A Measurement-based Gradient-descent Method to Optimally Dispatch DER Reactive Power

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***Abstract*—This paper proposes a measurement-based method to dispatch DER reactive-power output with the objective of minimizing distribution network loss and cost of DER reactive power. Central to the proposed method is the estimation of network loss and voltage sensitivities with respect to individual DER reactive-power output from synchronized power-injection data collected from distribution-level phasor measurement units. The estimated sensitivities are embedded within gradient-descent iterations that converge to DER reactive-power setpoints corresponding to the optimal operating point. The proposed method respects typical constraints on DER reactive-power outputs and bus voltage magnitudes. Numerical simulations involving the IEEE 33-bus distribution test system demonstrate that the proposed measurement-based method yields sufficiently accurate reactive-power setpoints compared to those obtained from a model-based benchmark solution to minimize network loss while regulating bus voltages.**

I. Introduction

The proliferation of distributed energy resources (DERs) like solar photovoltaic (PV) systems in the distribution network poses numerous challenges for grid operators. For example, solar PV generation is intermittent, variable, and uncertain, which may cause power quality issues related to nodal voltages, network loss, and line flows [1]. On the other hand, suitable control of DER reactive-power output can offer significant benefit to system operation with little added infrastructure. In this regard, a promising application for DER reactive-power control is to minimize network loss and regulate bus voltages (see, e.g., [2]). Optimal DER dispatch generally requires an accurate and up-to-date network model, but this may not be available to system operators in real time [3]. Also, standard model-based DER dispatch may lead to computationally challenging nonlinear optimization problems. In recent times, many measurement devices, such as smart meters and distribution-level phasor measurement units (D-PMUs), are being deployed in power distribution systems. This enables the development of measurement-based monitoring and control tools [4].

Existing measurement-based methods that optimize DER setpoints use estimated bus-voltage and injection sensitivities to improve the voltage profile across a distribution network, regulate the power exchange between the distribution and transmission systems, and minimize cost of generation [5]–[8]. These measurement-based methods rely on frequent re-optimization based on the estimated sensitivity models and recent measurements. Instead of frequently solving static optimization problems, another line of work uses so-called *feedback optimization* to design controllers that achieve distribution-level voltage regulation in centralized manner [9], [10] and distributed fashion [11]–[13]. Feedback optimization methods iteratively converge to optimal DER setpoints by taking gradient-descent steps of the objective function. Combined control of system voltages and substation power setpoints is achieved via primal-dual gradient methods to solve a saddle-point problem in a distributed fashion in [14]. The general framework is further extended to include aggregations of DERs, multi-phase systems, and discrete DER setpoints through a bilevel optimization formulation in [15]. The aforementioned feedback optimization methods all rely on some prior knowledge of the underlying power network structure to compute relevant gradients. Measurement-based loss minimization via reactive-power dispatch is developed using an extremum seeking approach in [16] that requires structured probing of DER injections with sinusoidal waveforms.

In this paper, we focus on the problem of optimally dispatching DER reactive-power outputs and propose a fully measurement-based approach that does not rely on any prior knowledge of the distribution-network topology or parameters. Our proposed method makes several contributions over the state of the art. First, we leverage synchronized measurements of nodal power injections obtained from D-PMUs to estimate linear sensitivities of the network loss and bus-voltage magnitudes with respect to reactive-power injections at DER buses. By repeatedly estimating the loss and voltage sensitivities, they readily adapt to operating-point and network-topology changes. Subsequently, we directly embed the estimated sensitivities into gradient-descent updates to enable DER reactive-power outputs to converge to setpoints that minimize an objective function consisting of network loss and DER costs. This obviates the need to formulate nonlinear functions of loss and voltage magnitudes in the optimization problem, as is typically the case in model-based methods (see, e.g., [2]). Furthermore, unlike the measurement- and sensitivity-based voltage control in [5], [6] and DER dispatch [7], [8], we achieve the optimal operating point with an iterative gradient-descent approach. As a direct consequence, computational burden incurred in updating DER setpoints is reduced. With respect to existing feedback optimization methods [9]–[15], our proposed method relies only on measurements, and numerical simulations demonstrate that DER reactive-power outputs converge to setpoints that correspond to the optimal operating point. Finally, compared to [16], the proposed method requires smaller injection perturbations for sensitivity estimation.

