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# A Performance Study of Some Transient Detectors

Zhen Wang and Peter Willett

***Abstract*—We present a simulation study of several different statistics applied to the detection of unknown transient signals in white Gaussian noise. The results suggest that relatively unsophisticated tests based on temporal localization of power, such as the Page test and a test based on a new statistic due to Nuttall, give reliable results.**

***Index Terms*—Transient detection.**

## I. Introduction

In many applications, it is desired to identify blocks of data that contain, in addition to noise, a signal of short duration. For example, in the passive sonar situation, a threat may betray itself by a single accidental report, and in the very different application of process-monitoring by acoustic emission (AE), transients may be due to the sudden release of stress (cracking). At any rate, were the transient signal known, the problem would be trivial. The interest, of course, is to develop an approach useful for signals regardless of their form, length, or location.

There are a number of techniques proposed for the detection of transients, some with simple implementation (a Page test), some more complex (for example, techniques based on the Gabor transformation), and some quite numerically-intensive indeed and, at present, not well suited to real-time application. In this correspondence, we compare numerically the performances of several of the first two types of schemes on a variety of simulated transients, each added to white Gaussian noise. We do not wish to represent that this study is exhaustive, either in terms of the transients used or the detectors tested—we have tried to make the former representative, and as to the latter, we apologize for omissions. We also have made no attempt at "tuning" beyond what has been suggested in the open literature; on the contrary, we wish to interrogate the various algorithms in situations to which they are not particularly well matched. The detectors are as follows.

*Gb: GLRT Based on the Gabor Transformation:* The signal model is $\boldsymbol{x} = \boldsymbol{S}\boldsymbol{a} + \boldsymbol{e} + \boldsymbol{w}$, in which $\boldsymbol{x}$ denotes the observations arranged in an $N$-dimensional column vector, $\boldsymbol{S}$ is a pre-chosen matrix of dimensions $N \times M_1$ whose columns consist a basis for the signal subspace, the vector $\boldsymbol{a}$ is unknown and is called the "signal descriptor," the mismatch $\boldsymbol{e}$ describes the residual, and $\boldsymbol{w}$ is white and Gaussian, with zero mean and unity variance (for convenience). Assume the signal can have frequencies in the range $[0, M_t]$, and let the sampling rate be $f_s$ and the observation interval be $N_t$ seconds. We have $N = N_t f_s$ and $M_1 = N_t(2M_t + 1)$. We let the transform matrix $\boldsymbol{R}$ be the left inverse of $\boldsymbol{S}$ and use $(\boldsymbol{R}\boldsymbol{R}^T)^{-1/2}\boldsymbol{R}$ instead, if necessary, to make $\boldsymbol{R}$ orthonormal. The generalized likelihood ratio test (GLRT) statistic is given by [4].

$$T_{GLR}(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{R}^T\boldsymbol{R}\boldsymbol{S}(\boldsymbol{S}^T\boldsymbol{R}^T\boldsymbol{R}\boldsymbol{S})^{-1}\boldsymbol{S}^T\boldsymbol{R}^T\boldsymbol{R}\boldsymbol{x} \quad (1)$$

in which the type of basis determines $\boldsymbol{S}$. Here, the basis is the Gabor transformation [9], which is most appropriate for decaying narrowband signals.

*Wd: GLRT Based on the Wavelet Transform:* The GLRT in (1) is based on the wavelet transformation [4], [1], [2]. The rows of $R$ are the sampled versions of the wavelet transform operator. Since we do not wish our results to be specific to any type of transient, we choose the Daubechies (order 4) basis. We scale to six levels.

*P2: Nuttall's Power-Law Detector:* A detector attracting interest uses the "power-law" [6]

$$T(\boldsymbol{X}) = \sum_{k=1}^{N} X_k^{\nu} \quad (2)$$

where the $\{X_k\}$ are the magnitude-squared FFT bins corresponding to the observations $x_n$. The choice $\nu = 2$ has been shown to have good robustness properties, and we refer to it as *P2*.

