Power Allocation for a Hybrid Energy Harvesting Relay System with Imperfect Channel and Energy State Information

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***Abstract*—In this paper, we consider both channel state uncertainty and harvested energy state uncertainty for a source–relay–destination communication link where the source and the relay are equipped with hybrid energy sources. Taking into account these uncertainties is of important for practical energy harvesting (EH) communication. While channel state uncertainties also affect conventional communication systems and have been widely studied, harvested energy state uncertainties are specific to energy harvesting systems and have not been considered in the literature before. The considered hybrid energy sources include a constant energy source and an energy harvester. Our objective is to maximize the worst case system throughput over a finite number of transmission intervals. We propose robust optimal offline, optimal online, and suboptimal online power allocation schemes. The offline power allocation design is formulated as an optimization problem which can be solved optimally. For the online case, we propose a dynamic programming (DP) approach to compute the optimal transmit power. To alleviate the prohibitively high complexity inherent to DP, we also propose several suboptimal low–complexity online power allocation schemes. Simulation results confirm the robustness of the proposed power allocation schemes to channel and energy state uncertainties.**

I. Introduction

Energy harvesting (EH) has attracted significant interest as an environmentally friendly supply of energy for the nodes of cooperative communications systems [1]–[4]. EH nodes harvest energy from their surroundings using solar, thermoelectric, and motion effects or by exploiting some other physical phenomena, and store the harvested energy in their batteries for future use. Thus, EH nodes can potentially work as a stand–alone energy source for transmission of data packets or as a supplement to a conventional constant energy source for increasing the transmission capacity.

In the literature, several new transmission strategies and power allocation policies for point–to–point EH communication systems have been reported [5], [6]. The use of EH relays in cooperative communication was introduced in [1], where a comprehensive performance analysis was provided for relay selection in a cooperative network employing EH relays. A deterministic EH model (assuming a priori knowledge of the energy arrival times and the amount of harvested energy) for the Gaussian relay channel was considered in [2], where delay and non–delay constrained traffic was studied. The concept of energy transfer in EH relay systems was studied in [3], where an offline power allocation scheme was proposed. In [4], offline and online power allocation schemes for a source-relay-destination link have been considered.

The above works on EH–assisted communication [1]–[6] assume that the channel state information (CSI) of all links and the harvested energy state information (HESI) of all nodes are perfectly known at the central node, which executes the resource allocation algorithm. In a realistic scenario, the CSI of all links has to be estimated using pilot/training symbols and the CSI and the HESI of other nodes have to be fed back to the central node through feedback channels. Therefore, the CSI may not be perfectly known to the central node due to different sources of errors in the estimation process such as noise, quantization errors, and outdated estimates [7]. Moreover, the HESI may not be perfectly known at the central node either due to feedback errors or outdated estimates. Thus, the power allocation algorithm of the EH system should take both the uncertainties in the CSI and the HESI explicitly into account in order to provide robust performance. Recently, CSI uncertainty has been considered for a two–way EH communication system in [8], where the source nodes harvest energy from the relay nodes. However, HESI uncertainty, which is equally important for the design of robust EH communications systems, has not been considered in the existing literature before.

Motivated by these practical considerations, in this paper, we consider channel and energy state uncertainties in a source–relay–destination communication link where the source and the relay are equipped with hybrid sources of energy. The hybrid sources of energy comprise a non–renewable constant energy source and an energy harvester [9]. In the literature, there are two prevalent methods to incorporate the effect of channel uncertainty: worst case optimization and probabilistically constrained optimization [10]. In this paper, we adopt the worst case optimization by assuming bounded uncertainties for CSI and HESI as this approach does not require any statistical information to model the uncertainty. Moreover, unlike probabilistic optimization, the worst case optimization with bounded CSI uncertainty ensures that channel outages do not occur. In this paper, we propose robust optimal offline, optimal online, and suboptimal but low complexity online power allocation schemes maximizing the end–to–end throughput. Offline schemes are of interest when the amounts of estimated harvested energy and the estimated channel signal–to–noise ratios (SNRs) are known a priori for all transmission intervals. The obtained results from offline schemes can only serve as a theoretical upper bound of the performance. However, in practice, the amounts of harvested energy and the channel SNRs are random and time varying in nature and cannot be predicted in advance. Therefore, in practice, online power allocation schemes, which require only causal knowledge of the channel SNRs and harvested energies, have to be employed.

II. System Model

System Description: We consider an EH relay system, where the source, S, communicates with the destination, D, via a half–duplex decode–and–forward (DF) relay, R. S and R have a hybrid energy source, respectively. The hybrid source includes a constant energy source, possibly connected through a cable to the power grid, and an EH module which harvests energy from the surroundings. The harvested energy can be of any form, e.g., solar, wind, or electro–mechanical energy. S and R are equipped with batteries, which have limited storage capacities to store the harvested energy for future use. In particular, the batteries of S and R can store at most $B_{S,max}$ and $B_{R,max}$ Joules of energy, respectively. Throughout this paper, we assume that S, which is the central node of the considered system, acquires the information about the channel SNRs and the harvested energies, calculates the optimal transmit power for S and R, and informs R about the optimal power allocation.

