Measurement-based Locational Marginal Pricing in Active Distribution Systems

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***Abstract*—This paper proposes a measurement-based method for calculating real-time distribution locational marginal prices (DLMPs) without the use of an offline network model. Instead, the proposed method relies *only* on online measurements collected at a subset of distribution system buses to estimate a linear sensitivity model mapping bus voltages to injections, which in turn is embedded in an optimal power flow (OPF) problem as an equality constraint. The proposed method completely obviates the need for an accurate distribution network model that may not be available, especially for active distribution networks with faster variations in operating point. Also, the proposed method renders the original OPF problem with nonlinear constraints a computationally efficient quadratic programming problem (with linear constraints) and provides sufficiently accurate DLMPs at buses where measurements are collected. Via numerical simulations involving a 33-bus test system, we demonstrate that the proposed method yields similar DLMPs as solving the OPF problem with an up-to-date model and greatly outperforms it when the model is out of date.**

***Index Terms*—Distribution system, electricity market, locational marginal pricing, measurements, synchrophasors**

I. Introduction

The proliferation of distributed energy resources (DERs) comprising distributed generation, energy storage, and flexible loads, is driving power distribution networks from being a collection of predominantly passive components to actively coordinating resources therein [1]. Imperative to this paradigm shift is to establish real-time distribution-level electricity trading practices to incentivize fast-responding DERs in providing grid support and to compensate them through a fair pricing scheme instead of the prevailing payments based on fixed or time-of-use rates [2]. Real-time markets (in the range of minutes to hours) are particularly pertinent in distribution networks as accurate longer-term forecasts of distributed generation and individual nodal loads needed for, e.g., day-ahead markets, are more difficult to obtain [3].

Inspired by wholesale electricity markets for the transmission level, the use of distribution locational marginal prices (DLMPs) represents a promising solution to establish distribution-level real-time markets [4]. However, the task of calculating DLMPs is patently more challenging than their transmission-level counterparts. Computational burden for the transmission-level problem is typically contained by leveraging the so-called DC power flow approximation [5], but the assumptions for the approximation do not hold in distribution networks [6]. Instead, an optimal power flow (OPF) problem constrained by the nonlinear power flow equations and various operational limits related to DER capacities, line flows, and voltage magnitudes needs to be solved repeatedly. This approach may be computationally burdensome for large distribution systems, calling to question its practical viability in real-time pricing applications, especially considering the expected growth in electricity demand and DER integration [7]. Moreover, while offline models for transmission systems may conceivably be accessible, accurate distribution network models that reflect the up-to-date operating point are often not available [8]. The use of an inaccurate or outdated offline model may lead to erroneous DLMPs that do not support the market equilibrium. Challenges in this regard are expected to amplify in the transition from the traditional passive distribution networks to active counterparts, leading to larger, faster, and more frequent variations in operating point [9].

In this paper, we extract DLMPs as the Lagrange multipliers associated with a linear sensitivity-based equality constraint, replacing the nonlinear power flow equations, in an OPF problem. The linear equality constraint approximates the relationship between nodal voltages and power injections. Instead of relying on an offline model, we estimate the linear voltage-to-power sensitivity model using *only* measurements of nodal voltages and power injections. We assume these measurements are available from synchrophasor technologies, e.g., distribution-level phasor measurement units (D-PMUs), equipped at buses with market participants in the distribution system. The measurement and communication capabilities of D-PMUs ensure that the estimated sensitivity model can be updated periodically to reflect the up-to-date system operating point [10]. As a direct consequence, the calculation of DLMPs does not rely on any prior offline knowledge related to a network model, and the resulting DLMPs adapt to the system's evolving operating point and even changes in the network topology. Moreover, compared to solving an optimization problem constrained by the nonlinear power flow equations, the proposed approach incurs lower computational burden to calculate sufficiently accurate DLMPs.

