Frequency Dynamics-aware Real-time Marginal Pricing of Electricity Under Uncertainty

Bo Chen†, Roohallah Khatami‡, Yu Christine Chen†
†The University of British Columbia, Vancouver, BC, Canada, {cbhp1993, chen}@ece.ubc.ca
‡Southern Illinois University, Carbondale, IL, United States, roohallah.khatami@siu.edu

***Abstract*—This paper presents a real-time electricity marginal pricing method that captures the impact of frequency dynamics in the presence of i) aggressive net-load variability and ii) substantial net-load forecast uncertainty, both made more likely by extensive renewable integration. The proposed method augments a traditional economic dispatch with constraints pertaining to system frequency dynamics and explicit models for the net-load forecast uncertainty, while minimizing the sum of expected operation cost and expected frequency deviations from the synchronous speed across the scheduling horizon of interest. Chance constraints in the modified problem dictate the tolerable probability of violating lower and upper limits of dynamic system frequency and generator outputs. To facilitate solution efficiency, the chance-constrained economic dispatch transforms into a deterministic optimization problem under the assumption that the uncertainty in the net-load forecast is Gaussian. The effectiveness of the proposed model is demonstrated through case studies based on the Western System Coordinating Council test system. The results show that the proposed pricing method can reflect the impact of transient frequency deviations, yield greater profits for generators, and hedge against net-load uncertainty.**

I. Introduction

Renewable energy sources (RESs) help to address mounting environmental concerns of fossil sources. Extensive integration of RESs, however, poses notable challenges in reliable and efficient real-time operation of the power system as it copes with larger and faster variations in the net load (system load minus non-dispatchable RES generation), a task made even more difficult by the considerable uncertainty inherent to forecasts of RES generation [1]. There is growing concern that existing wholesale electricity markets, which have been instrumental in fostering competition and promoting system efficiency, may be challenged to sustain the same benefits for both energy suppliers and consumers along the current trajectory of RES deployment [2]. In light of this, our work aims to improve over existing market designs to accurately compensate generating units for the energy they produce that contribute to i) maintaining the second-to-second supply-demand balance across potential frequency transients, and ii) limiting the risk imposed by uncertainty in the net-load forecast.

Fundamental to competitive electricity markets is the concept of marginal pricing that reflects the rate of change of optimal cost due to an incremental change in demand. Traditionally, marginal pricing is derived from a problem commonly known as the economic dispatch (ED), which is solved for a single operating point with two notable underlying assumptions: i) the system operates in synchronous steady state, and ii) the net-load forecast is known precisely [3]. These may be invalidated in the future as high-inertia dispatchable fossil-fuel generators in the present-day power system become increasingly displaced by low-inertia non-dispatchable RESs [2].

A promising approach to account for net-load forecast uncertainty in an ED is to formulate a chance-constrained economic dispatch (CCED) minimizing expected costs subject to tolerable probability of constraint violations. Prior work in CCED can be categorized by the choice of probabilistic models (non-parametric [4] or parametric [5]) and by the solution approach (approximate [6] or exact [7]). Non-parametric methods that favour more general empirical probability distributions may require more data to achieve the same level of precision as parametric methods that assume a specific distribution with its parameters being estimated from historical data [8], [9]. In terms of the solution approach, approximate methods tend to establish a set of scenarios representative of the probability distribution of the uncertainty [6], and exact methods aim to transform the chance-constrained problem into an equivalent deterministic one [7].

Common to all CCED efforts mentioned above is the assumption that the power system operates in steady state, which may not hold in the face of larger and more frequent transients expected in future power systems. To address this issue, [10] formulates a dynamics-aware ED by incorporating discrete-time primary and secondary frequency response dynamics, but it focuses only on optimal decision making, not electricity pricing. Recent research in pricing and compensating generation sources for faster dynamic response include procuring *new* services like rotational and virtual inertia to mitigate undesired frequency excursions [11]–[13]. More aligned with traditional ED-based formulations, our recent work in [14] proposes a multi-time-scale dynamics-aware ED focused on marginal pricing of electricity. This model integrates the relatively fast system dynamics along with slower decisions on generator set-points into a single optimization problem. However, it does not address the impact of uncertainty associated with the net-load forecast on the marginal price of electricity during transients.