Via numerical simulations involving the IEEE 33-bus test system, we demonstrate that the proposed method effectively achieves the optimal operating point and yields accurate DER reactive-power setpoints compared to a model-based benchmark solution, and it quickly adapts to operating-point and network-topology changes. Furthermore, by embedding DER costs in the objective function and considering bus voltage constraints, the proposed measurement-based approach effectively solves a general formulation of the volt-var optimization problem without relying on any knowledge of system topology or parameters.

II. Preliminaries

In this section, we establish notation, describe the network model, and motivate the need for a measurement-based approach to dispatch DERs.

A. Network Model

Consider a distribution system with N buses collected in the set $\mathcal{N} = \{1, \dots, N\}$. Also, consider a subset of D buses $\mathcal{D} \subseteq \mathcal{N}$ connected to DERs. Suppose D-PMU measurements of pertinent system variables are sampled at time $t = k\Delta t$, $k = 0, 1, \dots,$ where Δt is the time interval between consecutive samples. Let $V_{i,[k]}$ denote the voltage magnitude at bus i and time step k. Let $P_{i,[k]}$ denote the net active-power injections at bus i and time step k and collect them in vector $P_{[k]} = [P_{1,[k]}, \dots, P_{N,[k]}]^{\mathrm{T}}$. Similarly, let $Q^{\mathrm{gen}}_{i,[k]}$ denote the reactive-power injection from the DER at bus $i \in \mathcal{D}$ at time step k and collect them in vector $Q^{\mathrm{gen}}_{[k]} = [\{Q^{\mathrm{gen}}_{i,[k]}\}_{i\in\mathcal{D}}]^{\mathrm{T}}$. Also, collect bus voltage magnitudes at buses with DERs installed at time step k in vector $V_{[k]} = [\{V_{i,[k]}\}_{i\in\mathcal{D}}]^{\mathrm{T}}$. Furthermore, compactly express the network loss $P^{\mathrm{loss}}_{[k]}$ at time step k as a function of DER reactive-power injections $Q^{\mathrm{gen}}_{[k]}$:

$$P^{\mathrm{loss}}_{[k]} = h_{[k]}(Q^{\mathrm{gen}}_{[k]}), \tag{1}$$

where $h_{[k]} : \mathbb{R}^D \mapsto \mathbb{R}$. In (1), the dependence on active- and reactive-power loads, DER active-power injections, as well as network topology and associated parameters (such as line impedances), at time step k, are implicitly considered in $h_{[k]}(\cdot)$. Given the notation established above, and assuming all bus active-power injections $P_{i,[k]}$ are measured, the total network loss at time step k can be computed by

$$P^{\mathrm{loss}}_{[k]} = \sum_{i\in\mathcal{N}} P_{i,[k]}. \tag{2}$$

B. Problem Formulation

Typically, DER reactive-power setpoints that optimize operation in the distribution system are obtained by solving a problem that relies on an offline network model with accurate topology and line parameters. This approach formulates $f(\cdot) : \mathbb{R}^D \mapsto \mathbb{R}$ as an objective function subject to network power-flow equations, bus voltage limits, and DER operational constraints in the following optimization problem:

$$\underset{Q^{\mathrm{gen}}_{[k]}\in\mathcal{Q}}{\text{minimize}} \quad f(Q^{\mathrm{gen}}_{[k]}), \tag{3a}$$

$$\text{subject to} \quad V_{[k]} = \sigma_{[k]}(Q^{\mathrm{gen}}_{[k]}), \tag{3b}$$

$$V_{\min} \leq V_{[k]} \leq V_{\max}, \tag{3c}$$

where $\sigma_{[k]} : \mathbb{R}^D \mapsto \mathbb{R}^D$ maps DER reactive-power outputs to bus voltage magnitudes through the power flow constraint (the dependence on bus active-power injections is implicitly considered in $\sigma_{[k]}$), $V_{\min}$ and $V_{\max}$ respectively denote minimum and maximum bus voltage limits, and $\mathcal{Q}$ denotes the feasible region of DER reactive-power outputs.

