Performance analysis of the multi-user system

by

Wing Kwan, NG

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This is to certify that I have examined the above MPhil thesis
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Professor Vincent K. N. LAU, Research Supervisor

Professor Roger S.K. CHENG, Examination committee (Chairman)

Professor Kam Tim WOO, Examination committee

Professor Hoi-Sing KWOK, Acting Head of Department

The Department of Electronic and Computer Engineering

21st July 2008

Dedicated to my parents, elder sister, younger brother.

Special thanks to Yvonne Chan and Rachel Au-Yeung.

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Table of Contents

List of Tables

List of Figures

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The Department of Electronic and Computer Engineering

The Hong Kong University of Science and Technology

Abstract

Cross-layer design has been shown to offer high spectral efficiency which benefits from the inherent multi-user diversity in wireless fading channels. In cross-layer OFDMA systems with perfect CSIT, it is well known that the system throughput (ergodic capacity) scales in the order of $\mathcal{O}(\log\log K)$ due to the MuDiv gain. However, with imperfect CSIT, it is still not clear whether we can get the same performance as that of the perfect case.

In the first part of this thesis, we shall consider the cross-layer OFDMA scheduling design under various practical PHY layer and MAC layer constraints for a wireless system with one base station and K mobile users . We study the cross-layer scheduling design with imperfect channel state information (CSI) at the base station for delay-tolerant applications. The imperfectness of CSI is assumed to be the result of feedback or duplexing delay. With imperfect CSI at transmitter (CSIT), there exists a potential packet transmission error when the scheduled data rate exceeds the instantaneous channel capacity referring to packet outage. The OFDMA cross-layer design with delayed CSIT is modeled as an mixed integer and convex optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput (b/s/Hz successfully received by the mobiles). At the time same time, we are interested to

know the trade-off between packet outage diversity gain and multi-user diversity gain. Therefore, by using extreme value theorem, we are able to show the trade-off analytically.

In the second part of this thesis, we would like to evaluate the performance of a uplink multiaccess channel with successive interference cancellation receiver equipped in the base-station. We derive analytically the *per-user* packet outage probability and the total system goodput for multi-access systems using multiuser detector with adaptive successive interference cancellation (MUD-SIC). Slow fading channel is assumed where packet transmission error (outage) is the primary concern even if strong channel coding is applied. To capture the effect of potential packet error, goodput should be used as performance measure. Unlike previous works, our analysis focuses on the *error-propagation effects* in MUD-SIC detector where the packet outage event for a single user is depending on the other users. Also, we derive the optimal SIC decoding order (to maximize system goodput) and evaluate the closed-form *per-user* packet outage probabilities for the n users for MUD-SIC. Simulation results are used to verify the analytical expressions.

Abbreviations

AMPS	Advanced mobile phone service
AWGN	Additive white Gaussian noise
B3G	Beyond third generation cellular system
BER	Bit error rate
BPSK	Binary phase shift keying
BSC	Base station controller
cdf	Cumulative distribution function
CDMA	Code division multiple access
CSCG	Circularly symmetric complex Gaussian
CSI	Channel state information
CSIR	Channel state information at receiver
CSIT	Channel state information at transmitter
D-AMPS	Digital advanced mobile phone service
D-BLAST	Diagonal Bell layered space-time
DFE	Decision feedback equalization
DFT	Discrete Fourier transform
DS/SS	Direct sequence spread spectrum
DVB	Digital video broadcasting
EV-DO	Evolution-data optimized
EV-DV	Evolution-data and video
FDD	Frequency division duplex
FDMA	Frequency division multiple access
FER	Frame error rate

FFT	Fast Fourier transform
GSM	Global system for mobile communications
HSDPA	High-speed downlink packet access
i.i.d.	Independent and identically distributed
ISI	Inter-symbol interference
ISO	International Standard Organization
ICI	Inter-carrier interference
LAN	Local area network
LDPC	Low-density parity-check code
MAC	Media access control
MAI	Multiple access interference
MAN	Metropolitan area network
MC-CDMA	Multiple carrier-code division multiple access
ML	Maximum likelihood
MMSE	Minimum mean square error
MUD	Multiuser detection
NLOS	Non line of sight
OFDM	Orthogonal frequency division multiplexing
OFDMA	Orthogonal frequency division multiple access
OSI	Open System Interconnect
p.d.f	Probability density function
PER	Packet error rate
PHY	Physical layer
QoS	Quality of service

RRS	Round robin scheduler
SIC	Successive interference cancellation
SMS	Short Message Service
SNR	Signal to noise power ratio
TDD	Time division duplex
TDMA	Time division multiple access
UMTS	Universal mobile telecommunications system
UWB	Ultra-wideband
Wi-MAX	Worldwide interoperability for microwave access
WLAN	Wireless local area networks
WMAN	Wireless metropolitan area network

Notation

$\approx$	approximately equal to
$\log_x(y)$	the log, base x of y
$\mathbf{E}[.]$	expectation operator
$\mathbf{E}[.\vert.]$	Conditional expectation operator
$(.)^*$	complex conjugate
$(.)^\mathcal{T}$	transpose
$\mathcal{F}(.)$	Fourier transform operator
$\mathcal{F}^{-1}(.)$	Inverse Fourier transform operator
$\phi_x(s)$	Characteristic function of random variable X

Chapter 1

Introduction

1.1 Wireless Communication

Wireless Communication has been one of the most active research areas over the past decades, the fruits of these research works have benefited human beings through different wireless products. They penetrates our offices, homes and even in our pocket, they become an essential part of our life.

The first wireless communication system can be traced back to the last century. In 1895, Guglielmo Marconi [1] demonstrated the first radio transmission system by using a free propagating electromagnetic wave as a carrier. Since then, tere has been rapid progress in the wireless technology.

A century after the invention of Marconi, in the 1980s, the first generation (1G) cellular system, AMPS(advanced mobile phone service) system was developed in American and the other system TACS (Total Access Communication system) was developed in the European countries. These systems suffered from some weaknesses when compared to today's digital technologies. Since it is an analog standard, it is very susceptible to static noise and has no protection from eavesdropping using a scanner.

The second generation (2G) digital cellular systems development is due to the incompatibly of 1G system standards and the increasing demand of a mobile phone service. These factors lead to a quickly converged uniform standard (GSM) in Europe and was first implemented in Finland in 1991. Later on, IS-136 and IS-95 were implemented in the U.S which provided more options to terminal users. These kind of systems have significantly improved the spectral efficiency and network capacity to support more than voice service such as Short Message Service (SMS) and photo transmission.

Fueled by the exposition of demand for high data rate multimedia application such as video streaming and video conferencing, researcher developed a lot of high speed and high quality communication system. This includes the development of 3G systems(CDMA 2000, UMTS), 3.5G systems (HSDPA,EV-DO,EV-DV), B3G systems (Beyond 3G), wireless LAN (IEEE 802.11 a/b/g/n), ultra-wideband(UWB) systems, and WiMAX (IEEE 802.16) as well as Wi-Man (IEEE 802.20) systems. Although the listed above advance wireless technology provided much higher quality than that of the old days, the need for higher data rate grows ever faster, therefore new technology needs to be implemented in the future system.

In order to provide reliable and efficient communication over the wireless channel, researchers have been trying very hard to explore this topic since the 1950s. The reason of unreliable communication is mainly due to fading and interference. Fading refers to the time variation of the channel signal strength due to multi-path effect as well as large scaling fading [2]. On the other hand, unlike point to point wired channel, users communicate over the air and share the same channel , therefore significant interference does exist in the wireless channel.

Traditionally, various diversity techniques such as frequency diversity, time diversity are used to mitigate deep-fade situations. In addition, multiple access techniques such TDMA , FDMA together with cell sectoring are used to reduce co-channel interference in partial systems. However, these techniques can not satisfy our future need due to lack of spectral efficiently. Recent research works [3–7] have shown that cross-layer scheduling with OFDMA systems can boost up the capacity of downlink while using multi-user detection (MUD) in the uplink can achieve any points in the multiuser capacity region.

Therefore, we shall investigate the performance advantages of these two promising technologies throughout this thesis.

1.2 Literature Survey - Downlink

In [8, 9], a joint design of the MAC layer and link layer has been shown to achieve significant gains over the isolated design approach within each layer for single antenna systems as a result of multiuser diversity gain, which is achieved by scheduling transmissions (through power and rate adaption) to users when their instantaneous channel quality is near the peak. However, most of the existing literatures about cross-layer design, perfect channel state information (CSIT) is assumed to be available in the transmitter(basestation)[10–12]. The information is further assumed to be up-to-update of the scheduler finishes the scheduling of power,rate and user selection. In the above works, since CSIT is assumed to be perfect, by carefully adapting the power and rate, channel outage can be avoided as long as the error correcting code is sufficiently long and strong. As a result, ergodic capacity is a meaningful performance measure although it does not capture the outage effect.

However, with imperfect CSIT (channel state is a random variables to the transmitter) there is finite probability that the scheduled data exceeds the channel capacity and causes the packet corrupted. Furthermore, in slow fading channel, there is no significant channel variation across the encoding frame and there may be no classical Shannon meaning attached to the capacity in this situation. Therefore, ergodic capacity is no longer a suitable performance measure. Furthermore, the imperfectness of CSIT would cause 'mis-scheduling" which would degrade the system performance significantly. All in all, imperfectness of CSIT should be taken into consideration in the system design.

Besides, in the modern wireless communication system, it is a mix of real-time traffic (voice,multimedia teleconferencing, and games) and data traffic(file transfer and email). All of these applications require widely varying and very diverse quality of service (QoS) guarantees for the different types of offered traffic. In [13], a cross-layer scheduler desgin for multimedia applications in adaptive wireless networks is developed, novel admission and scheduling police is introduced to provide useful analytical results in terms of throughput, packet loss rate and average delay. In [14], the authors first introduce a theoretical framework for cross-layer optimization for orthogonal frequency division multiplexing(OFDM), necessary and sufficient conditions for optimal subcarrier assignment and power allocation are discussed. Then they further develop effective and practical algorithms for efficient and fair resource allocation in OFDM wireless networks[15] such as sorting-search dynamic subcarrier assignment, greedy bit loading, and power allocation, as well as objective aggregation algorithms, all of these algorithms provide useful tools in the future OFDM base scheduling based system.

On the other hand, scheduling have been implemented in practical 3G systems (CDMA based system), there are four basic types of traffic classes supported by UMTS Rel 99,the scheduler is responsible for the dynamic allocation of the radio resources in terms of data rate, time duration and power levels based on the class of users (QoS requirement) over a macroscopic time scale[16].

Thus, scheduling not only has great impact in academic areas, but also have evolutional effect in the practical system.

1.3 Literature Survey - uplink

Nowadays, the performance of cellular systems are limited by interference more than by any other single effect. Unlike traditional channel noise (thermal noise), interference is caused by human-designed device, mostly from devices designed to use the same spectrum. This kind of interference is called multiple access interference (MAI). It is a main factor which limits the capacity and performance. Unfortunately, the conventional detector does not take into account the existence of MAI. It follows a single-user detection strategy in which each user is detected separately without regard for other users. For example, in practical systems such as IS-95 and 3G cellular, an interference limited system is willfully created in order to achieve capacity and universal frequency reuse. The most common receiver uses match filter for the desired user and treats all the signal of other users as noise[17], however it is strictly suboptimal in most situations from an information theory perspective[4]. Therefore, multi-user receiver(detection) is needed to further improve system capacity.

Multiuser detection (MUD) have been studied over 20 years since the mid 1980.The optimal multi-user receiver for CDMA was first introduced by Verdu in his P.H.D thesis [18]. Although MUD provides a promising result in multiaccess channel, it has not found widespread acceptance in commercial systems because the major problem with multiuser detectors is the maintenance of simplicity. Over the past two decades of study, researchers have come up with the idea of interference cancellation [19]. In [20–23], iterative interference cancellation with turbo coding have been shown to have near-single-user performance which is very attractive. However, the complexity of order of turbo MUD increases with the number of users exponentially, which is not practical. On the other hand, successive interference cancellation (SIC) has complexity only proportional to the number of users which has the same complexity order as the conventional single user detector.Furthermore, it has been shown that commercial CDMA power control algorithm can be directly applied to SIC without any modification[24]. Therefore, SIC receivers have been considered as a candidate in the future communication system. In the early works of SIC receiver [25, 26], channel estimation is assumed to be perfect and the interference is completely cancellated in each decoding stage, but this assumption is never achievable in practice. In [24], researchers begin to consider the effect of error propagation together with power control. Nevertheless , decoding order in the receiver have not been considered.

To conclude, either in both uplink MUD and downlink cross-layer scheduling, we still need more research works to bridge the gap between theoretical and practical implementation such that the next generation of communication system can fulfill the future need of users.

1.4 Problem Statement

Cross-layer design is a revolutionary technology in wireless commination system which increases the system capacity substantially. However, most existing research works assume either perfect CSI estimation or perfect feedback. Scheduler can based on the perfect information to perform user selection, power adaption and rate adaption such that multi-user diversity is fully exploited. However, perfect CSIT is difficult to obtain in practice,especially in FDD systems in which explicit feedback is required. On the other hand, in TDD systems, feedback may be delayed and therefore the estimation no longer accurate since the channel is changing. With imperfect CSIT, channel outage would occur even strong error correction code is applied to the transmission frame. To take account of the potential packet outage, we define the average system goodput the performance measure metric, which measures the average total bit/s/Hz and successfully deliverers to the receivers.

In the cross-layer design, we would like to solve the following open issues in the downlink multiuser system:

- There are two important aspects of cross-layer gains in multiuser OFDMA systems with slow fading channels. They are the *system goodput gain* (by scheduling a strongest user per subband) as well as the *packet diversity gain* (by scheduling a user to transmit on multiple independent subbands). Due to the delayed CSIT, *packet outage diversity* is important to protect the packet from potential packet outage. Yet, it is not clear how the asymptotic system goodput gain and the packet outage diversity tradeoff with each other.

- How would the system goodput be affected by CSIT errors and the number of resolvable multipaths in the frequency selective fading channel?

On the other hand, in the uplink transmission, optimal MUD is a well-known technology that allows us to achieve any points in the multi-access capacity region from the information theory perspective. Due to the exponential complexity in the implementation of optimal MUD, successive interference cancellation have been proposed which has only linear complexity. Furthermore, it can be shown that MMSE-SIC is able to achieve corner points of the multi-access capacity region. However, this attractive technology comes with other research problems such as optimal power control, rate adaption , error propagation effect and decoding order.

In the uplink multiuser system with successive interference cancellation receiver , we would like to address the following issues:

- In slow fading channel without CSIT, outage occurs with non-zero probability. With the consideration of error-propagation effects and per user outage constraint, what is the maximum achievable capacity in the multiuser system?

1.5 Thesis Contributions

The central subject of interest of this thesis is the performance analysis in the future uplink and downlink transmission technologies. In the first part, we would like to focus on the downlink OFDMA scheduling system design, a systematic framework to address the scheduling where the presence of outdated CSIT is proposed. Because of the imperfect CSIT , outage occurs with non-zero probability. To address this problem, one way is to take into account the estimation error, and the other way is to schedule a user with multiple independent sub-bands such that the transmit information has diversity protection. This is further explained in the following:

- **Tradeoff between Cross-Layer Goodput Gain and Outage Diversity**: The OFDMA cross-layer design with delayed CSIT is modeled as an optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput (b/s/Hz successfully received by the mobiles). We derive simple closed-form expressions for the power and rate allocations as well as the asymptotic order of growth in system goodput for general CSIT error. From the analytical asymptotic analysis, tradeoff between the *system goodput gain* and the *packet outage diversity gain* in cross-layer OFDMA systems with delayed CSIT is illustrated.

In the second part, we aim at deriving a analytical expression to calculate the system performance for an uplink environment with multi-user detector equipped in the basestation. Optimal MUD is a promising technology which allows us to achieve any points in the multi-access capacity region from the information theory point of view. However, due to high complexity in the implementation, researchers have been searching for the past two decades for sub-optimal solutions but with guaranteed performance. In all of the substitutes, successive interference is shown to be optimal in the sense that it can achieve the corner point of the multi-access capacity region, and it only has linear complexity with the number of users. Therefore, in practice, we would like to find out the system performance when we consider some implementations details such as decoding order. The construction of the uplink parts is summarized as follows:

- **Per user outage analysis with Multiuser detection - successive interference cancellation :** We analytically derive the *per-user* packet outage probability and the total system goodput for multi-access systems using multiuser detector with adaptive successive interference cancellation (MUD-SIC). We consider a multiuser wireless system with n mobile users and a base station. We assume slow fading channels where packet transmission error (outage) is the primary concern even if strong channel coding is applied. To capture the effect of potential packet error, we consider the average packet error probability and the total system goodput of the n users, which measures the average b/s/Hz successfully delivered to the base station. Unlike previous works, our analysis focus on the *error-propagation effects* in MUD-SIC detector where the packet outage event for the i-th decoded user is coupled with that in the $i-1, .., 1$-th decoding attempts. We shall derive the optimal SIC decoding order (to maximize system goodput) and evaluate the closed-form *per-user* packet outage probabilities for the n users for MUD-SIC. Simulation results are used to verify the analytical expressions.

1.6 Thesis Outline

The remainder of the thesis is organized as follows.

Chapter 2 introduces the background material, including the basic theory of wireless communication, general cross-layer scheduling model and the multiuser detection technology.

Chapter 3 presents the downlink scheduling and rate adaptation and investigates the system performance with imperfect CSIT consideration. Asymptotic analysis is used to investigate the trade-off between system goodput gain and packet outage diversity in the cross-layer OFDMA design.

Chapter 4 describes a multiuser wireless system with n mobile users and a base station. We derive an analytical upper bound of per user outage probability and a lower bound of average system goodput which provides useful insight in the future system design.

Finally, we give some concluding remarks in Chapter 5.

1.7 Author's Publications List

Journal paper:

1. V. K. N. Lau, **W.K. Ng**," Per-User Packet Outage Analysis in Slow Multi-access Fading Channels with Successive Interference Cancellation for Equal Rate Applications " *IEEE Transactions on Wireless Communications*, accepted, Sep 2007.

2. V. K. N. Lau, **W.K. Ng** , David S. W. Hui , "Asymptotic Tradeoff between Cross-Layer Goodput Gain and Outage Diversity in OFDMA Systems with Slow Fading and Delayed CSIT",*IEEE Transactions on Wireless Communications*, accepted, January 2008.

3. M.Z, **W.K. Ng**, V.K.N Lau and C.T Lea, "Cross-Layer Scheduling under Mobility", submitted to *IEEE Transactions on Wireless Communication*, under review, March 2008.

4. **W.K. Ng**, V.K.N Lau, Design and Analysis of Outage-Limited Multi-access Cellular Systems with Macro-diversity. , drafting. May 2008.

Conference paper:

1. V. K. N. Lau, **W.K. Ng** , David S. W. Hui ,"Asymptotic Tradeoff between Cross-Layer Goodput Gain and Outage Diversity in OFDMA Systems with Slow Fading and Delayed CSIT", in Proceedings of the 2007 *IEEE International Symposium on Information Theory (ISIT 2007)*, Nice, France, June 2007, pp 2756-2760

2. V. K. N. Lau, **W.K. Ng** , David S. W. Hu, Bin Chen ,"Cross-Layer Optimization for OFDMA System with Imperfect CSIT in Quasi Static Channel", *International Conference on Communications and Networking in China 2008*, accepted March 2008.

Patent:

1. Ming Y. Tsang, Chi-Tin Luk, **Wing-Kwan Ng**, Vincent K. N. Lau , C.Y.Tsui and Roger S. K. Cheng, "Robust timing Synchronizations in MB-OFDM Frequency hopping System in SOP environment",filed in Sep. 2007

Chapter 2

Background

2.1 Introduction

The goal of this chapter is to give an overview of the wireless channel characterization and the modern technology to achieve high data rate communication. In particular, we would like to introduce the fundamental theory of OFDMA, cross-layer scheduling and successive interference cancellation as well as the difficulties in practical application.

2.1.1 Wireless Channel

The wireless channel poses a challenge as a medium for reliable high-speed-communication. Not only are noise and interference harmful to the transmission link, but also the unpredictable nature of channel variation. Modelling the radio channel has historically been one of the most difficult parts of mobile system design, and it is typically done in a statistical manner. To have a good understanding of the wireless channel, we would like to introduce the following two types of fading:

Large scale fading

Traditional path loss models are aimed on predicting the average received signal power at a given distance from the transmitter, and used to approximate the wave propagation according to Maxwell's equation. Conventional model such as free-space path loss does not consider the shadowing effect which is caused by obstacles between the transmitter and the receiver.

Measurements have shown that at any value d, the path loss at a practical location is random and with log-normal distribution above the mean path loss value. The statical equation which combines the path loss and shadowing effect is given in the following equation and can be found in reference [27]:

$$PL(d) = PL(d_0) + 10n \log \left(\frac{d}{d_0} \right) + X_\sigma, \tag{2.1.1}$$

where d_0 is the reference distance; n is the path loss exponent which indicate the rate of signal power drop beyond the reference distance; X_σ is a zero-mean Gaussian distributed random variable (in dB) with standard deviation σ (in dB), which is called *log-normal shadowing*. The log-normal distribution describes the random shadowing effects which occur over a larger number of measurement locations which have the same transmitter and receiver distance, but have different levels of clutter on the propagation path. The combined path loss and shadowing information provides the system designer with a useful reference to calculate the cell coverage and also the link budget (frequency reuse factor).

Small scale fading

In the small scale fading, the signal strength fluctuates over 30dB within a short period of time(in the order of milli-second) or travel distance (length of a few wavelengths). Small scale fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver ar slightly different times. These multipaths combined at the receiver antenna and result in either constructive or destructive interference and cause the fluctuation of received signal. There are two independent dimensions in small scale fading, they are listed below:

- **Multipath dimension:** To quantify the multipath dimension in microscopic fading, we can either look at the delay spread or coherence bandwidth. Root mean square (rms) delay spread (σ_τ) is defined as the range of multipath components with significant power. A linearly modulated signal with symbol period T_s experiences significant intersymbol interference (ISI) if $T_s << \sigma_\tau$. Conversely, when $T_s >> \sigma_\tau$ the system experiences negligible ISI.

  On the other hand, the multipath can also characterize in frequency domain by introducing the concepts of coherence bandwidth B_c.

  In general, if we are transmitting a narrow-band signal with bandwidth $BW << B_c$, then the fading across the entire signal bandwidth is highly correlated. This is usually referred to as the *flat fading*. On the other hand, if the signal bandwidth $BW >> B_c$, the the channel amplitude values at frequencies separated by more than the coherence bandwidth are roughly independent. In this case the fading is called *frequency selective fading* .It is found that for 0.5 correlation coefficient between two separated frequency amplitudes, the coherence bandwidth is related to delay spread by:

$$B_c = \frac{1}{\sigma_\tau} \tag{2.1.2}$$

- **Time-varying dimension:** To characterize the time variation dimension, we can have either the Doppler spread or the coherence time. The time variation nature of channel that arise form transmitter, receiver or mobility of the surrounding obstacles cause a doppler shift in the received signal and result in a doppler spread. The maximum Doppler spread is given by:

$$f_D = \frac{v}{\lambda} \tag{2.1.3}$$

where λ is the wavelength of the signal and v is the maximum speed between the mobile and the base station. Similar with the case of multipath dimension, we have an equivalent parameter to quantify the time variation dimension of microscopic fading which is coherence time T_c. It can be shown that, with 0.5 correlation coefficient, the coherence time can be approximated by:

$$T_c \approx \frac{9}{16\pi f_d} \tag{2.1.4}$$

A signal with symbol period T_s experiences fast fading if $T_s >> T_c$ and the signal experiences slow fading if $T_s << T_c$.