In this paper, similar to [14], we directly modify the traditional ED to include constraints pertinent to system frequency dynamics, yielding a dynamics-aware ED. Extending from [14], we further model probabilistic uncertainty in the net-load forecast as a multivariate Gaussian distribution. Chance constraints then dictate the tolerable probability of

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violating lower and upper limits of dynamic system frequency and generator outputs. The resulting dynamics-aware CCED minimizes the sum of the expected operation cost and expected frequency deviations from the synchronous speed across the scheduling horizon. Given the structure of the dynamics-aware CCED along with the Gaussian model for forecast uncertainty, we can transform the chance-constrained problem into a deterministic optimization problem, so as to facilitate solutions using standard optimization solvers. The marginal price of electricity under uncertainty is the (suitably scaled) Lagrange multiplier of the dynamic power balance constraint. It represents the cost of *energy* to regulate system frequency while satisfying tolerable risk specified by the probability of constraint violations. The proposed method admittedly deviates significantly from today's regulation markets where generators are paid for reserve *capacity*. However, as simulations involving the Western System Coordinating Council test system confirm, it offers a promising extension of existing real-time markets to provide greater compensation to generators for contributing to dynamic performance while mitigating risk brought about by net-load forecast uncertainty.

II. Preliminaries

In this section, we outline a traditional dynamics-oblivious CCED incorporating load forecast uncertainty. We also describe the system frequency dynamical model.

A. Traditional Chance-constrained Economic Dispatch

Consider a transmission system with G online generators in the set $\mathcal{G} = \{1, \ldots, G\}$ supplying forecasted net load $P_\circ^{\text{load}}$. Prevailing ED formulations assume synchronous steady-state operations, where generator g produces steady-state electrical power $P_{\circ,g}$ with cost function $C_g(P_{\circ,g})$. Uncertainty associated with predictions of the upcoming net load renders generator outputs to be random variables. Then the total expected cost of generation $\mathbb{E}[C(P_\circ)] = \mathbb{E}[\sum_{g\in\mathcal{G}} C_g(P_{\circ,g})]$, with $P_\circ = [P_{\circ,1}, \ldots, P_{\circ,G}]^{\text{T}}$, can be minimized in a CCED

$$\underset{P_\circ,\pi}{\text{minimize}} \quad \mathbb{E}[C(P_\circ)] \tag{1a}$$

$$\text{subject to} \quad \mathbb{1}_G^{\text{T}} P_\circ = P_\circ^{\text{load}}, \tag{1b}$$

$$P_\circ = \pi P_\circ^{\text{load}}, \tag{1c}$$

$$\mathbb{P}(P_\circ \geq P_{\min}) \geq (1-\varepsilon^P)\mathbb{1}_G, \tag{1d}$$

$$\mathbb{P}(P_\circ \leq P_{\max}) \geq (1-\varepsilon^P)\mathbb{1}_G, \tag{1e}$$

where participation factors collected in $\pi \in \mathbb{R}^G$ distribute system load amongst all generators, $\varepsilon^P \in (0,1)$ represents the tolerable probability of violation in chance constraints (1d)–(1e), and $\mathbb{1}_G$ is a G-dimensional vector of 1s (see, e.g., [15]).

Since the ED in (1) assumes steady-state operation, it does not offer any insights on the optimal dispatch or price of electricity during transients. This is well aligned with power systems dominated by high-inertia synchronous generators serving slow-varying loads. However, the displacement of fossil fuel-based synchronous generators by low-inertia sources is bringing about larger, faster, and more frequent frequency excursions away from synchronous steady state. Thus, there is a pressing need to update the dynamics-oblivious CCED in (1) so that resulting marginal prices capture the cost of electricity generation considering frequency transients.

B. Generator Frequency Dynamics

For each generator $g \in \mathcal{G}$, let ω_g denote its electrical angular frequency; also let P_g^{r}, P_g^{m}, and P_g respectively denote its reference set-point, turbine mechanical power, and electrical output. Each generator initially operates at the steady-state equilibrium point with $\omega_g(0) = \omega_{\text{s}}$, $P_g^{\text{r}}(0) = P_g^{\text{m}}(0) = P_g(0) = P_g^{\text{r}\circ}$. Let $\Delta\omega_g := \omega_g - \omega_{\text{s}}$ and assume that the electrical distances between geographically different parts of the power system are negligible, then all generator frequencies follow the same transient behaviour, i.e., $\Delta\omega_g = \Delta\omega$, $\forall\, g \in \mathcal{G}$ [16]. Also let $P = [P_1, \ldots, P_G]^{\text{T}}$, $P^{\text{m}} = [P_1^{\text{m}}, \ldots, P_G^{\text{m}}]^{\text{T}}$, and $P^{\text{r}} = [P_1^{\text{r}}, \ldots, P_G^{\text{r}}]^{\text{T}}$. Then system frequency dynamics can be modelled as

$$M\Delta\dot{\omega} = P^{\text{m}} - D\Delta\omega - P, \tag{2}$$

$$\tau\dot{P}^{\text{m}} = P^{\text{r}} - P^{\text{m}} - R^{-1}\mathbb{1}_G\Delta\omega, \tag{3}$$