We assume that the transmission is organized in equal duration time intervals and each interval is comprised of two time slots of duration T. In the following, we set $T = 1$ second for notational simplicity. The total transmission time is equal to K intervals. We assume there is no direct link between S and R due to heavy blockage. During the first time slot of an interval k, packet x_k, which contains Gaussian–distributed symbols, is transmitted by S and received at R. During the second time slot, the detected packet at R, $\hat{x}_k$, is transmitted by R and received at D. The transmit power of node $\mathcal{N} \in \{S, R\}$ in each transmission interval, $k \in \{1, 2, \cdots, K\}$, is the summation of the powers $P_{\mathcal{N},E,k}$ and $P_{\mathcal{N},H,k}$ drawn from the constant energy source and the energy harvester, respectively. We denote the extra amount of harvested energy, which cannot be stored in the battery in transmission interval k due to its limited storage capacity, by $\psi_{\mathcal{N},H,k}$. We assume that the powers required for signal processing at S and R, which are constant in each time interval, are supplied by the constant energy sources and are excluded from the power allocation algorithm design. Since the power amplifiers used for transmission are not ideal, the total powers drawn at node $\mathcal{N}$ from the constant energy source and the EH source are given by $\rho_{\mathcal{N}} P_{\mathcal{N},E,k}$ and $\rho_{\mathcal{N}} P_{\mathcal{N},H,k}$, respectively. Here, $\rho_{\mathcal{N}} \geq 1$ is a constant that accounts for the power amplifier inefficiency at node $\mathcal{N}$. For instance, if $\rho_{\mathcal{N}} = 2$, 100 Watts of power are consumed in the power amplifier for every 50 Watts of power radiated in the radio frequency, and the efficiency of the power amplifier in this case is $\frac{1}{\rho_{\mathcal{N}}} = 50\%$.

Channel Model: We assume the channels are quasi–static within each interval and the estimated complex valued channel gains of the S–R and the R–D links are denoted by $\hat{h}_{S,k}$ and $\hat{h}_{R,k}$, respectively. We assume $\hat{h}_{S,k}$ and $\hat{h}_{R,k}$ are independent of each other and independent and identically distributed (i.i.d.) over the time intervals. $\hat{h}_{S,k}$ and $\hat{h}_{R,k}$ can follow any distribution, e.g., Rayleigh, Rician, Nakagami–m, and Nakagami–q. We assume that the signals received at R and D are impaired by additive white Gaussian noise (AWGN) with zero mean and unit variance. Next, we model the uncertainty originating from estimating the channel gains. Thereby, the channel estimation errors are confined to some uncertainty regions. The sizes and the shapes of the uncertainty regions depend on the physical phenomena causing the errors [7]. The actual channel gains of the S–R and R–D links can be expressed as

$$h_{S,k} = \hat{h}_{S,k} + e_{S,k} \tag{1}$$

$$h_{R,k} = \hat{h}_{R,k} + e_{R,k}, \tag{2}$$

where $e_{S,k}$ and $e_{R,k}$ are the random estimation errors and are unknown to S. The actual channel SNRs of the S–R and R–D links are denoted as $\gamma_{S,k} = |h_{S,k}|^2$ and $\gamma_{R,k} = |h_{R,k}|^2$, respectively. By exploiting (1) and (2), $\gamma_{S,k}$ and $\gamma_{R,k}$ can be expressed as

$$\gamma_{S,k} = \hat{\gamma}_{S,k} + |e_{S,k}|^2 + 2\Re\{\hat{h}_{S,k} e^*_{S,k}\} \tag{3}$$

$$\gamma_{R,k} = \hat{\gamma}_{R,k} + |e_{R,k}|^2 + 2\Re\{\hat{h}_{R,k} e^*_{R,k}\}, \tag{4}$$

respectively, where $\hat{\gamma}_{S,k} = |\hat{h}_{S,k}|^2$ and $\hat{\gamma}_{R,k} = |\hat{h}_{R,k}|^2$. Here, $\Re(\cdot)$ and $(\cdot)^*$ represent the real part and the conjugate of the argument, respectively. Our goal is to maximize the system throughput for the worst case scenario to avoid outages due to the transmission rate exceeding the channel capacity. Therefore, we adopt the worst case channel SNRs of the S–R and R–D links for system design. As $|e_{\mathcal{N},k}|^2$ is always non–negative, for node $\mathcal{N} \in \{S, R\}$, $\gamma_{\mathcal{N},k}$ can be manipulated as

$$\begin{aligned}
\gamma_{\mathcal{N},k} &\geq [\hat{\gamma}_{\mathcal{N},k} + 2\Re\{\hat{h}_{\mathcal{N},k} e^*_{\mathcal{N},k}\}]^+ && (5)\\
&\geq [\hat{\gamma}_{\mathcal{N},k} - 2|\Re\{\hat{h}_{\mathcal{N},k} e^*_{\mathcal{N},k}\}|]^+ && (6)\\
&= [\hat{\gamma}_{\mathcal{N},k} - |\hat{h}_{\mathcal{N},k} e^*_{\mathcal{N},k} + \hat{h}^*_{\mathcal{N},k} e_{\mathcal{N},k}|]^+ && (7)\\
&\geq [\hat{\gamma}_{\mathcal{N},k} - (|\hat{h}_{\mathcal{N},k} e^*_{\mathcal{N},k}| + |\hat{h}^*_{\mathcal{N},k} e_{\mathcal{N},k}|)]^+ && (8)\\
&= [\hat{\gamma}_{\mathcal{N},k} - 2|\sqrt{\hat{\gamma}_{\mathcal{N},k}}||e_{\mathcal{N},k}|]^+, && (9)
\end{aligned}$$