Prior work to address computational challenges associated with calculating DLMPs includes efforts to formulate the pertinent pricing problem by assuming a radial topology and employing a relaxed convex branch-flow model [11]–[15]. However, the radial topology may be overly restrictive for future distribution networks with interconnected microgrids. Another general approach that is applicable for any network configuration embeds linearized power flow constraints instead of the nonlinear ones in the OPF problem [16]–[18]. Among these, [16] simply uses DC power flow equations while [17], [18] obtain line flows through a lossless model and approximate losses are lumped as additional loads on either end of the line afterward. While the methods in [11]–[18] tend to alleviate the computational burden of calculating DLMPs, they all depend on offline knowledge of (at least) the network topology, an up-to-date and accurate version of which may not be available in practice. At the same time, the advancement of online measurement technologies motivates the development of data-driven approaches for power system optimization applications [19], [20]. However, these data-driven approaches generally focus on derivation of power system security rules while still requiring an offline network model. By contrast, [21] develops a measurement-based security-constrained economic dispatch without relying on an offline model, but it is tailored for the transmission system and possible application to DLMP calculation is mentioned only in passing.

II. Preliminaries

In this section, we describe the distribution system model. We also formulate the full-blown OPF problem subject to non-linear power flow constraints, the solution of which includes standard model-based DLMPs.

A. Network and Power Flow Models

Consider a distribution system with N buses collected in the set $\mathcal{N} = \{1, \ldots, N\}$, G of which are connected to dispatchable DERs collected in the set $\mathcal{G} \subseteq \mathcal{N}$. Without loss of generality, assume bus 1 is the substation representing the slack bus. The substation bus voltage is fixed and known, and its power injection into the distribution feeder is modelled as the output of a dispatchable DER and included in $\mathcal{G}$. Further define the set $\mathcal{N}^- = \mathcal{N} \setminus \{1\}$. Let V_i and θ_i denote, respectively, the voltage magnitude and phase angle at bus $i \in \mathcal{N}^-$; and let $P_i^{\rm d}$ and $Q_i^{\rm d}$ denote, respectively, the active- and reactive-power demand arising from the aggregate non-dispatchable load at bus $i \in \mathcal{N}^-$. Also let $P_g^{\rm g}$ and $Q_g^{\rm g}$ denote, respectively, the active- and reactive-power generation arising from the aggregate dispatchable DER at bus $g \in \mathcal{G}$. Furthermore, collect voltage phase angles and magnitudes for all buses except the substation in vectors $\theta = [(\theta_i)_{i\in\mathcal{N}^-}]^{\rm T}$ and $V = [(V_i)_{i\in\mathcal{N}^-}]^{\rm T}$, respectively. Also collect active- and reactive-power demand (generation) arising from non-dispatchable loads (dispatchable DERs) in vectors $P^{\rm d} = [(P_i^{\rm d})_{i\in\mathcal{N}}]^{\rm T}$ ($P^{\rm g} = [(P_g^{\rm g})_{g\in\mathcal{G}}]^{\rm T}$) and $Q^{\rm d} = [(Q_i^{\rm d})_{i\in\mathcal{N}}]^{\rm T}$ ($Q^{\rm g} = [(Q_g^{\rm g})_{g\in\mathcal{G}}]^{\rm T}$), respectively. Then, power flow equations can be compactly expressed as

$$KP^{\rm g} - P^{\rm d} = f^P(\theta, V), \tag{1}$$
$$KQ^{\rm g} - Q^{\rm d} = f^Q(\theta, V), \tag{2}$$

where $K \in \mathbb{R}^{N\times G}$ is a matrix of 1s and 0s that maps entries related to buses with DERs in $\mathcal{G}$ to corresponding bus indices in $\mathcal{N}$, and $f^P : \mathbb{R}^{2N-2} \to \mathbb{R}^N$ and $f^Q : \mathbb{R}^{2N-2} \to \mathbb{R}^N$. The power injection from substation bus is modelled as a virtual DER, where its active- and reactive-power injections into the distribution feeder appear as the first entries in $P^{\rm g}$ and $Q^{\rm g}$, respectively. In (1)–(2), the dependence on network parameters (such as circuit breaker status and line impedances) is implicitly considered in the functions $f^P(\cdot)$ and $f^Q(\cdot)$.