Practical consideration

In this sub-section, we would like to give an overview on how the large scale fading as well as small scale fading affect the system design. Actually, system performance of wireless communication is heavily depends on the channel condition. For example, path loss exponent indicate the rate of signal decrease as the distance between transmitter and receiver increase. Intuitively, high path loss exponent is bad because the transmitter has to increase it's transmit power to main the received power level as the distance increases, which result in extra power usage and shortens the transmission range. This intuition is generally hold in *noise limited* communication such as point to point digital link. However, it's not the case in *interference limited* system such as cellular CDMA and GSM. In the interference limited system, high path loss exponent although results in weaker received signal in the deserved receiver. This also reduce the co-channel interference for the users in other cells. Because of this, aggressive frequency resource reuse can be realized in the cell planning which turns out to be of benefit in term of system capacity.

On the other hand, shadowing introduces randomness in the coverage of cellular systems. If the standard derivation of the shadowing component is large, the average received signal power at the cell edge will have large fluctuations. In order to maintain a certain QoS to the users, we have to transmit a higher power or shorten the cell radius to allow for some shadowing margins.

Finally, the effects of microscopic fading have high impact on the physical design of communication systems. In order to support high data rate transmission , signal may transmit with larger bandwidth in order to maintain the system performance. As the signal bandwidth increases, it will likely see a frequency selective fading channel rather than a flat fading channel. Frequency selective fading will introduce intersymbol interference (ISI) and this induces irreducible error floor. Hence, complex equalization at the receiver is needed.

To overcome the combined effect of fading, noise and signal interference, various

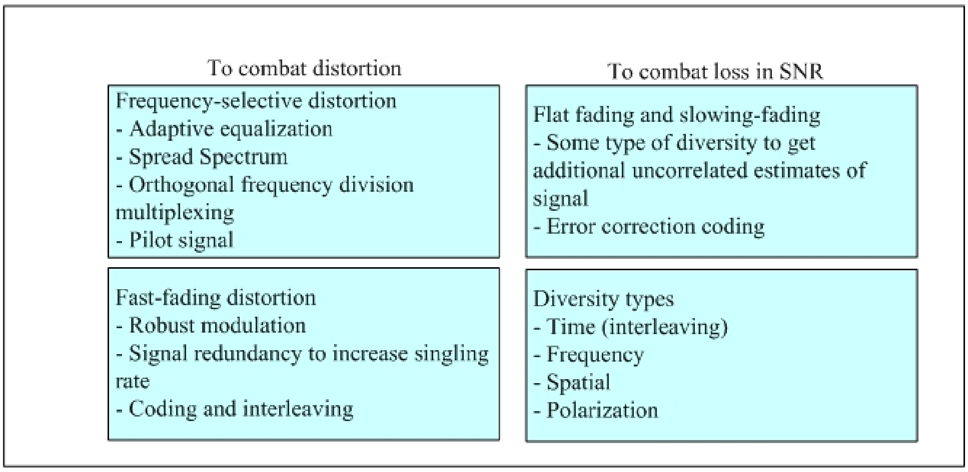

Figure 2.1: Basic mitigation types

techniques have been proposed [28]. The typical mitigation methods are summarized in Figure 2.1.

2.1.2 Orthogonal frequency division multiple access-OFDMA

In this sub-section, we would like to introduce a modern wireless communication technology - OFDMA. OFDMA is a multiaccess scheme which is based on Orthogonal frequency division multiplexing (OFDM). Nowadays, many applications have adopted OFDM(A) technique to improve spectral efficiency such as IEEE 802.11a/g/n and 802.16e (WiMax). Although the technologies are new to terminal users, the principle for multi-channel transmission over a bandlimited channel was proposed in 1966 [29]. Thanks to Weinstein and Ebert who introduced Discrete Fourier Transform (DFT) [30], nowadays OFDM can be implemented in a efficient way byadvance VLSI technology.

The idea of OFDM is to split a high-rate data stream into a number of low rate stream that are transmitted simultaneously over the number of subcarrier.

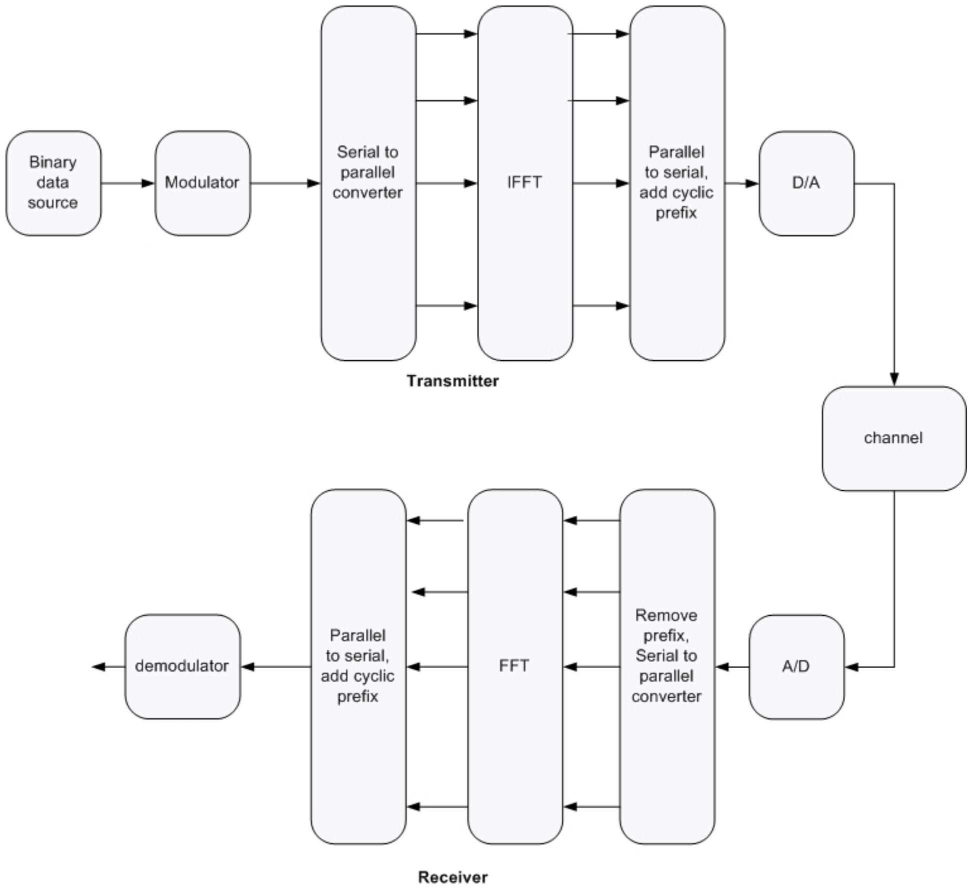

Figure 2.2: OFDM system base-band implementation

Unlike the traditional frequency division multiplexing, overlapping in the subcarriers(subbands) are allowed in the orthogonal way to improve spectral efficiency.

Since a wide-band frequency selected channel is divided into many small pieces of sub-channels, each sub-channel experiences a *flat fading channel*. In order to prevent inter-symbol interference (ISI) and inter-carrier interference(ICI), a cyclic prefix of suffix is usually attached to each OFDM symbol. A typical OFDM system with IFFT/FFT implementation is shown in Figure 2.2.

Based on the structure of OFDM, we can define the multiple access scheme by assigning subsets of subcarriers to individual users as shown in the Figure 2.3. This allows simultaneous low data rate transmission from several users.

The main advantages of OFDMA are summarized below:

- Enables adaptive modulation for every user.
- Frequency diversity can be achieved by spreading the carriers over all the used spectrum (OFDM-CDMA).
- Enables orthogonality in the uplink by synchronizing users in time and frequency.
- Multiuser diversity can be achieved if subcarriers allocation is based on the

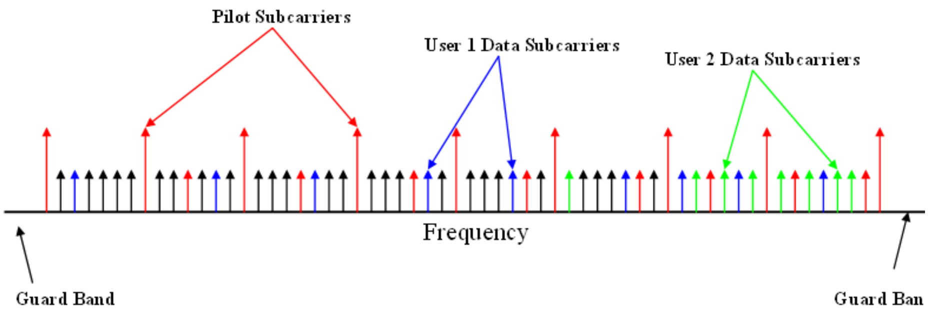

Figure 2.3: OFDMA scheduling block diagram

channel state information.

2.1.3 Cross-layer scheduling

A communication link can be viewed as a hierarchy of layers as shown in Figure2.4. Traditionally, each layer performs a well-defined task and communication system design is based on an isolated approach of each layer. This isolated approach work well for the time invariant channel. However, for the time-varying channel such as wireless channel, cooperation of each layers is needed to exploit the time-varying nature of the channel and enhance the wireless system performance.

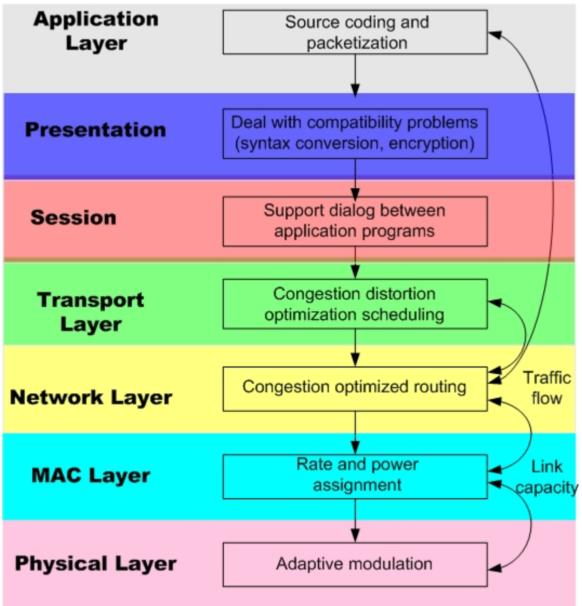

Figure 2.4: International Standard Organization (ISO)'s Open System Interconnect (OSI) reference model

In this thesis, we would like to investigate the cross-layer scheduling which involve physical layer and MAC layer joint design. The role of the physical layer is to deliver information bits across a wireless channel in an efficient and reliable manner given a limited resource. Resource in this context refers to the bandwidth

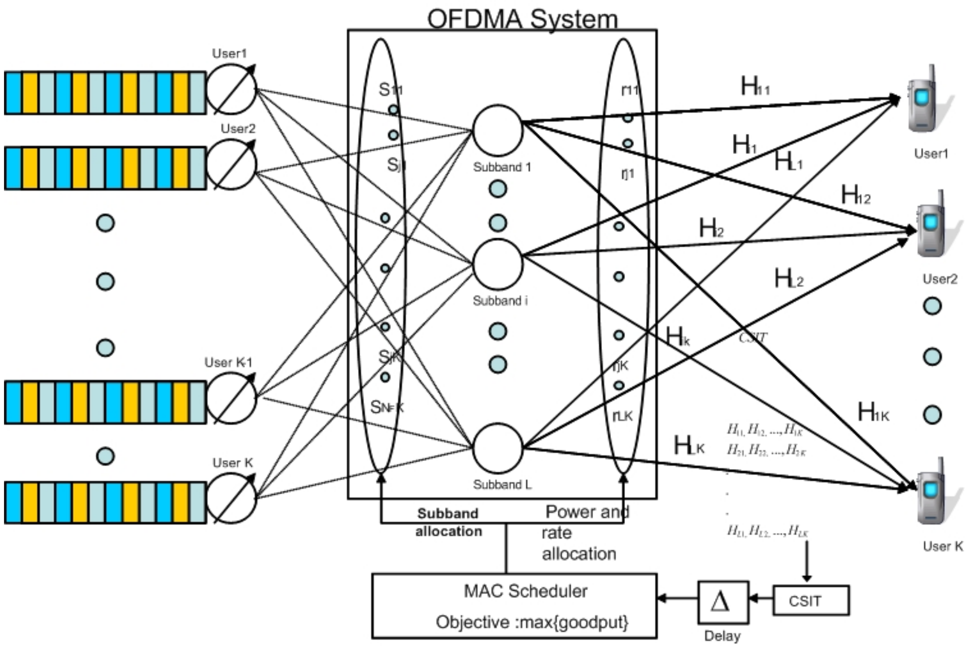

Figure 2.5: OFDMA scheduling block diagram

and transmit power; performance refers to the bit rate (bits per second) and the frame error rate. The design objective is generally to increase the bit rate at a given target frame error rate with fixed bandwidth and power budget. On the other hand, MAC layer is responsible for the rate and power allocation of each user. By the conventional optimization on MAC, the power and rate allocation is independent of the actual channel condition, which is obviously sub-optimal since the channel capacity is not fixed and not time-varying.

To illustrate the concept of scheduling, let's consider an OFDMA system with scheduling capability which is shown in Figure 2.5. We have different scheduling strategies, depending on the quality of channel state information available at the transmitter, buffer state of each user and system requirement. In general, cross-layer is typically designed to maximize the average system throughput or proportional fairness throughput with users's QoS requirement. Output of the scheduler consist of serval resource allocation policies which satisfy the objective and constraint(s). The sub-carriers allocation policy, is based on feedback information of the channel conditions and change the user-to-subcarrier assignment adaptively. If the assignment is done sufficiently fast, this further improves the OFDM robustness to fast fading and narrow-band co-channel interference, and makes it possible to achieve even better system spectral efficiency. Besides, the selected transmission power level and transmission rate policy will also be adapted to achieve the system objective. The magic behind the cross-layer scheduling is *multi-user diversity(Mu-Div)*. It improves system performance by exploiting channel fading[1][31], allocating all the system resources to the strongest user, the benefit of this strong channel is fully capitalized. Unlike the traditional diversity technique which increase the realizability of transmit information, *Mu-Div* provide a gain in the total system throughput. Furthermore, it is not a technique to reduce the effect of fading, but to take advantage on the fluctuation of channel.

2.1.4 Successive interference cancellation

Conventional single user detectors operate by enhancing a desired user while suppressing other users, considered as interference multiple access interference (MAI) or noise. A different viewpoint is to consider other users not as noise but to jointly detect all users' signals (multiuser detection). This has significant potential of increasing capacity and near/far resistance.[32]

Multi-user detection (MUD) is one of the most important recent advances in communication technology. This is used to deal with demodulation of the mutually

[1]Channel fluctuations due to fading assume that, if the fading condition of all users are independent, then with high probability there is a user with a channel strength much larger than the mean level.

interfering digital streams of information that occurs in wireless communication. The optimal design of CDMA multi-user receiver can be dated back to 1980. However, the complexity of this kind of multi-user detector is very high and it stills not practical to employ in commercial systems. Therefore, researcher have been working very hard over the past two decades and some important substitutes have been invented . They are listed in table 2.1 with illustration of complexity:

MUD type	Complexity	Latency	Error-correction code
Optimal max. likelihood	2^K	1	Separate
Linear	K to K^3	1	Separate
Turbo	PK to 2^K	$2P$	Integrated
Parallel IC	PK	P	Integrated
Successive IC	K	K	Integrated
Nonorth. matched fiter	K	1	Separate
Orth. matched filter	K	1	Separate

Table 2.1: general trends of different multiuser receivers, with spreading factor N, number of users K, and P receiver stages

Although there are a lot of multi-user detectors, we would like to focus on successive interference cancellation. SIC is a promising future multi-user detection technique. Unlike the exponential complexity in the optimal detector, it provides a linear complexity with the number of users. The approach of successive decoding is to decode a user first, then re-encoding the decoded bits and after making an estimate of the channel, the interfering signal is recreated at the receiver and subtracted from the received waveform. SIC benefits users who are in the later decoding stage as the MAI from the previous users are cancellated. Because users in the later stage do not experience MAI, they can transmit less power to maintain the same system performance and cause less MAI for the initial users. Therefore, SIC can reduce the interference from all perspective users .

Even though SIC provide a promising gain in terms of system performance, there are some technique issues that need to solved before we can apply them to practical system. First, we need to find out to implement SIC into existing system with existing power control. Convectional concept of power control, equal received power of all users in the receiver antenna, may not be optimal for SIC receiver since this kind of receiver takes advantage of the disparity of received powers. Second, we want to identify the error propagation. Decoding errors are cumulative, therefore users in the last decoding stage may face the largest decoding error and the imperfect cancellation may create extra interference . Last but not the least, the signal in each stage may induce correlations and colored noise because of uncancelled interference. Therefore, further research works are needed before applying this technology in practice.

Chapter 3

Cross-Layer optimization and analysis

In this chapter, we would like to present a cross-layer design as an optimization problem where rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system objective. Furthermore, we should discuss the trade-off between cross-layer goodput gain and packet outage diversity gain.

3.1 Introduction

In OFDMA systems, it is well-known [6, 7] that cross-layer scheduling (by selecting a set of users with the best channel condition for each subcarrier) can substantially increase the system spectral efficiency due to multiuser diversity gain (MuDiv) on system throughput.

However, in all these works, the channel state knowledge at the base station (CSIT) is assumed to be perfect. When we have perfect CSIT, packet errors can be ignored even in slow fading channels by careful rate adaptation as well as applying strong channel coding for the transmitted packets. Hence, system performance is usually evaluated based on *ergodic capacity*. In [33], it is shown that system throughput (ergodic capacity) in cross-layer systems scales with $\mathcal{O}(\log \log K)$ for multi-users systems with perfect knowledge of CSIT at the base station where K is the number of users in the system.

However, in practice, the CSIT can never be perfect due to either the CSIT estimation noise in Time Division Duplex(TDD) systems or the outdate of CSIT due to feedback delay. When the CSIT is imperfect, there will be potential packet transmission error because of channel outage (packet outage). This happens even if powerful error correction coding is applied. Because of delayed CSIT, the instantaneous mutual information is not known precisely at the base station and hence, there is finite probability that the scheduled data rate exceeds the instantaneous mutual information, causing the transmitted packet to be corrupted. Hence, conventional performance measure by throughput (*ergodic capacity*) fails to account for the penalty of packet outage. The cross-layer design with delayed CSIT is a relatively new topic. In [34], cross-layer scheduling for OFDMA systems is analyzed using limited feedback in the CSIT. The authors also show that system throughput scales in the order of $\mathcal{O}(\log \log K)$ with one bit feedback. In [35], an opportunistic scheduling approach is proposed with rate feedbacks from the mobiles. Yet, in all these cases, due to the perfect (but partial) feedback[1] assumption, packet error (packet outage) is not an issue as long as the error correction code is sufficiently strong and hence, these works also considered ergodic capacity as the performance objective. However, when we have *delayed CSIT* in slow fading

[1]Partial feedback here refers to the limited feedback. Perfect feedback here refers to the assumption that there is no feedback errors or feedback delay in the limited feedback.

channels, packet outage is a key issue and must not be ignored in the cross-layer design or performance analysis. In this case, the cross-layer *packet outage diversity* is important to protect the packet errors due to channel outage and there is a natural tradeoff between the *system goodput gain* and *packet outage diversity* in cross-layer systems. In [36], the authors established a theoretical framework for the fundamental tradeoff of spatial diversity and spatial multiplexing gain in point-to-point MIMO systems. In [37], the authors extended the framework to consider multiuser (uplink) systems. However, in all these works, no knowledge of CSIT is assumed at the base station. Furthermore, flat fading channel is considered and hence, the results cannot be applied in our case with delayed CSIT and frequency selective fading channels. As far as we are aware, the followings are some open fundamental questions remained to be answered for cross-layer OFDMA systems with delayed-CSIT.

- There are two important aspects of cross-layer gains in multiuser OFDMA systems with slow fading channels. They are the *system goodput gain* (by scheduling a strongest user per subband) as well as the *packet diversity gain* (by scheduling a user to transmit on multiple independent subbands). Due to the delayed CSIT, *packet outage diversity* is important to protect the packet from potential packet outage. Yet, it is not clear how the asymptotic system goodput gain and the packet outage diversity tradeoff with each other.

- How would the system goodput be affected by CSIT errors and the number of resolvable multipaths in the frequency selective fading channel?

In this chapter, asymptotic tradeoff analysis between the *system goodput gain* and the *packet outage diversity gain* in cross-layer OFDMA [2] systems with slow frequency selective fading and delayed CSIT are focused. The OFDMA cross-layer design with delayed CSIT is modeled as an optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput (b/s/Hz successfully received by the mobiles). We derived simple closed-form expressions for the power and rate allocations as well as the asymptotic order of growth in system goodput for general CSIT error $\sigma_e^2 \in [0, 1)$.

3.1.1 System model

In this chapter, we shall adopt the following convention. $\mathbf{X}$ denotes a matrix and $\mathbf{x}$ denotes a vector. $\mathbf{X}^\dagger$ denotes matrix transpose and $\mathbf{X}^H$ denotes matrix hermitian.

3.1.2 Frequency Selective Fading Channel Model and Delayed CSIT Model

We consider a downlink transmission in OFDMA system. The channel is assumed to be time-invariant, frequency selective channel model. The number of resolvable paths are approximately $L = \left\lfloor \frac{W}{\Delta f_c} \right\rfloor$, where W is the signal bandwidth and Δf_c is the coherence bandwidth. Consider a time-invariant L-tap delay line channel model, the channel impulse response between the base station and the k-th user is given by:

$$h(\tau; k) = \sum_{n=0}^{L-1} h_n^{(k)} \delta(\tau - \frac{n}{W}) \tag{3.1.1}$$

[2]The OFDMA system in our chapter is a concrete example to demonstrate the idea of the chapter. Actually, our analysis technique and concept in the trade-off between diversity and goodput can be generalized and applied to many systems which support scheduling.

where $\{h_n^{(k)}\}$ are modeled as independent identically distributed (i.i.d.) complex Gaussian circularly symmetric random variables with zero mean and variance $\frac{1}{L}$. Therefore, the received signal of the k-th user can be represented as the follow:

$$y_k(t) = \sum_{n=0}^{L-1} h_n^{(k)} x(t - \frac{n}{W}) + n(t) \tag{3.1.2}$$

where $x(t)$ is the transmitted signal from the base station and $n(t)$ is complex white Gaussian noise with density N_0.