*Mx: Nuttall's "Maximum" Detector:* In the time domain, when both the signal duration $M$ and the average signal power $s$ are known, the processor

$$T(\boldsymbol{x}) = \sum_{m=1}^{N+1-M} \exp\left(\frac{s}{1+s}\sum_{n=m}^{M-1+m} x_n^2\right) \quad (3)$$

is optimal under the assumption that a white Gaussian transient's start-point is uniformly distributed [7]. Clearly, the signal power must be known precisely; the "maximum" processor

$$T(\boldsymbol{x}) = \max_m \left\{\sum_{n=m}^{M-1+m} x_n^2\right\} \quad (4)$$

is introduced [7] as an approximation to the previous when only the duration $M$ is known, and in our experience, little is lost by its use. In our simulation, we use $M = 128$, which, admittedly, is well matched to the signals of interest.

*EM: EM Detector:* In [10] it is proposed to model DFT data as either a single population of independent exponential random vaxiables (the transient-free situation) or as two populations. Under the additional assumption that the membership of the DFT data in these two populations is i.i.d. Bernoulli, the parameters (in fact, a mean and a Bernoulli probability since it is assumed in this correspondence that the transient-free noise power is known) may be efficiently estimated via the EM algorithm. Insertion of the result to a GLRT is straightforward.

*Pg: Page Detector:* The Page test [8] is the comparison of the *cusum* statistic

$$Z_n = \max(0, Z_{n-1} + g(x_n)) \quad (5)$$

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to a threshold $h$, with $Z_0 = 0$. In the above, $x_n$ is the $n$th (time-domain) data sample, and $g(\cdot)$ is any memoryless transformation with the practical stipulation that its mean be negative and positive, respectively, under signal-absent and signal-present statistics. In the case that a test were between i.i.d. Gaussian random processes of variance $\sigma_0^2$ and of variance $\sigma_1^2$, the nonlinearity $g(\cdot)$ should ideally be

$$g(x_n) = x_n^2 \left( \frac{1}{2\sigma_0^2} - \frac{1}{2\sigma_1^2} \right) - \log \left( \frac{\sigma_1}{\sigma_0} \right). \tag{6}$$

Application of this test to the non-i.i.d. cases examined here must be considered a mismatch, and here, we fix $\sigma_1 = 1.1$. Further, the Page scheme is designed to be sequential, meaning that a threshold-crossing, whenever one occurs, signifies a change in distribution. In order to compare the Page processor to the others, we use it in a block mode, meaning that the test statistic is simply $\max\{Z_n\}$.

At heart, all transient detectors test for inhornogeneity under signal-present conditions. The first two (GLRT-based) schemes first project the data to a subspace in which signal and noise are likely to be separated and then calculate a clever metric of sameness. Since the subspace (for example, of coarse-scale features) must be preset somewhat subjectively, these detectors are parametric and may be expected to be efficient at finding the sort of transients for which they are designed. The third and fifth structures also employ a fixed projection—the FFT—which, it is hoped, will tend to agglomerate signal energy in a few elements. However, since each transformed datum is treated equally, it is reasonable to say that these tests are non-parametric. The fourth and sixth schemes also test for inhomogeneity but do so directly in the time domain.

## II. Simulations

We show the performance of the detectors in seven situations.[1] The transients are all of short duration compared with the observation intervals. In each case, the sampling rate $f_s$ is 32 Hz, the observation interval $N_t$ is 32 s (meaning 1024 samples), the transient duration is 4 s (from 14–18 s), and the important frequency bins are in the range 0–8 Hz ($M_t = 8$). The transient signals are the sampled versions of the following descriptions.

$S_1$: **– White burst.** See Fig. 1: $s_1(t)$ is white and Gaussian from 14–18 s, with mean zero and variance as described above.

$S_2$: **– Single exponentially-decaying sinusoid.** See Fig. 2:

$$s_2(t) = e^{-\lambda t} \cos(2\pi f t + \phi) \tag{7}$$

with $\lambda = .5$, $f = 6$, and $\phi = 0$.