where $[x]^+ = \max\{x, 0\}$. Here, $|\Re(x)| \geq \Re(x)$, $|x + y| \leq |x| + |y|$, and $|xy| = |x||y|$ are used in (6), (8), and (9), respectively. As the exact channel estimation errors are not known to S, we only assume here that the errors are bounded as $|e_{S,k}| \leq |\epsilon_S|$ and $|e_{R,k}| \leq |\epsilon_R|$, where ϵ_S and ϵ_R are the maximum channel estimation errors of the S–R and R–D links, respectively [10]. Note that ϵ_S and ϵ_R determine how far $h_{S,k}$ and $h_{R,k}$, respectively, can deviate in both real and imaginary part from the estimated values $\hat{h}_{S,k}$ and $\hat{h}_{R,k}$. Now, $\gamma_{S,k}$ and $\gamma_{R,k}$ can be represented as

$$\gamma_{S,k} \geq \gamma^W_{S,k} = [\hat{\gamma}_{S,k} - 2|\sqrt{\hat{\gamma}_{S,k}}||\epsilon_S|]^+ \tag{10}$$

$$\gamma_{R,k} \geq \gamma^W_{R,k} = [\hat{\gamma}_{R,k} - 2|\sqrt{\hat{\gamma}_{R,k}}||\epsilon_R|]^+, \tag{11}$$

where $\gamma^W_{S,k}$ and $\gamma^W_{R,k}$ represent the worst case SNRs of S–R and R–D links, respectively. For future reference, we introduce the estimated average SNRs of the S–R and the R–D links as $\bar{\hat{\gamma}}_S = \mathcal{E}\{\hat{\gamma}_{S,k}\}$ and $\bar{\hat{\gamma}}_R = \mathcal{E}\{\hat{\gamma}_{R,k}\}$, respectively, where $\mathcal{E}\{\cdot\}$ denotes statistical expectation.

System Throughput: When x_k is transmitted from S with transmit power $P_{S,E,k} + P_{S,H,k}$ during the first time slot of transmission interval k, $\xi_{S,k} \triangleq \log_2(1 + \gamma^W_{S,k}(P_{S,E,k} + P_{S,H,k}))$ bits of data are transmitted via the S–R link. Similarly, when $\hat{x}_k$ is transmitted from R with transmit power $P_{R,k}$, $\xi_{R,k} \triangleq \log_2(1 + \gamma^W_{R,k}(P_{R,E,k} + P_{R,H,k}))$ bits of data are transmitted via the R–D link. We assume R ensures error free detection by employing capacity achieving codes and hence $\hat{x}_k = x_k$. Therefore, the end–to–end (S–D) system throughput in interval k is given by $\frac{1}{2}\min\{\xi_{S,k},\xi_{R,k}\}$ bits/second where the factor $\frac{1}{2}$ is due to the half-duplex constraint on relaying information signal.

Constant and Renewable Energy Sources: We assume that $E_{\mathcal{N},k}$ is the maximum transmit energy that can be drawn from the constant energy source at node $\mathcal{N} \in \{S,R\}$ in each interval, excluding the required constant signal processing power. On the other hand, the energy harvester at S collects $H_{S,k} \leq B_{S,max}$ Joules of energy from its surroundings at the end of the kth interval and stores it in a battery. Due to the inefficiency of the battery, a fraction of the stored harvested energy may be lost. We adopt the energy loss model from [11] to incorporate the imperfections of the battery which stores the harvested energy. We assume that $(1-\mu_S)\times 100\%$ of the stored harvested energy is leaked per time interval, where $0 \leq \mu_S \leq 1$ represents the storage efficiency of the battery at S per time interval. Note that as $H_{S,k}$ represents the harvested energy at S at the end of the kth interval, it already includes a possibly present battery leakage during the kth interval. However, $H_{S,k}$ is modeled as a uniformly distributed random variable with average EH rate $H_{S,E} \triangleq \mathcal{E}\{H_{S,k}\}$. As S is the central node, therefore, $H_{S,k}$ is perfectly known for power allocation optimization. Similar to [6], we assume that the harvested energy stored in the battery increases and decreases linearly provided the maximum storage capacity $B_{\mathcal{N},max}$ is not exceeded. Therefore, the available harvested energy at S at the beginning of time interval $k+1$ can be represented as

$$B_{S,k+1}=\min\{\mu_S(B_{S,k}-\rho_S P_{S,H,k})+H_{S,k},B_{S,max}\},\forall k, \quad (12)$$

where $B_{S,1}=H_{S,0}\geq 0$ denotes the available harvested energy at S before the transmission starts.

Similar to S, R harvests $H_{R,k} \leq B_{R,max}$ Joules of energy from its surroundings. We denote the storage efficiency of the battery at R by μ_R, $0 \leq \mu_R \leq 1$. We assume that at the end of each time interval, k, R conveys the accumulated harvested energy, $\mathcal{H}_{R,k}=\sum\limits_{i=0}^{k}\mu_R^{k-i}H_{R,i}$ to S. As S obtains the information regarding the harvested energy from R required to execute the power allocation scheme through a feedback channel, this information can be erroneous. Note that in contrast to the channel estimation error, the harvested energy error is always real. The accumulated actual harvested energy at R is assumed to lie in the interval

$$[\hat{\mathcal{H}}_{R,k}-\Delta H_R]^{+} \leq \mathcal{H}_{R,k} \leq \hat{\mathcal{H}}_{R,k}+\Delta H_R. \quad (13)$$

where $\hat{\mathcal{H}}_{R,k} \leq B_{R,max}$ denotes the feedback information regarding the accumulated harvested energy obtained at S, $\Delta H_R \geq 0$ represents the maximum feedback error which determines how far $\mathcal{H}_{R,k}$ can deviate from the estimated value $\hat{\mathcal{H}}_{R,k}$. For future reference, the average harvested energy at R is denoted by $H_{R,E} \triangleq \mathcal{E}\{H_{R,k}\}$. Similar to the channel SNRs, we consider the worst case scenario regarding the accumulated harvested energy at R to perform worst case optimization. We denote the worst case accumulated harvested energy at R by $\mathcal{H}_{R,k}^{W}$ and define it as

