The University of New South Wales

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Energy-Efficient Simultaneous Wireless Information and Power Transmission for IoT Devices

by

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Abstract

The rapid advancement of technology in the field of wireless communication devices has unlocked the world of Internet of Things which dramatically increased the power consumption worldwide. Simultaneous Wireless information and Power Transfer (SWIPT) is a up-andcoming solution to the increase in demand of transmission power, by enabling sustainability of the wireless communication devices.

However, the SWIPT technology requires further improvement in balancing between the quality of service and saving the energy such that the full potential of SWIPT can be explored. To ensure optimum wireless transmission with high quality of service and sustainability, this paper addresses the optimization of energy efficiency for SWIPT multiuser multi-input single-output broadcast channel with separated receiver design.

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Chapter 1

Introduction

The rapid advancement of technology in the area of wireless communication devices has unlocked the world of the Internet of Things (IoTs) where the number of wireless communication devices easily surpass the number of human population worldwide [18], [28]. While mobile phones, laptops and tablets contribute to this enormous network of Internet of Things more noticeably, the less prominent everyday objects such as the smart watches, wall mounted temperature sensors and even the security tag from clothing stores consist of wireless communication chips which unconsciously fill up the modern society with a surprisingly vast number.

As the standard of living elevates with digitisation of everyday objects, the demand for wireless communication devices grows exponentially and in turn the cost invested in maintenance of these devices also snowball. A significant drawback in these mass-produced wireless communication sensors is the limited energy storage capacity. Frequent replacement of battery in large volume is not only time consuming, costly and produce waste, but it can even be impossible in cases of medically inserted devices such as advanced prosthetics. Also, the load on the conventional power generators to meet the increased power consumption is another challenge to be tackled.

A possible resolution is to integrate an energy harvesting technology to the wireless communication devices such that the devices do not require battery replacements and can be self-sustained. [29] This will mitigate the cost and labour invested maintenance of the networks, while reducing the load on the conventional power generators and finite resources.

Different researches [1] considered the traditional renewable energy resources such as solar and wind. However, most researches were concluded that the conventional natural energy harvesting is not ideal for serving wireless communication purposes due to their high dependence on weather and lack of mobility which cannot promise high quality of service.

So, in the more recent years, wireless power transfer methods which utilizes the electromagnetic waves in the surrounding environment. Wireless power transfer can be achieved using different technologies. For occasions where the energy harvesting devices are mobile, radio frequency based wireless power transfer is particularly preferred over other methods such as the inductive coupling or magnetic resonant coupling technologies, as they exploit the far-field properties of electromagnetic waves. [2] By ensuring transmission to mobile energy harvesting devices, information signal can be sent in the process of wireless energy transmission, unlocking the potential of simultaneous wireless information and power transfer. [3]

Although Simultaneous Wireless Information and Power Transfer (SWIPT) [30]-[32] is a relatively new concept, it has drawn much attention amongst researchers. Most existing researches focus on maximising the data rate of the harvested energy to ensure high quality of service in transmission [4]-[7]. However, sole maximisation of the throughput [8] requires enormous network power consumption, leading to small harvested energy to total energy consumption ratio. The high network power consumption inhibits the sustainability of the system, ultimately defeating the effort to create self-sustainable, economical system. On the contrary, only focusing on maximisation of harvested energy will downgrade the valuable throughput, and hence the quality of service [25].

Taking into account the two conflicting objectives of the existing researches, a promising approach to balance the trade-off is to evaluate the energy efficiency of the system [26]. Optimisation of energy efficiency has been studied in many research works over conventional information transmission systems, but only recently gained attention over SWIPT. There are some existing researches on the energy efficiency of SWIPT, such as the paper on multicarrier OFDMA systems [9], or papers on Multi Input Single Output systems based on fix precoder [10], [11]. Different from these existing works, this paper will address multiuser MISO SWIPT, using the separated receiver technique.

