]. More aligned with industry implementations, [21]–[23] propose various functional enhancements to improve the economic performance of the prevailing AGC. Alternatives to standalone economic dispatch and AGC that attempt to preserve the prevailing secondary control architecture include solutions of a steady-state frequency-aware economic dispatch problem [24], an economic dispatch problem constrained by discretized frequency dynamics [25], and a continuous-time economic dispatch problem with frequency dynamics [26]. While this work focuses on economic optimality of the industry-standard AGC system, other performance metrics are equally important. One that comes to mind foremost is closed-loop stability. We direct interested readers to [27] for a recent formal treatment of stability of the industry-standard AGC system. The literature on stability of retrofits and new variants of AGC is vast, but this is not aligned with the focus of this effort which is to examine the industry-standard AGC system.

Contributions. Unlike much of the previous work reviewed

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above, this paper is not centered on algorithm design or control synthesis. Our main contribution is to uncover conditions under which the standard industry-employed AGC, *without* any modifications, is economically optimal, i.e., it steers generator outputs to the solution of the economic dispatch problem in steady state. In light of the above, our *main contributions* can be summarized as follows:

- In the most general setting, we uncover necessary and sufficient conditions under which steady-state generator outputs realized by AGC action with standard feedforward gains—known as participation factors—correspond to *a Karush-Kuhn-Tucker (KKT) point* of the economic dispatch problem.
- We derive conditions under which such a KKT point is *a global minimum* of the economic dispatch problem. The generality of these results follow from directly analyzing KKT conditions at pre- and post-disturbance steady-state operating points. Interestingly, the analysis is predicated on generator-cost functions conforming to a particular algebraic constraint that involves loss penalty factors and derivatives of generator-cost functions up to second order.
- For the case of convex cost functions and a lossless network, we prove that steady-state generator outputs realized by the AGC action correspond to *the global minimum* of the economic dispatch problem if the generator-cost functions are quadratic.
- To validate our theoretical results, we simulate a differential-algebraic equation (DAE) model of a lossless version of the New England 39-bus test system involving models for the two-axis generator, governor and exciter controls, and a standard AGC system. We then compare the steady-state generator outputs under AGC action and the standard choice of participation factors with the ones obtained using participation factors that depend on generator governor droops and inertia constants. We show that, AGC leads to steady-state generator outputs that exactly match the solution of the economic dispatch problem with the standard choice of participation factors under the conditions mentioned above.

Our analysis and results are critical in establishing baselines for benchmarking performance of potential retrofits and upheavals proposed for AGC and economic dispatch. Through the above contributions, we conclusively reconcile the gap between the operation of the standard AGC system and a well-defined notion of economic optimality. Notably, our results are presented in the backdrop of an economic dispatch problem that minimizes total cost of generation subject to supply-demand constraint including losses, generator-capacity limits, and line-flow limits. Problems with this general form have been referenced routinely in the context of practical implementations [28], [29]. We conclude by reiterating that our main results (discussed above) are *necessary and sufficient*. This particular aspect underscores the technical contributions of the paper in examining economic optimality of the industry-standard AGC system in a comprehensive and mathematically rigorous manner. In a nutshell, the paper provides marked contributions over previous attempts at examining economic optimality of the AGC system that applied to stylized dynamics / economic-dispatch routines and only in the limiting regime of small load disturbances.

Paper Organization. The remainder of the paper is organized as follows. In Section II, we describe the system dynamical model and draw the connection to the solution of the economic problem. We also show how the standard choice of AGC participation factors is derived through a perturbation analysis of the economic dispatch problem. In Section III, we present our main results: conditions under which AGC guarantees economic optimality for lossy and lossless networks. Section IV contains numerical validation for the IEEE 39-bus system. Finally, we conclude this paper in Section V.

II. Background

In this section, we present the system model that combines dynamics arising from synchronous generators, the AGC system, and economic dispatch. With these, we characterize the system steady-state operating point as well as the KKT conditions of the economic dispatch problem. Finally, we discuss how the AGC participation factors are typically derived via a perturbation analysis of economic dispatch.

A. Generator and AGC Dynamical Models

Consider an AC electric power network with synchronous generators indexed in set $\mathcal{G}$ and transmission lines indexed in set $\mathcal{E}$. For each generator $g \in \mathcal{G}$, let δ_g, ω_g, P_g^{m}, P_g^{e}, and P_g^{r} denote the electrical angular position, angular speed, turbine mechanical power, electrical power output, and reference power input, respectively. Dynamics of generator g can be described by the swing equations augmented with a simplified turbine-governor model, as follows:

$$\dot{\delta}_g = \omega_g - \omega_{\mathrm{s}}, \tag{1}$$

$$M_g \dot{\omega}_g = P_g^{\mathrm{m}} - P_g^{\mathrm{e}}, \tag{2}$$

$$\tau_g \dot{P}_g^{\mathrm{m}} = P_g^{\mathrm{r}} - P_g^{\mathrm{m}} - \frac{1}{R_g \omega_{\mathrm{s}}}(\omega_g - \omega_{\mathrm{s}}), \tag{3}$$

where M_g, τ_g, and R_g denote its inertia constant, governor time constant, and droop constant, respectively; and $\omega_{\mathrm{s}} = 2\pi 60$ $[\mathrm{rad/s}]$ is the synchronous frequency of the system. The electrical power output, P_g^{e}, is a function of bus-voltage magnitudes and phase angles as solved from the network power balance. The generator g reference power input is given by

$$P_g^{\mathrm{r}} = P_g^{\star} + \alpha_g(\xi - \sum_{j \in \mathcal{G}} P_j^{\star}), \tag{4}$$

where $P_g^{\star}$ is the setpoint received from economic dispatch and α_g is the AGC participation factor with $\sum_{g \in \mathcal{G}} \alpha_g = 1$ [2], [30]. In practice, the participation factors are typically chosen according to the formula:

$$\alpha_g = \frac{(C_g^{''}(P_g^{\star}))^{-1}}{\sum_{j \in \mathcal{G}} (C_j^{''}(P_j^{\star}))^{-1}}, \tag{5}$$

where $\mathcal{G}$ is the set of generators participating in AGC and $C_g''(P_g^{\star})$ denotes the second derivative of the generator g cost function evaluated at the economic dispatch solution, $P_g^\star$ [2]. Implicit in (5) is that the participation factors are updated with each instance of economic dispatch [1], [2]. Moreover, ξ is the AGC state whose evolution is dictated by

$$\dot{\xi} = -\xi - \mathrm{ACE} + \sum_{g\in\mathcal{G}} P_g^{\mathrm{e}}, \tag{6}$$

where ACE denotes the *area control error* that accounts for deviations in frequency from the synchronous value. For a single-area system with no tie-line flows, the area control error is formulated as $\mathrm{ACE} = b(\omega - \omega_s)$ where $b > 0$ is the area bias factor and ω the prevailing frequency of the system [2]. (Without loss of generality, we assume ω is computed with real-time measurements as the average electrical frequency of all generators.)