where $M = [M_1, \ldots, M_G]^{\text{T}}$ and $D = [D_1, \ldots, D_G]^{\text{T}}$ respectively collect the generator inertia and damping constants, and $\tau = \text{diag}([\tau_1, \ldots, \tau_G])$ and $R^{-1} = \text{diag}([R_1^{-1}, \ldots, R_G^{-1}])$ collect the generator governor time constants and inverse-droop constants, respectively [17]. Summing (2) over all $g \in \mathcal{G}$, we define an aggregate mechanical power $\mathbb{1}_G^{\text{T}} P^{\text{m}}$ and get the following reduced-order dynamical model:

$$M_{\text{eff}}\Delta\dot{\omega} = \mathbb{1}_G^{\text{T}} P^{\text{m}} - D_{\text{eff}}\Delta\omega - P^{\text{load}}, \tag{4}$$

where the effective inertia constant M_{eff} and the effective damping constant D_{eff} are respectively given by $M_{\text{eff}} := \sum_{g\in\mathcal{G}} M_g$ and $D_{\text{eff}} := \sum_{g\in\mathcal{G}} D_g$, and $P^{\text{load}} = \mathbb{1}_G^{\text{T}} P$.

III. Dynamics-aware Economic Dispatch

This section formulates the dynamics-aware CCED followed by models pertinent to the uncertainty in net load and dynamic state variables. We then reformulate the CCED into a deterministic problem and show that the marginal price of electricity is the Lagrange multiplier associated with the dynamic power balance constraint.

A. Chance-constrained Problem Formulation

Consider the ED scheduling horizon from time t_0 to $t_0 + T$, subdivided by two pertinent time steps. The time step corresponding to faster system dynamics is denoted by $\Delta t^{\text{D}} = \frac{T}{N^{\text{D}}}$, which is sufficiently small to model generator dynamics (e.g., $0.05\,[\text{sec}]$). The scheduling horizon then divides into N^{D} intervals with end-points collected in the set $\mathcal{T}_{t_0}^{\text{D}} = \{t_0, t_0 + \Delta t^{\text{D}}, \ldots, t_0 + T\}$. Next, decisions on generator reference set-points are made over a longer time interval $\Delta t^{\text{S}} = \frac{T}{N^{\text{S}}}$ (e.g., $2.5\,[\text{sec}]$), which subdivides the scheduling horizon into N^{S} intervals with end-points collected in $\mathcal{T}_{t_0}^{\text{S}} = \{t_0, t_0 + \Delta t^{\text{S}}, \ldots, t_0 + T\}$. Further consider the net-load forecast over the scheduling horizon P_t^{load}, $t \in \mathcal{T}_{t_0}^{\text{D}}$. Due to forecast errors, the upcoming load is not known precisely and uncertainty is associated with the predictions.

We then formulate the following dynamics-aware CCED with fast decisions made every Δt^{D} interval and slower generator set-points determined every Δt^{S} interval:

$$\underset{P^{\mathrm{r}}_{t'},P^{\mathrm{m}}_t,\Delta\omega_t,P_t}{\text{minimize}} \sum_{t\in\mathcal{T}^{\mathrm{D}}_{t_0}} (\mathbb{E}[C(P^{\mathrm{m}}_t)] + \mathbb{E}[\kappa|\Delta\omega_t|])\Delta t^{\mathrm{D}} \tag{5a}$$

$$\text{subject to } M_{\text{eff}}\Big(\frac{\Delta\omega_{t+\Delta t^{\mathrm{D}}} - \Delta\omega_t}{\Delta t^{\mathrm{D}}}\Big) = \mathbb{1}^{\mathrm{T}}_G P^{\mathrm{m}}_t - D_{\text{eff}}\Delta\omega_t - P^{\text{load}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}\setminus\{t_0+T\}, \tag{5b}$$

$$\tau\Big(\frac{P^{\mathrm{m}}_{t+\Delta t^{\mathrm{D}}} - P^{\mathrm{m}}_t}{\Delta t^{\mathrm{D}}}\Big) = P^{\mathrm{r}}_{t'} - P^{\mathrm{m}}_t - R^{-1}\mathbb{1}_G\Delta\omega_t,\quad t'\in\mathcal{T}^{\mathrm{S}}_{t_0}\setminus\{t_0+T\},\quad t\in\{t',\ldots,t'+\Delta t^{\mathrm{S}}-\Delta t^{\mathrm{D}}\}, \tag{5c}$$

$$\mathbb{P}(P^{\mathrm{m}}_t \geq P^{\mathrm{m}}_{\min}) \geq \mathbb{1}_G(1-\varepsilon^P),\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{5d}$$

$$\mathbb{P}(P^{\mathrm{m}}_t \leq P^{\mathrm{m}}_{\max}) \geq \mathbb{1}_G(1-\varepsilon^P),\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{5e}$$

$$\mathbb{P}(\Delta\omega_t \geq \Delta\omega_{\min}) \geq 1-\varepsilon^\omega,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{5f}$$

$$\mathbb{P}(\Delta\omega_t \leq \Delta\omega_{\max}) \geq 1-\varepsilon^\omega,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}. \tag{5g}$$