The ultimate objective of this paper is to design a computational efficient beamformer which

maximises the energy efficiency of SWIPT systems, while guaranteeing the quality of service. This paper consists of four different chapters – Chapter 1 and 2 consist of the introduction and literature review. The system model and problem formulation will be outlined in Chapter 3 and 4 respectively. The proposed optimal resource allocation and the strategies involved in solution are discussed in chapter 5, and the simulation results of the designed system are elaborated in chapter 6. Finally, in Chapter 6, possible direction of future research works are identified.

Chapter 2

Literature Review

In this section, different design factors that can be considered in system model to increase the energy efficiency will be elaborated.

2.1 Types of Receivers

The ideal idea of SWIPT would be to use the same signal to yield both information data and energy without dedicating some portions of the signal for single purpose only. Nevertheless, the radio frequency wireless energy transfer in the process of energy harvesting damages the information data, hence impractical in the real-world application. Therefore, in practice, the signal must be split in the domain of power, time, space, etc., to dedicate one section as information data signal and the other as energy signal. Power splitting and separated receiver techniques are the two common techniques and will be reviewed in the following section.

Power Splitting

Figure 2.1: Power splitting technique with one transmitter and one receiver. [19]

In power splitting technique, a receiver is designed to split the received signal into two ways of different power levels. One of these split signals is rectified at the energy harvesting circuit and the other portion of split signal is dedicated for decoding information data.

Separated receiver / Time switching receiver

Figure 2.2: Separated receiver technique with the base station, one receiver for receiving information and one for harvesting energy. [19]

With separated receiver technique, the receivers are dedicated as either an information receiver or energy harvesting receiver and does not require a circuitry that splits the power for different purposes. Time switching technique has the same strategy, except one receiver can switch between an information receiver and an energy harvester in time.

There are some advantages and disadvantages in both these techniques – the power splitting method can achieve instantaneous SWIPT, therefore it is easier to manage applications with delay constraints and has better performance. But because power splitting requires high receiver complexity, it is more expensive and additional noise can be introduced in the process of power splitting.

So, although power splitting technique may be better in terms of performance, time switching is often preferred because they are much cheaper and suitable for mass production, and the relatively simple circuitry involved reduces the chance of mechanical failure and unlike the power splitting cuircuit, it will introduce as much noise. Drawback or limitations in the performance can be mitigated at employing other techniques at the base station. Therefore, this paper will employ the separated receiver technique, with addition of beamforming technique.

2.2 MIMO and Beamforming

The Multiple Input Multiple Output (MIMO) [41] is an antenna technology employed to increase the degrees of freedom in a wireless transmission. By utilising the degrees of freedom obtained from MIMO, the RF energy can be steered towards desired direction using beamforming [39], [40]. For example, with a base station or a transmitter generating and directing RF signals towards a specific device, a constructive combination of energy or destructive combination of energy can be obtained.

Taking advantage of the properties of constructive and deconstructive waves, beamforming can be integrated into SWIPT as a feasible solution to provide both high data throughput and sustainability of the system.

Chapter 3

System Model

3.1 Signal Model

Taking advantage of the design factors evaluated in the literature review, the system model constructed is a multi user Multiple Input Single Output (MISO) SWIPT, using the separated receiver technique.

Figure 3.1: A downlink SWIPT-based multiuser MISO system with separated receiver technique.

The system consists of one transmitter (or the base station) with N number of transmit antennas, one information receiver and J number of energy harvesting receivers each with a single receiving antenna. We assume that the number of antenna at the transmitter is greater than the sum of the receiving antennas to increase reliability by utilising the directional gain and interference cancellation of beamforming.