In the remainder of the paper, we denote values taken by variables at a new steady-state operating point by $\overline{X}$ (corresponding values at the initial operating point are denoted by X). Suppose the system net load P_{load} changes by amount ΔP_{load}, so that the new net load is $\overline{P}_{\mathrm{load}} = P_{\mathrm{load}} + \Delta P_{\mathrm{load}}$. In steady state after the load change, the dynamical system (1)–(6) converges to the following operating point:

$$\overline{\omega}_g = \omega_{\mathrm{s}},\ \forall\, g \in \mathcal{G}, \tag{7}$$

$$\overline{P}_g^{\mathrm{m}} = P_g^\star + \alpha_g(\Delta P_{\mathrm{load}} + \Delta P_{\mathrm{loss}}),\ \forall\, g \in \mathcal{G}, \tag{8}$$

$$\overline{\xi} = P_{\mathrm{load}} + \Delta P_{\mathrm{load}} + \Delta P_{\mathrm{loss}}, \tag{9}$$

where ΔP_{loss} is the change in system loss due to the change in operating point. It follows from (2) that generator g steady-state electrical power output with the new net load is

$$\overline{P}_g^{\mathrm{e}} = P_g^\star + \alpha_g(\Delta P_{\mathrm{load}} + \Delta P_{\mathrm{loss}}),\ \forall\, g \in \mathcal{G}. \tag{10}$$

The above expressions follow from a steady-state analysis of system dynamics (see [31] for details). In particular, we will find (10) central to the main results of this work, since it establishes the links between the closed-loop system dynamics (as dictated by generators and AGC) and economic optimality. The main objective of our work is to uncover conditions under which $\overline{P}_g^{\mathrm{e}}$ matches the optimizer $\overline{P}_g^\star$ of the economic dispatch problem solved with the new net load $\overline{P}_{\mathrm{load}}$. Taking a step in this direction, we outline the economic dispatch problem and characterize its solution next.

B. Economic Dispatch Problem

Let $C_g(P_g)$ denote the cost function for generator g, and collect P_g, $g \in \mathcal{G}$, in $P_{\mathcal{G}} \in \mathbb{R}^{|\mathcal{G}|}$. The economic dispatch problem takes the following form:

$$\min_{P_g,\, g\in\mathcal{G}}. \sum_{g\in\mathcal{G}} C_g(P_g) \tag{11a}$$

$$\text{s.t.} \sum_{g\in\mathcal{G}} P_g = P_{\mathrm{load}} + P_{\mathrm{loss}}(P_{\mathcal{G}}), \tag{11b}$$

$$P_{(m,n)}^{\min} \le \sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g \le P_{(m,n)}^{\max},\ \forall\, (m,n) \in \mathcal{E}, \tag{11c}$$

$$P_g^{\min} \le P_g \le P_g^{\max},\ \forall\, g \in \mathcal{G}, \tag{11d}$$

where P_{load} is the look-ahead net load, $P_{\mathrm{loss}}(P_{\mathcal{G}})$ is the system loss modeled as a function of $P_{\mathcal{G}}$, $P_g^{\min}$ ($P_{(m,n)}^{\min}$) and $P_g^{\max}$ ($P_{(m,n)}^{\max}$) denote the minimum and maximum limits for generator g capacity (line (m,n) flow), and $\Psi_{(m,n)}^g$ captures the sensitivity of line (m,n) active-power flow with respect to generator g active-power injection [2]. The solution to the economic dispatch problem, $P_g^\star$, provides the reference set-point for generator g in (4). The Lagrangian for problem (11) is given by:

$$\begin{aligned}
\mathcal{L} &:= \mathcal{L}(P_g, \lambda, \psi_{(m,n)}^+, \psi_{(m,n)}^-, \mu_g^+, \mu_g^-) \\
&= \sum_{g\in\mathcal{G}} C_g(P_g) + \lambda(P_{\mathrm{load}} + P_{\mathrm{loss}}(P_{\mathcal{G}}) - \sum_{g\in\mathcal{G}} P_g) \\
&+ \sum_{(m,n)\in\mathcal{E}} \psi_{(m,n)}^+ \Big(\sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g - P_{(m,n)}^{\max} \Big) \\
&+ \sum_{(m,n)\in\mathcal{E}} \psi_{(m,n)}^- \Big(P_{(m,n)}^{\min} - \sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g \Big) \\
&+ \sum_{g\in\mathcal{G}} \mu_g^+ (P_g - P_g^{\max}) + \sum_{g\in\mathcal{G}} \mu_g^- (P_g^{\min} - P_g),
\end{aligned} \tag{12}$$

where λ, $\psi_{(m,n)}^+$, $\psi_{(m,n)}^-$, μ_g^+, and μ_g^- are dual variables corresponding to supply-demand, line-flow, and generator-capacity constraints. We refer to primal and dual solutions that satisfy the KKT conditions as KKT points, and we denote them by $P_g^\star$ ($P_{\mathcal{G}}^\star$ in vector form) and $\lambda^\star$, $\psi_{(m,n)}^{+\star}$, $\psi_{(m,n)}^{-\star}$, $\mu_g^{+\star}$, $\mu_g^{-\star}$, respectively. The KKT conditions for problem (11) can be derived from the Lagrangian (12), and they are given by:

$$\begin{aligned}
C_g'(P_g^\star) - \frac{\lambda^\star}{\Lambda_g^\star} + \sum_{(m,n)\in\mathcal{E}} (\psi_{(m,n)}^{+\star} - \psi_{(m,n)}^{-\star}) \Psi_{(m,n)}^g \\
+ \mu_g^{+\star} - \mu_g^{-\star} = 0,\ \forall\, g \in \mathcal{G},
\end{aligned} \tag{13a}$$

$$P_{\mathrm{load}} + P_{\mathrm{loss}}(P_{\mathcal{G}}^\star) - \sum_{g\in\mathcal{G}} P_g^\star = 0, \tag{13b}$$

$$\begin{cases} \psi_{(m,n)}^{+\star} \Big(\sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g^\star - P_{(m,n)}^{\max} \Big) = 0, \\ \psi_{(m,n)}^{+\star} \ge 0, \quad \forall\, (m,n) \in \mathcal{E}, \end{cases} \tag{13c}$$

$$\begin{cases} \psi_{(m,n)}^{-\star} \Big(P_{(m,n)}^{\min} - \sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g^\star \Big) = 0, \\ \psi_{(m,n)}^{-\star} \ge 0, \quad \forall\, (m,n) \in \mathcal{E}, \end{cases} \tag{13d}$$

$$\begin{cases} \mu_g^{+\star}(P_g^\star - P_g^{\max}) = 0,\ \mu_g^{+\star} \ge 0, \\ \mu_g^{-\star}(P_g^{\min} - P_g^\star) = 0,\ \mu_g^{-\star} \ge 0, \end{cases} \forall\, g \in \mathcal{G}, \tag{13e}$$

$$\begin{cases} P_{(m,n)}^{\min} \le \sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g^\star \le P_{(m,n)}^{\max}, \forall (m,n) \in \mathcal{E}, \\ P_g^{\min} \le P_g^\star \le P_g^{\max},\ \forall\, g \in \mathcal{G}. \end{cases} \tag{13f}$$

Above, $\Lambda_g^\star$ is the *loss penalty factor* for generator g evaluated at $P_{\mathcal{G}}^\star$; it is defined as [2]

$$\Lambda_g^\star := \left(1 - \frac{\partial P_{\mathrm{loss}}(P_{\mathcal{G}}^\star)}{\partial P_g} \right)^{-1}. \tag{14}$$

The overall system architecture discussed thus far is illustrated in Fig. 1. Before proceeding further, we overview several details pertaining to the constituent models.