The nonlinear optimization problem (3) may lead to significant computational burden. Instead of solving (3) directly, we leverage a gradient-descent method, in which optimal DER setpoints are updated iteratively as follows:

$$Q^{\mathrm{gen}}_{[k+1]} = \mathrm{proj}_{\mathcal{Q}}(Q^{\mathrm{gen}}_{[k]} - \alpha_{[k]}(\nabla f(Q^{\mathrm{gen}}_{[k]}) + \Sigma^{\mathrm{T}}_{[k]} \nabla g(V_{[k]}))), \tag{4}$$

where $\mathrm{proj}_{\mathcal{Q}}(\cdot)$ denotes the projection onto $\mathcal{Q}$, $\Sigma_{[k]}$ is the first-order sensitivity matrix of $\sigma_{[k]}(Q^{\mathrm{gen}}_{[k]})$ with respect to the decision variable $Q^{\mathrm{gen}}_{[k]}$, $g : \mathbb{R}^D \mapsto \mathbb{R}^D$ is a convex and continuously differentiable penalty function to ensure constraint (3c) is satisfied, and $\alpha_{[k]}$ is the step size. We can evaluate $\Sigma_{[k]}$ by linearizing a suitable power flow model of the system or by estimating it using online measurements (as in our proposed framework).

The gradient-based approach in (4) reduces the computational burden in obtaining the optimal DER setpoints as compared to solving the offline optimization problem (3). In contrast to existing work on feedback optimization methods that rely on model-based gradient updates (see, e.g., [10]), we embed estimated sensitivities of network loss and bus voltage magnitudes with respect to DER reactive-power injections in updating the DER reactive-power setpoints. This enables the system to reach the optimal operating point without any prior knowledge of the system model.

III. Measurement-based DER Dispatch

In this section, we estimate sensitivities of the network loss and voltage magnitudes with respect to DER reactive-power outputs. We then incorporate the estimated sensitivities in a gradient-descent method to obtain optimal DER reactive-power setpoints while respecting DER reactive-power output limits and bus voltage constraints.

A. Estimation of Sensitivity Models

Suppose $h_{[k]}(\cdot)$ and $\sigma_{[k]}(\cdot)$ are continuously differentiable with respect to $Q^{\mathrm{gen}}_{[k]}$. We approximate network loss and bus voltage magnitudes at time step $k+1$, $P^{\mathrm{loss}}_{[k+1]}$ and $V_{[k+1]}$, with the following first-order Taylor series expansions:

$$P^{\mathrm{loss}}_{[k+1]} \approx h_{[k]}(Q^{\mathrm{gen}}_{[k]}) + \nabla h^{\mathrm{T}}_{[k]}(Q^{\mathrm{gen}}_{[k+1]} - Q^{\mathrm{gen}}_{[k]}), \tag{5}$$

$$V_{[k+1]} \approx V_{[k]} + \nabla \sigma^{\mathrm{T}}_{[k]}(Q^{\mathrm{gen}}_{[k+1]} - Q^{\mathrm{gen}}_{[k]}), \tag{6}$$

where $\nabla h_{[k]} \in \mathbb{R}^D$ and $\nabla \sigma_{[k]} \in \mathbb{R}^{D\times D}$ denotes the gradient of $h_{[k]}(\cdot)$ and $\sigma_{[k]}(\cdot)$ with respect to DER reactive-power injections evaluated at the operating point $Q^{\text{gen}}_{[k]}$, respectively. The linearized models in (5) and (6) can be rewritten as

$$P^{\text{loss}}_{[k+1]} \approx \kappa_{[k]} + \rho^{\text{T}}_{[k]} Q^{\text{gen}}_{[k+1]}, \tag{7}$$

$$V_{[k+1]} \approx \lambda_{[k]} + \Sigma^{\text{T}}_{[k]} Q^{\text{gen}}_{[k+1]}, \tag{8}$$

where $\rho_{[k]} = \nabla h_{[k]}$ and $\kappa_{[k]} = h_{[k]}(Q^{\text{gen}}_{[k]}) - \rho^{\text{T}}_{[k]} Q^{\text{gen}}_{[k]}$, and similarly $\Sigma_{[k]} = \nabla \sigma_{[k]}$ and $\lambda_{[k]} = \sigma_{[k]}(Q^{\text{gen}}_{[k]}) - \Sigma^{\text{T}}_{[k]} Q^{\text{gen}}_{[k]}$. We note that the linear model in (8) is similar to the LinDistFlow model widely used in the literature (its derivation in matrix form is provided in [17]). Furthermore, we equivalently express (7) and (8) as

$$P^{\text{loss}}_{[k+1]} \approx \begin{bmatrix} 1 & (Q^{\text{gen}}_{[k+1]})^{\text{T}} \end{bmatrix} \gamma_{[k]}, \tag{9}$$