$$\mathcal{H}_{R,k}^{W}=[\hat{\mathcal{H}}_{R,k}-\Delta H_R]^{+}. \quad (14)$$

Based on the feedback information regarding the accumulated harvested energy at R, S calculates the available harvested energy at R at the beginning of time interval $k+1$ as

$$\hat{B}_{R,k+1}=\min\left\{\left[\mathcal{H}_{R,k}^{W}-\sum_{i=1}^{k}\rho_R\mu_R^{k-i+1}P_{R,H,i}\right]^{+},B_{R,max}\right\},\ \forall k, \quad (15)$$

where $\hat{B}_{R,1}=\mathcal{H}_{R,0}^{W}\geq 0$ denotes the calculated harvested energy at S before transmission starts. Thus, we can conclude from (12) and (15) that $B_{S,k}$ and $\hat{B}_{R,k}$ follow a first–order Markov process which depends only on the past and current states of the battery.

III. Power Allocation Schemes

In this section, we propose an offline and several online power allocation schemes for the considered EH system with channel and energy state uncertainties.

A. Optimal Offline Power Allocation

For offline power allocation, we assume prior (offline) knowledge of the estimated CSI, the HESI at S, and the estimated HESI at R in each time interval and consider the maximization of the total number of bits transmitted from S to D that can be delivered by a deadline of K intervals over a fading channel. The offline optimization problem can be formulated as follows

$$\max_{\mathcal{T}\succeq 0} \quad \sum_{k=1}^{K}\frac{1}{2}\min\{\xi_{S,k},\xi_{R,k}\} \quad (16)$$

$$\text{s.t.}\sum_{k=1}^{l}\rho_S\mu_S^{l-k}P_{S,H,k} \leq \sum_{k=0}^{l-1}\mu_S^{l-k-1}(H_{S,k}-\psi_{S,H,k}),\ \forall l \quad (17)$$

$$\sum_{k=1}^{l}\rho_R\mu_R^{l-k}P_{R,H,k} \leq \mathcal{H}_{R,l-1}^{W}-\psi_{R,H,l-1},\ \forall l \quad (18)$$

$$\sum_{k=0}^{q}\mu_S^{q-k}(H_{S,k}-\psi_{S,H,k})-\sum_{k=1}^{q}\rho_S\mu_S^{q-k+1}P_{S,H,k}\leq B_{S,max},\forall q \quad (19)$$

$$\mathcal{H}_{R,q}^{W}-\psi_{R,H,q}-\sum_{k=1}^{q}\rho_R\mu_R^{q-k+1}P_{R,H,k}\leq B_{R,max}-\Delta H_R,\forall q \quad (20)$$

$$\rho_S P_{S,E,k} \leq E_{S,k},\ \rho_R P_{R,E,k} \leq E_{R,k},\ \forall k \quad (21)$$

$$\gamma_{S,k}^{W}(P_{S,E,k}+P_{S,H,k})=\gamma_{R,k}^{W}(P_{R,E,k}+P_{R,H,k}),\ \forall k, \quad (22)$$

where $\mathcal{T} \triangleq [P_{S,E,k}\ P_{R,E,k}\ P_{S,H,k}\ P_{R,H,k}\ \psi_{S,H,k}\ \psi_{R,H,k}]$, $k \in \{1,2,\cdots,K\}$, $l \in \{1,2,\cdots,K\}$, $q \in \{1,2,\cdots,K-1\}$, and $\psi_{S,H,0}=\psi_{R,H,0}=0$. The slack variables $\psi_{S,H,k}$ and $\psi_{R,H,k}$ ensure that constraints (19), (20), and (22) can be met for all realizations of $\hat{\gamma}_{S,k}$, $\hat{\gamma}_{R,k}$, $H_{S,k}$, and $\mathcal{H}_{R,k}^{W}$. In particular, these slack variables represent the power (possibly) wasted in each transmission interval. Constraints (17) and (18) stem from the causality requirement on the energy harvested at S and R, respectively. Moreover, (19) and (20) ensure that the harvested energy does not exceed the limited storage capacity of the batteries at S and R, respectively. Note that ΔH_R in the right hand side of (20) avoids the possibility of battery overflow at R due to uncertainty. The limitations on the energy drawn from the constant supply are reflected in constraints (21) for S and R. Note that for a given time interval, for the constant energy supply, any extra amount of energy which is not used for transmission cannot be transferred to the next interval. Constraint (22) ensures that the amount of information transmitted from S

to R is identical to that transmitted from R to D so as to avoid data loss at R. Constraint (22) is required since we assume individual power constraints for S and R. This is a reasonable assumption since S and R have independent power supplies. Problem (16)–(22) can be restated as follows

$$\max_{\mathcal{T}\succeq 0,\tau_k\geq 0} \quad \sum_{k=1}^{K}\tau_k \tag{23}$$

$$\text{s.t.} \quad \frac{1}{2}\xi_{S,k}\geq\tau_k,\ \ \frac{1}{2}\xi_{R,k}\geq\tau_k,\ \ \forall k \tag{24}$$

$$\text{Constraints } (17)-(22). \tag{25}$$

Problem (23)–(25) is a convex optimization problem and hence can be solved optimally and efficiently [12]. Moreover, as problem (23)–(25) satisfies Slater's constraint qualification, the duality gap between the optimum values of the original problem and its dual is zero [12]. Therefore, we solve our problem by solving its dual. For this purpose, we first provide the Lagrangian of problem (23)–(25) which can be written as