Remark 1 (Note on models)**.** *The AGC system described in* (4)–(6) *follows from transcribing a state-space description for block diagrams sketched in [2]. Another architecture that finds frequent mention in the literature involves determining*

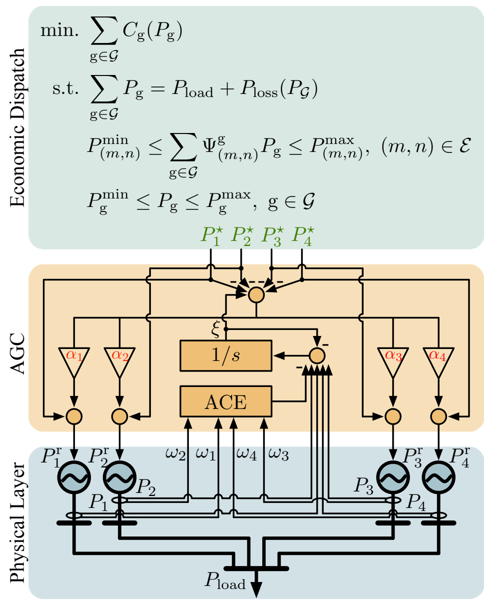

Fig. 1: Illustration of economic dispatch and AGC architectures.

the AGC state by directly integrating ACE *without feedback from generator electrical power outputs [1], [32], [33]. It turns out that this model yields the same steady-state operating point as in* (10)*, and so all subsequent developments apply.*

The economic dispatch problem (11) *is pieced together from [28], [29], [34], [35]. We emphasize these references as they are authored by industry stakeholders. See also [2], [32], [33] for textbook references with similar problem settings. We provide this collection of references to emphasize that problem* (11) *is a reasonable and representative formulation. The system loss $P_{\mathrm{loss}}(P_{\mathcal{G}})$ in* (11) *can be modeled as a quadratic function of $P_{\mathcal{G}}$ with Kron's loss formula:*

$$P_{\mathrm{loss}}(P_{\mathcal{G}}) = P_{\mathcal{G}}^{\mathrm{T}} B P_{\mathcal{G}} + P_{\mathcal{G}}^{\mathrm{T}} B_0 + B_{00}, \tag{15}$$

where $B \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{G}|}$, $B_0 \in \mathbb{R}^{|\mathcal{G}|}$, and $B_{00} \in \mathbb{R}$ consist of so-called B-coefficients [2], [36]. Although B-coefficients vary with operating point, they are typically assumed to be constant for the purpose of evaluating the loss penalty factors [2], [33]. The line-flow sensitivities, $\Psi^g_{(m,n)}$, $g \in \mathcal{G}$, $(m,n) \in \mathcal{E}$, are commonly referred to as injection shift factors. They can be computed from the power flow equations or estimated using measurements [2], [37].

C. Recovering Standard Participation Factors from a Perturbation Analysis

Before establishing our main results pertaining to the economic optimality of the standard AGC, we trace through typical analytical arguments that have been used to justify the choice of participation factors (5) in previous work [2], [38]–[42]. This is an important first step for our analysis as the economic optimality of the AGC and correspondingly, our main results, are intrinsically tied to the choice of participation factors. As we show below, the choice (5) has been frequently justified in the literature via a perturbation analysis of a companion economic dispatch problem with quadratic generator-cost functions and a linear supply-demand constraint (see, e.g., [2], [38]–[42]). Denote by ΔP_g, the deviation in electrical power output of generator g from the economic dispatch optimizer, $P_g^\star$. Suppose this results from a ΔP_{load} change in net load. Consider the first-order Taylor-series expansion of the generator g marginal cost around the economic dispatch solution:

$$C_g'(P_g^\star + \Delta P_g) \approx C_g'(P_g^\star) + C_g''(P_g^\star)\Delta P_g. \tag{16}$$

The above approximation is only accurate for sufficiently small load changes, since it is in this regime that we can assume ΔP_g is small. On the other hand, it is exact for the special case of quadratic cost functions regardless of the load-change magnitude. Neglecting losses, line-flow limits, and generator-capacity limits in (11), we recognize that the KKT condition (13a) simplifies as

$$C_g'(P_g^\star) = \lambda^\star,\ \forall\, g \in \mathcal{G}. \tag{17}$$

Let us *aspire* for optimality through AGC action by supposing that all generators operate with the same marginal cost at the new net load. This implies

$$C_g'(P_g^\star + \Delta P_g) = \lambda^\star + \Delta\lambda,\ \forall\, g \in \mathcal{G}, \tag{18}$$

where $\Delta\lambda$ denotes the change in marginal cost due to the net-load change. Substituting (17) and (18) into (16) we get

$$\Delta P_g = \Delta\lambda (C_g''(P_g^\star))^{-1}. \tag{19}$$

Summing (19) over all generators yields

$$\sum_{g\in\mathcal{G}} \Delta P_g = \Delta\lambda \sum_{g\in\mathcal{G}} (C_g''(P_g^\star))^{-1} = \Delta P_{\mathrm{load}}, \tag{20}$$

where the second equality holds when losses are neglected. From (19) and (20), we obtain

$$\frac{\Delta P_g}{\Delta P_{\mathrm{load}}} = \frac{(C_g''(P_g^\star))^{-1}}{\sum_{j\in\mathcal{G}} (C_j''(P_j^\star))^{-1}}. \tag{21}$$

This ratio naturally invites the interpretation of a participation factor as it captures the fraction of net-load change that ought to be allocated to each generator. The immediate impulse that follows is to set AGC participation factors based on the ratio in (21), or equivalently, per (5). However, notice the tenuous link to system dynamics in the above developments. Particularly, while (18) represents economic optimality at the new operating point, there is no indication that AGC implemented with participation factors in (5) would actually steer system dynamics to this operating point. Moreover, it is not clear under what precise conditions the choice (5) is *optimal* for a representative economic dispatch problem (such as the one in (11)), and with the underlying system dynamics (such as those described by (1)–(6)). We explore these aspects in detail next.