Using n_F-point IFFT and FFT in the OFDMA system, the equivalent discrete channel model in the frequency domain (after removing the cyclic prefix with length L) is:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k \tag{3.1.3}$$

where $\mathbf{x}$ and $\mathbf{y}_k$ are $n_F \times 1$ transmit and receive vectors and $\mathbf{n}_k$ is the $n_F \times 1$ i.i.d. complex Gaussian channel noise vector with zero mean and normalized covariance $E[\mathbf{n}_k \mathbf{n}_k^H] = 1/n_F$ (so that the total noise power across the n_F subcarriers is unity). $\mathbf{H}_k$ is the $n_F \times n_F$ diagonal channel matrix between the base station and the k-th user $\mathbf{H}_k = diag\left[H_0^{(k)}, ..., H_{n_F-1}^{(k)}\right]$, where $H_m^{(k)} = \sum_{l=0}^{L-1} h_l^{(k)} e^{\frac{-j2\pi lm}{n_F}}, \forall m \in \{0, ..., n_F - 1\}$ are the FFT of the time-domain channel taps $\{h_0^{(k)}, ..., h_{L-1}^{(k)}\}$. Since $H_m^{(k)}$ is a linear combination of Gaussian random variables, $\{H_0^{(k)}, .., H_{n_F-1}^{(k)}\}$ are circularly symmetric complex Gaussian random variables with zero mean and the correlation between $H_m^{(k)}$ and $H_n^{(k)}$ is

$$E\left[H_m^{(k)} H_n^{(k)H}\right] = \frac{1}{L} \frac{1 - e^{\frac{-2j\pi L(m-n)}{n_F}}}{1 - e^{\frac{-2j\pi(m-n)}{n_F}}} = \eta_{k,m,n} \tag{3.1.4}$$

Observe that $\eta_{k,m,n} = 0$ when $(m-n)L$ is integer multiple of n_F. Hence, we can divide $\{H_0^{(k)}, .., H_{n_F-1}^{(k)}\}$ into $L_s = n_F/L$ groups, where each group has L i.i.d.

elements, as follows:

$$\underbrace{\begin{bmatrix} H_0^{(k)} \\ H_{L_s}^{(k)} \\ \vdots \\ H_{(L-1)L_s}^{(k)} \end{bmatrix}}_{\mathbf{H}_0^{(k)}} \underbrace{\begin{bmatrix} H_1^{(k)} \\ H_{L_s+1}^{(k)} \\ \vdots \\ H_{(L-1)L_s+1}^{(k)} \end{bmatrix}}_{\mathbf{H}_1^{(k)}} \cdots \underbrace{\begin{bmatrix} H_{L_s-1}^{(k)} \\ H_{2L_s-1}^{(k)} \\ \vdots \\ H_{LL_s-1}^{(k)} \end{bmatrix}}_{\mathbf{H}_{L_s-1}^{(k)}}$$

In other words, there are L independent subbands (labelled as $m = 0, 1, 2, ..., L-1$) in the n_F-subcarriers with L_s correlated subcarriers in each subband.

The CSI at the base station transmitter (CSIT) is obtained from either explicit feedback (FDD systems) or implicit feedback (TDD systems) using channel reciprocity between uplink and downlink. Yet, in either case, the CSIT is outdated which resulted from feedback or duplexing delay. Hence, for simplicity, we consider TDD systems (with channel reciprocity) and assume the CSIR is perfect but the CSIT is outdated. The estimated CSIT (time domain) at the base station for the k-th user is given by:

$$\hat{h}_l^{(k)} = h_l^{(k)} + \Delta h_l^{(k)} \quad \Delta h_l^{(k)} \sim CN(0, \sigma_e^2) \;\; l \in \{0, 1, .., L-1\}$$

Hence, the estimated CSIT in frequency domain (m-th subcarrier) $\hat{H}_m^{(k)}$ after n_F-point FFT of $\{\hat{h}_0^{(k)}, ..., \hat{h}_{L-1}^{(k)}\}$ is given by:

$$\hat{H}_m^{(k)} = H_m^{(k)} + \Delta H_m^{(k)} \quad \Delta\mathbf{H}_m^{(k)} \sim CN(0, \sigma_e^2) \tag{3.1.5}$$

where $H_m^{(k)}$ is the actual CSIT of the m-th subcarrier for the k-th user, $\Delta H_m^{(k)}$ represents the CSIT error which is circular symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ_e^2. The correlation of the CSIT error between the m-th and n-th subcarriers of user k is given by:

$$E\left[\Delta H_m^{(k)} \Delta H_n^{(k)H}\right] = \sigma_e^2 \frac{1 - e^{\frac{-2j\pi L(m-n)}{n_F}}}{1 - e^{\frac{-2j\pi(m-n)}{n_F}}} \tag{3.1.6}$$

Finally, the CSI between the K users are i.i.d.

3.1.3 Instantaneous Mutual Information and System Goodput

The instantaneous mutual information between the base station and the $k-$th user is given by the maximum mutual information of the channel input $\mathbf{x}$ and channel output $\mathbf{y}_k$. Let B_k denotes the set of subband indices $m = \{0, 1, ..., L-1\}$ assigned to the k-th user. Hence, the instantaneous mutual information between the base station and the k-th mobile (given the CSIR $\mathbf{H}_k$) is given by:

$$C_k = \sum_{n=0}^{L_s-1} \sum_{m \in B_k} \log_2 \left(1 + \frac{n_F p_k \left| H_{mL_s+n}^{(k)} \right|^2}{L_s N_d} \right) \tag{3.1.7}$$

where L_s is the number of correlated subcarriers in one subband, N_d is the number of independent subbands allocated to the $k-$th user and p_k is the transmit power allocated to the k-th user.

In general, packet error is contributed by two factors, namely channel noise and the channel outage. In the former case, as long as we can provide sufficient strong channel coding (e.g. LDPC) with sufficiently long block length (e.g. 10Kbytes) to protect the information, it can be shown in [38] that Shannon's capacity can be approached to within 0.04 dB for a target FER of 10^{-6}. Hence, packet errors due to the first factor is practically negligible. On the other hand, the channel outage effect is systematic and cannot be eliminated by simply using strong channel coding. This is because the instantaneous mutual information[3] $C_k(\mathbf{H}_k)$ between the base station and k-th user is a function of actual CSI $\mathbf{H}_k$, which is unknown to the base station. Hence, the packet will be corrupted whenever the scheduled data rate r_k exceeds the instantaneous mutual information C_k. Hence, for simplicity,

[3]The instantaneous mutual information represents the maximum achievable data rate for error free transmissions.

we shall model the packet error solely by the probability that the scheduled data rate exceeding the instantaneous mutual information (i.e. packet error due to the channel outage only).

Traditionally, in most existing cross-layer designs, the system performance is mostly measured by ergodic capacity and the potential packet errors (due to channel outage) is completely ignored. While this is a meaningful measure when we have perfect CSIT or when we have fast fading channels (ergodic realizations of CSI within an encoding frame), ergodic capacity fails to capture the potential packet errors, which is a very critical issue in slow fading channels (non-ergodic channel) with outdated CSIT. In order to account for potential packet errors, we shall consider the *system goodput* (b/s/Hz successfully delivered to the mobile station) as our performance measure. Since packet errors (due to channel outage) is very important to the overall goodput performance, we shall require *diversity* to protect the information from channel outage to enhance the chance of successful packet delivery to the mobile receivers in the presence of outdated CSIT. By assigning N_d independent subbands to a mobile user, we sacrifice the cross-layer goodput gain to trade for N_d order diversity protection on the packet outage probability. We first define the instantaneous goodput [39] of a packet transmission for user k (b/s/Hz successfully delivered to the k-th mobile) as

$$\rho_k = \frac{r_k}{n_F}\mathbf{1}(r_k \leq C_k) \tag{3.1.8}$$

where 1(.) is an indicator function which is 1 when the event is true and 0 otherwise. The *average total goodput*[4] is defined as the total average b/s/Hz successfully

[4]The utility function can incorporate fairness, we can modify the system utility to be another function of average goodputs such as $U_{PF}(\overline{\rho_1}, \overline{\rho_2}, ..., \overline{\rho_k}) = \sum_{i=1}^{K} log(\overline{\rho_i})$ or $U_{weight}(\overline{\rho_1}, \overline{\rho_2}, ..., \overline{\rho_k}) = \sum_{i=1}^{K} \alpha_i \overline{\rho_i}$. Then we can follow the same procedure of this chapter to a derive the scheduling algorithm which consider fairness.

delivered to the K mobiles (averaged over multiple scheduling slots) and is given by:

$$
\begin{aligned}
U_{goodput}(\mathcal{A},\mathcal{B},\mathcal{P},\mathcal{R}) &= E\left[\sum_{k=1}^{K}\rho_k\right] = \frac{1}{n_F}E_{\hat{\mathbf{H}}}\left\{\sum_{k=1}^{K} r_k E_{\mathbf{H}}\left[\mathbf{1}(r_k \le C_k|\hat{\mathbf{H}})\right]\right\} \\
&= \frac{1}{n_F}E_{\hat{\mathbf{H}}}\left\{\sum_{k=1}^{K} r_k \underbrace{\Pr[r_k \le C_k|\hat{\mathbf{H}}]}_{\text{Conditional outage prob. } P_{out}}\right\}
\end{aligned}
$$

where $\mathcal{R} = \{r_1, ..., r_K\}$ is the rate allocation policy, $\mathcal{P} = \{p_1, ..., p_K : \sum_k p_k \le P_0\}$ is the power allocation policy , $\{\mathcal{A}\}$ is the user selection policy with respect to the outdated CSIT $\hat{\mathbf{H}}$, $\{\mathcal{B}\}$ is the set of subband allocation policy with respect to N_d independent subbands and $E_{\hat{\mathbf{H}}}\{X\}$ denotes the expectation of the random variable X w.r.t $\hat{\mathbf{H}}$. These policies are formally defined in the next section.

3.2 Cross-Layer Design for OFDMA Systems

In this section, we shall formulate the cross-layer scheduling design as an optimization problem. We shall first introduce the following definitions.

Definition 3.2.1 (Rate Allocation Policy $\mathcal{R}$). Let $r_k(\widehat{\mathbf{H}})$ be the scheduled data rate of the k-th user and $\mathcal{R} = \{r_k(\widehat{\mathbf{H}}) : k \in A(\widehat{\mathbf{H}})\}$ be the *rate allocation policy.*

Definition 3.2.2 (Power Allocation Policy $\mathcal{P}$). Let $p_k(\widehat{\mathbf{H}})$ be the transmitted power of the k-th user and $\mathcal{P} = \{p_k(\widehat{\mathbf{H}}) : \sum_{k\in A(\widehat{\mathbf{H}})} p_k(\widehat{\mathbf{H}}) = P_0\}$ be the *power allocation policy* with respect to a total transmit power P_0.

Definition 3.2.3 (Admitted User Set Policy $\mathcal{A}$). Let $A(\widehat{\mathbf{H}}) = \{k \in \{1, K\} : p_k > 0\}$ be the set of admitted users (users that are assigned downlink subbands for transmitting payload) and $\mathcal{A} = \{A(\widehat{\mathbf{H}})\}$ be the *admitted user set allocation policy.*

Definition 3.2.4 (Subcarrier Allocation Policy $\mathcal{B}$). Let $B_k(\widehat{\mathbf{H}}) \subset \{0,1,2,..,L-1\}$ be the set of subband indices assigned to the k-th user for $k \in A(\widehat{\mathbf{H}})$ such that each selected user is assigned N_d independent subbands[5] and $\mathcal{B} = \{B_k(\widehat{\mathbf{H}})\}$ be the *subcarrier allocation policy* with respect to N_d independent subbands.

Definition 3.2.5 (Exponential Equality). " $\doteq$ " denotes *exponential equality.* Specifically, $f(x) \doteq g(x)$ with respect to the limit $x \rightarrow a$,$a = \{0, \infty\}$, if $\lim_{x\rightarrow a} \frac{\log f(x)}{\log g(x)} = 1$." $\dot{\geq}$ " and " $\dot{\leq}$ " are defined in similar manner.

Definition 3.2.6 (Asymptotic Upper Bound). $\mathcal{O}(g(x))$ denotes *asymptotic upper bound.* Specifically, $f(x) = \mathcal{O}(g(x))$ if $f(x) \leq Mg(x)$ $\forall x > x_0$ for some x_0 and $M > 0$.

3.2.1 Cross-Layer Design Optimization Formulation

The cross-layer scheduling algorithm is responsible for the allocation of channel resource at every scheduling slot. The base station collects the delayed CSIT from the K mobile users at the beginning of the scheduling slot and deduces the user selection (*admitted set* $A(\widehat{\mathbf{H}})$), the subband allocation $\{B_k(\widehat{\mathbf{H}}), k \in A(\widehat{\mathbf{H}})\}$, the *power allocation* $\{p_k(\widehat{\mathbf{H}}) \geq 0, k \in A(\widehat{\mathbf{H}})\}$ and the *rate allocation* $\{r_k(\widehat{\mathbf{H}}), k \in A(\widehat{\mathbf{H}})\}$ so as to optimize the total average system goodput $U_{goodput}(\mathcal{A}, \mathcal{R}, \mathcal{P}, \mathcal{B})$ at a target packet outage probability ϵ. This can be written into the following optimization problem.

[5]In the optimization problem , we impose a fixed diversity order constraint N_d into the problem and study the asymptotic performance. This is because we are interested in the asymptotic performance rather than absolute performance, although the system performance of constrained N_d will be inferior to that with dynamic N_d (changing N_d on a frame-by-frame basis), they will have the same order of growth and that's why we impose N_d as a constraint to make the system analytically tractable and study how the system performance changes with N_d.

Problem 1 (Cross-Layer Optimization Problem). *The optimal power allocation policy $\mathcal{P}^*$, rate allocation policy $\mathcal{R}^*$, user selection policy $\mathcal{A}^*$ and subband allocation policy $\mathcal{B}^*$ are given by:*

$$\begin{aligned}&(\mathcal{P}^*, \mathcal{R}^*, \mathcal{A}^*, \mathcal{B}^*) = \arg\max_{\mathcal{P},\mathcal{R},\mathcal{A},\mathcal{B}} U_{goodput}(\mathcal{A}, \mathcal{R}, \mathcal{P}, \mathcal{B}) \text{ such that}\\ &P_{out}(k, \hat{\mathbf{H}}) = \Pr\left[r_k > \sum_{n=0}^{L_s-1} \sum_{m\in B_k} \log_2\left(1 + \frac{n_F p_k}{L_s N_d}|H^{(k)}_{mL_s+n}|^2\right) |\hat{\mathbf{H}}\right] = \epsilon\end{aligned} \tag{3.2.1}$$

where L_s is the number of correlated subcarriers in one subband.

The key to solve the above optimization problem is on the modeling of the conditional packet outage probability $P_{out}(k, \hat{\mathbf{H}})$. The cumulative distribution function (cdf) of the random variable $I_k = \sum_{n=0}^{L_s-1} \sum_{m\in B_k} \log_2\left(1 + \frac{n_F p_k}{N_d L_s}|H^{(k)}_{mL_s+n}|^2\right)$ (conditioned on the delayed CSIT $\hat{\mathbf{H}}$) is in general very tedious and it is virtually impossible to obtain closed-form rate and power solutions by brute force optimization on top of the complicated expression. To obtain first order design insight and simple closed-form solutions, we shall consider asymptotic $P_{out}(k, \hat{\mathbf{H}})$ for high and low SNR. We shall summarize the results in the following lemmas.

Lemma 1 (Asymptotic Packet Outage Probability for High and Low SNR). *For both high and low SNR ($P_0 \to \infty$ or $P_0 \to 0$), the asymptotic conditional packet outage probability $P_{out}(k, \hat{\mathbf{H}})$ is given by:*

$$\begin{aligned}P_{out}(k, \hat{\mathbf{H}}) &= \Pr\left[\frac{1}{L_s}\sum_{n=0}^{L_s-1} \sum_{m\in B_k} \log_2\left(1 + \frac{n_F p_k}{L_s N_d}|H^{(k)}_{mL_s+n}|^2\right) < r_k/L_s|\hat{\mathbf{H}}\right]\\ &\doteq F_{\chi^2_k;s^2(B_k);\sigma_e^2/N_d}\left(\frac{(2^{\frac{r_k}{L_s N_d}} - 1)L_s N_d}{n_F p_k}\right)\end{aligned} \tag{3.2.2}$$

where $F_{\chi^2_k;s^2(B_k);\sigma_e^2/N_d}(x)$ is the cdf of non-central chi-square random variable $\chi^2_k = \frac{1}{N_d}\sum_{m\in B_k} |H^{(k)}_{mL_s}|^2$ with $2N_d$ degrees of freedom, non-centrality parameter $s^2(B_k) =$

$\frac{1}{N_d}\sum_{m\in B_k}|\hat{H}^{(k)}_{mL_s}|^2$ *and variance* σ_e^2/N_d.

Proof 1. *Please refer to Appendix 6.0.1.*

The optimization Problem 1 consists of a mixture of combinatorial variables $(A,\{B_k\})$ and real variables $(\{r_k\},\{p_k\})$. We shall first obtain closed-form solution for rate and power allocation for a given admitted user set A and subcarrier allocation $\{B_k\}$.

3.2.2 Closed-form Solutions for Power and Rate Allocation Policies

In this section, we shall focus on deriving the asymptotically optimal power and rate allocation solution that optimize the system goodput for a given admitted user set A and subcarrier allocation $\{B_k\}$. Using Lemma 1, the target packet outage constraint in (3.2.1) for high and low SNR is equivalent to the following:

$$P_{out}(k,\hat{\mathbf{H}})=\epsilon \Longleftrightarrow r_k=L_sN_d\log_2\left(1+\frac{n_Fp_k}{N_dL_s}F^{-1}_{\chi^2_k;s^2(B_k);\sigma_e^2/N_d}(\epsilon)\right) \tag{3.2.3}$$

Substituting the equivalent constraint (3.2.3) into the system goodput, the objective function in (3.2.1) is given by:

$$U_{goodput}(\mathcal{A},\mathcal{R},\mathcal{P},\mathcal{B})=\frac{(1-\epsilon)}{n_F}E_{\hat{\mathbf{H}}}\left[\sum_{k\in A}L_sN_d\log_2\left(1+\frac{n_Fp_k}{N_dL_s}F^{-1}_{\chi^2_k;s^2(B_k);\sigma_e^2/N_d}(\epsilon)\right)\right]. \tag{3.2.4}$$

Taking into consideration of the total transmit power constraint P_0, the Lagrangian function of the optimization problem in (3.2.1) is given by:

$$L(\{p_k\},\lambda)=\frac{(1-\epsilon)L_sN_d}{n_F}\sum_{k\in A}\log_2\left(1+\frac{n_Fp_k}{N_dL_s}F^{-1}_{\chi^2_k;s^2(B_k);\sigma_e^2/N_d}(\epsilon)\right)-\lambda p_k$$

where $\lambda > 0$ is the Lagrange multiplier with respect to the total transmit power constraint. Using standard optimization techniques, the optimal power allocation is given by:

$$p_k^* = \frac{L_s N_d}{n_F} \left(\frac{1-\epsilon}{\lambda} - \frac{1}{F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)} \right)^+ \quad \forall k \in A(\hat{\mathbf{H}}) \tag{3.2.5}$$

Substituting (3.2.5) into the equivalent packet outage constraint in (3.2.3), the optimal rate allocation r_k^* is given by:

$$r_k^* = \left[L_s N_d \log_2 \left(\frac{(1-\epsilon) F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)}{\lambda} \right) \right]^+ \quad \forall k \in A(\hat{\mathbf{H}}) \tag{3.2.6}$$

3.2.3 Low Complexity User Selection and Subcarrier Allocation Policies

In this section, we focus on the combinatorial algorithm for user selection and subcarrier allocation given a delayed CSIT $\hat{\mathbf{H}}$. Using the optimal power allocation solution in (3.2.5) and for sufficiently large average SNR constraint P_0, the Lagrange multiplier λ is given by:

$$\lambda = \frac{|A|\,(1-\varepsilon)}{n_F P_0 / N_d L_s + \sum_{k \in A} \frac{1}{F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)}} \tag{3.2.7}$$

Substituting into the rate allocation solution in (3.2.6), the conditional system goodput is given by:

$$G^*_{goodput}(A, \{B_k\}) = \frac{(1-\epsilon) N_d L_s}{n_F} \sum_{k \in A} \log_2 \left(\frac{F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)}{|A|} \left(\frac{P_0 n_F}{N_d L_s} + \sum_{i \in A} \frac{1}{F^{-1}_{\chi_i^2; s^2(B_i); \sigma_e^2/N_d}(\epsilon)} \right) \right) \tag{3.2.8}$$

The conditional system goodput $G^*_{goodput}(A, \{B_k\})$ is a function of A and $\{B_k\}$ which are combinatorial variables. The optimal A^* and $\{B_k^*\}$ can be obtained by exhaustive search over all possible combinations that maximizes $G^*_{goodput}(A, \{B_k\})$. However, such procedure has huge complexity because of two factors. Firstly, the objective function $G^*_{goodput}(A, \{B_k\})$ in (3.2.8) is difficult to compute and with coupled dependency on A and $\{B_k\}$. Secondly, the combinatorial search itself is coupled between the n_F subcarriers.

Yet, we observe that for large average SNR P_0, the term $\frac{F^{-1}_{\chi^2_k; s^2(B_k); \sigma_e^2/N_d}(\epsilon) \sum_{i \in A} \frac{1}{F^{-1}_{\chi^2_i; s^2(B_i); \sigma_e^2/N_d}(\epsilon)}}{|A|}$ is of order $\mathcal{O}(1)$ (constant order) and does not scale with P_0. Hence, for large P_0, the first term shall dominate and the conditional system goodput can be approximated by:

$$G^*_{goodput}(A, \{B_k\}) \approx \frac{(1-\epsilon) N_d L_s}{n_F} \sum_{k \in A} \log_2 \left(\frac{F^{-1}_{\chi^2_k; s^2(B_k); \sigma_e^2/N_d}(\epsilon) P_0 n_F}{N_d L_s |A|} \right) \quad (3.2.9)$$

Observe that $F^{-1}_{\chi^2_k; s^2; \sigma_e^2/N_d}(x)$ is a increasing function of s^2 for a given x. Hence, the equivalent combinatorial search problem for A and $\{B_k\}$ is given by:

$$(A^*, \{B_k^*\}) = \arg \max_{\substack{A, \{B_k\} \\ |B_k| = N_d}} \prod_{k \in A} \left[\sum_{m \in B_k} |\widehat{\mathbf{H}}^{(k)}_{m L_s}|^2 \right] \quad (3.2.10)$$

However, even with the simplified searching objective in (3.2.10), the search for A and $\{B_k\}$ are still coupled among the n_F subcarriers due to the constraint that each B_k should contain N_d independent subbands. To address the complexity issue, we shall propose a low complexity *greedy* combinatorial search algorithm to obtain the admitted user set A^* and the subcarrier allocation sets $\{B_k^*\}$. The proposed algorithm is shown to achieve close-to-optimal performance by numerical simulation which is illuistrated in Figure 3.1. The *greedy* algorithm is summarized below with flow chart illustration in Figure 3.2.

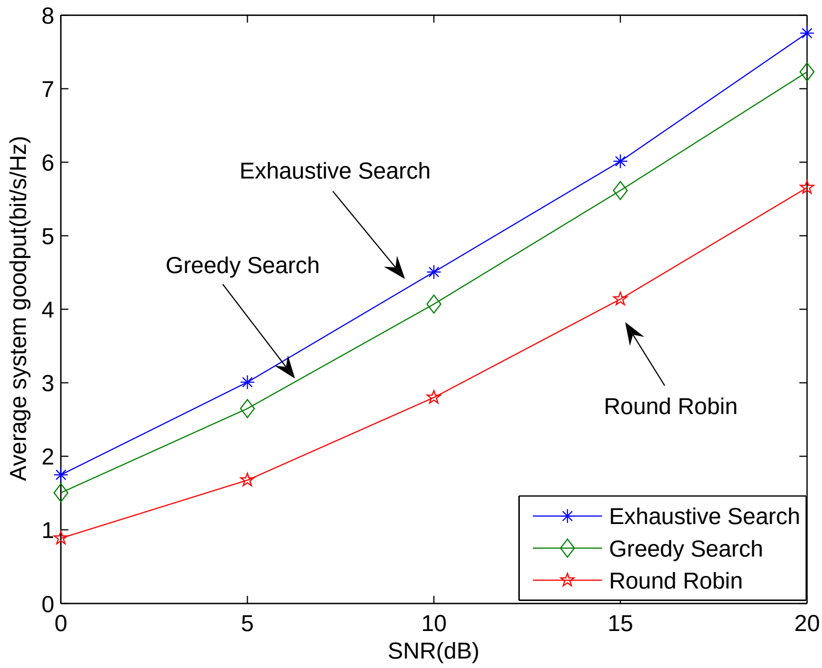

Figure 3.1: A comparison of the average system goodput versus SNR with CSIT error $\sigma_e^2 = 0.01$.