$S_3$: **– Sum of three exponentially-decaying sinusoids with differing onsets.** See Fig. 3:

$$s_3(t) = \sum_{k=1}^{K} a_k e^{-\lambda_k (t-\tau_k)} \cos(2\pi f_k (t-\tau_k) + \phi_k) \tag{8}$$

where

$$k = 3, \quad \lambda_k = .5, \quad a_1 = 1, \quad a_2 = 0.5, \quad a_3 = 0.8$$
$$f_1 = 3, \quad f_2 = 7, \quad f_3 = 4, \quad \tau_1 = 14, \quad \tau_2 = 15$$
$$\tau_3 = 16, \quad \phi_1 = \pi/6, \quad \phi_2 = \pi/4, \quad \phi_3 = \pi/2.$$

$S_4$: **– Sum of three modulated decaying exponentials, with identical onsets.** See Fig. 4:

$$s_4(t) = \sum_{k=1}^{K} a_k e^{-\lambda_k t} \cos(2\pi f_k (t-\tau) + \phi_k) \tag{9}$$

[1]We thank NUWC researchers [5] for their help in choosing representative transient signals.

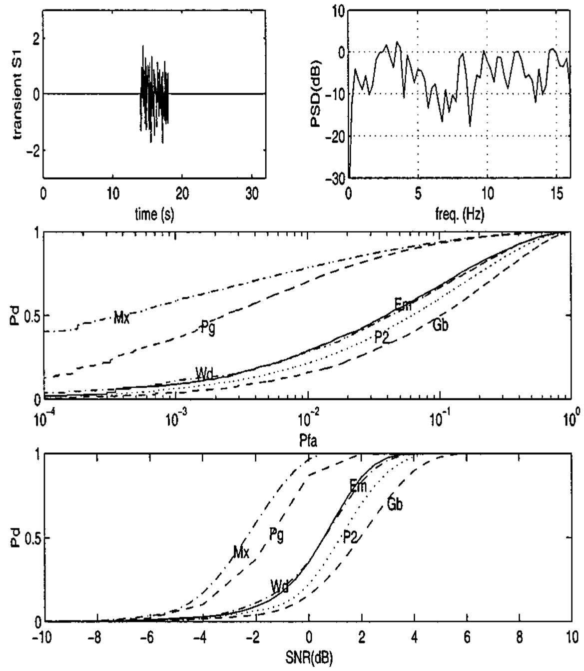


Fig. 1. Example of transient signal $S_1$: the white burst.

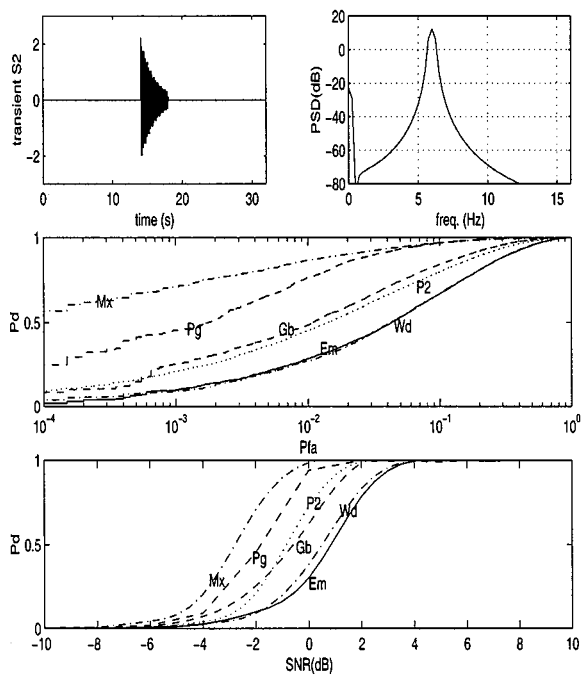


Fig. 2. Example of transient signal $S_2$: the single exponentially decaying sinusoid.

where

$$k = 3, \quad \lambda_k = .5, \quad a_1 = .5, \quad a_2 = 1, \quad a_3 = 0.7$$
$$f_1 = 8, \quad f_2 = 2, \quad f_3 = 5, \quad \tau = 14, \quad \phi_1 = \pi/6$$
$$\phi_2 = \pi/4, \quad \phi_3 = \pi/2.$$

$S_5$: **– Exponentially-enveloped white burst.** See Fig. 5:

$$s_5(t) = \exp(-\lambda t) s_1(t) \tag{10}$$

where $\lambda = 0.5$.