III. Main Results

In this section, we present the main results of the paper. We first examine the most general case where AGC with the choice of participation factors (5) merely steers generator electrical power outputs to a KKT point of the nonconvex economic dispatch problem (11). We then extend this result to show that under suitable convexity assumptions on the generator-cost and system-loss functions, AGC provably steers generator power outputs to a global minimum of the nonconvex economic dispatch problem (11). We also establish that the same holds for a convex economic dispatch problem neglecting losses under different—and easier to satisfy—assumptions. Finally, we consider the case with linear generator-cost functions.

A. The General Case

We present the following result centered on optimality of AGC for generic generator-cost functions and a generic system-loss function in the economic dispatch problem (11).

Theorem 1. *Suppose the system initially operates in steady state with net load, P_{load}, and generator electrical power outputs corresponding to a KKT point, $P_g^\star$, $g \in \mathcal{G}$, of the economic dispatch problem* (11)*. Consider AGC action triggered by a change in net load to $\overline{P}_{\text{load}} = P_{\text{load}} + \Delta P_{\text{load}}$. Denote by $\overline{P}_g^\star$, $g \in \mathcal{G}$, a KKT point of the economic dispatch problem solved with the new net load, $\overline{P}_{\text{load}}$. Further, denote the loss penalty factors corresponding to $P_g^\star$ and $\overline{P}_g^\star$, by $\Lambda_g^\star$ and $\overline{\Lambda}_g^\star$, respectively. Consider the following assumptions:*

- [A1] *The cost function $C_g(P_g)$ is twice differentiable.*
- [A2] *The KKT points $P_g^\star$, $\overline{P}_g^\star$ satisfy* (13) *with non-binding line-flow constraints* (11c) *and generator-capacity constraints* (11d)*.*
- [A3] *The generator-cost and system-loss functions are such that for the KKT points $P_g^\star$, $\overline{P}_g^\star$:*

$$\overline{\Lambda}_g^\star C_g'(\overline{P}_g^\star) - \Lambda_g^\star C_g'(P_g^\star) = (\overline{P}_g^\star - P_g^\star) C_g''(P_g^\star). \quad (22)$$

Under assumptions [A1]–[A3]*, the steady-state generator electrical power outputs following the net-load change correspond to KKT point, $\overline{P}_g^\star$, i.e.,*

$$\overline{P}_g^{\mathrm{e}} = \overline{P}_g^\star, \ \forall\, g \in \mathcal{G}, \quad (23)$$

if and only if the AGC participation factors are chosen as

$$\alpha_g = \frac{(C_g''(P_g^\star))^{-1}}{\sum\limits_{j\in\mathcal{G}} (C_j''(P_j^\star))^{-1}}, \ \forall\, g \in \mathcal{G}. \quad (24)$$

Proof. Under assumption [A2], the KKT condition (13a) reduces to the following constraints:

$$\lambda^\star = \Lambda_g^\star C_g'(P_g^\star), \quad \overline{\lambda}^\star = \overline{\Lambda}_g^\star C_g'(\overline{P}_g^\star). \quad (25)$$

For subsequent developments, we will find it useful to define (with slight abuse of notation adopted previously):

$$\Delta\lambda = \overline{\lambda}^\star - \lambda^\star. \quad (26)$$

Substituting for $\overline{\lambda}^\star$ and $\lambda^\star$ from (25) into (26), and leveraging property (22) in assumption [A3], we get

$$\Delta\lambda = \overline{\Lambda}_g^\star C_g'(\overline{P}_g^\star) - \Lambda_g^\star C_g'(P_g^\star) = (\overline{P}_g^\star - P_g^\star) C_g''(P_g^\star). \quad (27)$$

Rearranging terms in (27) yields

$$\overline{P}_g^\star - P_g^\star = \Delta\lambda (C_g''(P_g^\star))^{-1}. \quad (28)$$

Then, summing (28) across all generators, we get

$$\begin{aligned}\sum_{g\in\mathcal{G}} (\overline{P}_g^\star - P_g^\star) &= \Delta\lambda \sum_{g\in\mathcal{G}} (C_g''(P_g^\star))^{-1} \\ &= \Delta P_{\text{load}} + \Delta P_{\text{loss}}, \end{aligned} \quad (29)$$

where the second equality above follows from the supply-demand balance constraint (11b). From this equation, we obtain the following alternative expression for $\Delta\lambda$

$$\Delta\lambda = \frac{\Delta P_{\text{load}} + \Delta P_{\text{loss}}}{\sum_{g\in\mathcal{G}} (C_g''(P_g^\star))^{-1}}. \quad (30)$$

Suppose (23) holds. Rearranging terms in (10) along with (23) yields

$$\alpha_g (\Delta P_{\text{load}} + \Delta P_{\text{loss}}) = \overline{P}_g^\star - P_g^\star. \quad (31)$$

Replacing the right-hand side in (31) using (28), we get

$$\alpha_g (\Delta P_{\text{load}} + \Delta P_{\text{loss}}) = \Delta\lambda (C_g''(P_g^\star))^{-1}. \quad (32)$$

Finally, substituting $\Delta\lambda$ from (30) in (32), we get

$$\alpha_g (\Delta P_{\text{load}} + \Delta P_{\text{loss}}) = \frac{(C_g''(P_g^\star))^{-1}}{\sum\limits_{j\in\mathcal{G}} (C_j''(P_j^\star))^{-1}} (\Delta P_{\text{load}} + \Delta P_{\text{loss}}), \quad (33)$$

from which we readily obtain the AGC participation factors in (24).

Next, consider the other direction. Suppose that AGC participation factors are indeed chosen as in (24). From (10), we see that the steady-state generator g electrical power output with the choice of AGC participation factors (24) is given by

$$\overline{P}_g^{\mathrm{e}} = P_g^\star + \frac{(C_g''(P_g^\star))^{-1}}{\sum\limits_{j\in\mathcal{G}} (C_j''(P_j^\star))^{-1}} (\Delta P_{\text{load}} + \Delta P_{\text{loss}}). \quad (34)$$

Leveraging (22) from assumption [A3], we get

$$\overline{P}_g^{\mathrm{e}} = P_g^\star + \frac{\frac{\overline{P}_g^\star - P_g^\star}{\overline{\Lambda}_g^\star C_g'(\overline{P}_g^\star) - \Lambda_g^\star C_g'(P_g^\star)}}{\sum\limits_{j\in\mathcal{G}} \frac{\overline{P}_j^\star - P_j^\star}{\overline{\Lambda}_j^\star C_j'(\overline{P}_j^\star) - \Lambda_j^\star C_j'(P_j^\star)}} (\Delta P_{\text{load}} + \Delta P_{\text{loss}}). \quad (35)$$

Further, recognizing that (27) holds for all $g \in \mathcal{G}$, we can express the above as

$$\overline{P}_g^{\mathrm{e}} = P_g^\star + \frac{\frac{\overline{P}_g^\star - P_g^\star}{\Delta\lambda}}{\frac{1}{\Delta\lambda} \sum\limits_{j\in\mathcal{G}} (\overline{P}_j^\star - P_j^\star)} (\Delta P_{\text{load}} + \Delta P_{\text{loss}}), \quad (36)$$

which simplifies to

$$\overline{P}_g^{\mathrm{e}} = P_g^\star + \frac{\overline{P}_g^\star - P_g^\star}{\sum\limits_{j\in\mathcal{G}} (\overline{P}_j^\star - P_j^\star)} (\Delta P_{\text{load}} + \Delta P_{\text{loss}}). \quad (37)$$

Finally, since $\Delta P_{\text{load}} + \Delta P_{\text{loss}} = \sum_{j\in\mathcal{G}} (\overline{P}_j^\star - P_j^\star)$ from the supply-demand constraint (11b), we see that $\overline{P}_g^{\mathrm{e}} = \overline{P}_g^\star$, $\forall\, g \in \mathcal{G}$. This completes the proof. □

The main takeaway from Theorem 1 is the precise set of conditions under which AGC steers system dynamics to a KKT point of the standard economic dispatch problem (11).