Figure 3.1 represents the information data and energy transmission from a transmitter and the receivers, where the active receiver acts as the information decoding receiver. Perfect channel state information and linear harvested energy is assumed for the system model at this stage.

$$
y_{I_R} = \mathbf{h}^H \boldsymbol{\omega} x + n \tag{3.1}
$$

$$
y_{j_{E_R}} = \mathbf{g}_j^H \boldsymbol{\omega} x + z_j \tag{3.2}
$$

The signal received at the information receiver (3.1) and signal received at j^{t^h} energy receiver (3.2) are derived respectively as above, where $x \in C$ denotes the data symbol and $\omega \in C$ is the information beamforming vector. Here we assume that the expected value of received signal, $\varepsilon |x|^2$ equals to 1 because it is normalized. h^H is the channel quality vector between the transmitter and the information receiver, and g_i^H j^H is the channel quality between the transmitter and the j^{t^h} energy receiver. n $CN(\mu, \delta^2)$ and z_j represent the additive white Gaussian noises at the receivers, where δ^2 is the noise power on the receiving end.

3.2 Resource Allocation Algorithm Design

With the equation of the received information signal, the maximum rate R_s at which the information can be transmitted through a signal can be derived using the formula of Shannon's capacity.

$$
R_s = \log_2\left(1 + \frac{|\boldsymbol{h}^H \boldsymbol{\omega}|^2}{\delta^2}\right),\tag{3.3}
$$

where $\frac{|h^H\omega|^2}{\delta^2}$ $\frac{d^2\omega|^2}{\delta^2}$ is basically the signal to noise ratio, (SNR).

Figure 3.2: Transmission rate versus Bandwidth according to Shannon's capacity.

Figure 3.2 is a graph which shows the rate of transmission versus the signal to noise ratio. As can be seen on the graph, the maximum transmission rate reaches a limit where the increase in signal to noise ratio will no longer have much effect on the transmission rate due to the maximum error free data rate a channel can support.

3.3 Preliminary Simulation

The total power at the transmitter equals to the transmission power plus the circuit power, and the energy efficiency equation equals to the maximum error free data rate a channel can support, divided by the power consumption.

$$
P_{\text{input}} = \text{Transmission power} + \text{Circuit power} = ||\boldsymbol{\omega}||^2 + P_c \tag{3.4}
$$

Energy Efficiency =
$$
\frac{P_{\text{output}}}{P_{\text{input}}} = \frac{\log_2 \left(1 + \frac{|\boldsymbol{h}^H \boldsymbol{\omega}|^2}{\delta^2} \right)}{||\boldsymbol{\omega}||^2 + P_c}
$$
(3.5)

Assuming the channel quality vector, power consumption and noise power are a fixed value, the relationship between the energy efficiency and transmission power was simulated using MATLAB.

Figure 3.3: Preliminary simulation for a behaviour of Energy Efficiency against Total Transmission Power.

As can be seen from Fig 3.3, the energy efficiency of a SWIPT system initially increases very quickly with the increase of transmission power, then start to decrease fairly quickly after reaching a maximum point. This is because standalone application of spectral efficient communication technology to increase the signal to noise ratio cannot overcome the constraint imposed by Shannon's capacity as well as the non-negligible circuit power consumption. Therefore, the non-trivial trade-off between energy efficiency and transmission power is a major factor which contributes to the establishment of practical and feasible SWIPT system.

Chapter 4

Problem Formulation

To achieve the ultimate objective, the energy efficiency equation obtained from previous section is set as the objective function. It is assumed that the channel quality and noise power is given, hence the beamforming vector acts as the control variable for maximum energy efficiency of transmitted signal.

$$
\max_{\omega} \frac{\log_2 \left(1 + \frac{|\boldsymbol{h}^H \boldsymbol{\omega}|^2}{\delta^2} \right)}{||\boldsymbol{\omega}||^2 + P_c}
$$
\n(4.1)

Although energy efficiency of the energy harvester does not play direct role in the above objective function, there are two constraints that need to be considered in this system where the energy harvester is relevant.

$$
C1: ||\boldsymbol{\omega}||^2 \le P_{\text{max}} \tag{4.2}
$$

$$
C2: \zeta | \mathbf{g}_j^H \boldsymbol{\omega} |^2 \ge P_{\min_j} \tag{4.3}
$$

The first constraint that needs to be considered is that the transmission power $||\boldsymbol{\omega}||^2$ cannot be larger than the maximum power the transmitter can transmit – limiting the maximum beamforming factor possible.