Greedy Algorithm for A and $\{B_k\}$ at high SNR:

Step 1: Initialize $A^* = \emptyset, B_k^* = \emptyset$, a user selection list $A_{selection}$ which include all user indices and a subband selection list $B_{selection}$ which include all independent subband indices.

Step 2: Initialize a temporary list T_k for all user in $A_{selection}$ to store subband indices.

$$T_k = \arg \max_{|T_k| = N_d} \left(\sum_{m \in B_{selection}} |\widehat{\mathbf{H}}_{mL_s}^{(k)}|^2 \right)$$

Step 3: Select user $k = \arg \max_{k \in A_{selection}} \left(\sum_{m \in T_k} |\widehat{\mathbf{H}}_{mL_s}^{(k)}|^2 \right)$.

Step 4: Put the selected users into set A^* and the corresponding subbands into set B_k^*.

Step 5: Remove the selected users and the selected subbands from $A_{selection}$ and $B_{selection}$ and repeated step 2 until all the independent subbands are allocated to users.

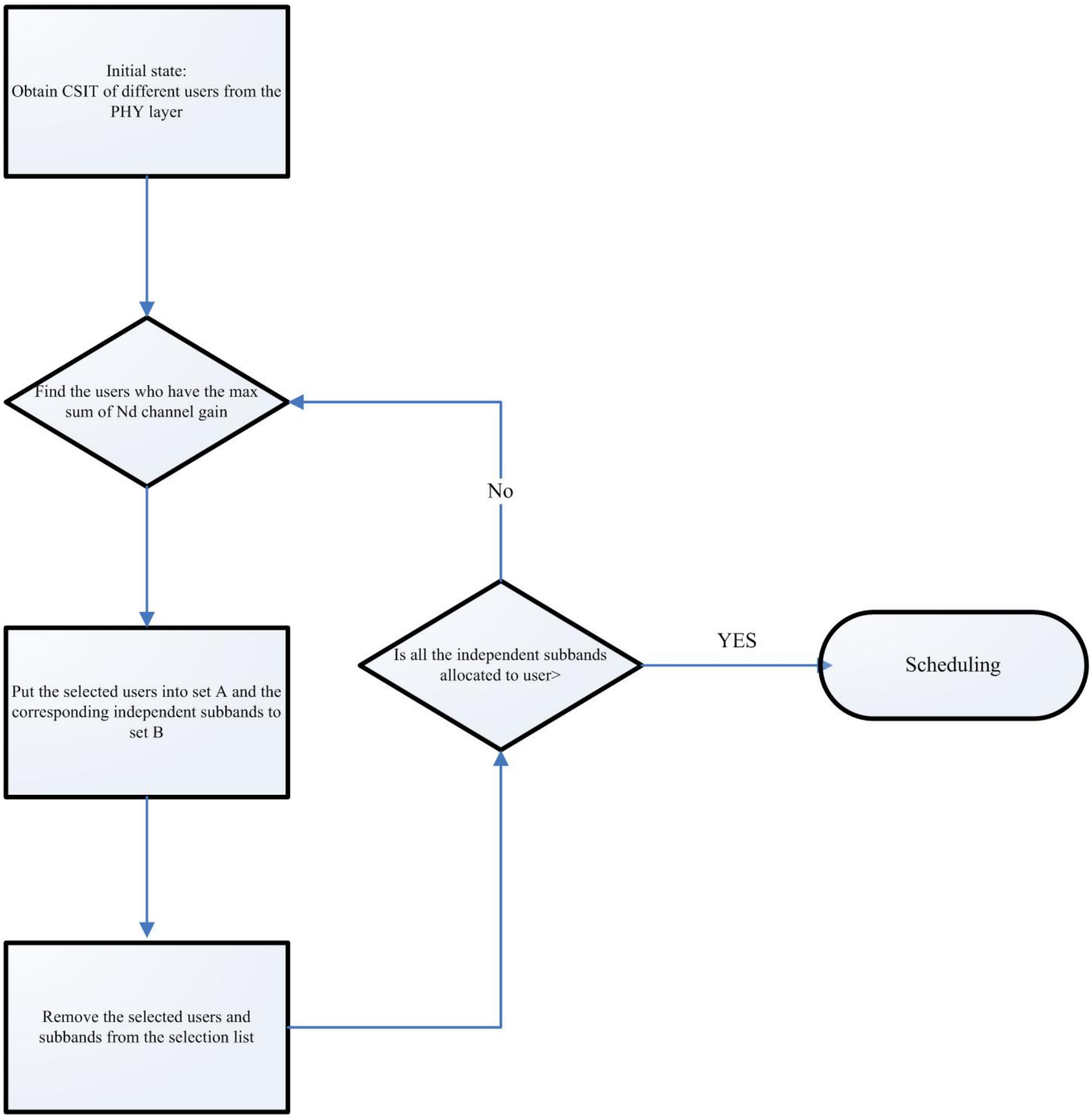

Figure 3.2: A flow chart of the Greedy cross-layer scheduling algorithm.

On the other hand, the water-filling solution in (3.2.5) for low SNR ($P_0 \to 0$) will give only one non-zero term for p_k^*. In other words, for low SNR, we have $|A| = 1$ only and the $p_k^* = P_0$ for some $k \in A$. The corresponding system goodput for low SNR is given by:

$$G^*_{goodput}(A, B_k) \approx \frac{(1-\epsilon)N_d L_s}{n_F} \log_2 \left(1 + \frac{F^{-1}_{\chi^2_k; s^2(B_k); \sigma^2_e/N_d}(\epsilon) P_0 n_F}{N_d L_s} \right) \quad \text{for } k \in A \tag{3.2.11}$$

Observe that $F^{-1}_{\chi^2_k; s^2(B_k); \sigma^2_e/N_d}(x)$ is a increasing function of s^2 for a given x. Hence, the equivalent combinatorial search problem for A and B_k is given by:

$$(A^*, B^*_k) = \arg \max_{\substack{k, B_k \\ |B_k| = N_d}} \left[\sum_{m \in B_k} |\widehat{\mathbf{H}}^{(k)}_{mL_s}|^2 \right] \tag{3.2.12}$$

In this case, the optimal combinatorial search algorithm for A and B_k in low SNR is similar to the one in high SNR, except that we only select one user with the corresponding subbbands and stop the algorithm after the first iteration.

3.3 Asymptotic Performance Analysis for Cross-Layer Design

In this section, we shall analysis asymptotically the order of growth of the average system goodput with respect to some important system parameters such as the average SNR P_0, the number of users K and the CSIT quality (CSIT error variance) σ^2_e. We shall first introduce the following important lemma based on *extreme value theorem.*

Lemma 2 (Extreme Value Theorem). *Let $\{X_1, ..., X_K\}$ be a set of K i.i.d. central chi-square random variables with $2n$ degrees of freedom and variance σ^2_X and $X^* = \max_k X_k$. We have*

$$\Pr\left\{\sigma^2_X \log K + \sigma^2_X (n-2) \log\log K \le X^* \le \sigma^2_X \log K + \sigma^2_X n \log\log K\right\} \ge 1 - \mathcal{O}\left(\frac{1}{\log K}\right) \tag{3.3.1}$$

for large K.

In other words, $X^* \approx \mathcal{O}\left(\sigma_X^2 \log K + \sigma_X^2 n \log\log K\right)$ with probability one for sufficiently large K.

Proof 2. *Please refer to appendix 6.0.2.*

As a result, the average system goodput is given by:

Theorem 1 (Asymptotic System Goodput for High and Low SNR).

$$\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] = \begin{cases} \mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi^2_{k^*};\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0\right)\right] & \textit{for high SNR,} \\ \mathcal{O}\left[(1-\epsilon)P_0 F^{-1}_{\chi^2_{k^*};\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)\right] & \textit{for low SNR.} \end{cases} \tag{3.3.2}$$

for sufficiently large K where $\tilde{s}^2 = \left(\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right)$.

Proof 3. *Please refer to appendix 6.0.3.*

Hence, the order of growth in the cross-layer throughput gain is contained entirely in the inverse non-central chi-square cdf via the non-centrality parameter s^2. Yet, there is no closed form for $F^{-1}_{\chi^2_k;s^2;\sigma_e^2/N_d}(x)$ in general case. We shall discuss the asymptotic tradeoff between cross-layer goodput gain and the packet outage diversity N_d in the following asymptotic cases.In addition to the asymptotic analysis, we shall also simulate the system performance in term of average system goodput and compare the result with asymptotic performance in different scenarios. In our simulation, frequency selective fading channel is considered with uniform power-delay profile for simplicity. The number of subcarriers N_f is 1024 and the total number of independent taps $L = 16$. Hence, the 1024 subcarriers are grouped into 16 subbands, each containing $L_s = 64$ correlated subcarriers. The target packet error probability ϵ is set to 0.01. Each point in the figure is obtained by 5000 realizations.

3.3.1 Frequency Diversity at Small Target Packet Outage Probability ϵ

We shall first introduce the following lemma about $F^{-1}_{\chi^2_k;s^2;\sigma_e^2/N_d}(x)$ for small x.

Lemma 3 (Order of Growth for small ϵ). *Let X be a non-central random variable with $2n$ degrees of freedom, noncentral parameter s^2 and variance σ_X^2. For a given s^2, the inverse cdf of X can be expressed as below for asymptotically small ϵ.*

$$F_X^{-1}(\epsilon) \doteq \epsilon^{1/n}\sigma_X^2(n!)^{1/n}\exp\left(\frac{s^2}{n\sigma_X^2}\right) \tag{3.3.3}$$

Proof 4. *Please refer to appendix 6.0.4.*

Thus, the average outage probability $\overline{P_{out}(k)}$ is given by the following theorem:

Theorem 2 (Frequency Diversity at Small Target Packet Outage Probability ϵ). *For sufficiently small ϵ, the average packet outage probability $\overline{P_{out}(k)}$ scales with the SNR P_0 (at a given average goodput) in the order of:*

$$\overline{P_{out}(k)} = E_{\widehat{\mathbf{H}}}\left[P_{out}(k, \widehat{\mathbf{H}})\right] = \mathcal{O}\left(P_0^{-N_d}\right) \tag{3.3.4}$$

Hence, N_d is the order of frequency diversity protection against packet outage.

Proof 5. *Please refer to Appendix 6.0.6.*

3.3.2 Cross-Layer Goodput Gains at Large K and fixed N_d

We have the following lemma about the order of growth of inverse non-central chi-square cdf $F^{-1}_{\chi^2_k;s^2;\sigma_X^2}(x)$ with respect to s^2 for large s^2.

Lemma 4 (Order of Growth for large s). *Let X be a non-central random variable with $2n$ degrees of freedom, noncentrality parameter $s^2 > 0$ and variance σ_X^2. For a given ϵ, the inverse cdf of X can be expressed as $F_X^{-1}(\epsilon) \doteq \mathcal{O}(s^2\sigma_X^2)$ asymptotically for large s^2.*

Proof 6. *Please refer to appendix 6.0.4.*

Using the results of Lemma 2 and Lemma 4 for large K and $\sigma_e^2 < 1$, we have the following Theorem:

Theorem 3 (Asymptotic System Goodput at Large K for High and Low SNR at fixed N_d).

$$\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] = \begin{cases} \mathcal{O}\left\{(1-\epsilon)\log\left[P_0\left(1-\sigma_e^2\right)(\log K)\right]\right\} & \textit{for high SNR, } \sigma_e^2 < 1, \\ \mathcal{O}\left\{(1-\epsilon)P_0(1-\sigma_e^2)\left(\log K\right)\right\} & \textit{for low SNR, } \sigma_e^2 < 1. \end{cases} \tag{3.3.5}$$

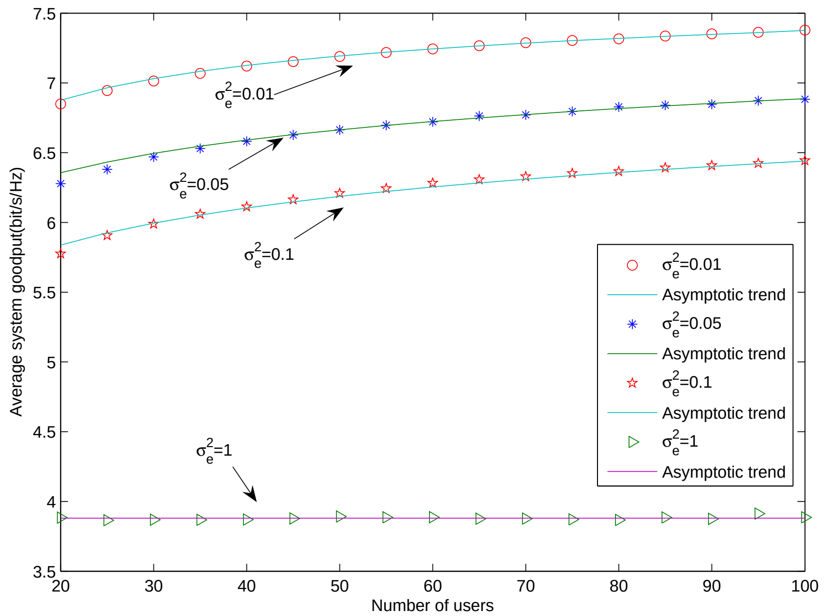

Figure 3.3: Average system goodput versus number of users with N_d=2, different CSIT error (σ_e^2=0.01,0.05,0.1,1) at high SNR(20dB).

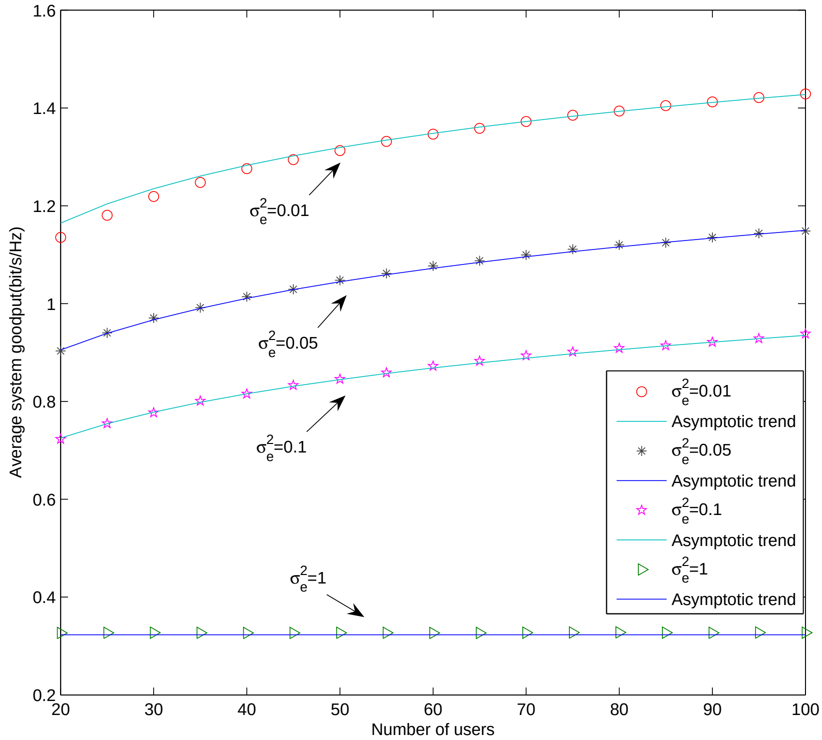

Figure 3.4: Average system goodput versus number of users with N_d=2, different CSIT error (σ_e^2=0.01,0.05,0.1,1) at low SNR(0dB).

Figure 3.3 depicts the average system goodput performance(bit/s/Hz) of the proposed scheduling schemes as a function of the number of users in high SNR (20 dB) and various frequency diversity order $N_d = 2$. It can be seen that when the number of user K increases, the system goodput grows as $\mathcal{O}\left\{\log\left[\left(1-\sigma_e^2\right)\log K\right]\right\}$ due to multi-user diversity. Figure 3.4 shows the average system goodput performance(bit/s/Hz) of the proposed scheduling schemes as a function of the number of users K in low SNR (0 dB) and $N_d = 2$. The average system goodput grows in the order of $\mathcal{O}\left\{\left(1-\sigma_e^2\right)\left(\log K\right)\right\}$ which matches the predicted asymptotic trend quite closely.

Remark 3.3.1. Theorem 3 is valid for estimation error $\sigma_e^2 \in [0, 1)$. When going from equation (3.3.2) to (3.3.5), we used Lemma 4: $F^{-1}_{\chi^2_{k^*};\widetilde{s^2};\sigma_e^2/N_d}(\epsilon) \doteq \mathcal{O}(s^2\sigma_e^2)$, but this holds only for non-zero and sufficiently large non central parameter s^2. Hence, the results in equation (3.3.5) holds only for $\sigma_e^2 < 1$. For the case when $\sigma_e^2 = 1$ and $s^2 = 0$,the $F^{-1}_{\chi^2_{k^*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)$ in Theorem 1 becomes inverse cdf of central chi square. In that case, the average goodput is given by equation (3.3.2). As a result, the average goodput does not growth with the number of users as illustrated in Figure 3.3 and 3.4.

3.3.3 Asymptotic System Goodput at Large N_d and fixed K

From equation (3.3.1) in Lemma 2, there exists $K_0 > 0$ such that for $K_0 > 0$, the non-central parameter

$$\left[\tfrac{1-\sigma_e^2}{N_d}\left(\log K + (N_d - 2)\log\log K\right)\right] \;\le\; \tilde{s}^2(\widehat{H}) \;\le\; \left[\tfrac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right]$$

with probability one for all N_d. As a result, consider the case for large N_d and fixed $K > K_0$[6]. From equation (3.3.2), the first term in the equation $\left(\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right)$ will trend to zero as N_d increases faster than $\log K$ while the second term will be bounded by $\log\log K$. In this case, we have the non central parameter $\tilde{s^2}$ which is bounded by:

$$\tilde{s^2} = \mathcal{O}\left\{\left[(1-\sigma_e^2)\left(\log\log K\right)\right]\right\} \tag{3.3.6}$$

for some $K > K_0 > 0$ such that $\frac{N_d}{\log K} \to \infty$.

The asymptotic goodput at Large N_d for High and Low SNR for $K > K_0$ is

[6]In general, the results will hold if we allow K to grow as N_d increase as long as $N_d/\log K \to \infty$.

given by :

$$\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] = \begin{cases} \mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)P_0\right)\right] & \text{for high SNR,} \\ \mathcal{O}\left[(1-\epsilon)P_0 F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)\right] & \text{for low SNR.} \end{cases} \tag{3.3.7}$$

There is a factor $(1-\sigma_e^2)$ in $\tilde{s}^2$ outside the $\log\log K$ in equation (3.3.6) and $F^{-1}_{\chi^2_k;s^2;\sigma^2_X}(x)$ in equation (3.3.7) is an increasing function of $\tilde{s}^2$. Hence, we need *double exponentially* more users K to compensate the penalty due to $(1-\sigma_e^2)$ in the system goodput (via $\tilde{s}^2$).

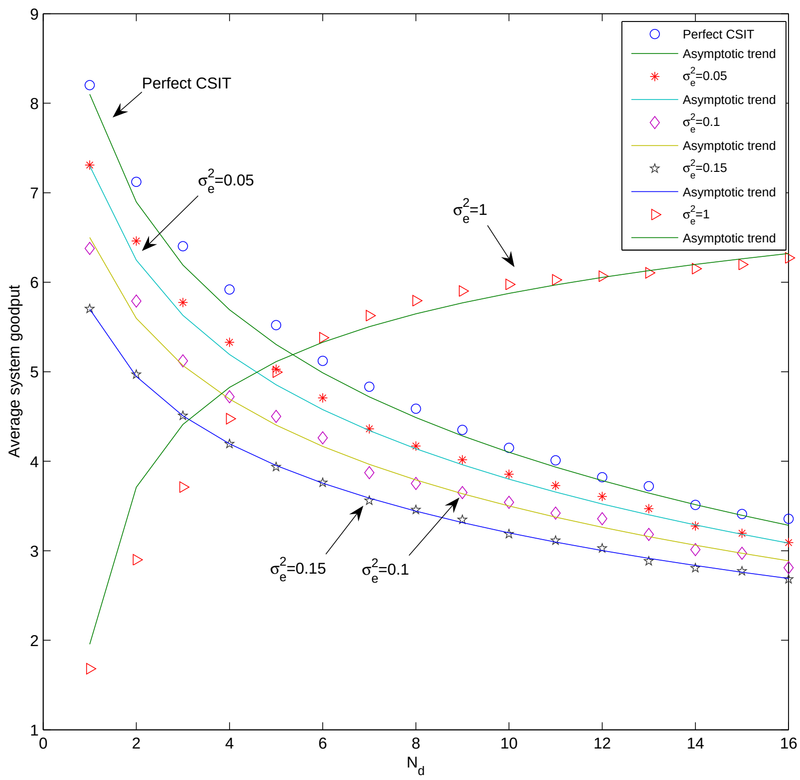

Figure 3.5: Average system goodput versus packet diversity order (N_d) with different CSIT error σ_e^2 at high SNR(20dB) and K=20.

Figure 3.5 illustrates the average system goodput performance versus N_d in high SNR (20dB) at different CSIT errors $\sigma_e^2 = 0, 0.05, 0.1, 0.15, 1$. Figure 3.6 shows the same scenario for low SNR (0 dB) regime. The system goodput is shown to be a

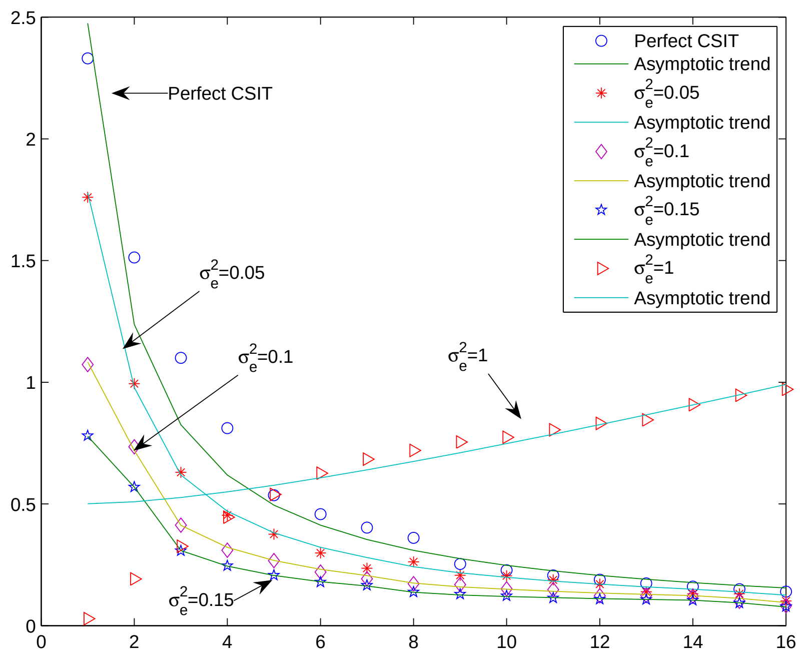

Figure 3.6: Average system goodput versus packet diversity order (N_d) with different CSIT error σ_e^2 at low SNR(0dB) and K=20.

decreasing function of N_d. For large N_d, the cross-layer goodput gain is decreased substantially. On the other hand, the average packet outage probability scales in the order of $\mathcal{O}(P_0^{-N_d})$. From these results, we can deduce that there is a natural tradeoff between packet outage diversity order N_d and the cross-layer goodput gain. Comparing with the well-known cross-layer throughput gain of $\mathcal{O}(\log\log K)$ when we have perfect CSIT, we observe that the efficiency of the multiuser selection diversity (goodput) is reduced to $\log\log\log K$ for large N_d.