As we have mentioned before, similar versions have been frequently referenced in the literature [2], [28], [29], [32]–[35]. The fact that line-flow and capacity constraints are assumed to be non-binding for the results to hold does *not* suggest that the theorem is applicable for a simpler (less constrained) version of the economic dispatch problem.

B. Convex Generator-cost & System-loss Functions

The result in Theorem 1 applies, in general, to arbitrary generator-cost and system-loss functions. We now consider the particular case of the loss model (15) and convex generator-cost functions. In this setting, we first characterize the solutions of the economic dispatch problem (11).

Lemma 1. *Consider the following assumptions:*

[A4] *The cost function $C_g(P_g)$ is strictly convex, and marginal cost $C_g'(P_g) > 0$.*

[A5] *The system-loss model* (15) *is strictly convex with $B \succ 0$.*

[A6] *For net-load values $P_{\rm load}$ and $\overline{P}_{\rm load}$, the respective KKT points $P_g^\star$, $\overline{P}_g^\star$, $g \in \mathcal{G}$, collected in $P_\mathcal{G}^\star$, $\overline{P}_\mathcal{G}^\star$ satisfy:*

$$P_\mathcal{G}^\star, \overline{P}_\mathcal{G}^\star < \frac{1}{2} B^{-1}(\mathbb{1}_{|\mathcal{G}|} - B_0). \tag{38}$$

(The above constraint is equivalent to requiring $\Lambda_g^\star > 0$, $\forall\, g \in \mathcal{G}$ for the particular loss model (15)*.)*

Under [A1], [A2], [A4]–[A6], *the KKT points $P_g^\star$, $\overline{P}_g^\star$ correspond to global minima of the nonconvex problem* (11).

The proof for the above statement is provided in the Appendix. With this characterization of the solution to problem (11), we now examine if (and when) AGC steers the system dynamics to a global minimum of the economic dispatch problem.

Theorem 2. *Suppose the system initially operates in steady state with net load, $P_{\rm load}$, and generator electrical power outputs corresponding to a global minimum of the economic dispatch problem* (11), *$P_g^\star$, $g \in \mathcal{G}$. Consider AGC action triggered by a change in net load to $\overline{P}_{\rm load} = P_{\rm load} + \Delta P_{\rm load}$. Denote by $\overline{P}_g^\star$, $g \in \mathcal{G}$, a global minimum of the economic dispatch problem solved with the new net load, $\overline{P}_{\rm load}$. Under assumptions* [A1]–[A6], *steady-state generator electrical power outputs following the net-load change correspond to a global minimum of the nonconvex economic dispatch problem solved with the new net load and corresponding system loss, i.e., $\overline{P}_g^{\rm e} = \overline{P}_g^\star$, $\forall\, g \in \mathcal{G}$, if and only if the AGC participation factors are chosen as in* (5).

Proof. The proof follows along the same line as that for Theorem 1, with the added realization that KKT points considered in Theorem 1 correspond to global minima per Lemma 1. □

C. Convex Generator-cost Functions and Lossless Network

In practice, condition (22) in assumption [A3] may be difficult to satisfy as it depends on individual generator-cost functions *and* network-wide power flow. Next, by neglecting losses, we make a similar statement about AGC optimality, albeit under an equivalent assumption that is easier to satisfy.

Theorem 3. *Suppose the generator-cost functions are strictly convex and assume the system is lossless; this setup renders problem* (11) *convex. Suppose the system initially operates in steady state with net load, $P_{\rm load}$, and generator electrical power outputs corresponding to the global minimum of the economic dispatch problem* (11), *$P_g^\star$, $g \in \mathcal{G}$. Consider AGC action triggered by a change in net load to $\overline{P}_{\rm load} = P_{\rm load} + \Delta P_{\rm load}$. Denote by $\overline{P}_g^\star$, $g \in \mathcal{G}$, the global minimum of the economic dispatch problem for the new net load, $\overline{P}_{\rm load}$. Consider also the following assumption:*

[A7] *The generator g cost function is such that for the economic dispatch solutions $P_g^\star, \overline{P}_g^\star$:*

$$C_g'(\overline{P}_g^\star) - C_g'(P_g^\star) = (\overline{P}_g^\star - P_g^\star) C_g''(P_g^\star). \tag{39}$$

Under [A1], [A2], *and* [A7], *the new steady-state generator electrical power outputs following the load change correspond to the global minimum of the economic dispatch problem solved with the new net load, i.e., $\overline{P}_g^{\rm e} = \overline{P}_g^\star$, $\forall\, g \in \mathcal{G}$, if and only if AGC participation factors are chosen as in* (5).

Proof. The proof follows along the same line as Theorem 1 while setting $\Lambda_g^\star = \overline{\Lambda}_g^\star = 1$, and with the added realization that a KKT point considered in Theorem 1 correspond to the unique global minimum per the assumptions listed above. □

Remark 2 (Generator-cost functions that satisfy [A7]). *Linear and quadratic generator-cost functions satisfy* (39) *while, e.g., cubic cost functions, do not. However, Theorem 3 does not apply for linear cost functions. Note that the optimal solution of the economic dispatch problem with linear cost functions enforces all generators with nonzero output (except one marginal generator) to operate at maximum capacity. (This is also in direct contradiction with assumption* [A2].*) Consequently, new steady-state generator electrical power outputs with the choice of AGC participation factors* (5) *correspond to the unique global minimum of the economic dispatch problem if and only if the generator-cost functions are* quadratic.

Remark 3 (Can [A3] be satisfied for small $\Delta P_{\rm load}$ and quadratic cost functions in lossy networks?). *This is an important question that governs the range of settings under which the general results highlighted above hold. For a lossy system, assumption* [A3] *need not be satisfied even if the cost functions are quadratic and we consider a small value of $\Delta P_{\rm load}$. To see why, note that a small $\Delta P_{\rm load}$ would only imply that $\overline{\Lambda}_g^\star \approx \Lambda_g^\star$* and not $\overline{\Lambda}_g^\star \approx \Lambda_g^\star \approx 1$.