B. Model-based Calculation of DLMPs

The DLMPs can be obtained alongside the solution of an OPF problem formulated as follows:

$$\underset{\theta,V,P^{\rm g},Q^{\rm g}}{\text{minimize}} \quad C(P^{\rm g}, Q^{\rm g}), \tag{3a}$$
$$\text{subject to} \quad KP^{\rm g} - P^{\rm d} = f^P(\theta, V),\ (\lambda^P), \tag{3b}$$
$$KQ^{\rm g} - Q^{\rm d} = f^Q(\theta, V),\ (\lambda^Q), \tag{3c}$$
$$\underline{V} \le V \le \overline{V},\ (\nu^-, \nu^+), \tag{3d}$$
$$\underline{P}^{\rm g} \le P^{\rm g} \le \overline{P}^{\rm g},\ (\phi^-, \phi^+), \tag{3e}$$
$$\underline{Q}^{\rm g} \le Q^{\rm g} \le \overline{Q}^{\rm g},\ (\rho^-, \rho^+), \tag{3f}$$

where the operation cost of the distribution system, $C(P^{\rm g}, Q^{\rm g})$, is minimized in the objective function (3a) subject to the operational constraints (3b)–(3f), and the Lagrange multipliers associated with each constraint are included in parentheses thereafter. The nodal active- and reactive-power balance constraints are respectively included as (3b) and (3c). Also, nodal voltage magnitudes along with the active- and reactive-power output of DERs are confined to their minimum/maximum limits through (3d)–(3f). For simplicity, we neglect line-flow limits by assuming ample capacity for the operating points considered. We instead focus on the impact of nodal location and voltage-magnitude limits on the DLMPs.

The optimality conditions for the OPF problem in (3) are established through Karush-Kuhn-Tucker (KKT) conditions. Below, we first formulate the Lagrangian of the problem in (3), from which KKT conditions are then derived. The Lagrangian of (3) is

$$\begin{aligned}\mathcal{L} &= C(P^{\rm g}, Q^{\rm g}) + (\lambda^P)^{\rm T}(f^P(\theta, V) - KP^{\rm g} + P^{\rm d}) \\ &+ (\lambda^Q)^{\rm T}(f^Q(\theta, V) - KQ^{\rm g} + Q^{\rm d}) \\ &+ (\nu^-)^{\rm T}(\underline{V} - V) + (\nu^+)^{\rm T}(V - \overline{V}) \\ &+ (\phi^-)^{\rm T}(\underline{P}^{\rm g} - P^{\rm g}) + (\phi^+)^{\rm T}(P^{\rm g} - \overline{P}^{\rm g}) \\ &+ (\rho^-)^{\rm T}(\underline{Q}^{\rm g} - Q^{\rm g}) + (\rho^+)^{\rm T}(Q^{\rm g} - \overline{Q}^{\rm g}).\end{aligned} \tag{4}$$

Denote the optimal Lagrangian as $\mathcal{L}^\star$ and distinguish the optimal values taken by decision variables and Lagrange multipliers of (15) with superscript $\star$. Then the KKT conditions include stationarity conditions, given by the following:

$$\frac{\partial \mathcal{L}^\star}{\partial \theta^\star} = (\lambda^{P\star})^{\rm T}\frac{\partial f^P(\theta^\star, V^\star)}{\partial \theta^\star} + (\lambda^{Q\star})^{\rm T}\frac{\partial f^Q(\theta^\star, V^\star)}{\partial \theta^\star} = 0, \tag{5}$$
$$\begin{aligned}\frac{\partial \mathcal{L}^\star}{\partial V^\star} &= (\lambda^{P\star})^{\rm T}\frac{\partial f^P(\theta^\star, V^\star)}{\partial V^\star} + (\lambda^{Q\star})^{\rm T}\frac{\partial f^Q(\theta^\star, V^\star)}{\partial V^\star} \\ &+ \nu^{+\star} - \nu^{-\star} = 0,\end{aligned} \tag{6}$$
$$\frac{\partial \mathcal{L}^\star}{\partial P^{\rm g\star}} = \frac{\partial C(P^{\rm g\star}, Q^{\rm g\star})}{\partial P^{\rm g\star}} - (\lambda^{P\star})^{\rm T}K + \phi^{+\star} - \phi^{-\star} = 0, \tag{7}$$
$$\frac{\partial \mathcal{L}^\star}{\partial Q^{\rm g\star}} = \frac{\partial C(P^{\rm g\star}, Q^{\rm g\star})}{\partial Q^{\rm g\star}} - (\lambda^{Q\star})^{\rm T}K + \rho^{+\star} - \rho^{-\star} = 0, \tag{8}$$

as well as the complementary slackness conditions, as follows:

$$\nu_i^{-\star}(\underline{V}_i - V_i^\star) = 0,\ \nu_i^{+\star}(V_i^\star - \overline{V}_i) = 0,\ i \in \mathcal{N}, \tag{9}$$
$$\phi_g^{-\star}(\underline{P}_g^{\rm g} - P_g^{\rm g\star}) = 0,\ \phi_g^{+\star}(P_g^{\rm g\star} - \overline{P}_g^{\rm g}) = 0,\ g \in \mathcal{G}, \tag{10}$$
$$\rho_g^{-\star}(\underline{Q}_g^{\rm g} - Q_g^{\rm g\star}) = 0,\ \rho_g^{+\star}(Q_g^{\rm g\star} - \overline{Q}_g^{\rm g}) = 0,\ g \in \mathcal{G}, \tag{11}$$
$$\nu_i^{-\star}, \nu_i^{+\star}, \phi_g^{-\star}, \phi_g^{+\star}, \rho_g^{-\star}, \rho_g^{+\star} \ge 0,\ i \in \mathcal{N},\ g \in \mathcal{G}. \tag{12}$$

Analogous to transmission locational marginal prices, the DLMPs represent the rate of change of distribution system operation cost due to incremental changes in demand at different buses in the system [22]. Mathematically, the active-power (reactive-power) DLMP at a particular bus is the first derivative of optimal Lagrangian (4) with respect to the active-power (reactive-power) load at that bus [23]. Using the chain rule in calculus and the optimality conditions in (5)–(12), it is straightforward to show that the i-th entries of $\lambda^{P\star}$ and $\lambda^{Q\star}$ indeed respectively represent the locational marginal prices attributed to active- and reactive-power at distribution bus i.

C. Problem Statement

Since the inclusion of the exact nonlinear power flow model in (1)–(2) leads to a nonconvex optimization problem in (3), the model-based DLMP calculation outlined above may incur considerable computational burden. Although solving the OPF problem formulated with relaxed or linearized power flow constraints (see, e.g., [11]–[18]) helps to address the computational challenges, accurate DLMPs still rely on an offline system model that comprises the up-to-date network topology, device parameters, and operating point, which is often not available in practice. Thus, aimed at practical calculation of DLMPs for real-time markets with respect to both computational burden and reliance on offline models, we replace the nonlinear power flow constraints in (3) by a linear sensitivity model, which is estimated from only online measurements, obviating the need for any knowledge of the underlying physical network that gave rise to the measurements.

III. Measurement-based Calculation of DLMPs

In this section, we describe the estimation of the linear sensitivity model, which is then embedded in an OPF problem. The solution of this problem comprises the optimal DER setpoints as well as measurement-based DLMPs.

A. Estimated Linear Power Flow Model

Let $\mathcal{E} \subseteq \mathcal{N}$ represent the set of E buses equipped with D-PMUs, and assume that the substation bus is measured. Also, in order to recover DLMPs at buses with dispatchable DERs, i.e., market participants, they must be equipped with D-PMUs, so we assume that $\mathcal{G} \subseteq \mathcal{E}$. Further define the set $\mathcal{E}^- = \mathcal{E} \setminus \{1\}$. Collect voltage phase angles and magnitudes of measured buses (except those of the substation where the voltage is assumed to be fixed and known) in vectors $\theta_\mathcal{E} = [(\theta_i)_{i\in\mathcal{E}^-}]^\mathrm{T}$ and $V_\mathcal{E} = [(V_i)_{i\in\mathcal{E}^-}]^\mathrm{T}$, respectively. Also collect active- and reactive-power demand at measured buses in vectors $P^\mathrm{d}_\mathcal{E} = [(P^\mathrm{d}_i)_{i\in\mathcal{E}}]$ and $Q^\mathrm{d}_\mathcal{E} = [(Q^\mathrm{d}_i)_{i\in\mathcal{E}}]$, respectively. Net active- and reactive-power injections are then respectively $P_\mathcal{E} = MP^\mathrm{g} - P^\mathrm{d}_\mathcal{E}$ and $Q_\mathcal{E} = MQ^\mathrm{g} - Q^\mathrm{d}_\mathcal{E}$, where $M \in \mathbb{R}^{E\times G}$ is a matrix of 0s and 1s that maps indices in $\mathcal{G}$ to corresponding ones in $\mathcal{E}$. Measured values of the variables defined above are distinguished by $\widehat{\cdot}$ placed over the corresponding variables.