The second constraint is that the total energy harvested at a receiver must be higher than the minimum power required at the j^{t^h} receiver. This is so that the harvested energy is high enough to power the receiver and allow it to sustain itself.

Figure 4.1: Graph of the Objective Function, Constraint 1 and Constraint 2. The coloured regions are visual representation of possible values which satisfies the equation.

Figure 4.1 is a graphical representation of the possible regions in the objective function and the constraints, respectively. It can be seen that the first constraint has finite number of points, as the possible points are bounded within are region. Therefore, it is clear that the first constraint is a convex function, and solving the equation is straight forward. On the other hand, both the objective function and the constraint are unbounded as can be seen in the figure. Hence there are infinite number of points that can be selected from these unbounded regions. i.e. they are non-convex functions.

The challenge lies in optimisation of these non-convex functions, as the proof of global optimality is much more difficult than that of a convex function. Some main techniques that can be used in optimization of non-convex functions are Dinkelbach method, convex optimization and semi-definite programming, which will be examined in the commencing research work of thesis.

4.1 Systematic Design

The objective of the system is to maximize the energy efficiency of the overall SWIPT system while ensuring reliable quality of service. The energy efficiency equation derived in the system model section, can be re-written as:

$$
\frac{\log_2\left(1+\frac{|\boldsymbol{h}^H\boldsymbol{\omega}|^2}{\delta^2}\right)}{||\boldsymbol{\omega}||^2 + P_c} \quad \Longrightarrow \quad \frac{\log_2\left(1+\frac{\tau}{\delta^2}\right)}{\text{Tr}(\boldsymbol{W}) + P_c} \tag{4.4}
$$

Where $\mathbf{W} = \boldsymbol{\omega} \boldsymbol{\omega}^H$.

Now the objective function is to maximize the EE with respect to W Considering the constraints and limitations established in the system model section, the resource allocation is developed as following:

$$
\max_{\mathbf{W}, \,\omega, \,\tau} \frac{\log_2\left(1 + \frac{\tau}{\delta^2}\right)}{\text{Tr}(\mathbf{W}) + P_c} \tag{4.5}
$$

$$
\text{s.t} \quad C1 \, : \, \text{Tr}(\boldsymbol{W}) \le P_{\text{max}}, \tag{4.6}
$$

$$
C2 \; : \; \zeta \; \operatorname{Tr}(\mathbf{g}_j \mathbf{g}_j^H \mathbf{W}) \ge P_{\min_j}, \tag{4.7}
$$

$$
C3: \tau \le \text{Tr}(\mathbf{W}h\mathbf{h}^H), \tag{4.8}
$$

$$
C4 \, : \, \mathbf{W} \, = \, \boldsymbol{\omega} \boldsymbol{\omega}^H \quad (Rank(\mathbf{W}) \le 1). \tag{4.9}
$$

The constraint C4 is imposed to guarantee that $W_k = w_k w_k^H$ holds after optimization.

Chapter 5

Optimal Solution

Figure 5.1: Flow Chart representation of the optimization algorithm.

5.1 Optimal Solution

As explained in the problem formulation section, the objective function (5.2) and the constraints C2, C3 and C4 are non-convex functions. The challenge in solving a non-convex problem is that there may be multiple local minimums or maximums, and the complexity in finding the global minimum or maximum is extremely high [35].

Since the formulated objective function is a nonlinear fractional programming problem with a concave numerator and a convex denominator where the set of control variables are convex, and the Dinkelbach method is adopted to effectively find the optimal solution.