D. Linear Generator-cost Functions and Lossless Network

We now examine the case where the generator-cost functions are linear. One can easily notice that the participation factors given by (5) are ill-defined in this setting since the second derivative of the cost functions would be identically zero. In light of this, we derive suitable participation factors that enable AGC to guarantee economic optimality. Let generator $g \in \mathcal{G}$ participating in frequency regulation have a linear cost function, i.e.,

$$C_g(P_g) = a_g P_g + b_g, \ \forall\, g \in \mathcal{G}, \tag{40}$$

for some constants $a_g, b_g > 0$. With the net load P_{load}, suppose generator m is the marginal generator in the system dictating the marginal cost of electricity, i.e., $\lambda^\star = C'_m(P^\star_m) = a_m$.

Consider a net-load change of $\Delta P_{\text{load}} > 0$ from the original value P_{load} so that the new net load is $\overline{P}_{\text{load}} = P_{\text{load}} + \Delta P_{\text{load}}$. Define set $\mathcal{G}' := \{j \in \mathcal{G} : a_j \geq a_m\}$ that collects generators in order of increasing linear-term coefficients in their cost functions, i.e., the generator with the lowest linear-term coefficient, which is still greater than or equal to a_m, is associated with the index $m+1$ and so on. Further suppose that the capacity constraints of the transmission lines are not binding. To ensure that the new steady-state generator electrical outputs with AGC, $\overline{P}_g = P^\star_g + \alpha_g \Delta P_{\text{load}}$, $g \in \mathcal{G}$, exactly match the optimizers of the economic dispatch problem solved with the new net load $\overline{P}_{\text{load}}$, i.e., $\overline{P}_g = P^\star_g + \alpha_g \Delta P_{\text{load}} = \overline{P}^\star_g,\ g \in \mathcal{G}$, the AGC participation factors should be picked as follows. If $\Delta P_{\text{load}} \leq P^{\max}_m - P^\star_m$, i.e., the marginal generator m can accommodate the entire net-load increase without violating its maximum capacity limit, then

$$\alpha_m = 1, \text{ and } \alpha_g = 0,\ \forall\, g \in \mathcal{G}'. \tag{41}$$

If, instead, $\Delta P_{\text{load}} > P^{\max}_m - P^\star_m$, where for some $m' \in \mathcal{G}'$,

$$\sum_{j \in \mathcal{G}',\, j < m'} P^{\max}_j - P^\star_j < \Delta P_{\text{load}} \leq \sum_{j \in \mathcal{G}',\, j \leq m'} P^{\max}_j - P^\star_j.$$

Then, generator $m' \in \mathcal{G}'$ becomes the new marginal generator, and the participation factors should be set as follows:

$$\begin{aligned} \alpha_g &= \begin{cases} 0, & \forall\, g > m', g \in \mathcal{G}', \\ \frac{(P^{\max}_g - P^\star_g)}{\Delta P_{\text{load}}}, & \forall\, g < m', g \in \mathcal{G}', \end{cases} \\ \alpha_{m'} &= 1 - \sum_{j \in \mathcal{G}',\, j < m'} \frac{P^{\max}_j - P^\star_j}{\Delta P_{\text{load}}}. \end{aligned} \tag{42}$$

The choices for the participation factors above can be justified by considering a simple procedure to arrive at the optimal solution of the economic dispatch problem with linear generator-cost functions for a net-load increase. One would first increase the output of the generator with the lowest marginal cost until its maximum capacity is reached. Then the output of the unit with the next lowest marginal cost would be raised until it reaches maximum capacity. Eventually, the repeated procedure arrives at a generator whose maximum capacity is not reached but the new net load is entirely served by generators in the system. This generator is designated as the new marginal generator in the system. In light of the above procedure, the AGC recruits the marginal generator and then generators with next lowest marginal costs to balance the net-load increase. In this way, the AGC guarantees economic optimality by provably steering generation to the optimizers of the economic dispatch problem. The case in which the net-load change $\Delta P < 0$ follows via similar logic, and we refrain from including details to avoid undue repetition.

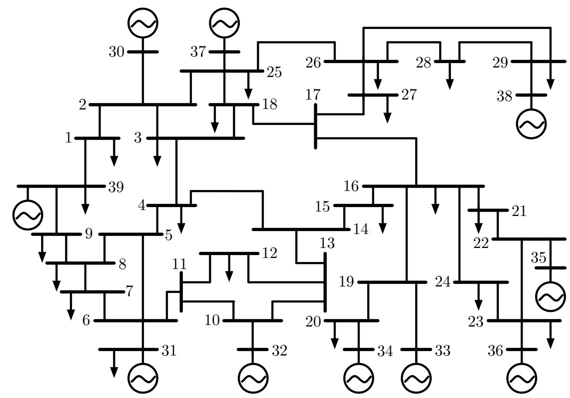

Fig. 2: One-line diagram for the New England test system.

IV. Case Studies

We focus on validating Theorem 3 with numerical simulations of the New England 39-bus 10-machine test system. While theoretical developments are based on simplified generator models, the DAE simulations performed in PSAT [43] involve detailed two-axis machine models along with governor and exciter controls [44]. All numerical values are reported to 4 significant digits.

A. Simulation Setup

The one-line diagram of the test system is depicted in Fig. 2. Generator parameters and nominal load values are sourced from the PSAT data file in [43], while line resistances are set to zero to model a lossless system. Cost function for generator g is assumed to be quadratic of the form $C_g(P_g) = a_g P^2_g + b_g P_g + c_g$, with coefficients a_g, b_g, and c_g for all generators in the system reported in Table I. The original PSAT data file for the New England test case does not contain generator-cost data. Thus, we choose a_g by sampling a uniform distribution between 0 and 10, b_g by sampling between 0 and 5, and c_g between 0 and 1. Each simulation begins with the system operating in steady state with generator electrical power outputs equal to the economic dispatch optimizer, i.e., $P^{\mathrm{e}}_g = P^\star_g,\ \forall\, g \in \mathcal{G}$. These are obtained by solving problem (11) neglecting losses for the nominal load (as specified in the PSAT data file), and numerical values are reported in Table I. We perform a total of 15 time-domain simulations for 5 different values of net-load change, ΔP_{load}, spanning -40%, -30%, -20%, -10%, and $+10\%$ (in each simulation, all loads in the system are varied uniformly). Notably, the net-load variations we consider do not lead to any violations in operating constraints, per assumption [A2]. For each of the 5 net-load change scenarios, 3 different choices of AGC participation factors are considered. The first choice is the optimal one (5). The other two are based on generator droop

TABLE I: Coefficients for generator g cost function of the form $C_g(P_g) = a_g P^2_g + b_g P_g + c_g$ and economic dispatch optimizers, $P^\star_g$, for initial load.

