Dynamics-aware Continuous-time Economic Dispatch and Optimal Automatic Generation Control

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Abstract— **In this work, we aim to minimize the cost of generation in a power system while meeting demand in near-to real time. The proposed architecture is composed of two sub-problems: continuous-time economic dispatch (CTED) and optimal automatic generation control (OAGC). In its original form, the CTED problem incorporates generator aggregate-frequency dynamics, and it is infinite-dimensional. However, we present a computationally tractable function space-based solution method for the proposed problem. We also develop an optimization-based control algorithm for implementing OAGC. Theoretical considerations for decoupling the two problems are explored. We validate the economic efficiency and frequency performance of the proposed method through simulations of a representative power network.**

I. Introduction

One of the main challenges for a power system operator is to continually schedule generation to meet demand [1], [2]. The prevailing practice involves two parts: i) an offline *economic dispatch* (ED) problem, in which the system operator minimizes the cost of generation based on load forecasts while enforcing various operational constraints in steady state, and ii) real-time *automatic generation control* (AGC), which regulates system frequency to the synchronous value and fixes power interchanges between different balancing areas to their scheduled quantities. Typically, ED is performed (approximately on the order of) every 5 minutes to dispatch generators to meet the forecasted load. During real-time operations, the proportional-integral control-based AGC adjusts generator outputs around their ED setpoints based on deviations in frequency and tie-line flows [3], [4].

The future grid will extensively integrate renewable generation, resulting in faster and less predictable frequency deviations away from synchronous operation [5], [6]. If the existing ED paradigm persists, significant AGC control effort would be needed in real time to maintain synchronous frequency and economic efficiency of the generators. Moreover, the existing AGC does not guarantee system cost minimization, especially if the real-time load deviates significantly from the forecast. To this end, we propose a combined architecture composed of: i) a continuous-time ED problem that acknowledges continuous load variations so that real-time control effort is reduced, and ii) an optimal AGC scheme that provides a mechanism to price AGC control effort while minimizing total cost of generation in real time.

Recognizing the need to improve economic and dynamic performance of power system operations in the face of challenges such as increasing intermittency and variability, a variety of approaches have been put forward to optimize ED and AGC. Some approaches have proposed improvements to classical AGC [7], [8]. Model predictive control approaches have been proposed for developing economic dispatch [9]–[11]. In [12], ED and AGC are connected by reverse engineering AGC from an optimization point of view. In [13], a joint problem is decomposed to a multi-period ED and the AGC from [12]. A frequency-aware ED has been proposed in [14]. Primal-dual gradient methods have also been proposed to design decentralized feedback control laws [15]–[19].

Our central idea is to include a continuous-time dynamic model of the generators in existing methods for ED, and also conceptualize an AGC approach that ensures economic efficiency in real time. We begin with an ideal optimal control problem for the system operator where total cost of generation is minimized across timescales currently pertinent to ED and AGC while dynamically enforcing operational constraints. However, we will find that this optimal control problem cannot be solved simultaneously for ED and AGC actions since the real-time load is not known when ED is solved. Recognizing this limitation, and the fact that the objectives and constraints are required to be satisfied at two different timescales, we define a combination of two problems that can be solved and the solutions of which are close to those of the ideal problem. We refer to these problems as: continuous-time economic dispatch (CTED) and optimal automatic generation control (OAGC). The CTED problem considers the dynamic constraints of generators and minimizes cost of generation over a scheduling horizon on the order of that considered for ED. We have developed a function space-based method to reformulate the infinite-dimensional CTED problem into a linear program [20]. (See also [21], [22] for related work.) With our previous effort in [20] covering the dispatch timescale, for real-time operation, the formulated OAGC problem minimizes the sum of well-defined cost functions for generators while ensuring frequency is restored to synchronous value. We develop an optimization-based control algorithm to solve OAGC. Importantly, the proposed approach embeds two different

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cost functions for scheduling generation during CTED and OAGC.

Salient features of the proposed architecture that distinguish it from similar approaches outlined in existing literature are as follows:

- As we incorporate the dynamic constraints of the generators, CTED yields continuously differentiable trajectories as solutions, which ensures that large-signal changes to AGC references are minimized.
- The proposed OAGC provides a systematic method to price the cost of restoring frequency to nominal in real time. In essence, it broadens the timescales over which economic optimality of generators can be guaranteed in power system operations.
- Unlike most of the existing literature, our method retains communication, control, and computation architectures currently in place for existing ED and AGC.
- Our method enables two separate cost functions for CTED and OAGC. This is appropriate since a generator might need extra control effort for adjusting power in real time versus scheduling beforehand.

The remainder of the paper is organized as follows. In Section II, we summarize the ED and AGC architecture that is currently in use, and introduce the aggregate dynamic model of the generators leveraged to develop our optimal control problem. In Section III, we introduce our ideal infinite-dimensional optimal control problem. We then define an approximate problem with a companion solution strategy that involves decomposition into CTED and OAGC components. In Section IV-B, we describe the function space-based method by which we convert the CTED problem into a linear programming problem. In Section IV-A, we develop an optimization-based control method for implementing OAGC. Section V reports simulation results that demonstrate economic efficiency of the proposed method across timescales in a representative power network. Finally, we conclude and outline directions for future work in Section VI.