$$V_{[k+1]} \approx \begin{bmatrix} 1 & (Q^{\text{gen}}_{[k+1]})^{\text{T}} \end{bmatrix} \Lambda_{[k]}, \tag{10}$$

with $\gamma_{[k]} = [\kappa_{[k]}, \rho^{\text{T}}_{[k]}]^{\text{T}}$, and $\Lambda_{[k]} = [\lambda^{\text{T}}_{[k]}, \Sigma^{\text{T}}_{[k]}]^{\text{T}}$. In order to remove the reliance on an accurate network model, we estimate the entries of $\gamma_{[k]}$ and $\Lambda_{[k]}$ using only online measurements. Suppose that M measurements of bus net active-power injections, $\widehat{P}_{[k-M+1]}, \ldots, \widehat{P}_{[k]}$, are available so that the network loss at time steps $k-M+1, \ldots, k$, $\widehat{P}^{\text{loss}}_{[k-M+1]}, \ldots, \widehat{P}^{\text{loss}}_{[k]}$, can be computed using (2). Further suppose measurements of DER reactive-power injections, $\widehat{Q}^{\text{gen}}_{[k-M+1]}, \ldots, \widehat{Q}^{\text{gen}}_{[k]}$, and DER bus voltage magnitudes, $\widehat{V}_{[k-M+1]}, \ldots, \widehat{V}_{[k]}$, are available. Assuming that active- and reactive-power loads as well as active-power DER injections remain relatively constant over the M measurement samples, we stack up M instances of (9) and (10) to yield

$$y_{[k]} = X_{[k]} \gamma_{[k]}, \tag{11}$$

$$Z_{[k]} = X_{[k]} \Lambda_{[k]}, \tag{12}$$

where $y_{[k]} \in \mathbb{R}^M$ and $X_{[k]} \in \mathbb{R}^{M\times(D+1)}$ are given by

$$y_{[k]} = \begin{bmatrix} \widehat{P}^{\text{loss}}_{[k-M+1]} \\ \vdots \\ \widehat{P}^{\text{loss}}_{[k]} \end{bmatrix}, \quad X_{[k]} = \begin{bmatrix} 1 & (\widehat{Q}^{\text{gen}}_{[k-M+1]})^{\text{T}} \\ \vdots & \vdots \\ 1 & (\widehat{Q}^{\text{gen}}_{[k]})^{\text{T}} \end{bmatrix}, \tag{13}$$

and $Z_{[k]} \in \mathbb{R}^{M\times D}$ is given by

$$Z_{[k]} = \begin{bmatrix} (\widehat{V}^{\text{gen}}_{[k-M+1]})^{\text{T}} \\ \vdots \\ (\widehat{V}^{\text{gen}}_{[k]})^{\text{T}} \end{bmatrix}. \tag{14}$$

Assume that $M > (D+1)$, then (11) and (12) are overdetermined systems of equations, and we can obtain the ordinary least-squares estimates for $\gamma_{[k]}$ and $\Lambda_{[k]}$ as

$$\widehat{\gamma}_{[k]} \approx (X^{\text{T}}_{[k]} X_{[k]})^{-1} X^{\text{T}}_{[k]} y_{[k]}, \tag{15}$$

$$\widehat{\Lambda}_{[k]} \approx (X^{\text{T}}_{[k]} X_{[k]})^{-1} X^{\text{T}}_{[k]} Z_{[k]}, \tag{16}$$

from which we can extract the estimated $\widehat{\kappa}_{[k]}$ and $\widehat{\rho}_{[k]}$ in the approximate network loss function (7), and similarly the estimated $\widehat{\lambda}_{[k]}$ and $\widehat{\Sigma}_{[k]}$ in the approximate linear voltage function (8). Note that in the proposed framework, we include voltage magnitudes only at buses in the set $\mathcal{D}$ to build the sensitivity model. However, this could easily be generalized by including measurements from all buses in $Z_{[k]}$, to incorporate voltage constraints at all buses in the set $\mathcal{N}$ in problem (3).