$$\begin{aligned}
\mathcal{L} &= \sum_{k=1}^{K}\tau_k+\sum_{k=1}^{K}\lambda_{S,k}\Big(\frac{1}{2}\log_2(1+\gamma_{S,k}^W(P_{S,E,k}+P_{S,H,k}))-\tau_k\Big)\\
&+\sum_{k=1}^{K}\lambda_{R,k}\Big(\frac{1}{2}\log_2(1+\gamma_{R,k}^W(P_{R,E,k}+P_{R,H,k}))-\tau_k\Big)\\
&-\sum_{l=1}^{K}\alpha_{S,l}\Big(\sum_{k=1}^{l}\rho_S\mu_S^{l-k}P_{S,H,k}-\sum_{k=0}^{l-1}\mu_S^{l-k-1}(H_{S,k}-\psi_{S,H,k})\Big)\\
&-\sum_{l=1}^{K}\alpha_{R,l}\Big(\psi_{R,H,l-1}-\mathcal{H}_{R,l-1}^W+\sum_{k=1}^{l}\rho_R\mu_R^{l-k}P_{R,H,k}\Big)\\
&-\sum_{q=1}^{K-1}\omega_{S,q}\Big(\sum_{k=0}^{q}\mu_S^{q-k}(H_{S,k}-\psi_{S,H,k})-\sum_{k=1}^{q}\rho_S\mu_S^{q-k+1}P_{S,H,k}\\
&-B_{S,max}\Big)-\sum_{q=1}^{K-1}\omega_{R,q}\Big(\mathcal{H}_{R,q}^W-\psi_{R,H,q}+\Delta H_R-\sum_{k=1}^{q}\rho_R\mu_R^{q-k+1}P_{R,H,k}\\
&-B_{R,max}\Big)-\sum_{k=1}^{K}\beta_{S,k}(\rho_S P_{S,E,k}-E_{S,k})-\sum_{k=1}^{K}\beta_{R,k}(\rho_R P_{R,E,k}-E_{R,k})\\
&-\sum_{k=1}^{K}\eta_k\Big(\gamma_{S,k}^W(P_{S,E,k}+P_{S,H,k})-\gamma_{R,k}^W(P_{R,E,k}+P_{R,H,k})\Big)
\end{aligned}\tag{26}$$

where $\lambda_{S,k}\geq 0$, $\lambda_{R,k}\geq 0$, $\alpha_{S,l}\geq 0$, $\alpha_{R,l}\geq 0$, $\omega_{S,q}\geq 0$, $\omega_{R,q}$, $\beta_{S,k}\geq 0$, $\beta_{R,k}\geq 0$, and η_k are the Lagrange multipliers associated with constraints (24), (17), (18), (20), (21), and (22), respectively. Note that the boundary conditions $P_{S,E,k}\geq 0$, $P_{S,H,k}\geq 0$, $P_{R,E,k}\geq 0$, $P_{R,H,k}\geq 0$, $\psi_{S,H,k}\geq 0$, $\psi_{R,H,k}\geq 0$ are absorbed into the Karush–Kuhn–Tucker (KKT) conditions for deriving the optimal $P_{S,E,k}$, $P_{S,H,k}$, $P_{R,E,k}$, $P_{R,H,k}$, $\psi_{S,H,k}$, and $\psi_{R,H,k}$. We adopt the Lagrange dual decomposition method and calculate the optimal $P_{S,E,k}$, $P_{S,H,k}$, $P_{R,E,k}$, $P_{R,H,k}$, $\psi_{S,H,k}$, and $\psi_{R,H,k}$ and the optimal Lagrange multipliers required in (23)–(25) via an iterative procedure [12]. The dual of problem (23)–(25) can be stated as

$$\min_{\mathcal{V}\geq 0,\eta_k} \quad \max_{\mathcal{T}\geq 0} \ \mathcal{L} \tag{27}$$

where $\mathcal{V}=[\lambda_{S,k}\ \lambda_{R,k}\ \alpha_{S,l}\ \alpha_{R,l}\ \omega_{S,q}\ \omega_{R,q}\ \beta_{S,k}\ \beta_{R,k}]$. Using standard optimization techniques and the KKT optimality conditions, the optimal $P_{S,E,k}$, $P_{S,H,k}$, $P_{R,E,k}$, $P_{R,H,k}$, $\psi_{S,H,k}$, $\psi_{R,H,k}$, and τ_k can be obtained as

$$P_{S,E,k}^{\mathrm{OPT}}=\left[\Xi_{S,k}-\frac{1}{\gamma_{S,k}^W}-P_{S,H,k}\right]^+, \tag{28}$$

$$P_{R,E,k}^{\mathrm{OPT}}=\left[\Xi_{R,k}-\frac{1}{\gamma_{R,k}^W}-P_{R,H,k}\right]^+, \tag{29}$$

$$P_{S,H,k}^{\mathrm{OPT}}=\left[\Lambda_{S,k}-\frac{1}{\gamma_{S,k}^W}-P_{S,E,k}\right]^+, \tag{30}$$

$$P_{R,H,k}^{\mathrm{OPT}}=\left[\Lambda_{R,k}-\frac{1}{\gamma_{R,k}^W}-P_{R,E,k}\right]^+, \tag{31}$$

$$\psi_{S,H,k}^{\mathrm{OPT}}=\left[\sum_{i=0}^{k-1}\mu_S^{k-i}(H_{S,i}-\psi_{S,H,i})-\sum_{i=1}^{k}\mu_S^{k-i+1}\rho_S P_{S,H,i}+H_{S,k}-B_{S,max}\right]^+, \tag{32}$$