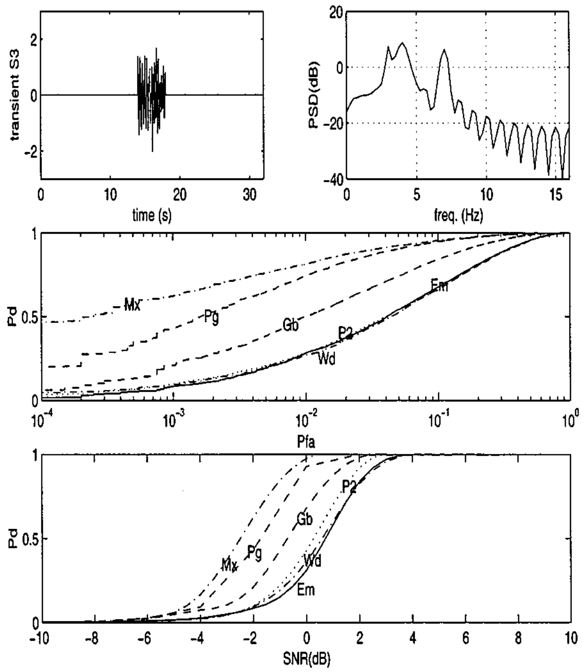


Fig. 3. Example of transient signal $S_3$: the sum of three exponentially decaying sinusoids with differing onsets.

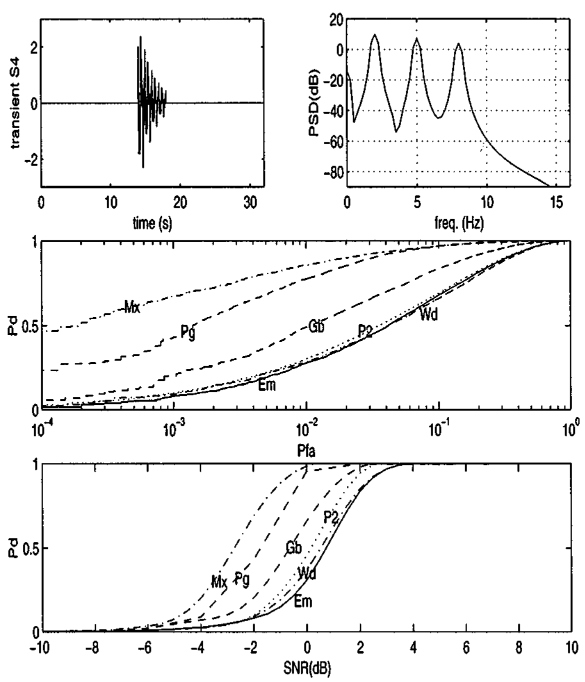


Fig. 4. Example of transient signal $S_4$: the sum of three exponentially decaying sinusoids with identical onsets.

$S_6$: – **White burst group.** See Fig. 6: $s_6(t)$ is the sum of four nonoverlapping white bursts.

$S_7$: – **Tone with a stepped frequency modulation.** See Fig. 7: $s_7(t)$ is made up of contiguous and phase-continuous sinusoids, with equal amplitude but different frequencies (these being $f_1 = 2, \quad f_2 = 6, \quad f_3 = 8, \quad f_4 = 4, \quad f_5 = 7$) and with durations 0.5, 1, .5, 1 and 1 s, respectively.