B. *Optimal Reactive-power Dispatch with Gradient Descent*

Instead of solving a potentially computationally burdensome model-based optimal reactive-power dispatch problem (3), we opt to embed the estimated loss and voltage sensitivities in successive measurement-based gradient-descent updates. We aim to minimize network loss and cost of DER outputs by defining the objective function as follows:

$$f(Q^{\text{gen}}_{[k]}) = h_{[k]}(Q^{\text{gen}}_{[k]}) + \frac{1}{2}(Q^{\text{gen}}_{[k]})^{\text{T}} \Upsilon Q^{\text{gen}}_{[k]}, \tag{17}$$

where $\Upsilon = \text{diag}(\upsilon_1, \ldots, \upsilon_D) \in \mathbb{R}^{D\times D}$ is a diagonal coefficient matrix with non-negative entries, i.e., $\upsilon_i \geq 0$, representing cost coefficients proportional to the *square* of DER reactive-power outputs. We opt to have quadratic costs for DER reactive-power output, but we note that proportional costs could easily be accommodated in the proposed framework.

In order to solve (3) with objective function (17) using a measurement-based gradient-descent method, we first derive the derivative of $f(\cdot)$. Recognizing that $\widehat{\rho}_{[k]}$ corresponds to the gradient of the loss function $h_{[k]}(\cdot)$ evaluated at the operating point, we have:

$$\nabla f(Q^{\text{gen}}_{[k]}) = \widehat{\rho}_{[k]} + \Upsilon Q^{\text{gen}}_{[k]} \tag{18}$$

Subsequently, we embed (18) and the estimated voltage sensitivities $\widehat{\Sigma}_{[k]}$ into (4) to obtain the following DER reactive-power setpoint updates:

$$Q^{\text{gen}}_{[k+1]} = \text{proj}_{\mathcal{Q}}(Q^{\text{gen}}_{[k]} - \alpha_{[k]}(\widehat{\rho}_{[k]} + \Upsilon Q^{\text{gen}}_{[k]} + \widehat{\Sigma}^{\text{T}}_{[k]} \beta S(V_{[k]}))), \tag{19}$$

where β is a weight parameter and $S(\cdot)$ is the soft-thresholding operator associated with the penalty function $g(\cdot)$ given by:

$$S(V_{[k]}) = \begin{cases} V_{[k]} - 1, & \text{if } V_{[k]} \geq V_{\max}, \\ 0, & \text{if } V_{\min} \leq V_{[k]} \leq V_{\max}, \\ V_{[k]} - 1, & \text{if } V_{[k]} \leq V_{\min}, \end{cases} \tag{20}$$

We emphasize that in (19) all variables and parameters are either directly measured or estimated from measurements, which makes the DER updates independent from an offline network model.

In the proposed framework, the DER reactive-power setpoints collected in $Q^{\text{gen}}_{[k+1]}$ are updated every M time steps using (19), after the estimated sensitivities of network loss $\widehat{\rho}_{[k]}$ and bus voltages $\widehat{\Sigma}_{[k]}$, are computed using (15) and (16), respectively. Repeated and iterative estimations of the loss and voltage sensitivities and dispatch of DER reactive-power injections achieve convergence towards the optimal operating point while adapting to changes in the network.

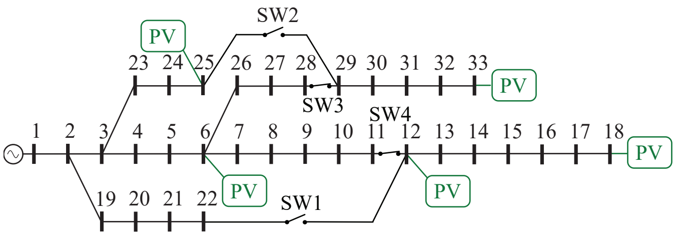

Fig. 1. IEEE 33-bus test system with DER systems at buses 6, 12, 18, 25, and 33. Switches SW1 and SW2 are normally open, and switches SW3 and SW4 are normally closed.

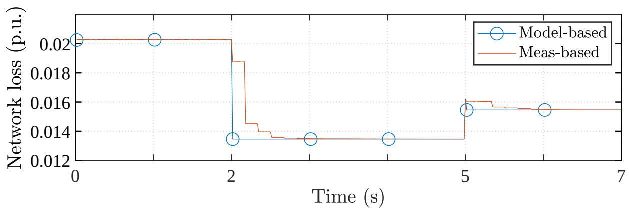

Fig. 2. Network loss resulting from the measurement-based DER reactive-power dispatch method compared to the model-based method.