3.4 Summary

In this chapter, we explore the asymptotic trade-off between cross-layer goodput gain and packet outage in OFDMA downlink system, with delayed CSIT in slow fading frequency selective channel. We formulate the cross-layer design as a mixed convex and combinational optimization problem. Due to the delayed CSIT, it is critical to account for potential packet errors (due to channel outage) and we consider total system goodput as our optimization objective. By allocating N_d independent subbands to a user, the packet outage probability drops in the order of SNR^{-N_d}. On the other hand, the *system goodput* scales in the order of $\mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi^2_{k*};\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0\right)\right]$ at high SNR where $\tilde{s}^2 = \mathcal{O}\left\{(1-\sigma_e^2)(\log\log K)\right\}$ and $\mathcal{O}\left\{(1-\sigma_e^2)(\log K)\right\}$ for large N_d $[K > K_0]$ and large K [fixed N_d] respectively.

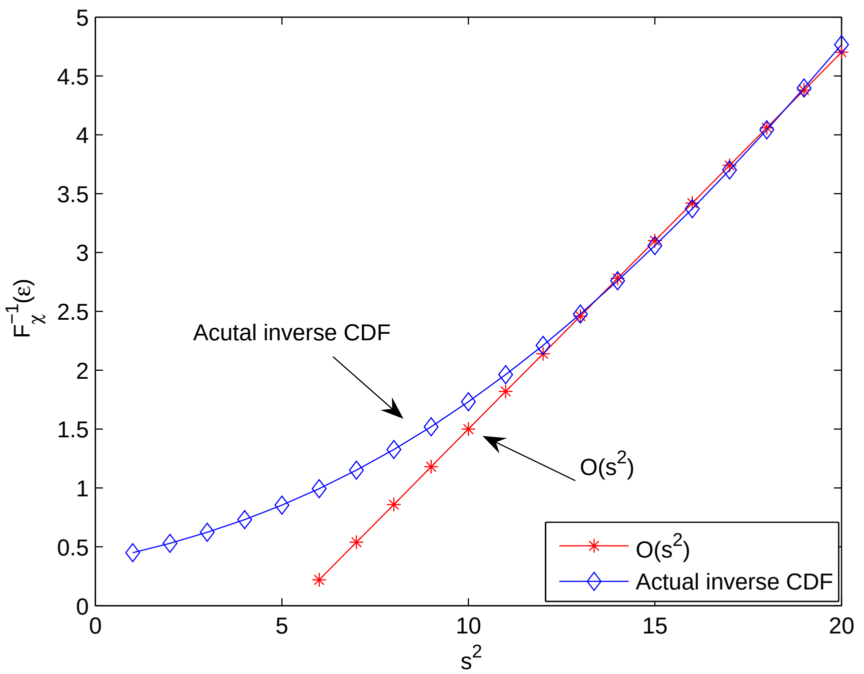

Figure 3.7: Inverse CDF of non central chi square random variable versus non-centrality parameter s^2 for ϵ=0.001 with degrees of freedom equal to 6.

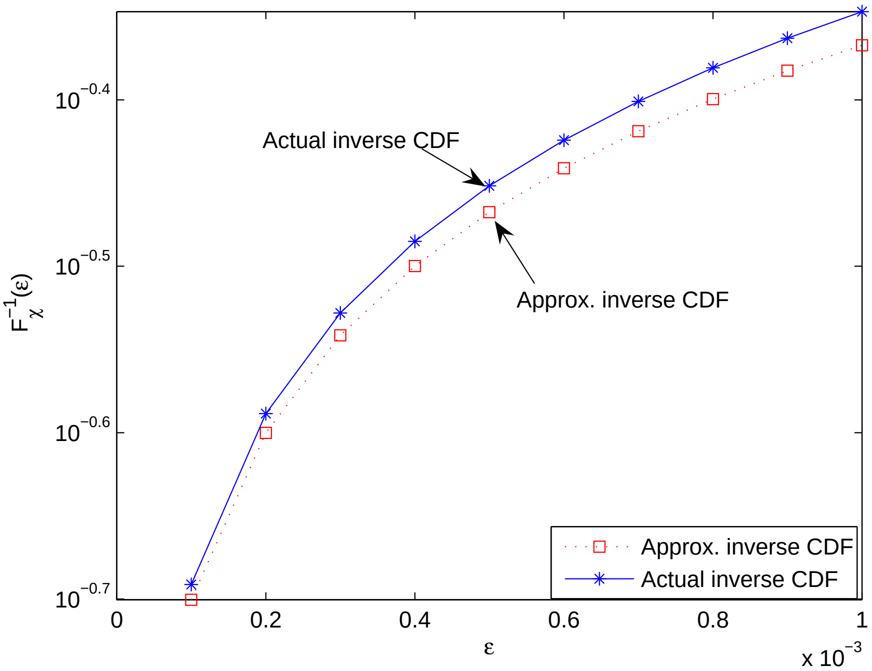

Figure 3.8: Inverse CDF of non central chi square random variable versus ϵ with degree of freedom 6 and non central parameter $s^2 = 1$.

Chapter 4

Uplink multi-user detection analysis

In this chapter, we will present an analytical framework on uplink multiple access channel with successive interference cancellation receiver. We would like to show how error propagation affect system performance and what is the optimal decoding order to maximize the achievable capacity.

4.1 Introduction

The uplink of wireless cellular system, where many mobile users communicate to a single base station, can be modeled by the multi-access channel. The multi-access channel is characterized by a capacity region, which is the set of achievable rate vector[40] and multi-user detection with successive interference cancellation (MUD-SIC) is one receiver scheme that can achieve the corner points in the dominant face of the multiaccess capacity region[1]. Most of the existing works on multiaccess

[1] In general, joint detection is needed to achieve the multi-access capacity region.

channel are either focused on signal processing algorithms or performance analysis for multi-user detection. In [41], signal design for multiaccess channel is discussed. In [42], multiuser detection algorithm for overloaded CDMA system is discussed. Conventional performance analysis of multi-access fading channel is usually based on the ergodic capacity[43, 44]. Uplink power adaptation for multiaccess channel is addressed in [9] where the transmit power of mobile users are optimized with respect to a system objective function of user capacities. In all these works, *ergodic capacity* is the key performance measure and optimization objective. However, the ergodic capacity is a reasonable performance measure only for fast ergodic fading channels where a transmitted packet spans across ergodic realizations of channel fading. In this case, the transmitted packets from the mobile users can be guaranteed to be successfully received by the base station as long as powerful channel coding such as LDPC code[45] with sufficiently long block length is applied and the transmitted data rate is less than the ergodic channel capacity. However, for slow fading channels (non-ergodic channels), in which the channel fading is quasi-static within the entire encoding frame, the transmitted packets cannot be guaranteed to be always successfully received even if powerful channel coding is applied. In this case, the instantaneous mutual information of the channel appears as a random variable to the transmitters. The packet transmitted will be corrupted if the data rate is larger than the instantaneous mutual information (despite the use of error correction code) and this is called *packet outage*. Hence, in slow fading channels, ergodic capacity is no longer a useful performance measure because it does not take into account potential packet outage. To include the effects of potential packet errors due to channel outage, we should analyze the *packet outage probability* and *system goodput* (which is defined as the average bits/sec/Hz successfully delivered to the receiver).

In this chapter, our focus is to evaluate the *per-user* packet error (outage) probabilities and the system goodput for multi-access slow fading channel with adaptive MUD-SIC. We consider a system with a base station and n mobile users where there is no channel state knowledge (CSIT) at the transmitters of the mobiles. We assume adaptive successive interference cancellation (MUD-SIC) processing at the base station where the decoding order among the n mobile users is adaptive based on the channel state information at the base station (CSIR) so as to maximize the total system goodput. In [9, 46, 47], the delay-limited capacity of the multi-access channel with perfect CSIT is analyzed without considering packet outage events. In [48, 49], the authors analyzed the system goodput for multiaccess channels with optimal (maximal likelihood (ML)) multiuser detection and linear multiuser detection (MMSE). Yet, the results from these works cannot be applied in our case with MUD-SIC in quasi-static fading channels. In the case of ML detector (joint detection), the outage event is defined as the event that the rate vector is outside the *instantaneous capacity region*[2] of the multiaccess channel and there is no "notion" of *per-user packet outage* or *error-propagation effects*. In the case of MMSE detectors, the outage event is completely decoupled among the n users. However, when we consider MUD-SIC detector, there is *mutual coupling* (error propagation) of the packet error events between the n users in the SIC decoding process. For example, the packet error event of the user decoded in the k-th iteration depends not only on the packet transmitted by user k but also on all the users decoded in

[2]Instantaneous capacity region refers to the multiaccess capacity region for a given channel fading realization.

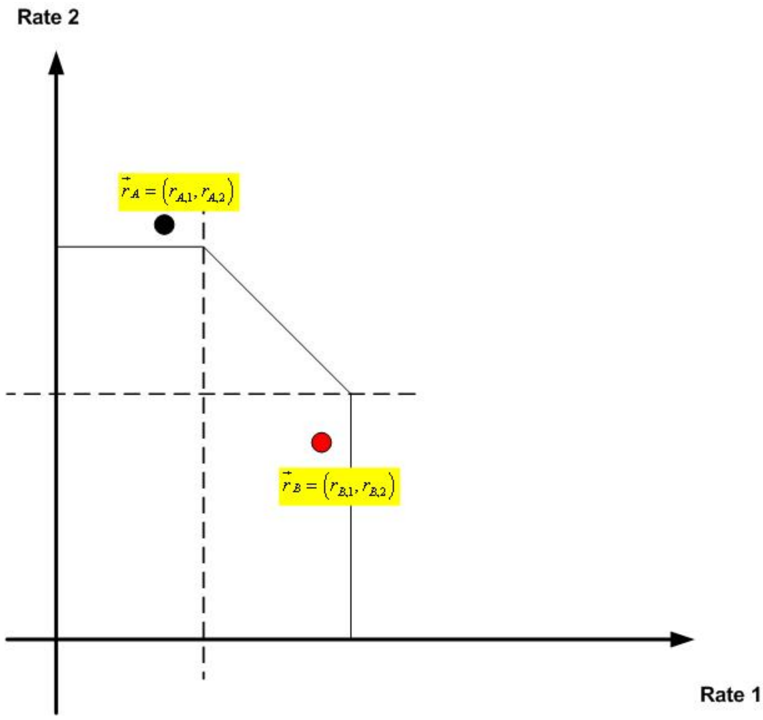

Figure 4.1: Illustration of the mutual coupling (error propagation) of packet outage events and the importance of decoding order in MUD-SIC for system goodput considerations in quasi-static multiaccess fading channels. Rate vector $\vec{r}_A$, which is outside the *instantaneous capacity region*, may contribute to non-zero system goodput if user 1 is decoded first. Rate vector $\vec{r}_B$, which is inside the instantaneous capacity region, may contribute to zero system goodput if a wrong decoding order is used.

the $(k-1)$-th, $(k-2)$-th,..., and the first round.

Furthermore, as illustrated in figure 4.1, the *per-user packet outage event* (for user 1) cannot be deduced from whether the rate pair is inside or outside the *instantaneous capacity region*. For the rate vector $\vec{r}_A$ (outside the instantaneous capacity region), packet from user 1 can still be successfully decoded if the right decoding order is used. In addition, the choice of decoding order is also very important to the overall system goodput. For rate vector $\vec{r}_B$ in Figure 4.1 (inside the instantaneous capacity region), if user 1 is decoded first, both packets from user 1 and user 2 will be corrupted and we will have zero system goodput. On the other hand, if user 2 is decoded first, both packets can be successfully received. Furthermore, the packet outage event of user 1 depends not only on the channel state of user 1 but also on that of user 2 as well due to the coupling of the adaptive MUD-SIC. As far as we are aware, the issues of coupled *per-user packet outage events* or error-propagation for MUD-SIC detection have not been addressed previously. In this chapter, we shall address two important issues associated with MUD-SIC detection in quasi-static multi-access fading channels where we have homogeneous users with equal data rate[3].

- **Optimal Decoding Order in MUD-SIC:** While there are some works on finding the optimal decoding order in MUD systems, they did not consider the *per-user outage event* (error propagation) in MUD-SIC. For example, the optimal decoding order is found in[9] to maximize a general utilize function of ergodic capacity. In [49], joint detection is used and hence, the outage event is defined by whether the rate vector is outside the instantaneous capacity region and this is very different from the *per-user packet outage event* we considered here.

- **Closed-Form Analysis of Per-user Packet Outage Probability and System Goodput:** Based on the optimal decoding order obtained, we shall derive the closed-form per-user packet outage probability for MUD-SIC, taking care of the coupled per-user outage event in the SIC decoding process.

The chapter is organized as follows. In Section 4.2, the multi-user system model is described. In Section 4.3, we shall derive the packet error probability of the n users and the overall system goodput. In Section 4.4, simulation results

[3]Equal data rate represents an important class of voice applications in wireless networks

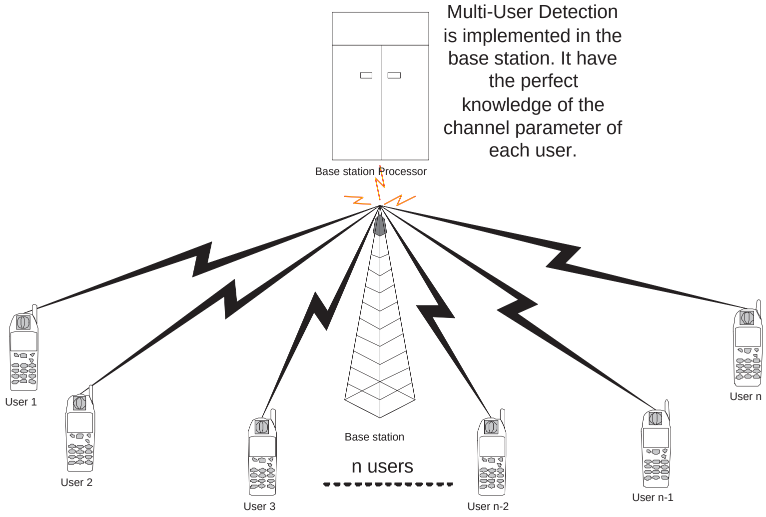

Figure 4.2: System model of multi-user network with multi-user detection

are obtained to verify the analytical expressions and to compare the performance gains of adaptive SIC in multiaccess channels. Finally, we shall conclude with a brief summary of results in Section 4.5.

4.2 System Model

In this section, we shall elaborate on the overall system models and the base station processing for the multi-access fading channel. In this chapter, capital letter represents random variable and small letter represents a realization of the random variable. $\mathcal{E}[X]$ denotes the expectation of the random variable X. π denotes a decoding order where $\pi(i)$ gives the user index in the i-th decoding iteration and $\pi^{-1}(k)$ gives the decoding order of the k-th user. X^* denotes the complex conjugate of a random variable X and $X_{[i]}$ represents the i-th ordered statistics in a sample size of n.

4.2.1 Multi-access Channel Model

Figure 4.2 illustrates the overall system model. We have a base station and n mobile users. The uplink transmissions of the n mobiles are synchronous so that successive interference cancellation (SIC) is applied at the base station with perfect channel state information (CSIR). On the other hand, the mobile transmitters do not have any channel state information (CSIT). We consider slow flat fading multi-access channels where the channel fading remains quasi-static within the entire transmitted packet. This is a realistic assumption for pedestrian mobility (10km/hr) in most systems such as HSDPA, 3G1X and WiFi, where the coherence time is around 20ms and the frame duration is less than 2ms.

Let X_i be the transmitted symbol from the i-th user with average transmit SNR $E[|X_i|^2] = \sigma_i^2$ and H_i be the channel fading coefficient between the $i-$th mobile and the base station (which is modelled as zero-mean complex Gaussian random variable with covariance $E[|H_i|^2] = 1$) and $\mathbf{H} = [H_1, ..., H_n]$ be the aggregate channel fading. The received signal at the base station is given by

$$Y = \sum_{i=1}^{n} H_i X_i + Z \tag{4.2.1}$$

where Z denotes channel noise, which is modelled as zero mean complex Gaussian with normalized variance $E[|Z|^2] = 1$.

4.2.2 MUD-SIC Processing and Per-User Packet Error Model

The base station has to detect the signal transmitted by the n users based on the received signals Y. In this chapter, we assume the base station is equipped with synchronous multi-user detection with successive interference cancellation (MUD-SIC) and perfect channel state information (CSIR). Given a particular decoding order $\pi = (\pi(1), ..., \pi(n))$ where $\pi(i)$ is the user index of the user decoded in the i-th decoding iteration, the instantaneous mutual information (using Gaussian random codebook) of the $\pi(i)-$th user is given by

$$C_{\pi(i)}(\mathbf{H}, \pi, i) = \log_2 \left(1 + \frac{|H_{\pi(i)}|^2 \sigma^2_{\pi(i)}}{\sum_{p=i+1}^{n} (|H_{\pi(p)}|^2 \sigma^2_{\pi(p)}) + 1} \right) \tag{4.2.2}$$

where σ_k^2 is the transmit SNR of the $k-$th user.

In this chapter, we consider a homogeneous system where the n users transmit information with the same data rate ($r_1 = ... = r_n = r$). This represents an important case such as voice applications in the cellular systems. Since the channel fading is quasi-static within the transmitted packets and the mobile transmitters do not have knowledge of the channel states $H_1, .., H_n$, the instantaneous mutual information of the n users $\{C_1, ..., C_n\}$ appears as random variables to the mobile transmitters. The transmitted packet of the $\pi(i)$-th user will be corrupted if the data rate of the transmitted packet r exceeds the instantaneous mutual information $C_{\pi(i)}$ of individual users. This refers to *packet outage*. In fact, the packet error probability is contributed by two factors, namely the *packet outage* and the *channel noise*. The second factor is due to the finite block length effect of channel coding. Suppose strong enough channel coding is applied and the channel coherence time is much longer than the symbol duration (so that sufficiently long block length can be used), the packet outage will be the dominant factor that contributes to packet error. Note that the packet outage is due to the slow fading channels (non-ergodic channels) and cannot be eliminated even if capacity achieving codes are used at the transmitter. Hence, we shall assume packet error probability is mainly due to the packet outage only and shall use the two words interchangeably in the chapter.

In this chapter, we consider multiaccess channel with adaptive SIC and hence, the decoding order π is a function of the CSIR $\mathbf{H}$. Let $\mathcal{P} = \{\pi_{\mathbf{H}}\}$ denotes the *decoding order policy*, which is a set of decoding order $\pi_{\mathbf{H}}$ with respect to every realization of CSIR $\mathbf{H}$. Given a decoding order policy $\mathcal{P}$, we are interested to find the average PER (averaged over ergodic realization of CSI) of the user k, $\overline{P_{out}}(r, \mathcal{P}, k)$. However, the analysis of per-user PER is not trivial due to the coupling of decoding events in SIC. For example, when user k is decoded in the 3-rd iteration, the success of packet delivery depends not only on the instantaneous mutual information of user k but also on the success/failure in the 1st and 2nd decoding iterations. In fact, the success or failure of a packet transmission of a user cannot be simply told from whether the rate vector is inside the multiaccess capacity region. As illustrated in Figure 4.1, rate vector $\vec{r}_A$, which is outside the capacity region, may contribute to non-zero system goodput if the correct decoding order is used. To take care of the intrinsic coupling of the adaptive SIC in the PER analysis, we define the *effective instantaneous mutual information*[4] of user $k = \pi(i)$ in the i-th decoding iteration as:

$$\widetilde{C_k}(\mathbf{H}, \pi, i) = \log_2 \left(\frac{1 + \sum_{j=i}^{n} \sigma_{\pi(j)}^2 |H_{\pi(j)}|^2 + \widetilde{W_i^\pi}}{1 + \sum_{j=i+1}^{n} \sigma_{\pi(j)}^2 |H_{\pi(j)}|^2 + \widetilde{W_i^\pi}} \right) \tag{4.2.3}$$

where $\widetilde{W_i}$ denotes the accumulated *undecodable interference* after $i-1$ decoding iterations and it is given by:

$$\widetilde{W_i^\pi} = \sum_{j=1}^{i-1} \sigma_{\pi(j)}^2 \left|H_{\pi(j)}\right|^2 \mathcal{I}[r \geq \widetilde{C_{\pi(j)}}(\mathbf{H}, \pi, j)] \tag{4.2.4}$$

where $\mathcal{I}[.]$ represents the indicator function[5], r is the transmitted data rate and $\widetilde{W_1^\pi} = 0$.

[4]The effective mutual information here is different from the mutual information in (4.2.2) in the sense that the success/failure events in the $i-1, i-2, ..., 1$ decoding attempts are taken care.

[5]$\mathcal{I}[A] = 1$ if the event A is true and zero otherwise.

Hence, the average PER of the user k (averaged over CSIR) is given by:

$$\overline{PER}(r,\mathcal{P},k) \approx \overline{P_{out}}(r,\mathcal{P},k) = 1 - \sum_{\pi\in\mathcal{P}} \Pr\left[r < \widetilde{C_k}\left(\mathbf{H},\pi,\pi^{-1}(k)\right)|\pi\right]\Pr[\pi] \quad (4.2.5)$$

In order to capture the effect of potential packet error due to channel outage, we define the *average system goodput* under a given decoding order policy ($\mathcal{P}$), $\rho(r,\mathcal{P})$, to be the total bits/sec/Hz that successfully delivered to the base station. That is,

$$\overline{\rho}(r,\mathcal{P}) = \sum_{k=1}^{n} r\left\{1 - P_{out}(r,\mathcal{P},k)\right\} \quad (4.2.6)$$

Note that both system goodput and the PER are functions of the decoding order policy $\mathcal{P}$. In the next section, we shall deduce the optimal decoding order policy to maximize the average system goodput.

4.2.3 Optimal Decoding Order Policy

Note that existing literature that discusses about the optimal decoding order are all based on some utility functions of ergodic capacity [9, 46] in which potential packet errors (outage) of the n users are not taken into consideration. In this section, we shall derive the optimal decoding order (per fading slot) to maximize the system goodput $\rho(r,\mathcal{P})$ as defined in (4.2.6). The results are summarized in the lemma below.