Suppose pertinent system variables are sampled at time $t = k\Delta t$, $k = 0, 1, \ldots$, where Δt is the sampling interval. Then we collect measured bus voltage phase angles and magnitudes at time step k in $\widehat{x}_{[k]} = [\widehat{\theta}^\mathrm{T}_{\mathcal{E},[k]}, \widehat{V}^\mathrm{T}_{\mathcal{E},[k]}]^\mathrm{T}$. Further collect the measured net bus power injections (include those of the substation) at time step k in $\widehat{y}_{[k]} = [\widehat{P}^\mathrm{T}_{\mathcal{E},[k]}, \widehat{Q}^\mathrm{T}_{\mathcal{E},[k]}]^\mathrm{T}$. We hypothesize a linear relationship between the measured injections and voltages given by

$$\widehat{y}_{[k]} = J_{[k]}\widehat{x}_{[k]} + c_{[k]} = [\widehat{x}^\mathrm{T}_{[k]}\ 1]H_{[k]}, \tag{13}$$

where $H_{[k]} = [J_{[k]}, c_{[k]}]^\mathrm{T}$. By collecting a minimum of $2E + 1$ most recent samples and stacking the corresponding instances of (13) while assuming $H_{[k]}$ remains constant across these samples, we can conceivably deploy the ordinary least squares (OLS) algorithm to obtain an estimate of $H_{[k]}$. However, due to correlation amongst voltages of various buses, the OLS algorithm tends to lead to ill-conditioned regressor matrices. In our recent work [24], [25], we have found the partial least squares (PLS) algorithm to be effective to combat the collinearity problem and provide meaningful estimates of $H_{[k]}$. We refer interested readers to [24] for further details on the estimation algorithm and its performance. Here, it suffices to assume that, at time step k, an updated estimate $\widehat{H}_{[k]} = [\widehat{J}_{[k]}, \widehat{c}_{[k]}]^\mathrm{T}$ is computed using recently obtained measurements via the PLS algorithm. Also, we will find it useful to decompose $\widehat{J}_{[k]}$ and $\widehat{c}_{[k]}$ as follows:

$$\widehat{J}_{[k]} = \begin{bmatrix} \widehat{J}^{P\theta}_{[k]} & \widehat{J}^{PV}_{[k]} \\ \widehat{J}^{Q\theta}_{[k]} & \widehat{J}^{QV}_{[k]} \end{bmatrix}, \quad \widehat{c}_{[k]} = \begin{bmatrix} \widehat{c}^{P}_{[k]} \\ \widehat{c}^{Q}_{[k]} \end{bmatrix}. \tag{14}$$

B. Measurement-based Optimal Power Flow Problem

We modify the OPF problem in (3) by replacing the nonlinear power flow constraints in (3b)–(3c) by a linear sensitivity model estimated from measurements of nodal voltages and injections. The resulting measurement-based OPF problem is then formulated as follows:

$$\underset{\theta_\mathcal{E}, V_\mathcal{E}, P^\mathrm{g}, Q^\mathrm{g}}{\text{minimize}}\ C(P^\mathrm{g}, Q^\mathrm{g}), \tag{15a}$$

$$\text{subject to } MP^\mathrm{g} - P^\mathrm{d}_\mathcal{E} = \widehat{J}^{P\theta}_{[k]}\theta_\mathcal{E} + \widehat{J}^{PV}_{[k]}V_\mathcal{E} + \widehat{c}^{P}_{[k]}, (\lambda^P_\mathcal{E}), \tag{15b}$$

$$MQ^\mathrm{g} - Q^\mathrm{d}_\mathcal{E} = \widehat{J}^{Q\theta}_{[k]}\theta_\mathcal{E} + \widehat{J}^{QV}_{[k]}V_\mathcal{E} + \widehat{c}^{Q}_{[k]}, (\lambda^Q_\mathcal{E}), \tag{15c}$$

$$\underline{V}_\mathcal{E} \le V_\mathcal{E} \le \overline{V}_\mathcal{E},\ (\nu^-_\mathcal{E}, \nu^+_\mathcal{E}), \tag{15d}$$

$$\underline{P}^\mathrm{g} \le P^\mathrm{g} \le \overline{P}^\mathrm{g},\ (\phi'^-, \phi'^+), \tag{15e}$$

$$\underline{Q}^\mathrm{g} \le Q^\mathrm{g} \le \overline{Q}^\mathrm{g},\ (\rho'^-, \rho'^+), \tag{15f}$$

where the operation cost of the distribution system, $C(P^\mathrm{g}, Q^\mathrm{g})$, is minimized in the objective function (15a) subject to the operational constraints in (15b)–(15f). In (15), voltage phase angles and magnitudes along with DER active- and reactive-power setpoints of only buses with measurements are decision variables, i.e., the problem does not optimize over buses without measurements in $\mathcal{N} \setminus \mathcal{E}$. As a consequence, the solution of (15) recovers DLMPs for only the monitored buses.