In each of the Figs. 1–7, the upper left and right plots show, respectively, the transient signal (without added noise) itself and its spectrum. The middle plot in each figure is a receiver operating characteristic (ROC) based on $10^5$ simulation runs. Here each transient signal has total energy 81, whereas the additive noise is white and Gaussian with zero mean and unity variance. Thus, the aggregate signal-to-noise ratio (SNR) is approximately $-11$ db, and if the transient's energy were uniformly distributed throughout its duration, the "while-active" SNR could be said to be $-2$ dB. The lowest plot in each figure is an alternative view of necessary SNR for a false alarm rate of $10^{-3}$. In each plot, the letters refer to the type of detector, as explained in Section I; for example, **Pg** is the Page detector. The performances are summarized in Table I. Fig. 7 is slightly different from the others in that it offers an additional example plot of the signal as used for the ROC, that is, with

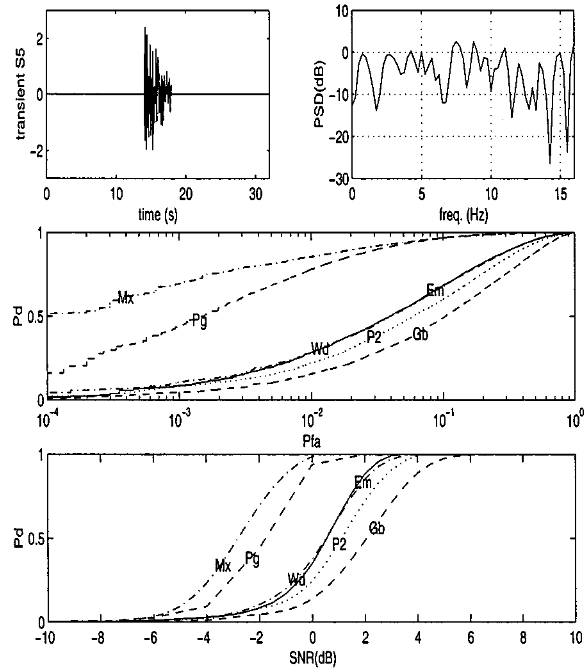


Fig. 5. Example of transient signal $S_5$: the exponentially enveloped white burst.

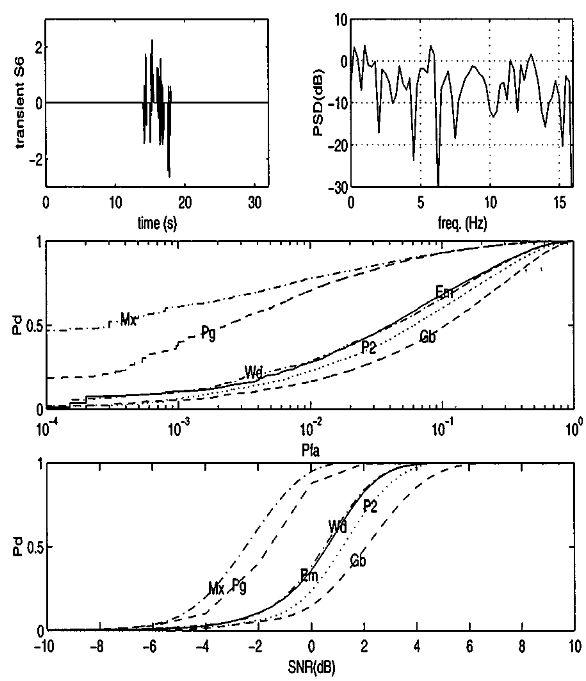


Fig. 6. Example of transient signal $S_6$: the white burst group.