Lemma 5 (Optimal Decoding Order). *Given the instantaneous receive SNR $\{\gamma_1,..,\gamma_n\}$ where $\gamma_k = \sigma_k^2|H_k|^2$, the optimal decoding order that maximize the system goodput $\rho(r,\mathcal{P})$ is given by*

$$\pi(j) = \arg\max_{k\in(S\setminus T)}\left(\gamma_k\right) \quad (4.2.7)$$

where $S = \{1..n\}, T = \{\pi(1), .., \pi(j-1)\}$ *and the pdf of* γ_k *is given by*

$$f_{\gamma_k}(x_k) = \frac{1}{\sigma_k^2} e^{-\frac{x_k}{\sigma_k^2}} \tag{4.2.8}$$

Proof 7. *Please refer to Appendix 6.0.7.*

Define $\xi_i \in \{0,1\}$ as the event that the i-th decoded user is decoded successfully ($\xi_i = 1$ denotes successful decoding and $\xi_i = 0$ denotes decoding failure). The event ξ_i is given by:

$$\xi_i = \mathcal{I}\left\{ r < \log\left(1 + \frac{\gamma_{\pi(i)}}{1 + \sum_{j<i} \gamma_{\pi(j)}(1-\xi_j) + \sum_{j>i} \gamma_{\pi(j)}}\right)\right\} \in \{0,1\} \tag{4.2.9}$$

where $\mathcal{I}(A)$ is the *indicator function.* Define

$$I_i = \mathcal{I}\left\{ r < \log\left(1 + \frac{\gamma_{\pi(i)}}{1 + \sum_{j>i} \gamma_{\pi(j)}}\right)\right\}. \tag{4.2.10}$$

Given the optimal decoding order policy $\mathcal{P}^*$ in (4.2.7) and the associated optimal decoding order π, we have $C_{\pi(i)}(\mathbf{H}, \pi, i) > C_u(\mathbf{H}, \pi, i)$ for all $u \neq \pi(i)$. Hence, for user $\pi(i)$ in the i-th decoding iteration, packet error for user $\pi(i)$ can be declared whenever packet error occurs in any of the $j = 1, 2, .., i$-th decoding iterations. In other words, we have

$$\xi_i = 0 \Rightarrow I_1 \cup I_2 \cup .... \cup I_i = 0 \tag{4.2.11}$$

Hence, the average packet outage probability of user k transmitting at a rate r in (4.2.5) can be simplified as:

$$\begin{aligned}\overline{P_{out}}(r, \mathcal{P}^*, k) &= \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{\pi^{-1}(k)} \Pr[\xi_i = 0|\pi] \right) \Pr[\pi] \\ &= \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{\pi^{-1}(k)} \Pr[I_1 \cup I_2 \cup .... \cup I_i = 0|\pi] \right) \Pr[\pi] \leq \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{\pi^{-1}(k)} \sum_{j=1}^{i} \Pr[I_j = 0|\pi] \right) \Pr[\pi]\end{aligned} \tag{4.2.12}$$

where the final upper bound is due to union bound and $\Pr[I_j = 0|\pi]$ is the conditional packet outage probability in the j-th iteration under the decoding order π. From (4.2.10), $\Pr[I_j = 0|\pi]$ is given by:

$$\Pr(I_j = 0|\pi) = \Pr[r > C_{\pi(i)}(\mathbf{H}, \pi, i)|\pi] \tag{4.2.13}$$

and $\Pr[\pi]$ is the probability for the decoding order π in the optimal policy $\mathcal{P}^*$ to be selected in the current time slot and is given by

$$\Pr(\pi) = \Pr(\gamma_{\pi(1)} \geq \gamma_{\pi(2)} \geq \gamma_{\pi(3)} .. \geq \gamma_{\pi(n)}). \tag{4.2.14}$$

In other words, the average outage probability is the average of the occurrence of all the packet outage events that happen prior to the current decoding iteration (average over every possible decoding order π in the policy $\mathcal{P}^*$).

Similarly, the average system goodput under the optimal decoding order policy $\mathcal{P}^*$ is given by:

$$\begin{aligned}
\overline{\rho}(r, \mathcal{P}^*) &= \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{n} r \Pr[\xi_i = 1|\pi] \right) \Pr(\pi) \\
&= \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{n} r \left(1 - \Pr[I_1 \cup I_2 \cup .... \cup I_i = 0|\pi]\right) \right) \Pr(\pi) \\
&\geq \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{n} r \left(1 - \sum_{j=1}^{i} \Pr[I_j = 0|\pi] \right) \right) \Pr(\pi)
\end{aligned} \tag{4.2.15}$$

4.3 Performance Analysis

In this section, we shall derive the analytical expressions on the *per-user packet outage probability* and the *system goodput* for MUD-SIC detector under the optimal decoding order policy $\mathcal{P}^*$.

4.3.1 System Goodput and Per-User Packet Outage Probability for MUD-SIC

From equations (4.2.15) and (4.2.12), both the average system goodput $\overline{\rho}$ and the average packet error probability of user k, $\overline{P_{out}(r, \mathcal{P}^*, k)}$, are determined by averaging over all the possible decoding order π under the optimal policy $\mathcal{P}^*$. To obtain the analytical expressions of the average system goodput and the average packet error probability, we have to determine the conditional outage probability of the j-th iteration $\Pr(I_j = 0|\pi)$ in (4.2.13) and the probability of choosing the decoding order $\Pr(\pi)$. Given a decoding order π and from (4.2.2), $\Pr(I_j = 0|\pi)$ can be expressed as

$$\Pr(I_j = 0|\pi) = 1 - \Pr[r \leq C_{\pi(j)}(\mathbf{H}, \pi, j)|\pi] = 1 - \Pr\left[\gamma_{\pi(j)} - \eta \sum_{p=j+1}^{n} \gamma_{\pi(p)} \geq \eta \middle| \pi\right] \tag{4.3.1}$$

where $\eta = 2^r - 1$. Without loss of generality, we consider a decoding order $\pi = (1, 2, .., n)$. From $\mathcal{P}^*$, the optimal decoding order is in descending order of γ_i. Hence, conditional on $\pi = (1, 2, ..., n)$, we have $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_n$. By [50], the joint pdf of the γ_j is then given by

$$f_{\gamma_1,\gamma_2,...\gamma_n}(x_1, x_2...x_n|\pi) = \begin{cases} \frac{1}{\Pr(\pi)\prod_{i=1}^{n}\sigma_i^2} \prod_{i=1}^{n} e^{\frac{-x_i}{\sigma_i^2}} & \text{if } 0 \leq x_n \leq x_{n-1} \leq x_{n-2} ... \leq x_1 < \infty, \\ 0 & \text{otherwise.} \end{cases} \tag{4.3.2}$$

where $\Pr(\pi) = \Pr(\gamma_1 \geq \gamma_2 \geq \gamma_3 .. \geq \gamma_{n)})$. Hence, from the above equations, the computation of $\Pr(I_j = 0|\pi)$ and $\Pr[\pi]$ involved multi-dimensional nested integrals which are cumbersome and complicated. To obtain a tractable analytical expression for the average system goodput, we shall derive the following lemma.

Lemma 6. *The ordered channel gains ($\gamma_{[1]} \geq \gamma_{[2]} \geq ... \geq \gamma_{[n]}$) can be transformed into independent (but not necessarily identical) exponential random variables $\{Z_1, ..., Z_n\}$ by the following transformation*

$$Z_i = i[\gamma_{[i]} - \gamma_{[i+1]}] \tag{4.3.3}$$

where $0 \leq Z_i < \infty$ for all $i \in \{1, 2, .., n\}$ and $\gamma_{[n+1]} = 0$. $\{Z_i\}$ is a set of independent exponential random variables with p.d.f. given by:

$$f_{Z_i}(z) = \phi_i e^{-z\phi_i}$$

where the parameter ϕ_i given by

$$\phi_i = \frac{\sum_{u=1}^{i} \sigma_u^{-2}}{i} \tag{4.3.4}$$

Proof 8. *Please refer to Appendix 6.0.8.*

The implication of the above lemma is that the original ordered random variables $\{\gamma_{[i]}\}$ can be transformed[6] into a set of "virtual user" statistics (Z_v) which is independent. By making use of this lemma, the joint pdf of Z_v is then given by

$$f_{z_1,z_2,...z_n}(z_1, z_2...z_n|\pi) = \frac{1}{n!} \frac{1}{\Pr(\pi) \prod_{i=1}^{n} \sigma_i^2} \prod_{i=1}^{n} e^{-z_i\phi_i} \tag{4.3.5}$$

where ϕ_i is given by the equation(4.3.4). Hence, from (4.3.1), the conditional outage probability $\Pr(I_j = 0|\pi)$ in $j-th$ iteration conditioned on a given decoding order π can be expressed as

$$\Pr(I_j = 0|\pi) = 1 - \Pr\left[\sum_{v=j}^{n} \lambda_v Z_v \geq \eta \middle| \pi\right] = 1 - \Pr\left[\Gamma_j \geq \eta|\pi\right] \tag{4.3.6}$$

[6]Yet, unlike the standard ordered-statistics transformation[50], the transformed variables $\{Z_v\}$ are independent but not necessarily identical due to potentially different transmit SNR σ_i^2 among the n users.

where $\eta = 2^r - 1$, $\Gamma_j = \sum_{v=j}^{n} \lambda_v Z_v$ is a linear combination of $(n-j+1)$ independent exponential random variables $(\{Z_v\})$ and $\lambda_v = \frac{1-(v-j)\eta}{v}$. Now, the conditional outage probability is expressed in terms of a single random variable Γ_j and nested multi-dimensional integration can be avoided.

Making use of the characteristic function of the exponential random variable and the partial fraction theorem, the p.d.f. of the random variable Γ_j is found and summarized in the following lemma.

Lemma 7. *The p.d.f. of Γ_j is given by*

$$f_{\Gamma_j}(x) = \sum_{v=j}^{n} \frac{A_v}{|\overline{\lambda_v}|} B_v \tag{4.3.7}$$

where $x_j \in \Re$ and

$$\overline{\lambda_v} = \lambda_v / \phi_v \quad , \quad A_v = \left(\prod_{u=j,u\neq v}^{n} \frac{\overline{\lambda_v}}{(\overline{\lambda_v} - \overline{\lambda_u})} \right), B_v = \begin{cases} e^{\frac{-x}{|\overline{\lambda_v}|}} u(x) & \text{if } \overline{\lambda_v} \geq 0 \\ e^{\frac{x}{|\overline{\lambda_v}|}} u(-x) & \text{otherwise} \end{cases} \tag{4.3.8}$$

where ϕ_v is given by the equation(4.3.4) and $u(x)$ is the unit step function.

Proof 9. *Please refer to Appendix 6.0.9.*

From above lemma, $\Pr(I_j = 0|\pi)$ is found to be

$$\begin{aligned} \Pr(I_j = 0|\pi) &= 1 - \int_{\eta}^{\infty} f_{\Gamma_j}(x_j) dx_j \\ &= 1 - \sum_{v=j}^{n} A_v e^{\frac{-\eta}{\overline{\lambda_v}}} I(\overline{\lambda_v} \geq 0) \end{aligned} \tag{4.3.9}$$

The indicator function $I(\overline{\lambda_v} \geq 0)$ is due to the integration over the region $0 \leq \eta < \infty$.

After obtaining the closed-form expression for $\Pr(I_j = 0|\pi)$, we have to obtain the closed form expression for $\Pr(\pi)$. From the p.d.f. expression in equation (4.3.2), the probability of the optimal decoding order π can be derived from the fact that the integration of the joint p.d.f., $f_{z_1,z_2,...z_n}(z_1, z_2...z_n|\pi)$, over the entire space of $Z_1, .., Z_n$ equals to 1. Hence, $\Pr(\pi)$ is given by

$$\Pr(\pi) = \frac{1}{n!}\prod_{i=1}^{n}\frac{1}{\phi_i\sigma_i^2} \tag{4.3.10}$$

Note that the probability of optimal decoding order depend on the average received SNR (σ_i^2) of every user. If all user have the same received SNR ($\sigma_1^2 = \sigma_2^2...\sigma_n^2$), the probability of decoding order become

$$\Pr(\pi) = \frac{1}{n!} \tag{4.3.11}$$

Hence, under the special case of equal SNR, every optimal decoding order is statistically equiprobable.

Based on the analytical expressions for $\Pr(I_j = 0|\pi)$ and $\Pr(\pi)$, the average system goodput and the average packet error probability of user k under the optimal decoding order policy $\mathcal{P}^*$ are summarized in the following two theorem.

Theorem 4 (Lower Bound for Average System Goodput of MUD-SIC with Optimal Decoding Order). *The average system goodput* $(\overline{\rho})(r, \mathcal{P}^*)$ *with optimal decoding order policy* $\mathcal{P}^*$ *is given by*

$$\overline{\rho}(r, \mathcal{P}^*) \geq \frac{1}{n!}\sum_{\pi\in\mathcal{P}^*}\left[\sum_{i=1}^{n} r\left(1-\sum_{j=1}^{i}\left(1-\sum_{v=j}^{n}A_v e^{\frac{-\eta}{\overline{\lambda_v}}} I(\overline{\lambda_v}\geq 0)\right)\right)\right]\prod_{i=1}^{n}\frac{1}{\phi_i\sigma_{\pi(i)}^2} \tag{4.3.12}$$

where $\eta = 2^r - 1$.

From the above expression, the first term inside the summation represent the system goodput corresponds to each decoding permutation in the optimal decoding policy. The second term outside the summation correspond to the probability of each permutation inside the decoding policy.

Theorem 5 (Upper Bound for Average Per-User Packet Outage Probability of MUD-SIC with Optimal Decoding Order). *The average packet error probability of user k under the optimal decoding order policy $\mathcal{P}^*$ is given by*

$$\overline{P_{out}}(r, \mathcal{P}^*, k) \leq \frac{1}{n!} \sum_{\pi \in \mathcal{P}^*} \left(\sum_{i=1}^{\pi^{-1}(k)} \sum_{j=1}^{i} \left[1 - \sum_{v=j}^{n} A_v e^{\frac{-\eta}{\overline{\lambda_v}}} I(\overline{\lambda_v} \geq 0) \right] \right) \prod_{i=1}^{n} \frac{1}{\phi_i \sigma^2_{\pi(i)}} \tag{4.3.13}$$

4.3.2 Asymptotic Expressions on Average System Goodput and Per-User Packet Error Probability

In this section, we consider the asymptotic expressions of the average system goodput and the packet error probability under the optimal decoding order policy at large SNRs. Specifically, when the average SNR of all the users $\sigma_1^2 = ... = \sigma_n^2 \to \infty$, the channel capacity of j-th iteration with the optimal decoding order π becomes

$$C_{\pi(j)}(\mathbf{H}, \pi, j) = log_2 \left(1 + \frac{|H_{\pi(j)}|^2}{\sum_{p=j+1}^{n} |H_{\pi(p)}|^2} \right) \tag{4.3.14}$$

From the above expression, we observe that the channel capacity $C_{\pi(j)}(\mathbf{H}, \pi, j)$ is independent of the average transmit SNRs. Hence, by symmetry, all the decoding order π in $\mathcal{P}^*$ is statistically equiprobable ($\Pr(\pi) = \frac{1}{n!}$). The analytical expression for the system goodput ($\overline{\rho}(r, \mathcal{P}^*)$) under the optimal decoding policy $\mathcal{P}^*$ is given by

$$\overline{\rho}(r, \mathcal{P}^*) \geq \left[\sum_{i=1}^{n} r \left(1 - \sum_{j=1}^{i} \left(1 - \sum_{v=j}^{n} A_v I(\overline{\lambda_v} \geq 0) \right) \right) \right] \tag{4.3.15}$$

Similarly, the average packet error probability of user k becomes

$$\overline{P_{out}}(r, \mathcal{P}^*, k) \leq \frac{1}{n!} \sum_{\pi \in \mathcal{P}^*} \sum_{i=1}^{\pi^{-1}(k)} \sum_{j=1}^{i} \left[1 - \sum_{v=i}^{n} A_v I(\overline{\lambda_v} \geq 0) \right] \tag{4.3.16}$$

where $\overline{\lambda_v}$ is given by the equations (4.3.4) and (4.3.8).

4.4 Results and Discussions

In this section, we shall present the numerical results obtained from the analytical expressions and verify them with respect to the simulation results on the average packet error probability and the average system goodput. In the simulation, we consider a single cell uplink wireless communicated system with n single-antenna users. All the channel fading coefficients $\{H_1, ..., H_n\}$ are generated as i.i.d. complex Gaussian random realizations with zero mean and unit variance.

To obtain the average system goodput, we count the number of successfully decoded packets for the n users and average it over multiple fading realizations. To obtain the average packet error probability, we count the number of packet errors of a user k and average it over multiple fading realizations. In the simulation, each point of the system goodput and packet error probability are obtained by 20000 fading realizations. We consider two different successive interference cancellation policies, namely the *optimal policy* and the *random policy*.

- **Adaptive SIC with Optimal Decoding Order:** For every CSIR realization, the optimal decoding order is given by the descending order of the user received SNR ($\gamma_i = \sigma_i^2|H_i|^2$). The decoding process stops and all undecoded packets are declared corrupted as soon as there is any packet error in any decoding iteration because the subsequent iterations will surely be failed.

- **SIC with Random Decoding Order:** For every fading realization, a random permutation order is obtained and used as the decoding order. On each iteration, the base station attempts to decode the user using different possible paths as illustrated in figure 4.3. Different from the SIC with optimal

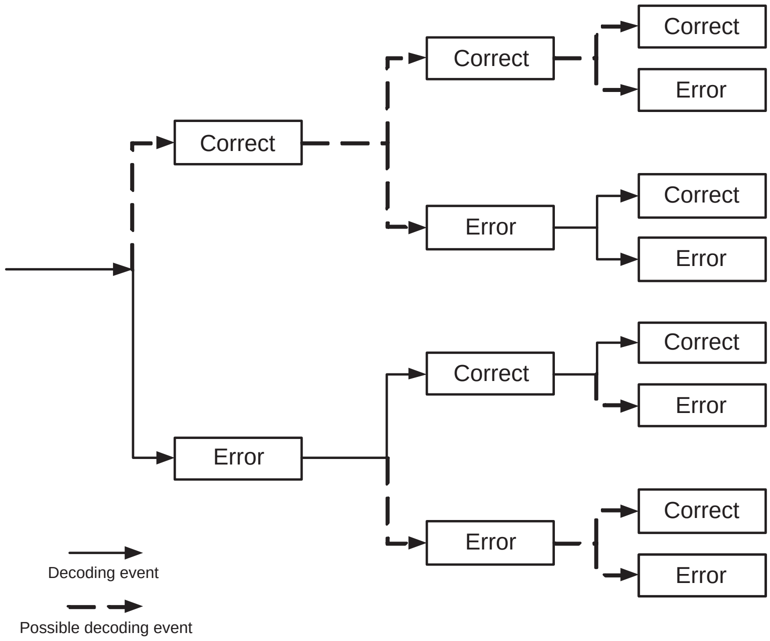

Figure 4.3: Illustration of the MUD-SIC decoding tree for random decoding order. The decoding process continues even there is packet error in the current iteration. This is because there is still a possibility that subsequent decoding iterations will be successful given the current decoding iteration fails.

decoding order, the decoding process continues even there is packet error in the current iteration. This is because in the tree processing as illustrated in figure 4.3, there is still a possibility that subsequent decoding iterations will be successful given the current decoding iteration fails. Finally, number of error packets for a user k will be counted and averaged over multiple fading realizations.

Under these two decoding order policies, we would like to compare the performance gains on the average system goodput and average packet error probability. Note that all solid lines represent the theoretical result and dotted markers represent the simulated result. Besides, units of all goodput measurements will be in bits/sec/Hz.

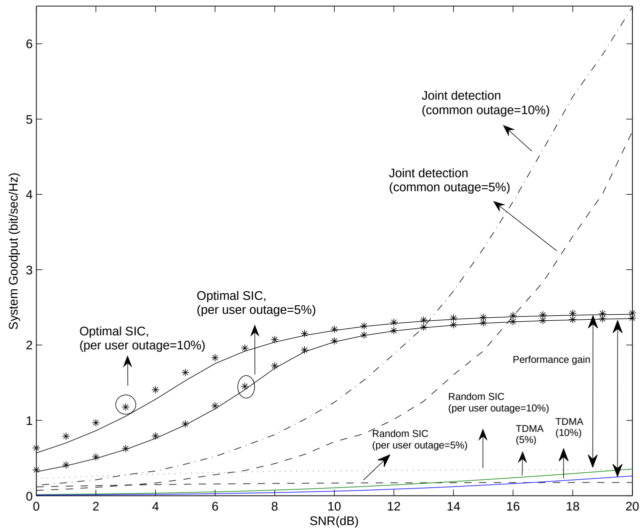

Figure 4.4: System goodput vs SNR(dB) with different outage (n=5). The solid line represent the theoretical expression and the dotted solid represent the simulated result of the system goodput respectively. The double sided arrow represent the performance gain of the optimal SIC over the random SIC.

4.4.1 Results on the Average System Goodput

Figure 4.4 shows the average system goodput versus the average user SNR (dB) for $n = 5$ users. Each curve in the graph represent different detection methods with same target average packet error probability (10% and 5%). It can be observed that the system goodput with optimal decoding order increases with SNR but with a diminishing return. This is because at high SNR, the SNR term in the MUD-SIC will cancel out each other in (4.2.2). As a result, the system goodput will be limited by the SINR rather than SNR[7]. Besides, a huge performance gain is found in SIC with the optimal decoding order over the SIC with random decoding order. With random SIC, the average system goodput suffers from the frequent packet decoding error. In order to achieve the same average packet error probability level, each user has to transmit at a lower data rate and this reduces the average

[7]Note that for joint detection, the system goodput will not be limited by SINR anymore. Yet, our focus in the chapter is to study the performance of MUD-SIC.

system goodput in the case of random decoding order. Furthermore, for the case with random decoding order, the average system goodput does not increase with the SNR[8]. Also, joint detection method which consider *common outage* is plotted for comparison. An interesting result can be observed from the figure. In low SNR regime, the optimal SIC outperforms the joint detection and vice versa in the high SNR regime. This is because joint detection consider *common outage* and optimal SIC consider *per user outage*. In low SNR regime, the performance of joint detection is limited by the decoding error in a weakest user, regardless any successful decoding in others. However, in the optimal SIC, as long as some users can be decoded correctly, it can contribute the system goodput. In high SNR regime, the performance of optimal SIC is degraded which due to strong interference from other users as we discussed previously. Nevertheless, the joint detection does not suffer from strong interference, therefore the system goodput still increase with SNR in high SNR regime which does not occur in SIC. Figure 4.5 shows the average system goodput versus the number of users n with different average packet error probabilities (10%) . Along all the curves, the same user SNR is fixed at 5dB and 10dB respectively. From the figure, the average system goodput increases as the number of the user increases. Besides, there is also diminishing return when number of users increases. This is because the packet error probability of each user depends on the other users which have been decoded. Similarly, there is a significant performance gain (indicated by the double arrow) between the optimal SIC and the random SIC (except when $n = 1$). As number of the users

[8]The goodput performance under random decoding order is limited not by the SNR but rather by the "packet outage" due to multiuser interference or SINR. For instance, with random decoding order, users decoded at later iterations will most likely suffer from non-zero *accumulated interference* $\widetilde{W_k^\pi}$ due to unsuccessful decoding in earlier iterations. Hence, the effective mutual information $\widetilde{C_k}$ is limited by the SINR which saturates at large SNR.

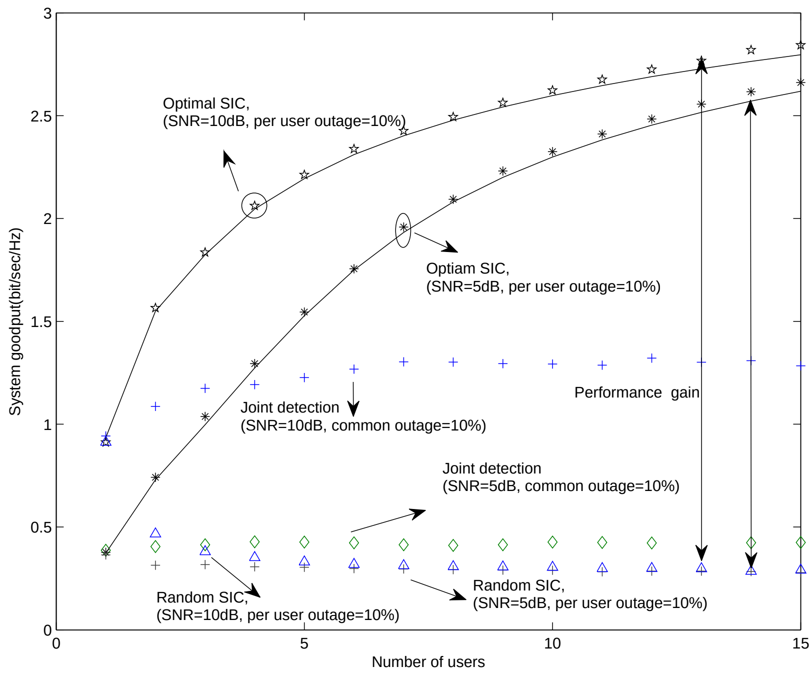

Figure 4.5: System goodput against number of users with different SNR (packet error probability=10%)

increases, the importance of the decoding order increases and this contributes to the performance gains. Similar to the above case, the average system goodput in the random SIC case does not scale with number of the users due to the failure of the interferers cancellation. Hence, the optimal decoding order for SIC is very important especially for high SNR cases. Furthermore, joint detection method which consider *common outage* is plotted for comparison. It is also very interesting that the system goodput of joint detection (which consider *common outage*) does not increase with the number of user for a fixed SNR. This counter intuitive result can be explained by the performance of joint detection is always limited by the weakest user. If we don't increase the SNR to help the weakest user, the system goodput can't increase with the number or user.