II. Preliminaries

In this section, we introduce fundamentals of conventional economic dispatch (ED), automatic generation control (AGC), and the synchronous-generator dynamical model. Leveraging the synchronous-generator dynamical model, we develop an aggregate representation of generator dynamics, which is then used to formulate the optimal control problem.

A. Current Practices: Economic Dispatch (ED) and Automatic Generation Control (AGC)

Consider a single-area power system with G generators connected to buses $\mathcal{G} = \{1, \ldots, G\}$ that serve L loads connected to buses $\mathcal{L} = \{G+1, \ldots, G+L\}$. Currently, ED involves the solution of an optimization problem of the general form:

$$\min_{P_{\mathcal{G}}^{\mathrm{ed}} \in \mathbb{R}^G} \quad \sum_{g \in \mathcal{G}} C_g(P_g^{\mathrm{ed}}) \tag{1a}$$

$$\text{s.t.} \quad 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}} = P_{\mathrm{load}}, \tag{1b}$$

$$\underline{P}_{\mathcal{G}}^{\mathrm{ed}} \leq P_{\mathcal{G}}^{\mathrm{ed}} \leq \overline{P}_{\mathcal{G}}^{\mathrm{ed}}. \tag{1c}$$

In the above problem, (1b) is the power balance constraint, P_{load} and $P_{\mathcal{G}}^{\mathrm{ed}} := [P_1^{\mathrm{ed}}, \ldots, P_G^{\mathrm{ed}}]^{\mathrm{T}}$ respectively denote the estimate of system load and the vector of dispatched power of generators. Furthermore, $C_g(\cdot)$ represents the cost function for generator $g \in \mathcal{G}$, and (1a) is the total cost of generation. Limits on power outputs are enforced by (1c).

Once the optimization problem (1) is solved, the optimal generator dispatch points, $P_g^{\mathrm{ed}\star}, \forall g \in \mathcal{G}$ serve as inputs to the AGC until the next solution instant. For a single-area power system, the generator reference-power setpoints (we discuss shortly how these relate to the generator dynamical model) are given by:

$$P_g^{\mathrm{ref}}(t) = P_g^{\mathrm{ed}\star} + \alpha_g \Big(\xi(t) - \sum_{\ell \in \mathcal{G}} P_\ell^{\mathrm{ed}\star} \Big), \tag{2}$$

where α_g denotes the area control error (ACE) participation factor for generator $g \in \mathcal{G}$. In (2), $\xi(t)$ is recovered from the following dynamics:

$$\dot{\xi}(t) = \epsilon(t) - \xi(t) + \sum_{g \in \mathcal{G}} P_g^{\mathrm{ele}}(t), \tag{3}$$

where $P_g^{\mathrm{ele}}(t)$ is the electrical power output of generator g. Furthermore, $\epsilon(t)$ is the ACE, defined as:

$$\epsilon(t) = -\beta \frac{1}{|\mathcal{G}|} \sum_{g \in \mathcal{G}} \Delta\omega_g(t), \tag{4}$$

where β is bias factor for the balancing area, and $\Delta\omega_g(t)$ denotes (measured) frequency offset of generator $g \in \mathcal{G}$ from nominal. Following are common choices for the bias factor and generator ACE participation factors:

$$\beta = \sum_{g \in \mathcal{G}} (R_g^{-1} + D_g), \ \alpha_g = \frac{(IC_g^{\star\prime})^{-1}}{\sum_{\ell \in \mathcal{G}} (IC_\ell^{\star\prime})^{-1}}, \tag{5}$$

where R_g denotes the speed-droop regulation constant, D_g is the damping constant, and $IC_g^{\star\prime}$ denotes the derivative of the optimal incremental cost of generation for generator g.

B. Synchronous-generator Model

For each generator $g \in \mathcal{G}$, $\theta_g(t)$, $\omega_g(t)$, $P_g^{\mathrm{mec}}(t)$, and $P_g^{\mathrm{ele}}(t)$ are the rotor angular position, electrical angular frequency, turbine mechanical power, and electrical power output, respectively. Assuming each generator initially operates at the steady-state equilibrium point $\omega_g(0) = \omega_{\mathrm{s}} = 2\pi 60\,\mathrm{rad/s}$, and defining $\Delta\omega_g := \omega_g - \omega_{\mathrm{s}}$, dynamics of generator g are expressed by the swing equations (along with a simplified turbine-governor model):

$$\begin{aligned}
\dot{\theta}_g(t) &= \Delta\omega_g(t), \\
M_g \Delta\dot{\omega}_g(t) &= P_g^{\mathrm{mec}}(t) - D_g \Delta\omega_g(t) - P_g^{\mathrm{ele}}(t), \\
\tau_g \dot{P}_g^{\mathrm{mec}}(t) &= -P_g^{\mathrm{mec}}(t) + P_g^{\mathrm{ref}}(t) - R_g^{-1} \Delta\omega_g(t),
\end{aligned} \tag{6}$$

where M_g, D_g, and τ_g denote the inertia constant, damping constant, and governor time constant, respectively.