$$\psi_{R,H,k}^{\mathrm{OPT}}=\left[\mathcal{H}_{R,k}^W-\mu_R(\mathcal{H}_{R,k-1}^W-\psi_{R,H,k-1})-B_{R,max}+\Delta H_R\right]^+, \tag{33}$$

$$\begin{aligned}\tau_k^{\mathrm{OPT}}&=\frac{1}{2}\log_2\left(1+\gamma_{S,k}^W(P_{S,E,k}+P_{S,H,k})\right)\\&=\frac{1}{2}\log_2\left(1+\gamma_{R,k}^W(P_{R,E,k}+P_{R,H,k})\right),\end{aligned} \tag{34}$$

$$\text{where } \Xi_{S,k}=\frac{\lambda_{S,k}}{2\ln(2)\left(\beta_{S,k}\rho_S+\eta_k\gamma_{S,k}^W\right)}, \tag{35}$$

$$\Lambda_{S,k}=\frac{\lambda_{S,k}/(2\ln(2))}{\sum_{j=k}^{K}\rho_S\alpha_{S,j}\mu_S^{j-k}-\sum_{j=k}^{K-1}\rho_S\omega_{S,j}\mu_S^{j-k+1}+\eta_k\gamma_{S,k}^W}, \tag{36}$$

$$\Xi_{R,k}=\frac{\lambda_{R,k}}{2\ln(2)\left(\beta_{R,k}\rho_R-\eta_k\gamma_{R,k}^W\right)}, \tag{37}$$

$$\Lambda_{R,k}=\frac{\lambda_{R,k}/(2\ln(2))}{\sum_{j=k}^{K}\rho_R\alpha_{R,j}\mu_R^{j-k}-\sum_{j=k}^{K-1}\rho_R\omega_{R,j}\mu_R^{j-k+1}-\eta_k\gamma_{R,k}^W}. \tag{38}$$

Here, $(\cdot)^{\mathrm{OPT}}$ denotes the optimal solutions in (28)–(34). We define t as the iteration index. For a given set of Lagrange multipliers and given values of $P_{S,H,k}^{\mathrm{OPT}}(t-1)$ and $P_{R,H,k}^{\mathrm{OPT}}(t-1)$, we obtain $P_{S,E,k}^{\mathrm{OPT}}(t)$ and $P_{R,E,k}^{\mathrm{OPT}}(t)$ using (28) and (29), respectively, and then calculate $P_{S,H,k}^{\mathrm{OPT}}(t)$ and $P_{R,H,k}^{\mathrm{OPT}}(t)$ based on (30) and (31), respectively. We also calculate $\psi_{S,H,k}^{\mathrm{OPT}}(t)$ and $\psi_{R,H,k}^{\mathrm{OPT}}(t)$ based on (32) and (33), respectively. However, to calculate $P_{S,E,k}(1)$ and $P_{R,E,k}(1)$ for $t=1$, $P_{S,H,k}(0)\geq 0$ and $P_{R,H,k}(0)\geq 0$ are chosen such that (17)–(20) are satisfied. We update the Lagrange multipliers using the standard gradient method [12]. With the updated Lagrange multipliers, we solve $P_{S,E,k}^{\mathrm{OPT}}(t+1)$, $P_{S,H,k}^{\mathrm{OPT}}(t+1)$, $P_{R,E,k}^{\mathrm{OPT}}(t+1)$, $P_{R,H,k}^{\mathrm{OPT}}(t+1)$, $\psi_{S,H,k}^{\mathrm{OPT}}(t+1)$, and $\psi_{R,H,k}^{\mathrm{OPT}}(t+1)$ again and the same procedure continues until convergence. Note that, due to the convexity of problem (23)–(25), convergence to the optimal solution is guaranteed as long as the step sizes satisfy the infinite travel condition [12].

B. Optimal Online Power Allocation by DP

In practice, only causal CSI and HESI are available at node S for power allocation. Therefore, the offline power allocation scheme is not readily applicable as in a given time interval k, the future CSI and the upcoming harvested energy are not known in advance. We propose to employ a stochastic DP approach for optimum online power allocation [6], [13]. Like the optimal online power allocation scheme in [4], we follow the standard procedure of stochastic DP according to Bellman's equations [13] to obtain $P_{S,E,k}^{\mathrm{OPT}}$, $P_{S,H,k}^{\mathrm{OPT}}$, $P_{R,E,k}^{\mathrm{OPT}}$, and $P_{R,H,k}^{\mathrm{OPT}}$, $k \in \{1, 2, \cdots, K\}$. Due to space limitations, we do not show the problem formulation in detail. In practice, the optimal results are calculated for different realizations of $\hat{\gamma}_{S,k}$, $\hat{\gamma}_{R,k}$, $B_{S,k}$, and $\hat{B}_{R,k}$ and are stored in a look–up table. This is done before transmission starts. When transmission starts, for a given realization of $\hat{\gamma}_{S,k}$, $\hat{\gamma}_{R,k}$, $B_{S,k}$, and $\hat{B}_{R,k}$ in time interval k, those values of $P_{S,E,k}^{\mathrm{OPT}}$, $P_{S,H,k}^{\mathrm{OPT}}$, $P_{R,E,k}^{\mathrm{OPT}}$, and $P_{R,H,k}^{\mathrm{OPT}}$ that correspond to that realization are picked from the look–up table.