In all cases, the simulation results match with the analytical results. This verifies the analytical expressions on the average system goodput.

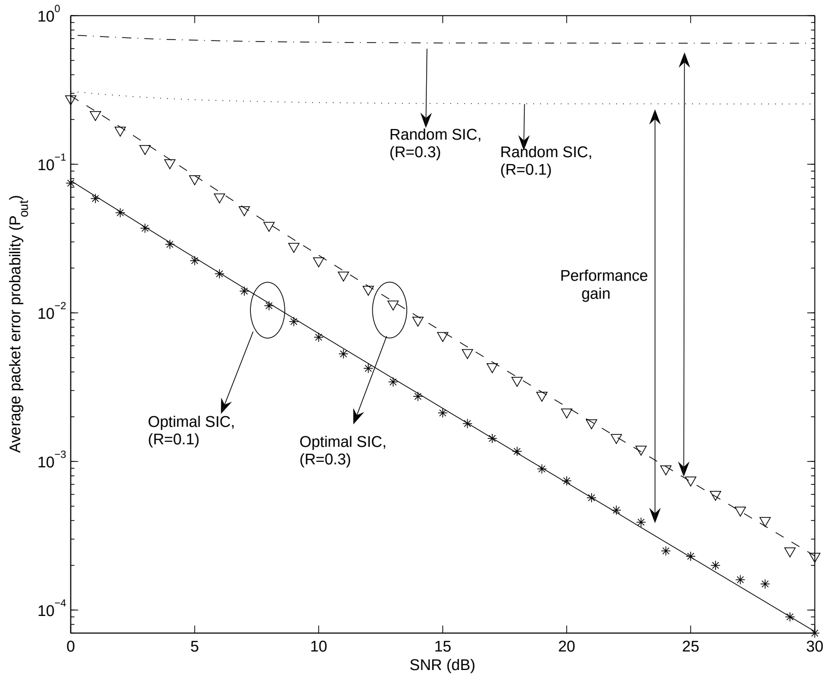

Figure 4.6: Average packet error probability against SNR with different transmitted rate(r) (Number of users(n)=5). The solid line represent the theoretical expression and the dotted solid represent the simulated packet error expression respectively. Different curve represent different transmitted rate with the same user. The double sided arrow represent the performance gain of the optimal SIC over the random SIC.

4.4.2 Results on Average Per-User Packet Error Probability

Similarly, the average packet error probability versus the average user SNR has been simulated and shown in figures 4.6 and 4.7 for $n = 5$ and $n = 10$ number of users respectively. From the figures, the average packet error probability corresponding to the optimal decoding order policy decreases with the average user SNR. However, with random SIC, the packet error probability does not decrease with increasing SNR.

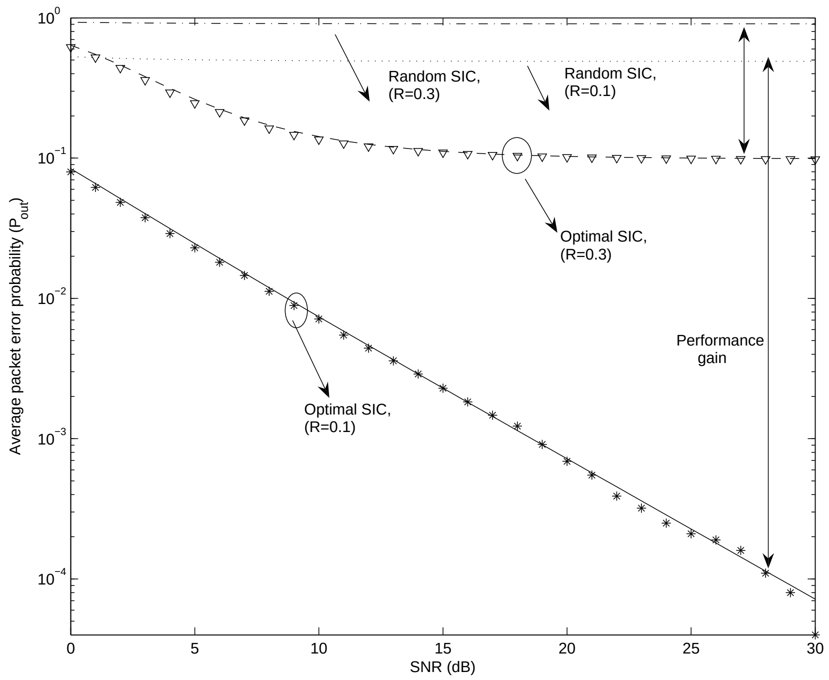

Figure 4.7: Average packet error probability against SNR with different transmitted rate(r) (Number of users(n)=10)

On the other hand, the average packet error probability versus the number of users at the same average user SNR (5dB and 10dB) and the same transmit data rate R is shown in figures 4.8 and 4.9. It can be observed that with the optimal decoding order policy, the packet error probability increases at a slower rate as the number of users increases. Similarly, in all cases, the simulation results match the analytical results closely, verifying the analytical expressions on the average outage probabilities.

4.5 Summary

In this chapter, we have derived the analytical expressions for the average system goodput and the *per-user packet outage* probability for multiacces channel with MUD-SIC. We consider a system with n users and a base station and derive the

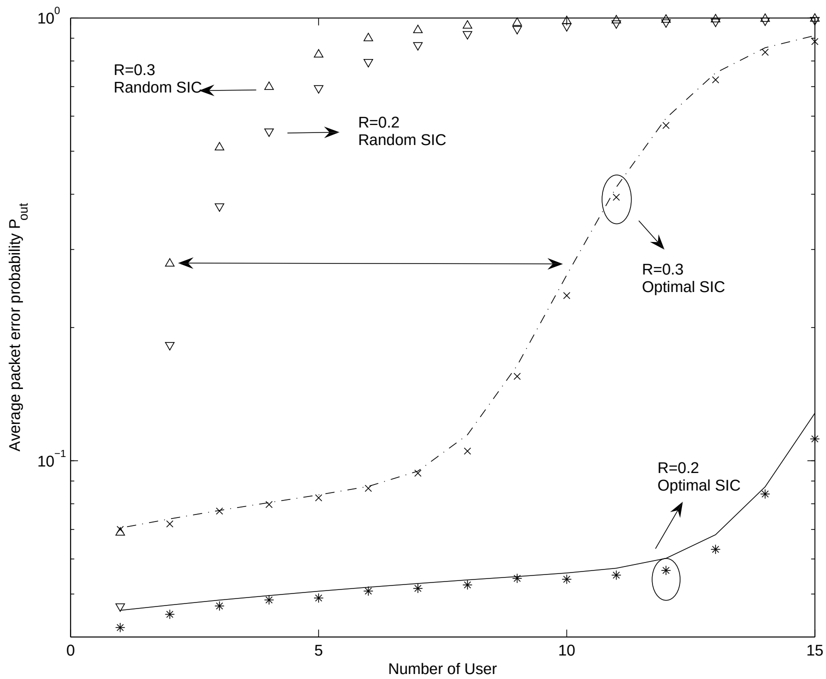

Figure 4.8: Average packet error probability against number of users with different transmitted rate(r) (SNR=5dB). The solid line represent the theoretical expression and the dotted solid represent the simulated packet error expression respectively. Different curve represent different transmitted rate with the same SNR(dB). The double sided arrow represent the performance gain of the optimal SIC over the random SIC.

optimal decoding order at the base station so as to maximize the total average goodput (which measures the b/s/Hz successfully delivered to the base station). Based on the optimal decoding order, we obtain the per-user packet outage probability and the system goodput based on ordered statistics. Numerical result and simulation results are obtained to verify the analytical expressions. The analytical expressions are found to be of close match with the simulations.

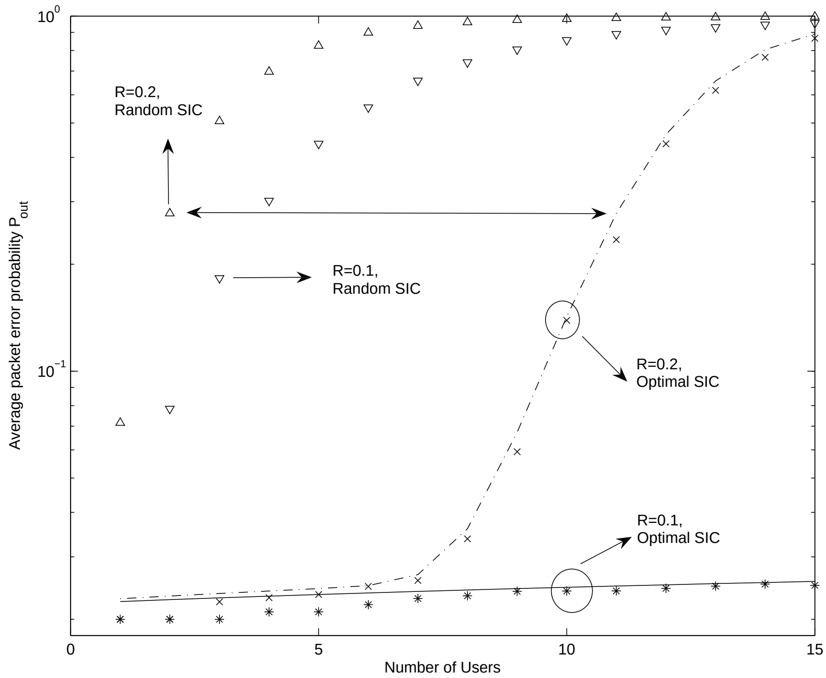

Figure 4.9: Average packet error probability against number of users with different transmitted rate(r) (SNR=10dB)

Chapter 5

Conclusions

In this thesis, we first investigate the performance of cross-layer scheduling with imperfect CSIT consideration in downlink OFDMA system. We formulate the cross-layer design as a mixed convex and combinational optimization problem. With imperfect CSIT, packet outage occurs even powerful error correction code is used for protection. To account for the packet outage effect, we define average system goodput, which measures the average b/s/Hz successfully delivered to the K mobiles, as the performance objective. Since the system performance suffers a significant loss on the system throughput in the presence of outdated CSI, we introduce certain degrees of diversity to protect the transmitted information. We are interested to find out the asymptotic performance on how the diversity order improve the system performance when there is CSIT error. By allocating N_d independent subbands to a user, the packet outage probability drops in the order of SNR^{-N_d}. On the other hand, the *system goodput* scales in the order of $\mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi^2_{k*};\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0\right)\right]$ at high SNR where $\tilde{s}^2 = \mathcal{O}\left\{(1-\sigma_e^2)(\log\log K)\right\}$ and $\mathcal{O}\left\{(1-\sigma_e^2)(\log K)\right\}$ for large N_d $[K > K_0]$ and large K [fixed N_d] respectively.

In the second part of thesis, we shifted our focus to uplink multi-access channel with successive interference cancellation receiver. We consider a system with n users and a base station and derive the optimal decoding order in the MUD based on the CSIR per fading slot at the base station. Based on the optimal decoding order, the analytical expressions for the packet outage probability and the system goodput are derived based on ordered statistics. From the results, the system goodput increases with the average SNR and the number of users with diminishing returns since interference is dominated in each decoding stage and results in uncancellated error.

Chapter 6

Appendix

6.0.1 Proof of Lemma 6.0.1 in Chapter 3

Consider the low SNR case when $P_0 \to 0$. The mutual information between the base station and the k-th mobile user with perfect CSIR is given by:

$$\frac{1}{L_s}\sum_{n=0}^{L_s-1}\sum_{m\in B_k}\log_2\left(1+\frac{\left|H_{mL_s+n}^{(k)}\right|^2 p_k n_F}{L_s N_d}\right) = \frac{1}{L_s}\sum_{n=0}^{L_s-1}\log_2\left(\prod_{m\in B_k}\left(1+\frac{|H_{mL_s+n}^{(k)}|^2 p_k n_F}{L_s N_d}\right)\right)$$

$$\doteq \frac{1}{L_s}\sum_{n=0}^{L_s-1}\log_2\left(1+\frac{\sum_{m\in B_k}\left|H_{mL_s+n}^{(k)}\right|^2 p_k n_F}{L_s N_d}\right) \stackrel{(a)}{=} N_d\log_2\left(1+\frac{\sum_{m\in B_k}\left|H_{mL_s}^{(k)}\right|^2 p_k n_F}{L_s N_d^2}\right)$$

where the $\doteq$ is due to the fact that $\prod_{m\in B_k}\left(1+\frac{|H_{mL_s+n}^{(k)}|^2 p_k n_F}{L_s N_d}\right) \doteq 1+\frac{p_k n_F}{L_s N_d}\sum_{m\in B_k}\left|H_{mL_s+n}^{(k)}\right|^2$

and the equality in (a) is due to the fact that:

$$\sum_{m\in B_k}\left|H_{mL_s+n}^{(k)}\right|^2 = tr\left(\mathbf{H}_m^{(k)}\mathbf{H}_m^{(k)H}\right) = tr\left\{\frac{1}{N_d}\mathcal{F}_L\mathcal{D}_n\mathcal{F}_L^H\mathbf{H}_0^{(k)}\mathbf{H}_0^{(k)H}\left(\frac{1}{N_d}\mathcal{F}_L\mathcal{D}_n\mathcal{F}_L^H\right)^H\right\}$$

$$= tr\left(\mathbf{H}_0^{(k)}\mathbf{H}_0^{(k)H}\right) = \sum_{m\in B_k}\left|H_{mL_s}^{(k)}\right|^2 \qquad (6.0.1)$$

where $\mathcal{F}_L$ is the $L \times L$ L-point FFT matrix (unitary) and $\mathcal{D}_n = diag[1, e^{-\frac{j2\pi n}{n_F}}, \ldots, e^{-\frac{j2\pi n(L-1)}{n_F}}]$. Hence, the packet outage probability for low SNR is given by:

$$\begin{aligned} P_{out}(k, \hat{\mathbf{H}}) &= \Pr\left[\frac{1}{L_s}\sum_{n=0}^{L_s-1}\sum_{m\in B_k}\log_2\left(1+\frac{n_F p_k}{L_s N_d}|H^{(k)}_{mL_s+n}|^2\right) < r_k/L_s|\hat{\mathbf{H}}\right] \\ &\doteq \Pr\left[N_d\log_2\left(1+\frac{n_F p_k}{L_s N_d^2}\sum_{m\in B_k}|H^{(k)}_{mL_s}|^2\right) < r_k/L_s|\hat{\mathbf{H}}\right] \end{aligned}$$

On the other hand, for high SNR, we first consider a lower bound of the packet outage probability in (3.1.9). From (6.0.1), the mutual information can be expressed as:

$$\begin{aligned} &\frac{1}{L_s}\sum_{n=0}^{L_s-1}\sum_{m\in B_k}\log_2\left(1+\frac{\left|H^{(k)}_{mL_s+n}\right|^2 p_k n_F}{L_s N_d}\right) = \frac{1}{L_s}\sum_{n=0}^{L_s-1}\log_2\left(\prod_{m\in B_k}\left(1+\frac{\left|H^{(k)}_{mL_s+n}\right|^2 p_k n_F}{L_s N_d}\right)\right) \\ &\overset{(a)}{\le} \frac{1}{L_s}\sum_{n=0}^{L_s-1}\log_2\left(\frac{1}{N_d}\sum_{m\in B_k}\left(1+\frac{\left|H^{(k)}_{mL_s+n}\right|^2 p_k n_F}{L_s N_d}\right)\right)^{N_d} \\ &\overset{(b)}{=} N_d\log_2\left(1+\frac{\sum_{m\in B_k}\left|H^{(k)}_{mL_s}\right|^2 p_k n_F}{L_s N_d^2}\right) \end{aligned} \tag{6.0.2}$$

where (a) is due to geometric mean less than or equal to the arithmetic mean and (b) is due to (6.0.1). Hence, we have

$$P_{out}(k, \hat{\mathbf{H}}) \doteq \Pr\left[N_d\log_2\left(1+\frac{\sum_{m\in B_k}\left|H^{(k)}_{mL_s}\right|^2 p_k n_F}{L_s N_d^2}\right) < r_k/L_s|\hat{\mathbf{H}}\right] \tag{6.0.3}$$

Next, we shall consider an upper bound of the packet for the packet outage probability in (3.1.9). Let $I^{(k)}_n = \sum_{m\in B_k}\log_2\left(1+\frac{|H^{(k)}_{mL_s+n}|^2 p_k n_F}{L_s N_d}\right)$. Since the outage event $\left\{\frac{1}{L_s}\sum_{n=0}^{L_s-1} I^{(k)}_n \le r_k/L_s\right\}$ is a subset of $\bigcup_{n=0}^{L_s-1}\left\{I^{(k)}_n \le r_k/L_s\right\}$, we have

$$\begin{aligned} P_{out}(k, \hat{\mathbf{H}}) &= \Pr\left[\frac{1}{L_s}\sum_{n=0}^{L_s-1} I^{(k)}_n \le r_k/L_s|\hat{\mathbf{H}}\right] \le \Pr\left[\bigcup_{n=0}^{L_s-1}\left\{I^{(k)}_n \le r_k/L_s\right\}|\hat{\mathbf{H}}\right] \\ &\le \sum_{n=0}^{L_s-1}\Pr\left[I^{(k)}_n \le r_k/L_s|\hat{\mathbf{H}}\right] \overset{(a)}{=} L_s\Pr\left[I^{(k)}_0 \le r_k/L_s|\hat{\mathbf{H}}\right] \end{aligned} \tag{6.0.4}$$

where (a) is because $I_n^{(k)}$ are identically distributed. Given the CSIT $\hat{\mathbf{H}}$, the random variables $\mathbf{H}_{k,n}$ inside the probability operator in (6.0.4) are non-central chi-square distributed with $2N_d$ degrees of freedom, variance $1-\sigma_e^2$ and non-centrality parameter $s^2 = \|\hat{\mathbf{H}}_n^{(k)}\|^2$. Let $\gamma_{k,n} = |H_n^{(k)}|^2$, $\alpha\gamma_n^{(k)}$ be the SNR of the n-th subcarrier and define a transformation $y = \frac{\log(1+\alpha\gamma)}{\log\alpha}$ where $\alpha = p_k n_F/(N_d L_s)$. The joint p.d.f. of the random variables $\{y_n^{(k)}\}$ after transformation from the random variables $\{\gamma_n^{(k)}\}$ is given by:

$$f(\mathbf{y}_k) = \frac{(\log\alpha)^{N_d}\alpha^{\sum_{m\in B_k} y_{mL_s}^{(k)}}}{\alpha^{N_d}(1-\sigma_e^2)^{N_d}} \exp\left\{ -\frac{\sum_{m\in B_k}\left(\alpha^{y_{mL_s}^{(k)}-1} - \frac{1}{\alpha} + s_{mL_s}^{(k)}{}^2\right)}{2(1-\sigma_e^2)} \right\} \times \prod_{m\in B_k} I_0\left(\frac{s\sqrt{\alpha^{y_{mL_s}^{(k)}-1} - 1/\alpha}}{1-\sigma_e^2} \right)$$

where $\mathbf{y}_k = \{y_m^{(k)}\}_{m\in B_k}$.

From (6.0.4), the upper bound can be expressed as:

$$\Pr\left[I_0^{(k)} \le r_k/L_s|\hat{\mathbf{H}}\right] = \Pr\left[\sum_{m\in B_k} y_{mL_s}^{(k)} \le \frac{r_k}{L_s\log\alpha}|\hat{\mathbf{H}}\right] = \int_{\sum_{m\in B_k} y_{mL_s}^{(k)} \le r_k/(L_s\log\alpha)} f(\mathbf{y})d\mathbf{y}$$
$$\overset{(a)}{\doteq} \frac{(\log\alpha)^{N_d}}{\alpha^{N_d}(1-\sigma_e^2)^{N_d}} \int_{\mathcal{G}} \alpha^{\sum_{m\in B_k} y_{mL_s}^{(k)}} d\mathbf{y}_k \le \frac{(\log\alpha)^{N_d}}{\alpha^{N_d}(1-\sigma_e^2)^{N_d}} \int_{\mathcal{G}} \alpha^{r_k/(L_s\log\alpha)} d\mathbf{y}_k \le \frac{(\log\alpha)^{N_d}}{(1-\sigma_e^2)^{N_d}} \frac{e^{r_k/L_s}}{\alpha^{N_d}}$$
$$\dot{\le} \frac{e^{r_k/L_s}}{(p_k n_F/(NdL_s))^{N_d}} \doteq \Pr\left[N_d\log_2\left(1 + \frac{\sum_{m\in B_k}\left|H_{mL_s}^{(k)}\right|^2 p_k n_F}{L_s N_d^2}\right) < r_k/L_s|\hat{\mathbf{H}}\right] \tag{6.0.5}$$

where (a) is due to the fact that when $y_{k,mL_s} > 1$, $\exp\left\{ -\frac{\sum_{m\in B_k}\left(\alpha^{y_{mL_s}^{(k)}-1} - \frac{1}{\alpha} + s_{mL_s}^{(k)}{}^2\right)}{2(1-\sigma_e^2)} \right\} \prod_{m\in B_k} I_0\left(\frac{s\sqrt{\alpha^{y_{mL_s}^{(k)}-1} - 1/\alpha}}{1-\sigma_e^2} \right)$ decays with α exponentially. Hence, at high SNR, we can ignore the integration region with any $y_{k,mL_s} > 1$ and replace with the integration region $\mathcal{G} = \{\sum_{m\in B_k} y_{mL_s}^{(k)} \le r_k/(L_s\log\alpha)\}\bigcap\{y_{mL_s}^{(k)} \le 1, \forall m \in B_k\}$. Thus, using equation (6.0.4) and (6.0.5), we have

$$P_{out}(k,\hat{\mathbf{H}}) \doteq \Pr\left[N_d\log_2\left(1 + \frac{\sum_{m\in B_k}\left|H_{mL_s}^{(k)}\right|^2 p_k n_F}{L_s N_d^2}\right) < r_k/L_s|\hat{\mathbf{H}}\right] \text{ for large SNR.}$$

6.0.2 Proof of Lemma 2 in Chapter 3

Consider a sequence of i.i.d. random variable x_k, having central chi-square distribution with degree of freedom $2n$. Formally, x_k is characterized by the CDF of $F(x) = 1 - e^{-\frac{x}{\sigma_X^2}} \sum\limits_{m=0}^{n-1} \frac{1}{m!} \left(\frac{x}{\sigma_X^2} \right)^m$; the PDF of $f(x) = \frac{1}{\sigma_X^{2n} \Gamma(n)} x^{n-1} e^{-\frac{x}{\sigma_X^n}}, x \geq 0$, where σ_X^2 is the variance of the underlying complex Gaussian random variables. Define the growth function $g(x) = \frac{1-F(x)}{f(x)}$. It is obvious that

$$\lim_{x \to \infty} g(x) = 1 \tag{6.0.6}$$

From [51] and [52], we have the following expression

$$\begin{aligned} &\log[-\log F^K(b_K + yg(b_K)) = -y + \tfrac{y^2}{2!} g'(b_K) + \tfrac{y^3}{3!} \left[g(b_K) g^{(2)}(b_K) - 2g'^2(b_K) \right] ... + ... \\ &+ \tfrac{e^{-y}+...}{2K} + \tfrac{5e^{-2y+...}}{2K} + ... - \tfrac{e^{-3y}}{8K^3} + ... + ... \end{aligned} \tag{6.0.7}$$

where b_K is given by $F(b_K) = 1 - \frac{1}{K}$, i.e. $e^{-\frac{b_K}{\sigma_X^2}} \sum\limits_{m=0}^{n-1} \frac{1}{m!} \left(\frac{b_K}{\sigma_X^2} \right)^m = \frac{1}{K}$.