The objective function (5a) comprises the expected values of two terms, each weighted by time step Δt^{D} and summed over the scheduling horizon. The ED minimizes the expected cost of generation $\mathbb{E}[C(P^{\mathrm{m}}_t)] = \mathbb{E}[\sum_{g\in\mathcal{G}} C_g(P^{\mathrm{m}}_t)]$ and the expected cost of regulating system frequency $\mathbb{E}[\kappa|\Delta\omega_t|]$ with $\kappa > 0$ being the coefficient of cost of absolute frequency deviation. Constraints in (5b)–(5c) represent the discretized system frequency dynamics, and chance constraints in (5d)–(5e) and (5f)–(5g) respectively impose the tolerable probability of violating limits in mechanical power and system frequency.

B. Uncertainty Models

We decompose the net-load forecast as $P^{\text{load}}_t = \overline{P}^{\text{load}}_t + \widetilde{P}^{\text{load}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}$, where $\overline{P}^{\text{load}}_t$ is the nominal (or mean) value and $\widetilde{P}^{\text{load}}_t$ represents the uncertain component. Then dynamic state variables $\Delta\omega_t$ and P^{m}_t can be similarly decomposed as

$$\Delta\omega_t = \Delta\overline{\omega}_t + \Delta\widetilde{\omega}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{6}$$

$$P^{\mathrm{m}}_t = \overline{P}^{\mathrm{m}}_t + \widetilde{P}^{\mathrm{m}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{7}$$

where $\Delta\overline{\omega}_t$ and $\overline{P}^{\mathrm{m}}_t$ are respectively the mean frequency deviation and turbine mechanical powers, at time t, and $\Delta\widetilde{\omega}_t$ and $\widetilde{P}^{\mathrm{m}}_t$ represent the associated uncertainty. Then we can express (5b)–(5c) as the sum of the following:

$$\begin{bmatrix}\Delta\overline{\omega}_{t+\Delta t^{\mathrm{D}}}\\ \overline{P}^{\mathrm{m}}_{t+\Delta t^{\mathrm{D}}}\end{bmatrix} = A\begin{bmatrix}\Delta\overline{\omega}_t\\ \overline{P}^{\mathrm{m}}_t\end{bmatrix} + B\overline{P}^{\text{load}}_t + CP^{\mathrm{r}}_{t'},\quad t'\in\mathcal{T}^{\mathrm{S}}_{t_0}\setminus\{t_0+T\},\ t\in\{t',\ldots,t'+\Delta t^{\mathrm{S}}-\Delta t^{\mathrm{D}}\}, \tag{8}$$

$$\begin{bmatrix}\Delta\widetilde{\omega}_{t+\Delta t^{\mathrm{D}}}\\ \widetilde{P}^{\mathrm{m}}_{t+\Delta t^{\mathrm{D}}}\end{bmatrix} = A\begin{bmatrix}\Delta\widetilde{\omega}_t\\ \widetilde{P}^{\mathrm{m}}_t\end{bmatrix} + B\widetilde{P}^{\text{load}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}\setminus\{t_0+T\}, \tag{9}$$

where matrices A, B, and C are given by

$$A = \begin{bmatrix} 1-\frac{D_{\text{eff}}}{M_{\text{eff}}}\Delta t^{\mathrm{D}} & \frac{1}{M_{\text{eff}}}\mathbb{1}^{\mathrm{T}}_G\Delta t^{\mathrm{D}} \\ -\tau^{-1}R^{-1}\mathbb{1}_G\Delta t^{\mathrm{D}} & I_G - \tau^{-1}\Delta t^{\mathrm{D}}\end{bmatrix},\quad B = \begin{bmatrix}-\frac{1}{M_{\text{eff}}}\Delta t^{\mathrm{D}}\\ \mathbb{0}_G\end{bmatrix},\ C = \begin{bmatrix}\mathbb{0}^{\mathrm{T}}_G\\ \tau^{-1}\Delta t^{\mathrm{D}}\end{bmatrix}. \tag{10}$$