C. Aggregate System Dynamical Model

We assume that the effective impedances between nodes in the network are approximately the same, so that all generator speeds follow the same transient behaviour [23], i.e.,

$$\Delta\omega_g = \Delta\omega, \quad \forall g \in \mathcal{G}. \tag{7}$$

The angular-frequency dynamics of each generator g in (6) are then expressed as:

$$M_g \Delta\dot{\omega}(t) = P_g^{\text{mec}}(t) - D_g \Delta\omega(t) - P_g^{\text{ele}}(t). \tag{8}$$

The system dynamics can be written by summing (8) over all $g \in \mathcal{G}$ as follows:

$$\begin{aligned}\underbrace{\sum_{g\in\mathcal{G}} M_g}_{=:M_{\text{eff}}} \Delta\dot{\omega}(t) &= \sum_{g\in\mathcal{G}} P_g^{\text{mec}}(t) \\ &- \underbrace{\sum_{g\in\mathcal{G}} D_g}_{=:D_{\text{eff}}} \Delta\omega(t) - \underbrace{\sum_{g\in\mathcal{G}} P_g^{\text{ele}}(t)}_{=:P_{\text{load}}(t)},\end{aligned} \tag{9}$$

where, M_{eff} is the *effective inertia constant*, D_{eff} is the *effective damping constant*, and the sum of electrical power outputs of the generators is equal to the total system load, $P_{\text{load}}(t)$ along with losses. Then, we can express the overall system dynamics with all the generators supplying the total load as follows:

$$M_{\text{eff}} \Delta\dot{\omega}(t) = 1_G^{\text{T}} P_{\mathcal{G}}^{\text{mec}}(t) - D_{\text{eff}} \Delta\omega(t) - P_{\text{load}}(t), \tag{10}$$

where $P_{\mathcal{G}}^{\text{mec}}(t) := [P_1^{\text{mec}}(t), \ldots, P_G^{\text{mec}}(t)]^{\text{T}}$ and 1_G is a length-G vector of ones. We will leverage the above dynamic power balance constraint to ensure supply-demand adequacy in the proposed optimization problems. Note that the governor equations are not captured in this model to preserve computational tractability and given that for the timescales of interest, the above model suffices.

III. Optimal Control Problem: ED + AGC

In this section, we outline an ideal and fundamental optimal control problem that combines ED and AGC. Recognizing the difficulty in solving such a problem, we then pass on—through appropriate simplifications—to abstractions that can be solved within typical computational constraints.

A. Fundamental Optimal Control Problem

The fundamental control task is to schedule generation to ensure AGC action as well as economically operate the system. Let the vector $P_{\mathcal{G}}^{\text{ref}}(t) := [P_1^{\text{ref}}(t), \ldots, P_G^{\text{ref}}(t)]^{\text{T}}$ represent the reference power input of generators. Also, let $V_g(P_g^{\text{ref}}(t))$ denote the cost function of providing this reference power. Note that $V_g(\cdot)$ is not the same as $C_g(\cdot)$, since $V_g(\cdot)$ also includes the cost of providing AGC action from the generators. The problem of interest is the minimization of total cost over scheduling horizon $\mathcal{T}$ (with length denoted by $|\mathcal{T}|$) subject to generator dynamic constraints:

$$\min_{P_{\mathcal{G}}^{\text{ref}}(t), t\in\mathcal{T}} \quad \int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} V_g(P_g^{\text{ref}}(t))dt \tag{11a}$$

$$\text{s.t.} \quad 1_G^{\text{T}} P_{\mathcal{G}}^{\text{ref}}(t) - D_{\text{eff}}\Delta\omega(t) - M_{\text{eff}}\Delta\dot{\omega}(t) = P_{\text{load}}(t), \tag{11b}$$

$$\underline{P}_{\mathcal{G}} \leq P_{\mathcal{G}}^{\text{ref}}(t) \leq \overline{P}_{\mathcal{G}}, \tag{11c}$$

$$\Delta\underline{\omega} \leq \Delta\omega(t) \leq \Delta\overline{\omega}, \tag{11d}$$

where (11b) is the dynamic power-balance constraint for the system that acknowledges generator dynamics as described in (10). Box constraints for reference power input and frequency deviation are enforced by (11c) and (11d) respectively, where $\underline{P}_{\mathcal{G}}$ and $\overline{P}_{\mathcal{G}}$ are vectors of generator capacity limits.

Though this problem is ideal for the operator, it cannot be solved in practice for a variety of reasons. First, note that $P_{\text{load}}(t)$ can only be known in real time and current forecasting practices do not yield accurate load samples even in near-to real time. In addition, generators cannot change their power outputs over a large range on very fast timescales, and some level of prescient scheduling close to where they would be operated in real time would be wise.

B. Problem Decomposition Strategy

Given the above considerations, our basic idea is to schedule generation—acknowledging generator dynamics—for the best estimate of look-ahead (i.e., forecasted) load, $P_{\text{load}}^{\text{la}}(t)$, and then compensate for load variation in real time, denoted by $P_{\text{load}}^{\text{rt}}(t)$, with an *optimal AGC* scheme. To this end, we decompose the actual load as follows:

$$P_{\text{load}}(t) = P_{\text{load}}^{\text{la}}(t) + P_{\text{load}}^{\text{rt}}(t). \tag{12}$$

Similarly, let us decompose the generator reference into constituent parts attributable to dispatch and AGC:

$$P_g^{\text{ref}}(t) = P_g^{\text{ed}}(t) + P_g^{\text{agc}}(t), \quad g \in \mathcal{G}. \tag{13}$$