	$g=30$	$g=31$	$g=32$	$g=33$	$g=34$	$g=35$	$g=36$	$g=37$	$g=38$	$g=39$
a_g [\$/hr $\cdot$ p.u.^{2}]	7.842	7.314	4.542	3.864	7.756	7.343	4.303	6.938	9.452	3.600
b_g [\$/hr $\cdot$ p.u.]	4.319	3.528	1.950	2.955	2.297	0.2515	1.144	4.171	0.0780	0.5465
c_g [\$/hr]	0.05630	0.07810	0.5002	0.2180	0.5716	0.1222	0.6712	0.5996	0.05600	0.6690
$P^\star_g$ [p.u.]	4.364	4.733	7.795	9.033	4.542	4.937	8.322	4.943	3.845	10.03

TABLE II: For each load change, ΔP_{load}, we report: i) the global optimizers from economic dispatch with new net-load in rows labelled "$\overline{P}_g^\star$: Dispatch;" ii) electrical power outputs from generators with AGC participation factors picked to be the optimal (5), governor-based $\alpha_g^{\mathrm{Governor}}$, and inertial-based $\alpha_g^{\mathrm{Inertia}}$ in rows labelled "$\overline{P}_g^{\mathrm{e}}$: Optimal," "$\overline{P}_g^{\mathrm{e}}$: Governor," and "$\overline{P}_g^{\mathrm{e}}$: Inertial," respectively; and iii) total cost of dispatch in the column labelled "$\sum_{g\in\mathcal{G}} C_g(\cdot)$."

ΔP_{load}	$\overline{P}_g^\star$, $\overline{P}_g^{\mathrm{e}}$ [p.u.]	$g=30$	$g=31$	$g=32$	$g=33$	$g=34$	$g=35$	$g=36$	$g=37$	$g=38$	$g=39$	$\sum_g C_g(\cdot)$
-40%	$\overline{P}_g^\star$: Dispatch	2.560	2.799	4.680	5.372	2.719	3.011	5.034	2.904	2.348	6.100	\$875.0/hr
	$\overline{P}_g^{\mathrm{e}}$: Optimal	2.560	2.798	4.680	5.371	2.720	3.011	5.034	2.904	2.348	6.100	\$875.0/hr
	$\overline{P}_g^{\mathrm{e}}$: Governor	0.934	2.312	5.374	6.612	2.122	2.516	5.901	2.522	1.625	7.608	\$927.5/hr
	$\overline{P}_g^{\mathrm{e}}$: Inertial	2.114	2.170	5.232	6.470	1.979	2.374	5.758	2.379	1.432	7.617	\$913.1/hr
-30%	$\overline{P}_g^\star$: Dispatch	3.011	3.282	5.459	6.287	3.174	3.492	5.856	3.414	2.722	7.082	\$1175/hr
	$\overline{P}_g^{\mathrm{e}}$: Optimal	3.011	3.282	5.459	6.287	3.175	3.492	5.856	3.413	2.722	7.082	\$1175/hr
	$\overline{P}_g^{\mathrm{e}}$: Governor	1.792	2.917	5.979	7.217	2.727	3.122	6.506	3.127	2.180	8.214	\$1205/hr
	$\overline{P}_g^{\mathrm{e}}$: Inertial	2.677	2.810	5.873	7.110	2.620	3.015	6.399	3.020	2.036	8.220	\$1197/hr
-20%	$\overline{P}_g^\star$: Dispatch	3.462	3.766	6.238	7.202	3.630	3.974	6.678	3.923	3.096	8.065	\$1520/hr
	$\overline{P}_g^{\mathrm{e}}$: Optimal	3.462	3.765	6.238	7.202	3.631	3.974	6.678	3.923	3.096	8.065	\$1520/hr
	$\overline{P}_g^{\mathrm{e}}$: Governor	2.649	3.522	6.584	7.822	3.332	3.727	7.111	3.732	2.735	8.819	\$1533/hr
	$\overline{P}_g^{\mathrm{e}}$: Inertial	3.239	3.451	6.513	7.751	3.261	3.656	7.040	3.661	2.639	8.823	\$1529/hr
-10%	$\overline{P}_g^\star$: Dispatch	3.913	4.249	7.016	8.117	4.086	4.456	7.500	4.433	3.471	9.047	\$1908/hr
	$\overline{P}_g^{\mathrm{e}}$: Optimal	3.913	4.249	7.016	8.118	4.087	4.456	7.500	4.433	3.471	9.047	\$1908/hr
	$\overline{P}_g^{\mathrm{e}}$: Governor	3.506	4.128	7.190	8.428	3.937	4.332	7.716	4.337	3.290	9.424	\$1912/hr
	$\overline{P}_g^{\mathrm{e}}$: Inertial	3.801	4.092	7.154	8.392	3.902	4.296	7.681	4.302	3.242	9.427	\$1911/hr
$+10\%$	$\overline{P}_g^\star$: Dispatch	4.815	5.216	8.574	9.948	4.998	5.419	9.143	5.453	4.219	11.01	\$2819/hr
	$\overline{P}_g^{\mathrm{e}}$: Optimal	4.815	5.217	8.573	9.948	4.998	5.419	9.144	5.453	4.219	11.01	\$2819/hr
	$\overline{P}_g^{\mathrm{e}}$: Governor	5.221	5.338	8.400	9.638	5.148	5.542	8.927	5.549	4.400	10.63	\$2822/hr
	$\overline{P}_g^{\mathrm{e}}$: Inertial	4.926	5.374	8.436	9.673	5.183	5.578	8.963	5.584	4.448	10.63	\$2821/hr

and inertia constants, given by $\alpha_g^{\mathrm{Governor}} = R_g^{-1}/\sum_{j\in\mathcal{G}} R_j^{-1}$ and $\alpha_g^{\mathrm{Inertia}} = M_g/\sum_{j\in\mathcal{G}} M_j$, respectively, where R_g and M_g are the droop and inertia constants for generator g.

B. Simulation Results

For each load-change scenario considered, we report the global optimizer $\overline{P}_g^\star$, $g \in \mathcal{G}$, of the economic dispatch problem (11) with the new load, and the steady-state generator electrical power outputs $\overline{P}_g^{\mathrm{e}}$, $g \in \mathcal{G}$, with optimal, governor-based, and inertia-based AGC participation factors in Table II. The generator electrical power outputs are recorded in steady state once AGC action has subsided. The total cost of dispatch corresponding to each set of generator setpoints or outputs is provided in the last column.

In each scenario, the steady-state generator outputs that result from the optimal AGC participation factors match the optimizer of the economic dispatch problem solved with the new load. The maximum error across all simulations is 0.001 p.u. and this is likely attributable to numerical integration errors and the higher-order generator models used in the simulations. It is worth noting that these errors do not lead to perceptible differences in total dispatch cost. Furthermore, steady-state generator electrical power outputs resulting from both the governor- and the inertia-based AGC participation factors yield higher costs than those from the optimal choice. Although the inertia-based participation factors appear to perform better than the governor-based ones, this is clearly a system-specific numerical artifact. The system may operate with the new load that is unaccounted for in the pre-load-change dispatch for a significant period of time. The accumulated cost of operation—which we do not report to preserve generality of findings—would be notably lower with the optimal AGC participation factors over some operating horizon. In addition to validating the theoretical results reported in Theorem 3, the simulations justify the reduced-order generator dynamical model used to establish the theoretical results.