C. Suboptimal Online Power Allocation

In the proposed DP–based optimal online power allocation algorithm, for a certain transmission time interval k, we take into account the average effect of all the succeeding time intervals. Due to the recursive nature of DP, the computational complexity of this approach increases alarmingly with increasing K. For this reason, in the following, we propose two different suboptimal but less complex online power allocation schemes.

1) Suboptimal Harvesting Rate (HR) Assisted Power Allocation ("HR Assisted" Scheme): In this scheme, we constrain the transmit powers $P_{S,H,k}$ and $P_{R,H,k}$ by the average energy harvesting rates $H_{S,E}$ and $H_{R,E}$, respectively. This scheme is referred to as "HR Assisted" power allocation. For a given time interval $k \in \{1, 2, \cdots, K-1\}$, the resulting optimization problem can be stated as

$$\max_{P_{S,E,k},\, P_{S,H,k},\, P_{R,E,k},\, P_{R,H,k}} \quad \frac{1}{2}\min\{\xi_{S,k}, \xi_{R,k}\} \tag{39}$$

$$\text{s.t.} \quad 0 \le \rho_S P_{S,E,k} \le E_{S,k}, \;\; 0 \le \rho_R P_{R,E,k} \le E_{R,k} \tag{40}$$

$$(1-\mu_S) B_{S,k} \le \rho_S P_{S,H,k} \le \min\{B_{S,k}, H_{S,E}\}, \tag{41}$$

$$(1-\mu_R) \hat{B}_{R,k} \le \rho_R P_{R,H,k} \le \min\{\hat{B}_{R,k}, H_{R,E}\}, \tag{42}$$

$$\gamma_{S,k}^{W}(P_{S,E,k} + P_{S,H,k}) = \gamma_{R,k}^{W}(P_{R,E,k} + P_{R,H,k}). \tag{43}$$

Problem (39)–(43) is a convex optimization problem. Therefore, as for the offline power allocation scheme, we can adopt the Lagrange dual method to solve problem (39)–(43) optimally and efficiently [12]. For the Kth time interval, we replace the right hand side of (41) and (42) by $B_{S,k}$ and $\hat{B}_{R,k}$, respectively, and solve modified problem (39)–(43).

2) Suboptimal Naive Power Allocation ("Naive" Scheme): In this suboptimal "naive" approach, for each time interval, k, only the stored energies at hand determine the transmit power, i.e., this approach does not take into account the statistical effect of the future time intervals. For each time interval k,

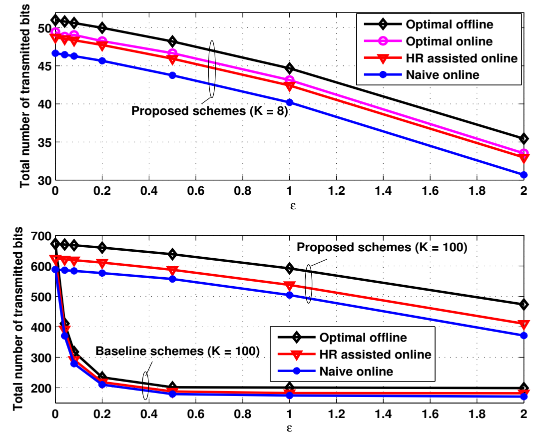

Fig. 1. Total number of transmitted bits vs. $|\epsilon_S| = |\epsilon_R| = \epsilon$ for $\Delta H_R = 0$ Joules and different K.

the optimization problem can be stated as

$$\max_{P_{S,E,k},\, P_{S,H,k},\, P_{R,E,k},\, P_{R,H,k}} \quad \frac{1}{2}\min\{\xi_{S,k}, \xi_{R,k}\} \tag{44}$$

$$\text{s.t.} \quad (1-\mu_S) B_{S,k} \le \rho_S P_{S,H,k} \le B_{S,k}, \tag{45}$$

$$(1-\mu_S) \hat{B}_{R,k} \le \rho_R P_{R,H,k} \le \hat{B}_{R,k}, \tag{46}$$

$$\text{Constraints (40) and (43).} \tag{47}$$

Problem (44)–(47) is a convex optimization problem and thus we can adopt the Lagrange dual method to solve the problem optimally and efficiently [12].

IV. Simulation Results

In this section, we evaluate the performance of the proposed offline and online power allocation schemes by simulations. We assume that in each time interval, $H_{S,k}$ ($H_{R,k}$) independently takes a value from the set $\{0, H_{S,E}, 2H_{S,E}\}$ ($\{0, H_{R,E}, 2H_{R,E}\}$), where all elements of the set are equiprobable [6]. For all presented simulation results, we assume that $\hat{\gamma}_{S,k}$ and $\hat{\gamma}_{R,k}$ follow an exponential distribution with mean $\bar{\hat{\gamma}}_S = \bar{\hat{\gamma}}_R = \bar{\hat{\gamma}} = 20$ dB. We also assume that $e_{S,k}$ and $e_{R,k}$ are uniformly distributed in discs of radius $|\epsilon_S|$ and $|\epsilon_R|$, respectively. We adopt $B_{S,max} = B_{R,max} = 300$ Joules, $\mu_S = \mu_R = 0.99$, and $\rho_S = \rho_R = 2.5$. To show the robustness of the proposed power allocation schemes, we also consider a baseline power allocation scheme and compare its performance with that of the proposed power allocation schemes. In the considered baseline power allocation scheme, the worst case uncertainties for CSI and HESI are not considered and instead the transmit powers are optimized for the estimated channel SNRs and harvested energies. In particular, for the baseline scheme, we set $|\epsilon_S| = |\epsilon_R| = 0$ for $k = \{1, 2, \cdots, K\}$ for all power allocation schemes and take into account the outage of the system if $\xi_{S,k} > \log_2(1 + \gamma_{S,k}(P_{S,E,k} + P_{S,H,k}))$ or $\xi_{R,k} > \log_2(1 + \gamma_{R,k}(P_{R,E,k} + P_{R,H,k}))$. In our simulations, when a link is in outage, we set the corresponding throughput to zero. Note that the proposed schemes do not suffer from outages as they ensure $\xi_{S,k} \le \log_2(1 + \gamma_{S,k}(P_{S,E,k} + P_{S,H,k}))$ and $\xi_{R,k} \le \log_2(1 + \gamma_{R,k}(P_{R,E,k} + P_{R,H,k}))$ due to the consideration of the worst case uncertainty. For all simulation results, 10^4 randomly generated realizations of the channel SNRs and the harvested energies are evaluated to obtain the average throughput.