We then get the following problem from (11):

$$\min_{P_{\mathcal{G}}^{\text{ed}}(t), P_{\mathcal{G}}^{\text{agc}}(t), t\in\mathcal{T}} \int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} V_g(P_g^{\text{ed}}(t) + P_g^{\text{agc}}(t))dt \tag{14a}$$

$$\begin{aligned}\text{s.t.} \quad & 1_G^{\text{T}}(P_{\mathcal{G}}^{\text{ed}}(t) + P_{\mathcal{G}}^{\text{agc}}(t)) - D_{\text{eff}}\Delta\omega(t) \\ & - M_{\text{eff}}\Delta\dot{\omega}(t) = P_{\text{load}}^{\text{la}}(t) + P_{\text{load}}^{\text{rt}}(t),\end{aligned} \tag{14b}$$

$$\underline{P}_{\mathcal{G}} \leq P_{\mathcal{G}}^{\text{ed}}(t) \leq \overline{P}_{\mathcal{G}}, \tag{14c}$$

$$\underline{P}_{\mathcal{G}} \leq P_{\mathcal{G}}^{\text{agc}}(t) + P_{\mathcal{G}}^{\text{ed}}(t) \leq \overline{P}_{\mathcal{G}}, \tag{14d}$$

$$\Delta\underline{\omega} \leq \Delta\omega(t) \leq \Delta\overline{\omega}. \tag{14e}$$

C. Continuous-time ED and Optimal AGC

We assume that the total cost of generation (V_g) can be disaggregated into a sum of two functions: dispatch cost C_g (corresponding to $P_g^{\text{ed}}(t)$ which is scheduled beforehand), and real-time cost F_g (corresponding to $P_g^{\text{agc}}(t)$ intended for real-time operation). In particular, we assume we can express the total cost as

$$V_g(P_g^{\text{ed}}(t) + P_g^{\text{agc}}(t)) = C_g(P_g^{\text{ed}}(t)) + F_g(P_g^{\text{agc}}(t)). \tag{15}$$

Furthermore, we approximate the supply-demand balance in (14b) as the sum of two parts:

$$1_G^{\text{T}} P_{\mathcal{G}}^{\text{ed}}(t) - D_{\text{eff}}\Delta\omega^{\text{la}}(t) - M_{\text{eff}}\Delta\dot{\omega}^{\text{la}}(t) = P_{\text{load}}^{\text{la}}(t), \tag{16}$$

$$1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = P_{\mathrm{load}}^{\mathrm{rt}}(t). \tag{17}$$

Note that $\Delta\omega^{\mathrm{la}}(t)$ in the above problem differs from the actual frequency deviation $\Delta\omega(t)$ in (11b). In essence, $\Delta\omega^{\mathrm{la}}(t)$ is the frequency deviation that results by only considering the look-ahead load in the power balance constraint. Substituting (15) into (14a) and replacing (14b) with (16) and (17), the problem in (14) becomes:

$$\min_{P_{\mathcal{G}}^{\mathrm{ed}}(t), P_{\mathcal{G}}^{\mathrm{agc}}(t), t\in\mathcal{T}} \int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} \Big(C_g\big(P_g^{\mathrm{ed}}(t)\big) + F_g\big(P_g^{\mathrm{agc}}(t)\big)\Big) dt \tag{18a}$$

$$\begin{aligned}
\text{s.t.}\quad & 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}}(t) - D_{\mathrm{eff}}\Delta\omega^{\mathrm{la}}(t) \\
& \quad - M_{\mathrm{eff}}\Delta\dot{\omega}^{\mathrm{la}}(t) = P_{\mathrm{load}}^{\mathrm{la}}(t), && \text{(18b)}\\
& 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = P_{\mathrm{load}}^{\mathrm{rt}}(t), && \text{(18c)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{ed}}(t) \le \overline{P}_{\mathcal{G}}, && \text{(18d)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{agc}}(t) + P_{\mathcal{G}}^{\mathrm{ed}}(t) \le \overline{P}_{\mathcal{G}}, && \text{(18e)}\\
& \Delta\underline{\omega} \le \Delta\omega^{\mathrm{la}}(t) \le \Delta\overline{\omega}. && \text{(18f)}
\end{aligned}$$

We next propose to decompose problem (18) into two separate optimization problems, namely continuous-time economic dispatch (CTED) and optimal automatic generation control (OAGC). The CTED problem is the continuous-time alternative to conventional ED, and it is formulated to schedule generators for supplying the look-ahead load $P_{\mathrm{load}}^{\mathrm{la}}(t)$ over $\mathcal{T}$ at minimum cost. It is given by:

$$\begin{aligned}
\min_{P_{\mathcal{G}}^{\mathrm{ed}}(t), \Delta\omega^{\mathrm{la}}(t), t\in\mathcal{T}} \quad & \int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} C_g(P_g^{\mathrm{ed}}(t)) dt && \text{(19a)}\\
\text{s.t.}\quad & 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}}(t) - D_{\mathrm{eff}}\Delta\omega^{\mathrm{la}}(t) \\
& \quad - M_{\mathrm{eff}}\Delta\dot{\omega}^{\mathrm{la}}(t) = P_{\mathrm{load}}^{\mathrm{la}}(t), && \text{(19b)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{ed}}(t) \le \overline{P}_{\mathcal{G}}, && \text{(19c)}\\
& \Delta\underline{\omega} \le \Delta\omega^{\mathrm{la}}(t) \le \Delta\overline{\omega}. && \text{(19d)}
\end{aligned}$$