Via visual inspection of (8)–(9), we note that the dynamics in the nominal values of states are decoupled from those in the uncertain components. Furthermore, the latter dynamics are driven only by the uncertainty in net load because generator set-points are deterministic decision variables. Now, suppose $\widetilde{P}^{\text{load}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}$, are characterized as independent and identically distributed Gaussian random variables with zero mean and standard deviation σ^{load}_t, i.e., $\widetilde{P}^{\text{load}}_t \sim \mathcal{N}(0,(\sigma^{\text{load}}_t)^2)$. Since (9) is linear, we can conclude that $\Delta\widetilde{\omega}_t$ and $\widetilde{P}^{\mathrm{m}}_t$ are also Gaussian random variables with zero mean, i.e.,

$$\begin{bmatrix}\Delta\widetilde{\omega}_t\\ \widetilde{P}^{\mathrm{m}}_t\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}0\\ \mathbb{0}_G\end{bmatrix}, \begin{bmatrix}(\sigma^\omega_t)^2 & \Sigma^{\omega,P}_t\\ \Sigma^{P,\omega}_t & \Sigma^P_t\end{bmatrix}\right),\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}, \tag{11}$$

where the variance is calculated as

$$\begin{bmatrix}(\sigma^\omega_t)^2 & \Sigma^{\omega,P}_t\\ \Sigma^{P,\omega}_t & \Sigma^P_t\end{bmatrix} = \sum_{k=0}^{\frac{t-t_0}{\Delta t^{\mathrm{D}}}-1} A^{\frac{t-t_0}{\Delta t^{\mathrm{D}}}-1-k}B\cdot(\sigma^{\text{load}}_{t_0+k\Delta t^{\mathrm{D}}})^2 \cdot (A^{\frac{t-t_0}{\Delta t^{\mathrm{D}}}-1-k}B)^{\mathrm{T}},\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}\setminus\{t_0\}, \tag{12}$$

with $\sigma^\omega_{t_0}=0$, $(\Sigma^{\omega,P}_{t_0})^{\mathrm{T}} = \Sigma^{P,\omega}_{t_0} = \mathbb{0}_G$, and $\Sigma^P_{t_0} = \mathbb{0}_{G\times G}$ as initial conditions are assumed to be known precisely.

C. Deterministic Problem Formulation

Bearing in mind that dynamic state variables are decomposed as in (6)–(7), we upper bound the objective function in (5a) via the triangle inequality as

$$\mathbb{E}[C(P^{\mathrm{m}}_t)] + \mathbb{E}[\kappa|\Delta\omega_t|] \leq \mathbb{E}\big[C(\overline{P}^{\mathrm{m}}_t + \widetilde{P}^{\mathrm{m}}_t)\big] + \kappa\mathbb{E}[|\Delta\overline{\omega}_t|] + \kappa\mathbb{E}[|\Delta\widetilde{\omega}_t|]. \tag{13}$$

Now consider a typical quadratic total generation cost function

$$C(P^{\mathrm{m}}_t) = P^{\mathrm{m}\,\mathrm{T}}_t\text{diag}(q)P^{\mathrm{m}}_t + r^{\mathrm{T}}P^{\mathrm{m}}_t + \mathbb{1}^{\mathrm{T}}_G c, \tag{14}$$

where $q = [q_1,\ldots,q_G]^{\mathrm{T}}$, $r = [r_1,\ldots,r_G]^{\mathrm{T}}$, and $c = [c_1,\ldots,c_G]^{\mathrm{T}}$ respectively represent the quadratic-, linear-, and constant-term coefficients. Furthermore, since the uncertain components in (13) are modelled by the Gaussian distribution in (11), the right-hand side of (13) evaluates as

$$C(\overline{P}^{\mathrm{m}}_t) + \kappa|\Delta\overline{\omega}_t| + (\sigma^P_t)^{\mathrm{T}}\text{diag}(q)\sigma^P_t + \kappa\sqrt{2/\pi}\sigma^\omega_t, \tag{15}$$

where σ^P_t denotes the element-wise square root of the vector of diagonal entries in Σ^P_t, which, along with σ^ω_t can be evaluated using (12). With the updated objective function in (15), we can reformulate the CCED in (5) into the following deterministic counterpart:

$$\underset{\Omega}{\text{minimize}} \sum_{t\in\mathcal{T}^{\mathrm{D}}_{t_0}}\Big(C(\overline{P}^{\mathrm{m}}_t) + \kappa(\Delta\overline{\omega}^+_t + \Delta\overline{\omega}^-_t) + (\sigma^P_t)^{\mathrm{T}}\text{diag}(q)\sigma^P_t + \kappa\sqrt{2/\pi}\sigma^\omega_t\Big)\Delta t^{\mathrm{D}} \tag{16a}$$

$$\text{subject to } M_{\text{eff}}\Big(\frac{\Delta\overline{\omega}_{t+\Delta t^{\mathrm{D}}} - \Delta\overline{\omega}_t}{\Delta t^{\mathrm{D}}}\Big) = \mathbb{1}^{\mathrm{T}}_G\overline{P}^{\mathrm{m}}_t - D_{\text{eff}}\Delta\overline{\omega}_t - \overline{P}^{\text{load}}_t,\ t\in\mathcal{T}^{\mathrm{D}}_{t_0}\setminus\{t_0+T\},\ (\lambda_t), \tag{16b}$$