In the other words, b_K is the solution of $\frac{b_K}{\sigma_X^2} - \log \sum\limits_{m=0}^{n-1} \frac{1}{m!} \left(\frac{b_K}{\sigma_X^2} \right)^m = \log K$.

So $\frac{b_K}{\sigma_X^2} - \log \left(\frac{1}{(n-1)!} \left(\frac{b_K}{\sigma_X^2} \right)^{n-1} \right) - \mathcal{O}\left(\log \left(\frac{1}{(n-2)!} \left(\frac{b_K}{\sigma_X^2} \right)^{n-2} \right) \right) \doteq \log K$ and $\frac{b_K}{\sigma_X^2} - (n-1)\log \left(\frac{b_K}{\sigma_X^2} \right) - (n-2)\, \mathcal{O}\left(\log \left(\frac{b_K}{\sigma_X^2} \right) \right) \doteq \log K$.

Thus, $b_K = \sigma_X^2 (\log K + (n-1) \log \log K)$ satisfies the above equation for large K. Note that the CDF of $\tilde{x} = \max\limits_{1 \leq k \leq K} x_k$ is given by $F^K(\tilde{x})$ substituting y as $\pm \log \log K$ in equation (6.0.7) and from equation (6.0.6),

$$\Pr\left\{ -\log \log K \leq \max_{1 \leq k \leq K} x_k - b_K \leq \log \log K \right\} \geq 1 - \mathcal{O}\left(\frac{1}{\log K} \right).$$

Therefore,

$$\begin{aligned} &\Pr\left\{ \sigma_X^2 \log K + \sigma_X^2 (n-2) \log \log K \leq \max_{1 \leq k \leq K} x_k \leq \sigma_X^2 \log K + \sigma_X^2 n \log \log K \right\} \\ &\geq 1 - \mathcal{O}\left(\frac{1}{\log K} \right) \end{aligned} \tag{6.0.8}$$

6.0.3 Proof of Theorem 1 in Chapter 3

Given the CSIT $\widehat{\mathbf{H}}$, the conditional average goodput of the k-th user ($k \in A^*(\widehat{\mathbf{H}})$) for high SNR P_0 after cross-layer scheduling is given by:

$$G^{**}_{goodput}(\hat{\mathbf{H}}) = \frac{(1-\epsilon)L_s N_d}{n_F} \sum_{k \in A^*} \log_2 \left(\frac{F^{-1}_{\chi^2_k; s^2(B_k); \sigma_e^2/N_d}(\epsilon) P_0 n_F}{N_d L_s |A^*|} \right) \tag{6.0.9}$$

where $s^2(\widehat{\mathbf{H}}; B_k^*) = \frac{1}{N_d} \sum_{m \in B_k^*} |\hat{H}^{(k)}_{mL_s}|^2$. The average system goodput is given by $\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})]$. Observe that $F^{-1}_{\chi^2_k; s^2; \sigma_e^2/N_d}(x)$ is an increasing function of s^2 for a given x.Consider selecting one user with the largest $s^2(\hat{\mathbf{H}}; B_k^*)$ from K users. Using the result in Lemma 2, we have $s^2(\widehat{\mathbf{H}}; B_k^*) = \mathcal{O}\left(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)\right)$ with probability 1 (for sufficiently large K). Assume that $K \gg |A|$ and if we ignore the inter-dependency (or coupling constraint) in the user selection result between different users, we have $s^2(\widehat{\mathbf{H}}; B_k^*) = \mathcal{O}\left(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)\right)$ with probability 1 for all other users $k \in A^*$. Hence, the result follows by direct substitution into (6.0.9).

Similarly, for low SNR ($P_0 \to 0$), the conditional average goodput of the k-th user ($k \in A^*(\widehat{\mathbf{H}})$) for low SNR P_0 after cross-layer scheduling is given by:

$$G^{**}_{goodput}(\hat{\mathbf{H}}) = \frac{(1-\epsilon)L_s N_d}{n_F} \log_2 \left(1 + \frac{F^{-1}_{\chi^2_{k^*}; s^2(B_k^*); \sigma_e^2/N_d}(\epsilon) P_0 n_F}{N_d L_s} \right) \doteq (1-\epsilon) F^{-1}_{\chi^2_{k^*}; s^2(B_k^*); \sigma_e^2/N_d}(\epsilon) P_0$$

where k^* is obtained by selecting one user with the largest $s^2(\hat{\mathbf{H}}; B_k)$ from the K users. Using the result in Lemma 2, we have $s^2(\widehat{\mathbf{H}}; B_k^*) = \mathcal{O}\left(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)\right)$ with probability 1.

6.0.4 Proof of Lemma 4 in Chapter 3

For a non-central chi-square random variable X (with $2n$ degrees of freedom, non-centrality parameter s^2 and variance σ^2). Given a constant x, the asymptotic CDF of X for large s is given by:

$$
\begin{aligned}
F_x(x) \quad &= \int_0^x \frac{1}{2\sigma^2}\left(\frac{u}{s^2}\right)^{(2n-2)/4} \exp\left(\frac{-(s^2+u)}{2\sigma^2}\right) I_n\left(\frac{s\sqrt{u}}{\sigma^2}\right) du \\
&\doteq \frac{1}{2\sigma^2}\int_0^x \left(\frac{u}{s^2}\right)^{(2n-2)/4} \exp\left(-\left[s-\sqrt{u}\right]^2/2\sigma^2\right) du \doteq \exp\left[-\frac{(s-\sqrt{x})^2}{2\sigma^2}\right]
\end{aligned}
$$

where $I_n(x)$ is the $n-$th order modified Bessel function of the first kind . Therefore the inverse cdf of a non-central chi square random variable X can be approximated asymptotically as

$$
F_X^{-1}(x) \doteq \mathcal{O}(s^2\sigma^2) \tag{6.0.10}
$$

Figure 3.7 illustrates a comparison between the actual and asymptotic $F^{-1}_{\chi^2;s^2;\sigma^2_X}(x)$ versus s^2 for a given x.

6.0.5 Proof of Lemma 3 in Chapter 3

The CDF of a non-central chi-square random variable X (with $2n$ degrees of freedom, non-centrality parameter s^2 and variance σ^2)for small x can be expressed as:

$$
\begin{aligned}
F_x(x) &= \int_0^x \frac{1}{2\sigma^2}\left(\frac{u}{s^2}\right)^{(2n-2)/4} \exp\left(\frac{-(s^2+u)}{2\sigma^2}\right) I_n\left(\frac{s\sqrt{u}}{\sigma^2}\right) du \\
&= \sum_{i=0}^{\infty} a_i x^i \doteq \frac{1}{\sigma^{2n} n!} \exp\left(\frac{-s^2}{\sigma^2}\right) x^n
\end{aligned} \tag{6.0.11}
$$

where $I_n(x)$ is the $n-$th order modified Bessel function of the first kind and $a_i = \frac{1}{i!}\left.\frac{\partial F_X^{(i)}(x)}{\partial x^i}\right|_{x=0}$ is the Taylor series coefficient. The $\doteq$ line is obtained for asymptotically small x by taking the first non-zero term in the Taylor series, which is a_n. Then the inverse CDF of X can be approximated as :

$$
F_x^{-1}(x) \doteq x^{1/n}\sigma^2(n!)^{1/n} \exp\left(\frac{s^2}{n\sigma^2}\right) \tag{6.0.12}
$$

for small x. Figure 3.8 illustrates the actual and asymptotic $F^{-1}_{\chi^2;s^2(B_k);\sigma_X^2}(x)$ versus x for a given s^2. We can observe that the asymptotic inverse CDF in (3.3.3) matches the actual inverse CDF closely for small x.

6.0.6 Proof of Theorem 2 in Chapter 3

Using the result of Lemma 3, the average system goodput $\overline{\rho}^*$ in (3.3.2) can be expressed as:

$$\overline{\rho}^* = \mathcal{O}\left((1-\epsilon)\log_2\left(\frac{\sigma_e^2}{N_d}(\epsilon N_d!)^{1/N_d}\exp\left(\frac{s^2}{\sigma_e^2}\right)P_0\right)\right)$$

As a result, the average packet outage probability $\overline{P_{out}(k)}$ scales with the SNR P_0 (at a given average goodput) in the order of:

$$\overline{P_{out}(k)} = E_{\hat{\mathbf{H}}}\left[P_{out}(k,\widehat{\mathbf{H}})\right] = \mathcal{O}\left(P_0^{-N_d}\right)$$

for sufficiently small ϵ.

6.0.7 Proof of Lemma 5 in Chapter 4

Consider a given CSIR realization $\mathbf{H}$, the optimal decoding order (w.r.t. goodput) is the one that has the largest number of successfully decoded users because the transmit data rate of all the n users are the same. Consider the first iteration, the accumulated *undecodable interference* $\widetilde{W_1^\pi} = 0$ and hence, the *effective instantaneous mutual information* in (4.2.3) for the first iteration is given by:

$$\widetilde{C_{\pi(1)}}(\mathbf{H},\pi,1) = C_{\pi(1)}(\mathbf{H},\pi,1) = \log_2\left(1+\frac{\gamma_{\pi(1)}}{1+\sum_{j=2}^{n}\gamma_{\pi(j)}}\right)$$

Let $\pi^*(1) = \arg\max_{k\in[1,n]}\gamma_k$ be the user with the largest instantaneous SNR and $j \neq \pi^*(1)$ be some other user. If the j-th user can be decoded in the first iteration (i.e. $r < \widetilde{C_j}(\mathbf{H},\pi,1)$), so can the $\pi^*(1)-$th user because $\widetilde{C_j}(\mathbf{H},\pi,1) \leq$

$\widetilde{C_{\pi^*(1)}}(\mathbf{H}, \pi, 1)$. Hence, we should decode user $\pi^*(1)$ in the first iteration because otherwise, (say decoding user j rather than user $\pi^*(1)$ in the first iteration), such decoding order will result in potentially higher accumulated *undecodable interference* $\widetilde{W_2^\pi} = \gamma_{\pi(1)}\mathcal{I}[r \geq \widetilde{C_{\pi(1)}}(\mathbf{H}, \pi, 1)]$. As a result, the optimal decoding order (given a CSIR realization) is given by always picking users with the highest SNR. i.e.

$$\pi^*(i) = \arg \max_{k \in [1,n] \setminus \{\pi^*(1), \pi^*(2), .. \pi^*(i-1)\}} \gamma_k. \tag{6.0.13}$$

6.0.8 Proof of Lemma 6 in Chapter 4

Without loss of generality, consider a particular optimal decoding order $\pi = (1, ..., n)$. Hence, we have $\gamma_1 \geq \gamma_2 \cdots \geq \gamma_n$. Applying probability transformation theory for the $1-1$ transformation in (4.3.3), the joint pdf of $\{Z_1, ..., Z_n\}$ is given by:

$$\begin{aligned} f_{Z_1,...,Z_n}(z_1, ..., z_n|\pi) &= \mathbf{J} \quad f_{\gamma_1,\gamma_2,...\gamma_n}(x_1, ..., x_n|\pi)|_{x_i = \sum_{j=i}^{n} Z_j/j, i \in \{1,2,..,n\}} \\ &= \mathbf{J} \quad \frac{1}{\Pr(\pi)\prod_{i=1}^{n}\sigma_i^2}\prod_{i=1}^{n} e^{-\phi_i z_i} \end{aligned} \tag{6.0.14}$$

where $\mathbf{J}$ denotes the Jacobian of the transformation which is given by $\frac{1}{n!}$ and ϕ_i is defined in (4.3.4). From equation (6.0.14), the joint p.d.f. can then be expressed as:

$$\begin{aligned} f_{Z_1,...,Z_n}(z_1, ..., z_n|\pi) &= \frac{1}{n!}\frac{1}{\Pr(\pi)\prod_{i=1}^{n}\sigma_i^2}\prod_{i=1}^{n} e^{-\phi_i z_i} \\ &= \prod_{i=1}^{n} G(z_i) \end{aligned} \tag{6.0.15}$$

where $G(z_i) = k_i e^{-\phi_i z_i}$ and k_i can be chosen to satisfy $\int_0^\infty G(z_i) = 1$. Hence, $Z_1, Z_2 ... Z_n$ are independent (not necessary identical) random variable. By choosing k_i equal to ϕ_i, all Z_i are exponential random variables but with a different parameter ϕ_i which is given by the equation(4.3.4).

6.0.9 Proof of Lemma 7 in Chapter 4

The characteristic function of the random variable $\Gamma_m = \sum_{v=m}^{n} \lambda_v Z_v$ is given by:

$$\Phi_{\Gamma_m}(\omega) = \prod_{v=m}^{n} \frac{1}{(1-\overline{\lambda}_v j\omega)} \tag{6.0.16}$$

By the partial fraction theorem[53], equation (4.3.7) can be expressed as:

$$\Phi_{\Gamma_m}(\omega) = \sum_{v=m}^{n} \frac{A_v}{(1-\overline{\lambda_v} j\omega)} \qquad \text{where} \quad A_v = \prod_{u=m,u\neq v}^{n} \frac{\overline{\lambda_v}}{\overline{\lambda_v}-\overline{\lambda_u}} \tag{6.0.17}$$

Hence, the characteristic function can be further expressed as the sum of characteristic function of several exponential random variable. After doing the inverse Fourier transform[54], the probability density function is given by:

$$f_{\Gamma_j}(x) = \sum_{v=j}^{n} \frac{A_v}{|\overline{\lambda_v}|} B_v \tag{6.0.18}$$

where $x_j \in \Re$ and

$$\overline{\lambda_v} = \lambda_v/\phi_v \quad , \quad A_v = \left(\prod_{u=j,u\neq v}^{n} \frac{\overline{\lambda_v}}{(\overline{\lambda_v}-\overline{\lambda_u})}\right), B_v = \begin{cases} e^{\frac{-x}{|\overline{\lambda_v}|}} u(x) & \text{if } \overline{\lambda_v} \geq 0 \\ e^{\frac{x}{|\overline{\lambda_v}|}} u(-x) & \text{otherwise} \end{cases} \tag{6.0.19}$$

for $x \in \Re$ and the results in the equation (4.3.7) follows.

References

[1] G. Corazza, "Proceedings of the ieee," *Studies in Mathematics and Mechanics Presented to Richard von Mises*, pp. 1307–1311, Jul 1998.

[2] T. Rappaport, *Wireless Communication: Principle and practice.* Prentice Hall, 2002.

[3] V. K. Lau and R. Y. Kwok, *Channel Adaptive Technologies and Cross Layer Designs for Wireless Systems with Multiple Antennas*, 1st ed. John Wiley Sons, 2006.

[4] J. Andrews, "Interference cancellation for cellular systems: A contemporary overview," *IEEE Wireless Commun. Mag.*, vol. 12, pp. 19–29, Apr. 2005.

[5] Y. Kim, B. J. Jeong, J. Chung, C.-S. Hwang, J. Ryu, K.-H. Kim, and Y. K. Kim, "Beyond 3g: vision, requirements, and enabling technologies," *IEEE Commun. Mag.*, vol. 41, pp. 120– 124, Mar. 2003.

[6] P. Svedman, "Multiuser diversity orthogonal frequency division multiple access systems," Master's thesis, Signal Processing Laboratory KTH Signals, Sensors and Systems Royal Institute of Technology, 2004.

[7] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiusercommunications," in *Proc. IEEE Int. Conf. Comm.1995*, vol. 1, June 1995, pp. 331–335.

[8] K. N. Lau and Y. K. Kwok, "On the synergy between adaptive physical layer and multiple-access control for integrated voice and data services in cellular networks," *IEEE Trans. on Vehicular Technologies*, pp. 1338–1351, November 2002.

[9] D. Tse and S. Hanly, "Multi-access fading channels: Part i: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, pp. 2796–2815, 1998.

[10] M. Realp, A. Perez-Neira, and C. Mecklenbrauker, "A cross-layer approach to multi-user diversity in heterogeneous wireless systems," in *Proc. IEEE International Conference on Communications, 2005. ICC '05*, vol. 4, 2005, pp. 2791– 2796.

[11] A. Sali, A. Widiawan, S. Thilakawardana, R. Tafazolli, and B. Evans, "Cross-layer design approach for multicast scheduling over satellite networks," in *Proc. IEEE International Symposium on Wireless Communication Systems*, Sep 2005, pp. 701– 705.

[12] Q. Liu, S. Zhou, and G. Giannakis, " Cross-layer scheduling with prescribed QoS guarantees in adaptive wireless networks," *IEEE J. Select. Areas Commun.*, pp. 1056– 1066, Nov 2005.

[13] S. Z. Qingwen Liu and G. Giannakis, "Cross-layer scheduling with prescribed QoS guarantees in adaptive wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 23, pp. 1056–1066, May 2005.

[14] G. Song and Y. Li, "Cross-layer optimization for OFDM wireless networks-part I: theoretical framework," *IEEE Trans. Wireless Commun.*, pp. 614–624, Mar 2005.

[15] ——, "Cross-layer optimization for OFDM wireless networks-part II: Algorithm Development," *IEEE Trans. Wireless Commun.*, pp. 625–634, Mar 2005.

[16] R. Y. Kwok and V. K. Lau, *Wireless Internet and Mobile Computing: Interoperability and Performance*, 1st ed. Wiley-IEEE Press, 2007.

[17] A. J. Viterbi, *CDMA: principles of spread spectrum communication.* Redwood City, CA, USA: Addison Wesley Longman Publishing Co., Inc., 1995.

[18] P. Svedman, "optimal multiuser signal detection," Ph.D. dissertation, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Aug 1984.

[19] T. M.Cover and J. A. Thomas, *Elements of Information Theory.* Wiley, 1991.

[20] M. Reed, C. Schlegel, P. Alexander, and J. Asenstorfer, "Iterative multiuser detection for CDMA with FEC: near-single-user-performance," *IEEE Trans. Commun.*, pp. 1693–1699, Jun 1998.

[21] P. Alexander, M. Reed, J. Asenstorfer, and C. Schlegel, " Iterative multiuser interference reduction: turbo CDMA," *IEEE Trans. Commun.*, pp. 1008–1014, Jul 1999.

[22] X. Wang and H. Poor, " Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, pp. 1046–1061, Jul 1999.

[23] P. Jung and M. Nasshan, " Results on turbo-codes for speech transmission in a joint detectionCDMA mobile radio system with coherent receiver antenna diversity," IEEE *Transactions on Vehicular Technology*, pp. 862–870, Nov 1997.

[24] J. C. J. T. M. Agrawal, A. Andrews, "iterative power control for imperfect successive interference cancellation," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 878– 884, May 2005.

[25] D. Warrier and U. Madhow, "The capacity of cellular cdma with controlled power disparities," in *Proc. EEE Vehicular Technology Conference*, May 1998, pp. 1873–1878.

[26] R. Siu Man Shum Cheng, "Power control for multirate cdma systems with interferencecancellation," in *Proc. IEEE Global Telecommunications Conference, 2000. GLOBECOM '00*, vol. 2, 2000, pp. 895–900.

[27] W. C. Jakes and D. C. Cox, Eds., *Microwave Mobile Communications.* Wiley-IEEE Press, 1994.

[28] B. Sklar, "Rayleigh fading channels in mobile digital communication systems.ii. mitigation," *IEEE Commun. Mag.*, vol. 35, pp. 102–109, July 1997.

[29] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell System Tech. J.*, vol. 45, pp. 1775–1796, Dec. 1966.

[30] S. Weinstein and P. Ebert, "Data transmission by frequency-division multiplexing using the discrete fourier transform," *IEEE Trans. Commun.*, vol. 19, pp. 628– 634, Oct. 1966.

[31] D. Tse, Ed., *Fundamentals of Wireless Communication.* Cambridge University Press, 2005.

[32] J. Holtzman, "Ds/cdma successive interference cancellation," in *Proc. IEEE Third International Symposium on Spread Spectrum Techniques and Applications, 1994. IEEE ISSSTA '94*, Jul 1994, pp. 69–78.

[33] J. Chen, A.Berry, and L. Honig, "Large system performance of downlink OFDMA," *Proc. IEEE Int. Symposium on Information Theory ISIT*, pp. 1399–1403, July 2006.

[34] S. Sanayei and A.Nosratinia, "Exploiting multiuser diversity with only 1-bit feedback," in *IEEE Wireless Communications and Networking Conference*, vol. 2, 2005, pp. 978–983.

[35] P. Svedmane, S. Wilson, L. Cimini, and B. Ottersten, "Opportunistic beamforming and scheduling for OFDMA systems," *IEEE Trans. Commun.*, Oct. 2004.

[36] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Personal Commun. Mag.*, vol. 49, pp. 1073– 1096, May 2003.

[37] D. Tse, P.Viswanath, and L.Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 10, pp. 1859–1874, Sept. 2004.

[38] S. Chung, G.D.Jr.Forney, T. Richardson, and R. Urbanke, "On the design of low-density parity-check codes within 0.0045 db of the shannon limit," *IEEE Commun. Lett.*, vol. 5, pp. 58–60, Feb. 2001.

[39] Z. K. M. Ho, V. K. N.Lau, and R. S. Cheng, "Closed loop cross layer scheduling for goodput maximization in frequency selective environment with no csit," in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2007, pp. 299–303.

[40] T.Cover, *Elements of information theory.* John Wiley and Sons Inc.,, 1991.

[41] T. Guess and M. K. Varanasi, "Signal design for bandwidth efficient multiple-access communications based on eigenvalue optimization," *IEEE Transactions on Information Theory*, vol. 46, pp. 2045–2058, 2000.

[42] A. Kapur and M. K. Varanasi, "Multiuser detection for overloaded cdma systems," *IEEE Transactions on Information Theory*, vol. 49, pp. 1728–1742, 2003.

[43] S. Shamai and A. Wyner, "Information theoretic considerations for symmetric cellular, multiple-access fading channels- part i," *IEEE Transaction on Information Theory*, vol. 43, pp. 1877–1894, 1997.

[44] ——, "Information-theoretic consideration for symmetric, cellular, multiple-access fading channels-part ii," *IEEE Transaction on Information Theory*, vol. 43, pp. 1895–1911, 1997.

[45] R. Gallager, "Low density parity check codes," *Number 21 in Research Monograph Series, MIT Press*, 1962.

[46] D. Tse and S. Hanly, "Multi-access fading channels: Part ii: Delay-limited capacities," *IEEE Transactions on Information Theory*, vol. 44, pp. 2816–2831, 1998.

[47] L. Li, N. Jindal, and A. Goldsmith, "Outage capacities and optimal power allocation for fading multiple-access channels," *IEEE Transactions on Information Theory*, vol. 41, no. 4, pp. 1326 – 1347, Apr 2005.

[48] I. Bettesh and S. Shamai, "Outages, expected rates and delays in multiple-users fading channels," *Proceedings Conference on Information Science and Systems*, vol. I, Mar 2000.

[49] D. Tuninetti and G. Caire, "The throughput of some wireless multiaccess systems," *IEEE Transactions on Information Theory*, vol. 48, no. 10, Oct 2002.

[50] N. B. Barry C. Arnold and H. Nagaraja, *A first course in order statistics.* John Wiley and Sons, INC., 1992.

[51] Q. Zhou and H.Dai, "Asymptotic analysis in MIMO MRT/MRC systems," *European Journal on Wireless Communications and Networking*, pp. 1–8, Aug. 2006.

[52] N. T. Uzgoren, “The asymptotic development of the distribution of the extreme values of a sample,” *Studies in Mathematics and Mechanics Presented to Richard von Mises*, pp. 346–353, Academic Press, New York, NY, USA, 1954.

[53] Oppenheim and Wilsky, *Signal and System.* Prentice-Hall, 1993.

[54] A. Leon-Garcia, *Probability and random processess for electrical engineering.* Addison Wesley, 1994.