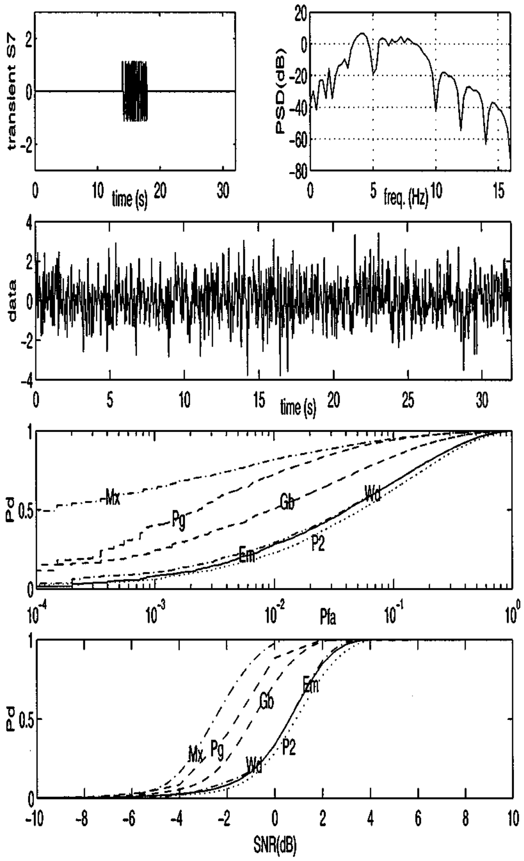


Fig. 7. Example of transient signal $S_7$: the tone with a stepped frequency modulation. The "data" trace in this plot exemplifies for all transient types the signal as it would look, with noise added, at the SNR used for the ROC simulation.

TABLE I
RANK, FROM 1 (BEST) TO 6 (WORST) OF THE VARIOUS DETECTORS IN SITUATIONS 1 THROUGH 7. DETECTORS WITH CLOSE PERFORMANCE ARE GIVEN THE SAME RANKS

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ |
|---|---|---|---|---|---|---|---|
| **Gb** | 6 | 3 | 3 | 3 | 6 | 6 | 3 |
| **Wd** | 3 | 5 | 5 | 5 | 3 | 3 | 4 |
| **P2** | 5 | 4 | 4 | 4 | 5 | 5 | 6 |
| **Mx** | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **EM** | 3 | 5 | 5 | 6 | 3 | 3 | 4 |
| **Pg** | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

a "while-active" SNR of $-2$ dB. It is, we hope, clear from this figure that the transient signals are not easy to spot.

## III. DISCUSSION

There is a remarkable consistency among the simulations. Nuttall's "maximum" detector is in all cases the best. The Page processor, despite its model of a "white" transient to be detected, also performs well, regardless of the signal's form. The performance of the Gabor detector is reasonably good and offers significant improvement over the EM-based, power-law, and wavelet-based schemes when the transient is narrowband $(S_2,\ S_3,\ S_4,\ S_7)$; however, it is inferior in "white" situations $(S_1,\ S_5,\ S_6)$. [The performance of the GLRT-based detector based on the short-time Fourier transform (STFT) is similar to Gabor.] For those white-transient situations in which the Gabor-based detector does poorly, the EM and wavelet detectors perform well and comparably to each other. The power-law detector generally has performance between those of the Gabor and EM/wavelet approaches, which is perhaps a statement of its relative robustness to the transient type—in fact, from the lowest plots of each figure, it can be seen that the performances of the power-law, Page, and "maximum" processors vary mildly against transient type. That the power-law detector is the worst among those studied when faced with the (frequency modulated) transient of type $S_7$ is not an indictment of the power law but rather an indication that the wideband-type detectors (EM and wavelet) tend to do well in this case.

Many transient detection algorithms are well suited to detecting a particular type of transient signal. This implies that these may be "tuned" for good performance in situations for which the form (shape, frequency, duration, etc.) of the possible short-duration signal is at least approximately known. Further, it is possible that a *bank* of detectors, each tuned to a particular type of signal, would offer good performance over a wide range of transients. We have not attempted this here and, indeed, have tried to be as fair as possible to the various detectors through a quite deliberate lack of such tuning.

Our goal has been to make statements about which detector performs well over a wide range of situations; our hope was that the preferred detector be simple in its implementation. Our conclusion is that both Nuttall's "maximum" processor and the variance-based Page processor are satisfactory. The former appears to be consistently superior to the latter, and since it is clear from its form that it is an intelligent modification on the Page idea, our opinion is that it should be investigated vigorously. However, we note that the parameter $M$ (the expected transient signal length) is somewhat "tuned" to the correct value. Further investigation, which is not discussed here, has indicated that performance at least equaling that of the Page detector is available, provided this parameter is within a factor of two of the correct value.

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