Similar to the procedure outlined in Section II-B, we can derive the optimality conditions for the problem in (15) by first formulating its Lagrangian as follows:

$$\begin{aligned} \mathcal{L} &= C(P^\mathrm{g}, Q^\mathrm{g}) \\ &+ (\lambda^P_\mathcal{E})^\mathrm{T}(\widehat{J}^{P\theta}_{[k]}\theta_\mathcal{E} + \widehat{J}^{PV}_{[k]}V_\mathcal{E} + \widehat{c}^{P}_{[k]} - MP^\mathrm{g} + P^\mathrm{d}_\mathcal{E}) \\ &+ (\lambda^Q_\mathcal{E})^\mathrm{T}(\widehat{J}^{Q\theta}_{[k]}\theta_\mathcal{E} + \widehat{J}^{QV}_{[k]}V_\mathcal{E} + \widehat{c}^{Q}_{[k]} - MQ^\mathrm{g} + Q^\mathrm{d}_\mathcal{E}) \end{aligned}$$

$$+ (\nu_{\mathcal{E}}^{-})^{\mathrm{T}}(\underline{V}_{\mathcal{E}} - V_{\mathcal{E}}) + (\nu_{\mathcal{E}}^{+})^{\mathrm{T}}(V_{\mathcal{E}} - \overline{V}_{\mathcal{E}})$$
$$+ (\phi'^{-})^{\mathrm{T}}(\underline{P}^{\mathrm{g}} - P^{\mathrm{g}}) + (\phi'^{+})^{\mathrm{T}}(P^{\mathrm{g}} - \overline{P}^{\mathrm{g}})$$
$$+ (\rho'^{-})^{\mathrm{T}}(\underline{Q}^{\mathrm{g}} - Q^{\mathrm{g}}) + (\rho'^{+})^{\mathrm{T}}(Q^{\mathrm{g}} - \overline{Q}^{\mathrm{g}}). \quad (16)$$

The stationarity conditions for voltage phase angles and magnitudes as well as DER active- and reactive-power outputs are given by, respectively,

$$\frac{\partial \mathcal{L}^{\star}}{\partial \theta_{\mathcal{E}}^{\star}} = (\lambda_{\mathcal{E}}^{P\star})^{\mathrm{T}} \widehat{J}_{[k]}^{P\theta} + (\lambda_{\mathcal{E}}^{Q\star})^{\mathrm{T}} \widehat{J}_{[k]}^{Q\theta} = 0, \quad (17)$$

$$\frac{\partial \mathcal{L}^{\star}}{\partial V_{\mathcal{E}}^{\star}} = (\lambda_{\mathcal{E}}^{P\star})^{\mathrm{T}} \widehat{J}_{[k]}^{PV} + (\lambda_{\mathcal{E}}^{Q\star})^{\mathrm{T}} \widehat{J}_{[k]}^{QV} + \nu_{\mathcal{E}}^{+\star} - \nu_{\mathcal{E}}^{-\star} = 0, \quad (18)$$

$$\frac{\partial \mathcal{L}^{\star}}{\partial P^{\mathrm{g}\star}} = \frac{\partial C(P^{\mathrm{g}\star}, Q^{\mathrm{g}\star})}{\partial P^{\mathrm{g}\star}} - (\lambda_{\mathcal{E}}^{P\star})^{\mathrm{T}} M + \phi'^{+\star} - \phi'^{-\star} = 0, \quad (19)$$

$$\frac{\partial \mathcal{L}^{\star}}{\partial Q^{\mathrm{g}\star}} = \frac{\partial C(P^{\mathrm{g}\star}, Q^{\mathrm{g}\star})}{\partial Q^{\mathrm{g}\star}} - (\lambda_{\mathcal{E}}^{Q\star})^{\mathrm{T}} M + \rho'^{+\star} - \rho'^{-\star} = 0. \quad (20)$$

Complementary slackness conditions are formulated analogously to those in (9)–(12), but for only measured buses.