Fig. 1 shows the total number of transmitted bits vs. the maximum error of the channel gain, $|\epsilon_S| = |\epsilon_R| = \epsilon$, for $K = 8$ and $K = 100$. Here, we assume $\Delta H_R = 0$ Joule, i.e., no feedback error for the HESI to observe the sole impact of CSI uncertainty. Moreover, we assume $E_{S,1} = \cdots = E_{S,K} = E_{R,1} = \cdots = E_{R,K} = 1$ Joule and $H_{S,E} = H_{R,E} = 6$ Joules. Results for the proposed offline and online power allocation schemes are provided. Note that due to the high computational complexity, the performance of the optimal DP based online power allocation scheme is not shown for $K = 100$. We have also included the performances of the baseline power allocation scheme.

We observe from Fig. 1 that the total number of transmitted bits decreases with increasing ϵ for all proposed power allocation schemes for both $K = 8$ and $K = 100$. We also observe that the proposed offline scheme performs better than the proposed online power allocation schemes for all ϵ. This is due to the fact that in the optimal offline scheme, we assume that both causal and noncausal information regarding the estimated CSI and the feedback HESI are available whereas the online schemes are based only on causal information regarding the estimated CSI and the feedback HESI. Moreover, as expected, the optimal DP based online scheme outperforms the considered suboptimal online schemes and performs close to the optimal offline scheme for $K = 8$. It is worth noting that when $\epsilon = 0$, the proposed and the baseline schemes for each power allocation scheme have the same performance, as expected. However, the performance of the baseline schemes degrades significantly with increasing ϵ. On the other hand, the performance of the proposed scheme degrades gradually. This finding reveals that our proposed power allocation schemes are much more robust to uncertainty in comparison to the baseline schemes.

In Fig. 2, we show the total number of transmitted bits vs. the number of time slots, K for $E_{S,1} = \cdots = E_{S,K} = E_{R,1} = \cdots = E_{R,K} = 0.2$ Joules and $H_{S,E} = H_{R,E} = 8$ Joules. To observe the impact of the uncertainty in harvested energy, we have assumed that the available amount of constant energy is small. Three different scenarios for $|\epsilon_S| = |\epsilon_R| = \epsilon$ and ΔH_R have been considered. In Scenario 1, we assume $\epsilon = 0$ and $\Delta H_R = 0$ Joule, i.e., the channel and energy states are perfectly known. In Scenario 2, we assume $\epsilon = 0.7$ and $\Delta H_R = 2$ whereas in Scenario 3, we assume $\epsilon = 0.7$ and $\Delta H_R = 5$. We show the performances of the optimal offline and suboptimal online power allocation schemes for the considered scenarios. We observe that for all proposed power allocation schemes, there is relatively large performance gap between Scenarios 1 and 2 in comparison to the gap between Scenarios 2 and 3. This is becasue Scenario 2 differs from Scenario 1 by both CSI and HESI errors whereas Scenario 3 differs from Scenario 2 by only HESI error. To show the robustness of the proposed power allocation schemes, we have also included the performance of the HR assisted online baseline scheme. We observe that for Scenario 1, where there is no error due to channel or energy state uncertainties, both the proposed and the baseline HR assisted schemes yield identical results, as expected. However, for Scenarios 2 and 3, we observe large performance gaps between the proposed and the baseline HR assisted schemes for all K and this gap increases with increasing K.

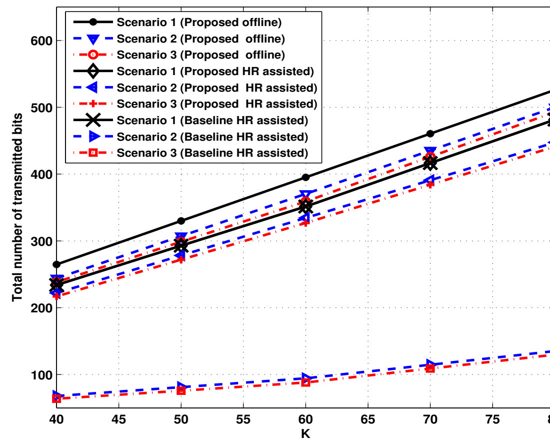

Fig. 2. Total number of transmitted bits vs. K for 3 different scenarios for $|\epsilon_S| = |\epsilon_R| = \epsilon$ and ΔH_R Joules.

V. Conclusions

In this paper, we have considered the problem of transmit power allocation for a hybrid EH single relay network with channel and energy state uncertainties. We have proposed robust optimal offline, optimal online, and suboptimal online power allocation schemes for worst case optimization by incorporating bounded uncertainties for the CSI and the HESI. While the effect of CSI uncertainties has been studied for conventional non–EH systems in the literature, our results show that the impact of HESI uncertainties is equally important for the robust design of EH relaying systems.

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