V. Concluding Remarks & Future Work

This paper establishes necessary and sufficient conditions under which a widely accepted choice of participation factors guarantee that steady-state generator electrical power outputs with AGC align with a global minimum of a prototypical economic dispatch problem. We validate our main results and compare the economic performance of AGC under different choices of participation factors through numerical case studies involving a modified version of the New England test system. Our analysis and results are important in establishing benchmark baselines to assess the performance of new or retrofit AGC and economic dispatch designs.

An exhaustive examination of operation with binding constraints is excellent grounds for future work. Other pertinent directions include investigating the performance of AGC with the nominal participation factors (5) in lossy networks where the optimal solution(s) of economic dispatch is (are) well characterized. This would also suggest avenues for optimal design of participation factors for a system with losses.

Appendix

Proof of Lemma 1. We begin by relaxing the constraint (11b) to obtain the following convex version of problem (11):

$$\min_{P_g,\, g\in\mathcal{G}} \sum_{g\in\mathcal{G}} C_g(P_g) \tag{43a}$$

$$\text{s.t.} \sum_{g\in\mathcal{G}} P_g \geq P_{\mathrm{load}} + P_{\mathrm{loss}}(P_{\mathcal{G}}), \tag{43b}$$

$$P_{(m,n)}^{\min} \leq \sum_{g\in\mathcal{G}} \Psi_{(m,n)}^g P_g \leq P_{(m,n)}^{\max},\ \forall\,(m,n)\in\mathcal{E}, \tag{43c}$$

$$P_g^{\min} \leq P_g \leq P_g^{\max},\ \forall\, g\in\mathcal{G}. \tag{43d}$$

The Lagrangian of problem (43) is given by

$$\mathcal{L} = \sum_{g\in\mathcal{G}} C_g(P_g) + \nu(P_{\mathrm{load}} + P_{\mathrm{loss}}(P_{\mathcal{G}}) - \sum_{g\in\mathcal{G}} P_g)$$

$$+\sum_{(m,n)\in\mathcal{E}}\psi^{+}_{(m,n)}\Big(\sum_{g\in\mathcal{G}}\Psi^{g}_{(m,n)}P_g-P^{\max}_{(m,n)}\Big)$$
$$+\sum_{(m,n)\in\mathcal{E}}\psi^{-}_{(m,n)}\Big(P^{\min}_{(m,n)}-\sum_{g\in\mathcal{G}}\Psi^{g}_{(m,n)}P_g\Big)$$
$$+\sum_{g\in\mathcal{G}}\mu^{+}_g(P_g-P^{\max}_g)+\sum_{g\in\mathcal{G}}\mu^{-}_g(P^{\min}_g-P_g). \tag{44}$$

The KKT conditions of problem (43) are as follows:

$$C'_g(P^\star_g)-\frac{\nu^\star}{\Lambda^\star_g}+\sum_{(m,n)\in\mathcal{E}}(\psi^{+\star}_{(m,n)}-\psi^{-\star}_{(m,n)})\Psi^{g}_{(m,n)}$$
$$+\mu^{+\star}_g-\mu^{-\star}_g=0,\ \forall g\in\mathcal{G}, \tag{45a}$$

$$\begin{cases}\nu^\star(P_{\text{load}}+P_{\text{loss}}(P^\star_{\mathcal{G}})-\sum_{g\in\mathcal{G}}P^\star_g)=0,\\ \nu^\star\geq 0,\end{cases} \tag{45b}$$

$$\begin{cases}\psi^{+\star}_{(m,n)}\Big(\sum_{g\in\mathcal{G}}\Psi^{g}_{(m,n)}P^\star_g-P^{\max}_{(m,n)}\Big)=0,\\ \psi^{+\star}_{(m,n)}\geq 0,\quad\forall(m,n)\in\mathcal{E},\end{cases} \tag{45c}$$

$$\begin{cases}\psi^{-\star}_{(m,n)}\Big(P^{\min}_{(m,n)}-\sum_{g\in\mathcal{G}}\Psi^{g}_{(m,n)}P^\star_g\Big)=0,\\ \psi^{-\star}_{(m,n)}\geq 0,\quad\forall(m,n)\in\mathcal{E},\end{cases} \tag{45d}$$

$$\begin{cases}\mu^{+\star}_g(P^\star_g-P^{\max}_g)=0,\ \mu^{+\star}_g\geq 0,\\ \mu^{-\star}_g(P^{\min}_g-P^\star_g)=0,\ \mu^{-\star}_g\geq 0,\end{cases}\quad\forall g\in\mathcal{G}, \tag{45e}$$

$$\begin{cases}P^{\min}_{(m,n)}\leq\sum_{g\in\mathcal{G}}\Psi^{g}_{(m,n)}P^\star_g\leq P^{\max}_{(m,n)},\forall(m,n)\in\mathcal{E},\\ P^{\min}_g\leq P^\star_g\leq P^{\max}_g,\ \forall g\in\mathcal{G}.\end{cases} \tag{45f}$$

Per assumptions [A4]–[A5] on convexity, the KKT conditions (45) are *necessary and sufficient* for the optimal solution. Furthermore, if $\nu^\star>0$, the KKT conditions (45) for the *convex* problem (43) are identical to those in (13) for the *nonconvex* problem (11). We therefore set out to uncover conditions under which $\nu^\star>0$ holds. Given assumption [A2], we set $\psi^{+\star}_{(m,n)}=\psi^{-\star}_{(m,n)}=\mu^{+\star}_g=\mu^{-\star}_g=0$ in (45b) to get: $\nu^\star=C'_g(P^\star_g)\Lambda^\star_g$. Assumption [A4] implies $C'_g(P^\star_g)>0$ for $P^{\min}_g\leq P^\star_g\leq P^{\max}_g$. Also, given the model adopted for system loss in (15), it follows that (38) in assumption [A6] enforces $\Lambda^\star_g>0,\forall g\in\mathcal{G}$. Thus, under assumptions [A4] and [A6], it follows that $\nu^\star>0$. The inequality (43b) is therefore binding at the optimal solution of the convex problem (43). Subsequently, this also corresponds to the optimal solution of the nonconvex problem (11). Since $C_g(P_g)$ is *strictly convex*, the optimal solution $P^\star_{\mathcal{G}}$ that satisfies the KKT conditions (45) is unique. That is, it corresponds to *the global minimum* of the *convex problem* (43) and *a global minimum* of the *nonconvex problem* (11). The arguments above also apply to show that $\overline{P}^\star_{\mathcal{G}}$ (corresponding to net load $\overline{P}_{\text{load}}$) is a global minimum of problem (11) under the same assumptions. □

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