Remark 1 (Connection to Model-based DLMPs)**.** Suppose measurements are available at all buses so that $\mathcal{E} = \mathcal{N}$. Then it is straightforward to deduce that the optimality conditions of the measurement-based problem in (15) coincide precisely with those of the model-based problem in (3) if and only if

$$\widehat{J}_{[k]}^{P\theta} = \frac{\partial f^{P}(\theta^{\star}, V^{\star})}{\partial \theta^{\star}}, \quad \widehat{J}_{[k]}^{Q\theta} = \frac{\partial f^{Q}(\theta^{\star}, V^{\star})}{\partial \theta^{\star}}, \quad (21)$$

$$\widehat{J}_{[k]}^{PV} = \frac{\partial f^{P}(\theta^{\star}, V^{\star})}{\partial V^{\star}}, \quad \widehat{J}_{[k]}^{QV} = \frac{\partial f^{Q}(\theta^{\star}, V^{\star})}{\partial V^{\star}}. \quad (22)$$

In other words, the optimality conditions of the two problems are equivalent if the estimated sensitivity matrices exactly equal the first-order derivatives of the nonlinear power flow equations with respect to voltage phase angles and magnitudes. Practically speaking, sufficiently accurate DLMPs can be estimated even with relatively sparse measurement coverage, as we will show next in Section IV. ■

IV. Case Studies

In this section, we demonstrate the effectiveness of the measurement-based method described in Section III to calculate DLMPs. Numerical simulations are performed with a 33-bus test system (see, e.g., [26]), as shown in Fig. 1, where DERs are connected to buses in the set $\mathcal{G} = \{1, 6, 7, 12, 18, 22, 25, 33\}$. Note that, by definition, $\mathcal{G}$ includes the substation bus modelled as a virtual DER. The operation cost function is described in Appendix A. The active-power (reactive-power) injection of the substation bus is constrained within $P_1^{\mathrm{g}} \in [-1, 1]$ p.u. ($Q_1^{\mathrm{g}} \in [-1, 1]$ p.u.) while the active-power (reactive-power) output of the g-th DER is limited to $P_g^{\mathrm{g}} \in [0, 0.25]$ p.u. ($Q_g^{\mathrm{g}} = 0$ p.u.). Assume that voltage phasors and active- and reactive-power injections at buses in $\mathcal{E}$ are sampled at 1-second intervals. The measurement-based OPF problem in (15) is solved every minute where the sensitivity model is estimated via the PLS algorithm with the previous 60 measurement sets. We use the MATLAB Interior Point solver for quadratic programming to obtain the DLMPs from the measurement-based OPF problem in (15). The simulations are conducted in MATLAB R2020b on a personal computer with

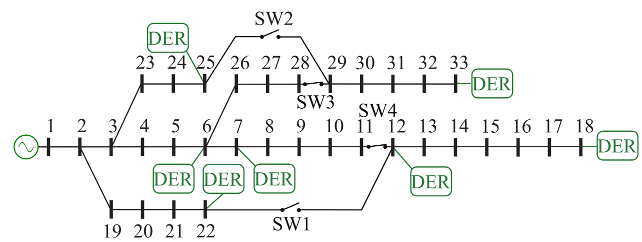

Fig. 1. One-line diagram for 33-bus test system with DERs at buses $6, 7, 12, 18, 22, 25$, and 33 and a virtual DER at bus 1. Switches SW1 and SW2 are normally open, and switches SW3 and SW4 are normally closed.

Intel Core i7-10610U processor at 1.80 GHz and 16 GB RAM. Estimating the sensitivity model and solving the measurement-based OPF problem respectively take 0.048 and 0.035 seconds (on average) for case studies presented in this section. Readers may refer to [24], [25] for execution times in larger test systems. For comparison, we solve the model-based OPF problem (with nonlinear power flow constraints) in (3) via the MATPOWER Interior Point Solver [27].

A. Benchmark Scenario

We benchmark the measurement-based DLMPs with the ones obtained from the model-based problem in (3). Here, we make the admittedly artificial assumption that the linear sensitivity model is estimated at the optimal operating point. In Fig. 2, we plot DLMPs resulting from the OPF problem obtained via (i) model-based approach assuming that the topology and operating-point are accurately captured by the model, (ii) measurement-based approach with 100% measurement coverage, and (iii) measurement-based approach with 25% measurement coverage (measuring only buses connected to DERs). The DLMPs obtained from the proposed measurement-based method and the model-based approach indeed match, validating comments in Remark 1.