The power balance constraint (19b) acknowledges aggregate generator dynamics, however, the frequency that results, i.e., $\Delta\omega^{\mathrm{la}}(t)$, is not the actual frequency measured in real-time. The CTED problem (19) is solved over the look-ahead horizon and once the optimal generator dispatch $P_{\mathcal{G}}^{\mathrm{ed}\star}(t)$ schedule is determined, the OAGC problem is solved in real-time to economically operate generators in real-time and compensate for load fluctuations from the look-ahead load:

$$\begin{aligned}
\min_{P_{\mathcal{G}}^{\mathrm{agc}}(t), t\in\mathcal{T}} \quad & \int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} F_g(P_g^{\mathrm{agc}}(t)) dt && \text{(20a)}\\
\text{s.t.}\quad & 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = P_{\mathrm{load}}^{\mathrm{rt}}(t), && \text{(20b)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{agc}}(t) + P_{\mathcal{G}}^{\mathrm{ed}\star}(t) \le \overline{P}_{\mathcal{G}}. && \text{(20c)}
\end{aligned}$$

Theorem 1. *The Problem (18) can be decomposed into the CTED (19) and OAGC (20) problems.*

Proof. The objective function of problem (18) is the sum of the objective functions of CTED (19) and OAGC (20). The constraints of CTED and OAGC are also decoupled and if we add them we get the constraints of the problem (18). Thus, by construction, the joint problem (18) can be decomposed into CTED (19) and OAGC (20). □

IV. Solution Strategy for OAGC and CTED

In this section, we present a solution strategy for the OAGC problem in (20) and overview a function space-based strategy to reformulate the infinite-dimensional CTED problem in (19) to a linear programming problem.

A. An Optimization-based Controller to Implement OAGC

A cursory examination of the optimal control problem (20) reveals that it is decoupled across time. Indeed, for each time instant, it suffices to solve the following:

$$\begin{aligned}
\min_{P_{\mathcal{G}}^{\mathrm{agc}}(t)} \quad & \sum_{g\in\mathcal{G}} F_g(P_g^{\mathrm{agc}}(t)) && \text{(21a)}\\
\text{s.t.}\quad & 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = P_{\mathrm{load}}^{\mathrm{rt}}(t), && \text{(21b)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{agc}}(t) + P_{\mathcal{G}}^{\mathrm{ed}\star}(t) \le \overline{P}_{\mathcal{G}}. && \text{(21c)}
\end{aligned}$$

However, it is not possible to solve (21) because measurements of $P_{\mathrm{load}}^{\mathrm{rt}}(t)$ are not available in practice. Furthermore, there is no guarantee that the optimizers of (21) would eliminate frequency offset in steady state. To obtain a realizable alternative that also ensures zero steady-state frequency error, we revert to the AGC dynamics in (2) and (3) which indicate how the generator references are determined. Summing both sides of (2) over all generators, recognizing that $\sum_{g\in\mathcal{G}} \alpha_g = 1$, and considering the decomposition in (13), we get:

$$1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ref}}(t) = \xi(t) = 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}\star}(t) + 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t). \tag{22}$$

Thus we can write

$$1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = \xi(t) - 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}\star}(t) \tag{23}$$

where $\xi(t)$ is obtained from (3) and $P_{\mathcal{G}}^{\mathrm{ed}\star}(t)$ is obtained by solving the CTED problem (updated every $|\mathcal{T}|$ units of time).

Hence, the OAGC problem can be written as follows:

$$\begin{aligned}
\min_{P_{\mathcal{G}}^{\mathrm{agc}}(t)} \quad & \sum_{g\in\mathcal{G}} F_g(P_g^{\mathrm{agc}}(t)) && \text{(24a)}\\
\text{s.t.}\quad & 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{agc}}(t) = \xi(t) - 1_G^{\mathrm{T}} P_{\mathcal{G}}^{\mathrm{ed}\star}(t), && \text{(24b)}\\
& \underline{P}_{\mathcal{G}} \le P_{\mathcal{G}}^{\mathrm{agc}}(t) + P_{\mathcal{G}}^{\mathrm{ed}\star}(t) \le \overline{P}_{\mathcal{G}} && \text{(24c)}\\
& \dot{\xi}(t) = \epsilon(t) - \xi(t) + \sum_{g\in\mathcal{G}} P_g^{\mathrm{ele}}(t), && \text{(24d)}
\end{aligned}$$

The problem (24) can be solved continually upon receival of AGC signal $\epsilon(t)$. In essence, the above strategy retains the AGC dynamics in (3) from current industry practice but provides a systematic and optimal alternative to the determination of corrective AGC action. This is in contrast to how AGC control is currently determined as in (2). In traditional AGC, the allocation of net AGC action to individual generators through the ACE participation factors only aspires toward—but does not ensure—operation close to the economically optimal dispatch point.

B. A Linear Programming Reformulation for CTED

The problem (19) is analytically intractable since it is patently infinite dimensional. We use a scalable and efficient function space-based solution method to solve the problem. An overview of this approach is provided next, and readers are referred to [20] for details.