IV. Case Studies

We perform simulations involving the IEEE 33-bus test system in MATPOWER [18] (see Fig. 1), in which we set the power base to 10 MVA. Five DER (e.g., solar PV) systems are connected to buses in $\mathcal{D} = \{6, 12, 18, 25, 33\}$. Assume that synchronized measurements of net active-power injections at all buses and DER reactive-power outputs as well as voltage magnitudes at buses in $\mathcal{D}$ are available from D-PMUs at intervals of $1/60$ s, which is reasonable as D-PMUs are capable of streaming up to 120 samples per second [4]. Also assume that the DER active-power outputs remain approximately constant over the time interval in which $M = 10$ sets of measurements are collected. Each DER reactive-power output is constrained within $\mathcal{Q}_i = [-0.1, 0.1]$ p.u., $i \in \mathcal{D}$, and $\mathcal{Q} = \Pi_{i\in\mathcal{D}}\mathcal{Q}_i$. In order to estimate the network sensitivities in (7) and (8), we perturb the reactive-power outputs of DER systems with random Gaussian distributed variations of zero mean and 0.01% standard deviation relative to the their nominal values.

A. Loss Minimization

In the first case study, we simulate a scenario, in which we focus on the capability of the method to minimize network loss. To this end, we set entries in Υ to 0 and use voltage limits of $1{\pm}0.15$ p.u. (we find that voltage constraints are not binding with these limits). We benchmark the measurement-based DER reactive-power dispatch against the model-based method assuming that an accurate network model is available. In a time-domain simulation, the system initially operates without DER reactive-power contribution. At time $t = 2$ s, DER reactive-power dispatch capability is activated. Subsequently, at time $t = 5$ s, the system topology is reconfigured by closing switches SW1 and SW2 and opening switches SW3 and SW4, and simultaneously, active- and reactive-power loads at all buses in the system grow by 25%. The simulation runs until

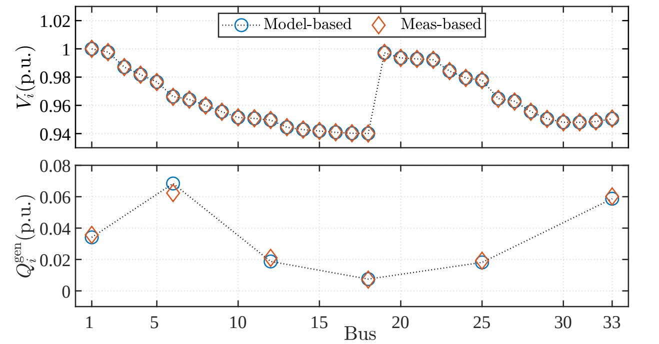

Fig. 3. Bus voltage magnitude V_i and DER reactive-power generation Q_i^{gen} at $t = 4.9$ s prior to changes in network topology and system load obtained via (i) model-based method, and (ii) proposed measurement-based method.

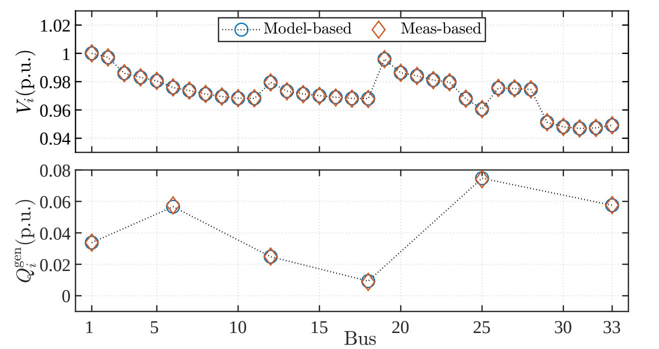

Fig. 4. Bus voltage magnitude V_i and DER reactive-power generations Q_i^{gen} at $t = 7$ s after changes in network topology and system load obtained via (i) model-based method, and (ii) proposed measurement-based method.

time $t = 7$ s. The step size $\alpha_{[k]}$ is set using the BarzilaiBorwein method, in order to take steepest-descent steps: [19]

$$\alpha_{[k]} = \frac{\left|(Q^{\text{gen}}_{[k]} - Q^{\text{gen}}_{[k-1]})^{\mathrm{T}}(\widehat{\rho}_{[k]} - \widehat{\rho}_{[k-1]})\right|}{\left|\left|\widehat{\rho}_{[k]} - \widehat{\rho}_{[k-1]}\right|\right|_2^2}. \tag{21}$$

Time evolution of the network loss resulting from simulating the scenario described above is plotted in Fig. 2. Once DER reactive-power dispatch is activated at time $t = 2$ s, the proposed measurement-based dispatch effectively converges to the minimum-loss operating point obtained by the model-based approach. Also, after network-topology and operating-point changes at time $t = 5$ s, the proposed method updates DER reactive-power setpoints to minimize loss. Figures 3 and 4 visualize bus voltages V_i and DER reactive-power outputs Q_i^{gen} at $t = 4.9$ s before network reconfiguration, and at $t = 7$ s after network reconfiguration, respectively. Results from the proposed method match very closely to those from the model-based dispatch with accurate system model.