$$\tau\Big(\frac{\overline{P}^{\mathrm{m}}_{t+\Delta t^{\mathrm{D}}} - \overline{P}^{\mathrm{m}}_t}{\Delta t^{\mathrm{D}}}\Big) = P^{\mathrm{r}}_{t'} - \overline{P}^{\mathrm{m}}_t - R^{-1}\mathbb{1}_G\Delta\overline{\omega}_t,\quad t'\in\mathcal{T}^{\mathrm{S}}_{t_0}\setminus\{t_0+T\},$$

$$t \in \{t', \dots, t' + \Delta t^{\mathrm{S}} - \Delta t^{\mathrm{D}}\},\ (\beta_t), \quad (16c)$$

$$\overline{P}_t^{\mathrm{m}} + \sigma_t^P \Phi^{-1}(1-\varepsilon^P) \leq P_{\max}^{\mathrm{m}}, \quad t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\mu_t^+), \quad (16d)$$

$$\overline{P}_t^{\mathrm{m}} - \sigma_t^P \Phi^{-1}(1-\varepsilon^P) \geq P_{\min}^{\mathrm{m}}, \quad t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\mu_t^-), \quad (16e)$$

$$\Delta\overline{\omega}_t + \sigma_t^\omega \Phi^{-1}(1-\varepsilon^\omega) \leq \Delta\omega_{\max}, \quad t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\nu_t^+), \quad (16f)$$

$$\Delta\overline{\omega}_t - \sigma_t^\omega \Phi^{-1}(1-\varepsilon^\omega) \geq \Delta\omega_{\min}, \quad t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\nu_t^-), \quad (16g)$$

$$\Delta\overline{\omega}_t = \Delta\overline{\omega}_t^+ - \Delta\overline{\omega}_t^-,\ t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\zeta_t), \quad (16h)$$

$$\Delta\overline{\omega}_t^+, \Delta\overline{\omega}_t^- \geq 0,\ t \in \mathcal{T}_{t_0}^{\mathrm{D}},\ (\rho_t^{\omega+}, \rho_t^{\omega-}), \quad (16i)$$

where $\Omega = \{\Delta\overline{\omega}_t, \Delta\overline{\omega}_t^+, \Delta\overline{\omega}_t^-, \overline{P}_t^{\mathrm{m}}, P_{t'}^{\mathrm{r}}\}_{t\in\mathcal{T}_{t_0}^{\mathrm{D}}, t'\in\mathcal{T}_{t_0}^{\mathrm{S}}}$ collects decision variables of the optimization problem, and $\Phi^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the standard normal distribution.

D. Marginal Price of Electricity

The marginal price represents the rate of change of the system operation cost due to an incremental change in electrical load, while satisfying generator and system static and dynamic constraints. Mathematically, the marginal price is expressed as the first derivative of optimal Lagrangian with respect to net load at time $t \in \mathcal{T}_{t_0}^{\mathrm{D}}$. By definition, given the optimal solution of the problem in (16), $\{\Delta\overline{\omega}_t^\star, \Delta\overline{\omega}_t^{+\star}, \Delta\overline{\omega}_t^{-\star}, \overline{P}_t^{\mathrm{m}\star}, P_{t'}^{\mathrm{r}\star}\}_{t\in\mathcal{T}_{t_0}^{\mathrm{D}}, t'\in\mathcal{T}_{t_0}^{\mathrm{S}}}$ and the optimal Lagrangian $\mathcal{L}^\star$, the marginal price is calculated as

$$\frac{1}{\Delta t^{\mathrm{D}}} \frac{d\mathcal{L}^\star}{d\overline{P}_t^{\mathrm{load}}} \quad (17)$$

where the division by Δt^{D} ensures that the marginal price applies for arbitrary Δt^{D} and results in consistent units aligned with the cost function. Particularly, for the cost function with units of $[\$/\mathrm{hr}]$, division by Δt^{D} yields marginal price in units of $[\$/\mathrm{MWh}]$ regardless of the length of Δt^{D}. Applying the chain rule in calculus, (17) simplifies as

$$\frac{1}{\Delta t^{\mathrm{D}}} \frac{d\mathcal{L}^\star}{d\overline{P}_t^{\mathrm{load}}} = \frac{1}{\Delta t^{\mathrm{D}}} \frac{\partial\mathcal{L}^\star}{\partial\overline{P}_t^{\mathrm{load}}} = \frac{1}{\Delta t^{\mathrm{D}}} \lambda_t^\star =: \lambda_t'^\star. \quad (18)$$