The basic strategy is to model all pertinent time-domain trajectories in the CTED problem in a finite-order function space spanned by so-called Bernstein polynomials. Bernstein polynomials of degree Q, having $Q+1$ polynomials, are defined as follows for $t \in [0,1]$:

$$b_{q,Q}(t) = \binom{Q}{q} t^q (1-t)^{Q-q}, \; q \in \mathcal{Q} := \{0, ..., Q\}. \quad (25)$$

(See, e.g., [21], [22] for further details.) We consider the horizon for dispatch $\mathcal{T}$ and divide it into N intervals of equal length T. Next, we construct a set of basis functions in each interval n using Bernstein polynomials of degree Q. Thus, the vector of basis functions spanning $\mathcal{T}$:

$$\mathbf{e}^{(Q)}(t) = [e_1^{(Q)}(t), \ldots, e_P^{(Q)}(t)]^{\mathrm{T}} \quad (26)$$

contains $P = (Q+1)N$ components defined as:

$$e_{n(Q+1)+q+1}^{(Q)}(t) = b_{q,Q}\left(\frac{t-t_n}{T}\right), t \in [t_n, t_{n+1}), \quad (27)$$

for $n \in \{0, \ldots, N-1\}$. Let us project $\Delta\omega(t)$, $P_{\mathrm{load}}^{\mathrm{la}}(t)$, and entries of $P_{\mathcal{G}}^{\mathrm{ed}}(t) := [P_1^{\mathrm{ed}}(t), \ldots, P_G^{\mathrm{ed}}(t)]^{\mathrm{T}}$ in a function space spanned by basis functions (26) as follows:

$$\Delta\omega(t) = \boldsymbol{\Delta}\omega \mathbf{e}^{(Q)}(t), \quad \forall t \in \mathcal{T}, \quad (28)$$

$$P_{\mathrm{load}}^{\mathrm{la}}(t) = \mathbf{P}_{\mathrm{load}}^{\mathrm{la}}(t) \mathbf{e}^{(Q)}(t), \quad \forall t \in \mathcal{T}, \quad (29)$$

$$P_g^{\mathrm{ed}}(t) = \mathbf{P}_g^{\mathrm{ed}} \mathbf{e}^{(Q)}(t), \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \quad (30)$$

where $\boldsymbol{\Delta}\omega$, $\mathbf{P}_{\mathrm{load}}^{\mathrm{la}}$, and $\mathbf{P}_g^{\mathrm{ed}}$ are P-dimensional row vectors of Bernstein coefficients. Then, CTED problem (19) converts into the following linear programming problem as detailed in Appendix I:

$$\min_{\mathbf{P}_g^{\mathrm{ed}}, \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}}, \boldsymbol{\Delta}\omega} T \sum_{g \in \mathcal{G}} \Big(C_g(\underline{P}_g^{\mathrm{ed}}) + \sum_{s \in \mathcal{S}_g} \frac{\mu_{g,s} \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}} 1_P}{Q+1} \Big) \quad (31a)$$

$$\text{s.t.} \sum_{g \in \mathcal{G}} \mathbf{P}_g^{\mathrm{ed}} - (D_{\mathrm{eff}} + M_{\mathrm{eff}} \mathcal{M}\mathcal{N}) \boldsymbol{\Delta}\omega = \mathbf{P}_{\mathrm{load}}^{\mathrm{la}}, \quad (31b)$$

$$\mathbf{P}_g^{\mathrm{ed}} = \underline{P}_g^{\mathrm{ed}} 1_P^{\mathrm{T}} + \sum_{s \in \mathcal{S}_g} \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}}, \quad \forall g \in \mathcal{G}, \quad (31c)$$

$$0 \leq \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}} \leq |\widehat{P}_{g,s}^{\mathrm{ed}}| 1_P^{\mathrm{T}}, \quad \forall g \in \mathcal{G}, \, s \in \mathcal{S}_g, \quad (31d)$$

$$\underline{P}_g^{\mathrm{ed}} 1_P^{\mathrm{T}} \leq \mathbf{P}_g^{\mathrm{ed}} \leq \overline{P}_g^{\mathrm{ed}} 1_P^{\mathrm{T}}, \quad g \in \mathcal{G}, \quad (31e)$$

$$\Delta\underline{\omega} 1_P^{\mathrm{T}} \leq \boldsymbol{\Delta}\omega \leq \Delta\overline{\omega} 1_P^{\mathrm{T}}. \quad (31f)$$

In Appendix A, we provide a brief overview of how the cost function and constraints in (19) are transformed into corresponding ones above as well as definitions of all variables and parameters. The problem also includes constraints that capture interconnections between different intervals to ensure continuous differentiability of trajectories across adjacent intervals. These are omitted for brevity.

V. Simulation Results

We consider the single-area 9-bus 3-generator transmission system with one-line diagram shown in Fig. 1. The network and associated parameters are from the Western System Coordinating Council (WSCC) network [24]. Generators are connected at buses $\mathcal{G} = \{1, 2, 3\}$, and loads are connected at buses $\mathcal{L} = \{4, 5, 6, 7, 8, 9\}$. The generator cost function in the ED/CTED problem (19) is piece-wise linear, and the cost function in the AGC/OAGC problem in (20) is quadratic. We opt for different cost functions as generators may require different control effort for power adjustment in real time. The data for the network- and generator-model parameters as well as cost functions are available online at [25].