B. Loss and DER Cost Minimization

Here, we simulate a scenario that minimizes a combined cost function comprising network loss and DER costs. To this end, we set the diagonal entries in Υ between 0.22 and 0.3 in order to give different costs to different DERs, and use voltage limits $V_{\min}$ and $V_{\max}$ of 1 ± 0.05 p.u. (we find some of the voltage constraints binding with these limits). Instead of the

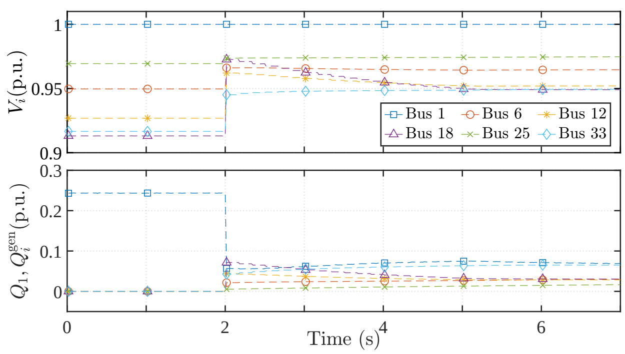

Fig. 5. Time-domain simulation via proposed measurement-based optimal DER reactive-power dispatch. DER reactive-power dispatch is activated at time $t = 2\,\text{s}$. Top pane: bus voltages V_i at buses with DERs; Bottom pane: substation reactive-power injection Q_1 and DER reactive-power outputs Q_i^{gen} at controllable buses.

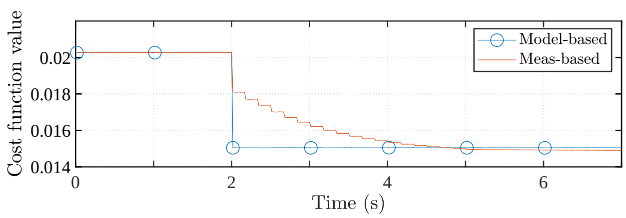

Fig. 6. Evaluation of cost function value over time for the measurement-based DER reactive-power dispatch method considering DER costs and voltage constraints, compared to the model-based method.

value in (21), we fix the step size $\alpha_{[k]}$ to 0.04. We further set the parameter $\beta = 50$ to appropriately weigh the penalty term for voltage constraint violations. We simulate the same scenario as in the previous case study, except that there are no changes in network topology or operating point.

In Fig. 5, we plot bus voltage magnitudes and DER reactive-power outputs resulting from simulating the scenario described above with the proposed measurement-based DER dispatch method. Indeed, once DER reactive-power dispatch is activated at time $t = 2\,\text{s}$, we observe that the proposed measurement-based framework effectively achieves the objective of minimizing network loss and DER costs. After activation of DERs, the substation reactive-power injection decreases sharply since DERs collectively provide voltage support by injecting reactive power, as shown in the bottom pane of Fig. 5. Furthermore, the evaluated cost function value at each time step for the above scenario is plotted in Fig. 6. We observe that the measurement-based method achieves a slightly lower value compared to the model-based benchmark at steady state, which is due to the "soft" voltage constraints implemented in the measurement-based framework.

V. Concluding Remarks

In this paper, we present a measurement-based method to optimally dispatch DER reactive power. We estimate linear sensitivities of the network loss and bus voltages with respect to DER reactive-power injections from synchronized bus injection and voltage measurements. Subsequently, we use the estimated sensitivities in gradient-descent iterations to obtain optimal DER reactive-power setpoints. Simulations of the IEEE 33-bus test system demonstrate that the proposed measurement-based method yields accurate DER setpoints compared to the model-based method, and achieves combined objectives of minimizing network loss and DER costs while considering bus voltage constraints without relying on any knowledge of an offline system model.

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