B. Adaptability to Topology Changes with Load Forecast

We reconfigure the test system topology by closing switches SW1 and SW2 and opening switches SW3 and SW4 simultaneously. We collect 60 seconds worth of measurements after the reconfiguration to estimate an updated linear sensitivity model. We then ascribe a 2% increase at all buses as the load forecast for the next 1-minute interval. In Fig. 3, we plot DLMPs resulting from the OPF problem obtained via (i) model-based approach with an up-to-date model capturing the topology reconfiguration, (ii) model-based approach assuming that model is out of date and still reflects the network topology before reconfiguration, (iii) measurement-based approach with 100% measurement coverage, and (iv) measurement-based approach with 25% measurement coverage (measuring only buses connected to DERs). The DLMPs obtained from the proposed measurement-based method closely match the model-based DLMPs with up-to-date network topology for both measurement coverage levels (with maximum absolute percent error being less than 1%). In contrast, when the network reconfiguration is not captured in the model-based OPF problem, the resulting out-of-date DLMPs do not reflect the true market clearing conditions.

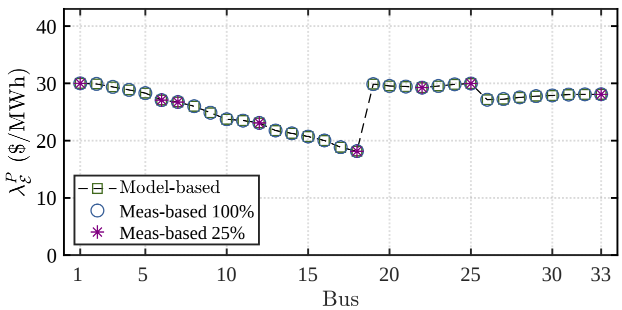

Fig. 2. Comparison of DLMPs obtained via (i) model-based method, (ii) measurement-based method with 100% measurement coverage, and (iii) measurement-based method with 25% measurement coverage.

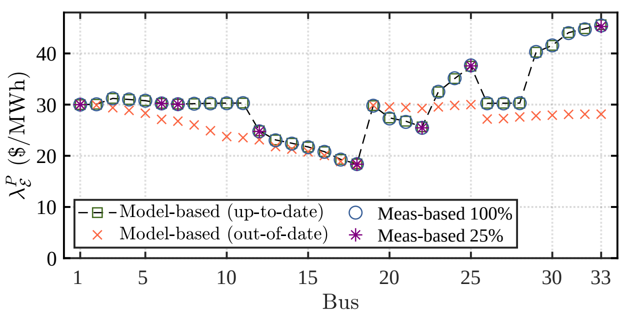

Fig. 3. Comparison of DLMPs obtained after system reconfiguration via model-based method with (i) up-to-date system model, (ii) out-of-date system model, and measurement-based method with (iii) 100% measurement coverage, (iv) 25% measurement coverage. DLMPs resulting from the proposed measurement-based method match the ones from model-based method with up-to-date system model.

V. Concluding Remarks

In this paper, we proposed a measurement-based method to calculate DLMPs where, in the pertinent OPF problem, the nonlinear power flow equations are replaced with a linear sensitivity model estimated from only online measurements of bus voltages and power injections collected at a subset of buses. The DLMPs attributed to active and reactive power are respectively obtained as the optimal Lagrange multipliers of the estimated active- and reactive-power balance constraints. Simulation results highlight the effectiveness of the proposed method to provide accurate DLMPs at the measured subset of distribution buses. Future work includes measurement-based pricing of combined energy and flexibility provided by DERs.

Appendix

A. Cost Function in (3) and (15)

In (3) and (15), we assume that the quadratic operation cost function takes the form $C(P^{\mathrm{g}}, Q^{\mathrm{g}}) = {P^{\mathrm{g}}}^{\mathrm{T}}\mathrm{diag}(a)P^{\mathrm{g}} + bP^{\mathrm{g}} + c$, where $a = [0, 4, 1, 2, 1.5, 2.5, 3.5, 4.5]$ [\$/(MWh)^2], $b = [30, 40, 10, 20, 15, 25, 35, 45]$ [\$/MWh], and $c = 0$ [\$].

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