Optimization problems are solved in GAMS using CPLEX 12.6.2 solver [26]. The optimizers of the ED/CTED problem are passed to Power System Toolbox (PST) [27] in Matlab where AGC/OAGC is performed alongside dynamic simulations. Although the CTED only considers generator dynamics as described by (19b), the PST simulations include a detailed system model that considers lossy lines, turbine governor model, and a detailed two-axis machine model. A simulation period of 1 minute is considered where load takes a constant value of 230 MW during the first 20 seconds and 430 MW during the last 20 seconds intervals and grows linearly in between.

We compare the performance of the following five simulation setups that include a mix of conventional ED, conventional AGC, CTED, and OAGC:

- Case I (2-point ED + AGC): Conventional ED performed at two instances, $t = 10, 50$ secs and dispatch signals are sent to traditional AGC.
- Case II (3-point ED + AGC): Conventional ED performed at three instances, $t = 10, 30, 50$ secs and dispatch signals are sent to traditional AGC.
- Case III (3-point ED + OAGC): Conventional ED performed at three instances, $t = 10, 30, 50$ secs and dispatch signals are sent to OAGC.
- Case IV (CTED + AGC): CTED performed with the simulation period divided into 5 intervals and the dispatch signal is sent to traditional AGC.
- Case V (CTED + OAGC): CTED performed with the simulation period divided into 5 intervals and the dispatch signal is sent to OAGC.

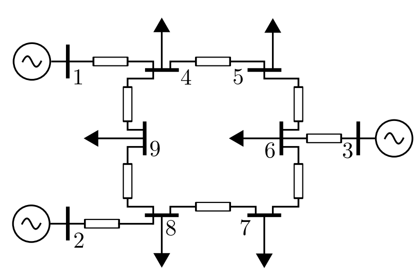

Fig. 1. Single-area 3-generator 9-bus test system [20].

A. Economic Performance

We report the dispatch and AGC costs for the five strategies in Table 1. The following are key observations that validate the proposed combined CTED+OAGC architecture.

- As we move from 2-point ED to 3-point ED to CTED, the generator dispatch cost increases, while AGC and total costs decrease. This demonstrates that as the generator dispatch resulting from the ED/CTED solution becomes more closely aligned with the load, the economic performance of the system improves.
- For all types of ED/CTED, integration with OAGC is more economical in comparison to traditional AGC.
- As we have a separate cost function for AGC that increases quadratically with respect to the real-time generation-load mismatch, the cost can be very high if the real-time power mismatch is large. We can see this from the simulation results. Particularly, for the 2-point ED case, the AGC cost is even greater than the ED cost.

TABLE I
BREAKDOWN OF TOTAL COST INTO DISPATCH AND AGC COSTS

Case	Dispatch Cost	AGC Cost	Total Cost
I	77.06	127.41	204.47
II	87.10	39.84	126.94
III	87.10	34.98	122.08
IV	96.92	3.10	100.02
V	96.92	2.61	99.53

B. Dynamic Performance

As the dispatch schedule is more closely aligned with the load in CTED, the dynamic performance of the system congruently improves. Particularly, trajectories of generator frequencies and power outputs are more smooth than those resulting from classical ED. We plot the dispatch signal resulting from ED/CTED and the actual electrical power output from generator 1 for Case II and Case IV in Fig. 2. We note that $P_g^{\mathrm{ed}\star}(t)$ has sharp changes in case of 3-point ED whereas for CTED the changes are more gradual. Also there is close match between $P_g^{\mathrm{ed}\star}(t)$ and $P_g^{\mathrm{ele}}(t)$ in CTED as compared to 3-point ED.

VI. CONCLUSIONS AND FUTURE WORK

Keeping in mind the large-scale renewable integration in the power grid, this paper proposed a dynamics-aware continuous time economic dispatch integrated with an optimal automatic generation control for power systems operations. A function space-based method was used to reduce the infinite-dimensional CTED problem to a finite-dimensional LP problem. An optimization-based control algorithm was developed to implement OAGC. The real-time dynamic simulation including OAGC was implemented in a power network with accurate nonlinear differential algebraic equation models of the system. The results show significant improvement in economic and dynamic performance of the system compared to standard ED and AGC architectures. Future work includes

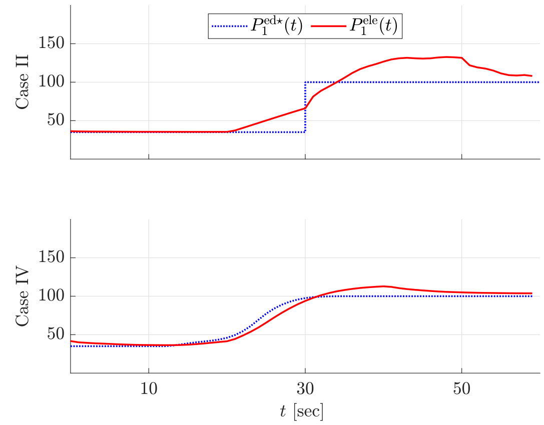

Fig. 2. Reference power input ($P_g^{\mathrm{ed}\star}(t)$) and electrical power output ($P_g^{\mathrm{ele}}(t)$) of generator 1 in MWs.

a thorough theoretical examination of CTED and OAGC. We aim to include additional constraints in CTED, uncover the interpretation of dual variables, and validate the overall CTED+OAGC strategy in a network with multiple balancing areas and under different contingencies.