IV. Case Studies

Simulations presented in this section use the Western System Coordinating Council test system where the system power base is $100\,[\mathrm{MVA}]$. Generator dynamic model and cost function parameters are respectively provided in Tables I and II. The scheduling horizon spans $T = 300\,[\mathrm{sec}]$ from $t_0 = 0\,[\mathrm{sec}]$. The shorter time interval capturing system dynamics is $\Delta t^{\mathrm{D}} = 0.05\,[\mathrm{sec}]$, while the generator set-point decisions are made every $\Delta t^{\mathrm{S}} = 2.5\,[\mathrm{sec}]$. The mean net load is forecasted to take a constant value of $315\,[\mathrm{MW}]$ for $t \in [0, 15]\,[\mathrm{sec}]$, followed by a 15% increase for $t \in (15, 300]\,[\mathrm{sec}]$. The dynamics-aware ED in (16) is modelled in the MATLAB YALMIP toolbox and solved using the GUROBI solver on a desktop computer with a $3.6\,[\mathrm{GHz}]$ i7 processor and $32\,[\mathrm{GB}]$ RAM.

TABLE I
Dynamic Model Parameters of Generators and Governors

Generator	M_g [sec]	D_g	τ_g [sec]	$\frac{1}{R_g}$
$g=1$	23.64	20	2	100
$g=2$	6.4	20	2	100
$g=3$	3.01	20	2	100

TABLE II
Generator Quadratic Cost Function Parameters

Generator	q_g [\$/(MW^{2}h)]	r_g [\$/MWh]	c_g [\$/h]
$g=1$	0.5500	25	150
$g=2$	0.0850	1.2	600
$g=3$	1.225	10	335

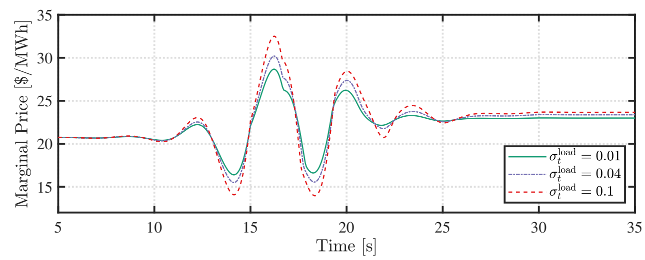

Fig. 1. Dynamics-aware marginal prices under load forecast uncertainty evaluated at the optimal solution of the dynamics-aware CCED in (16).

A. Dynamics-aware Marginal Price Under Uncertainty

We obtain the dynamics-aware marginal price of electricity under uncertainty by evaluating $\lambda_t'^\star$ in (18) at the optimal solution of (16). The marginal prices are plotted in Fig. 1 for three different net-load forecast uncertainty levels with σ_t^{load}, $t \in \mathcal{T}_{t_0}^{\mathrm{D}}$, being 0.01, 0.04, and $0.1\,[\mathrm{p.u.}]$. At the initial steady-state operating point, inequality constraints in (16) are not binding, and the marginal prices for the three cases are equal and constant. They later diverge due to the forecasted load change. The price differences observed across the three cases arise because, following a load change at $t = 15\,[\mathrm{sec}]$, the maximum power limit of generator 2 becomes binding. Higher levels of forecast uncertainty lead to larger marginal price because it necessitates dispatching more power from a more expensive unit to hedge against the same tolerable probability of unit 2 violating its maximum limit.

B. Revenues, Costs, and Profits

We now turn our attention to the comparison of costs and profits for generators. These are determined by employing the marginal price resulting from the proposed dynamics-aware CCED under various levels of uncertainty. The same simulation setup outlined in Section IV-A is used for this analysis. The total revenue is calculated as $\sum_{t\in\mathcal{T}_{t_0}^{\mathrm{D}}} \lambda_t'^\star \mathbb{1}_G^{\mathrm{T}} P_t^{\mathrm{m}\star} \Delta t^{\mathrm{D}}$. The revenue in this context is a random variable evaluated as a linear function of $P_t^{\mathrm{m}\star} \sim \mathcal{N}(\overline{P}_t^{\mathrm{m}\star}, \Sigma_t^P)$. This leads to the probability distribution of the total revenue to be expressed as the following sum of Gaussian distributions:

$$\sum_{t\in\mathcal{T}_{t_0}^{\mathrm{D}}} \lambda_t'^\star \mathbb{1}_G^{\mathrm{T}} P_t^{\mathrm{m}\star} \Delta t^{\mathrm{D}} \sim \sum_{t\in\mathcal{T}_{t_0}^{\mathrm{D}}} \mathcal{N}(\lambda_t'^\star \mathbb{1}_G^{\mathrm{T}} \overline{P}_t^{\mathrm{m}\star} \Delta t^{\mathrm{D}}, (\lambda_t'^\star)^2 \mathbb{1}_G^{\mathrm{T}} \Sigma_t^P \mathbb{1}_G (\Delta t^{\mathrm{D}})^2). \quad (19)$$