APPENDIX I

A. Details of Bernstein Polynomial Projection

The details pertaining to the Bernstein polynomial representation of the CTED problem provided below are adopted from [20].

1) Cost function (31a)*:* We consider a piece-wise linear cost function for generator g with S_g linearization segments. Positive auxiliary variable trajectories $\widehat{P}_{g,s}^{\mathrm{ed}}(t)$ are associated to each of the segments $s \in \mathcal{S}_g := \{1, \ldots, S_g\}$. The dispatch trajectories of generators and the associated cost functions are expressed in linear form as below $\forall t \in \mathcal{T}, \ \forall g \in \mathcal{G}$:

$$P_g^{\mathrm{ed}}(t) = \underline{P}_g^{\mathrm{ed}} + \sum_{s \in \mathcal{S}_g} \widehat{P}_{g,s}^{\mathrm{ed}}(t), \tag{32}$$

$$\widehat{C}_g(P_g^{\mathrm{ed}}(t)) = C_g(\underline{P}_g^{\mathrm{ed}}) + \sum_{s \in \mathcal{S}_g} \mu_{g,s} \widehat{P}_{g,s}^{\mathrm{ed}}(t), \tag{33}$$

where $\widehat{C}_g(P_g^{\mathrm{ed}}(t))$ and $\mu_{g,s}$ are linearized cost function of generator g and the slope of linearization segment s, respectively. Auxiliary variables $\widehat{P}_{g,s}^{\mathrm{ed}}(t)$ are projected into the Bernstein function space as

$$\widehat{P}_{g,s}^{\mathrm{ed}}(t) = \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}} \mathbf{e}^{(Q)}(t), \forall s \in \mathcal{S}_g, t \in \mathcal{T}, g \in \mathcal{G}, \tag{34}$$

where $\widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}}$ is a vector of Bernstein coefficients. It is confined to the lengths of linearization segments $|\widehat{P}_{g,s}^{\mathrm{ed}}|$ as follows $\forall g \in \mathcal{G}, \forall s \in \mathcal{S}_g$: $0 \leq \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}} \leq |\widehat{P}_{g,s}^{\mathrm{ed}}| 1_P^{\mathrm{T}}$. Substituting the function space representation of $P_g^{\mathrm{ed}}(t)$ from (30) and of $\widehat{P}_{g,s}^{\mathrm{ed}}(t)$ from (34) in (32) and eliminating the vectors of basis functions from both sides, we recast (32) as follows $\forall g \in \mathcal{G}$:

$$\mathbf{P}_g^{\mathrm{ed}} = \underline{P}_g^{\mathrm{ed}} 1_P^{\mathrm{T}} + \sum_{s \in \mathcal{S}_g} \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}}. \tag{35}$$

With the developments above, we can simplify (20a) as follows:

$$\int_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} \widehat{C}_g \left(P_g^{\mathrm{ed}}(t)\right) dt \tag{36}$$

$$= T \sum_{g\in\mathcal{G}} C_g(\underline{P}_g^{\mathrm{ed}}) + \sum_{g\in\mathcal{G}} \sum_{s\in\mathcal{S}_g} \mu_{g,s} \widehat{\mathbf{P}}_{g,s}^{\mathrm{ed}} \int_{t\in\mathcal{T}} \mathbf{e}^{(Q)}(t) dt. \tag{37}$$

The cost function in (31a) follows from applying the identity [28]: $\int_{t\in\mathcal{T}} \mathbf{e}^{(Q)}(t)dt = \frac{T}{Q+1} 1_P$.

2) Power balance constraint (31b)*:* The time derivatives of Bernstein polynomials of degree Q can be expressed as a linear combination of Bernstein polynomials of degree $Q-1$ [21]. In the context of our problem, this allows us to define $\Delta\dot{\omega}(t)$ in the space spanned by Bernstein polynomials of degree $Q-1$ for $t \in \mathcal{T}$ as:

$$\Delta\dot{\omega}(t) = \mathbf{\Delta}\omega \dot{\mathbf{e}}^{(Q)}(t) = \mathbf{\Delta}\omega \mathcal{M} \mathbf{e}^{(Q-1)}(t) =: \mathbf{\Delta}\dot{\omega} \mathbf{e}^{(Q-1)}(t),$$

where $\mathcal{M}$ is the $P \times (P-N)$ matrix that relates $\dot{\mathbf{e}}^{(Q)}(t)$ and $\mathbf{e}^{(Q-1)}(t)$ [21], and $\mathbf{\Delta}\dot{\omega}$ is the $(P-N)$-dimensional row vector of the Bernstein coefficients of $\Delta\dot{\omega}(t)$. From here, we see that $\mathbf{\Delta}\dot{\omega} = \mathbf{\Delta}\omega\mathcal{M}$. This relationship, along with (28)–(30) allows us to simplify the constraint (19b) into (31b), with $\mathcal{N}$ denoting the $(P-N) \times P$ degree-raising matrix that relates $\mathbf{e}^{(Q-1)}(t)$ to $\mathbf{e}^{(Q)}(t)$ [28].

3) Inequality constraints (31d)–(31f)*:* These follow from leveraging the convex hull property of Bernstein polynomials [21] to confine the Bernstein coefficients of (19c) and (19d) to their minimum and maximum limits.

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