Moreover, since we consider quadratic generator cost functions, the total generation cost is the sum of the squares of

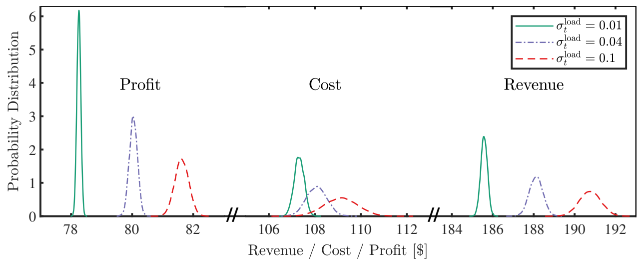

Fig. 2. Probability distributions of total revenues, costs, and profits of generators resulting from dynamics-aware pricing for different levels of load forecast uncertainty.

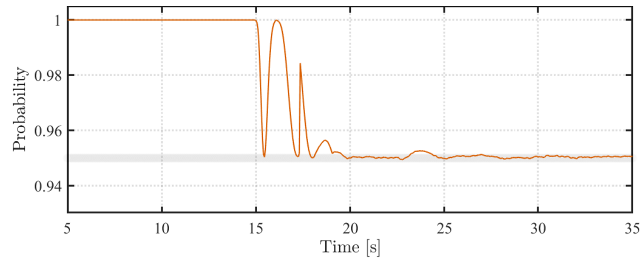

Fig. 3. The empirical probability with which the maximum power limit for generator 2 is violated obtained from 500,000 repeated dynamic simulations using net-load forecast sampled from $\mathcal{N}(\overline{P}_t^{\mathrm{load}}, (0.1)^2)$ with P_t^{r} extracted from the optimal solution of the dynamics-aware ED in (16).

Gaussian random variables, and it can be represented as the sum of generalized chi-squared distributions given by

$$\sum_{t \in \mathcal{T}_{t_0}^{\mathrm{D}}} C(P_t^{\mathrm{m}\star})\Delta t^{\mathrm{D}} \sim \sum_{t \in \mathcal{T}_{t_0}^{\mathrm{D}}} \widetilde{\chi}^2(w_t, k_t, h_t, m_t, s_t), \tag{20}$$

where the parameters w_t, k_t, h_t, m_t, and s_t can be obtained in closed form [18]. Then total profit is obtained by subtracting the cost from the revenue. It is worth noting that $\overline{P}_t^{\mathrm{m}\star}$ and $\lambda_t'^{\star}$ needed to obtain the probability distributions of total revenue and cost are readily available from the optimal solution of the CCED in (5). In Fig. 2, for each load forecast uncertainty scenario considered in Section IV-A, we plot the probability distributions of total revenue, cost, and profit. Visual examination of Fig. 2 reveals that greater load forecast uncertainty yields higher revenues and profits for generators due to the elevated marginal price to hedge against the greater risk of violating limits, as depicted in Fig. 1. Also, greater load forecast uncertainty leads to correspondingly wider distributions in revenues and costs, and consequently profits.

C. Dynamic Performance

Consider net-load forecast $P_t^{\mathrm{load}} = \overline{P}_t^{\mathrm{load}} + \widetilde{P}_t^{\mathrm{load}}$, where $\overline{P}_t^{\mathrm{load}}$ is predicted to increase by 15% at $t = 15\,[\mathrm{sec}]$ as above and $\widetilde{P}_t^{\mathrm{load}} \sim \mathcal{N}(0, (0.1)^2)$, $t \in \mathcal{T}_{t_0}^{\mathrm{D}}$. We sample this load probability distribution and simulate the system frequency dynamics 500,000 times using the model consisting of (3)–(4) with P_t^{r} extracted from the optimal solution of the dynamics-aware ED in (16). In Fig. 3, we plot the probability with which the maximum power limit for generator 2, $P_{\max}^{\mathrm{m}} = 1.9\,[\mathrm{p.u.}]$, is violated. We find that the probability remains above 0.95, consistent with the tolerance set in the chance constraint (5e).

V. Concluding Remarks

In this paper, we presented a dynamics-aware marginal pricing scheme that incorporates constraints pertaining to system frequency dynamics and Gaussian models for the net-load forecast uncertainty. Chance constraints delineate the tolerable probability of violating lower and upper limits of dynamic generator outputs and system frequency. The dynamics-aware marginal price thus embeds the impact of frequency dynamics under uncertainty in the net-load forecast. Numerical results confirm the benefits of the proposed marginal price in providing broader revenue opportunities for generators contributing to system performance under uncertainty. Future work will address non-Gaussian correlated random variables from RESs.

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