By re-writing the fractional form into the algorithm given by Dinkelbach method

 $(F(q_k) = max{N(x) - q_kD(x)|x \in S})$ [13], the objective function can be written in the form:

$$
\max_{\mathbf{W}, \tau \in \mathbf{H}^T} \frac{\log_2 \left(1 + \frac{\tau}{\delta^2} \right)}{\text{Tr}(\mathbf{W}) + P_c} \tag{5.1}
$$

$$
\textbf{<<} \text{=}\n\text{max}_{\mathbf{W},\tau \in \mathbf{H}^T} \log_2 \left(1 + \frac{\tau}{\delta^2} \right) - q^* \left(\text{Tr}(\mathbf{W}) + P_c \right) \tag{5.2}
$$

given the equation, a sequence of linear program can be generated, then their solutions are converged to the solution of the original fractional form. (Further elaborated in Section 5.2 Dinkelbach Method)

Taking advantage of the equation obtained using the Dinkelbach algorithm, the Semi-Definite Programming relaxation [24] is applied to establish more efficient design:

$$
\min_{\mathbf{W}, \tau \in \mathbf{H}^T} \quad -\log_2\left(1 + \frac{\tau}{\delta^2}\right) + q\left(\text{Tr}(\mathbf{W}) + P_c\right) \tag{5.3}
$$

$$
s.t \quad C1 : \text{Tr}(\boldsymbol{W}) - P_{\text{max}} \le 0 \tag{5.4}
$$

$$
C2 \div \zeta \operatorname{Tr}(\mathbf{g}_j \mathbf{g}_j^H \mathbf{W}) - P_{\min_j} \ge 0 \tag{5.5}
$$

$$
C3 \, : \, \text{Tr}(\boldsymbol{W} \boldsymbol{h} \boldsymbol{h}^H) \, - \, \tau \ge 0 \tag{5.6}
$$

$$
C4: Rank(\mathbf{W}) \le 1 \tag{5.7}
$$

(5.8)

Note that the objective function is obtained by reversing the maximization function (5.1) to meet the formulation standard of Lagrangian dual problem. Now, the only non-convexity of the problem arises from constraint C4.

To mitigate this problem, the achievable optimal value can be bounded using a class of approximate method, convex relaxation [20]. By applying the Semi-Definite Programming, we obtain:

$$
\min_{\mathbf{W}, \tau \in \mathbf{H}^T} \ -\log_2\left(1 + \frac{\tau}{\delta^2}\right) + q\left(\text{Tr}(\mathbf{W}) + P_c\right) \tag{5.9}
$$

$$
s.t \quad C1 : \text{Tr}(\boldsymbol{W}) - P_{\text{max}} \le 0 \tag{5.10}
$$

$$
C2 \div \zeta \operatorname{Tr}(\mathbf{g}_j \mathbf{g}_j^H \mathbf{W}) - P_{\min_j} \ge 0 \tag{5.11}
$$

$$
C3 \, : \, \text{Tr}(\boldsymbol{W} \boldsymbol{h} \boldsymbol{h}^H) \, - \, \tau \ge 0 \tag{5.12}
$$

(5.13)

The rank-one constraint C4 can be removed for more effective optimization, by proving the tightness of the SDP relaxation. This can be achieved by referring to the following theorem:

Theorem 1. The rank of optimal beamforming matrix W is less than or equal to one if channels H and G_j are statistically independent and the objective function is feasible.

Please refer to the appendix for the proof of Theorem 1.

As demonstrated in the proof of Theorem 1, the dual problem of the objective function (5.3) fulfills the Karush-Kuhn-Tucker conditions and Slater's condition as long as the conditions in theorem 1 are satisfied. Therefore the Semi-Definite Programming relaxation is tight and the beamforming matrix W established in (5.3) is optimal for the maximization of energy efficiency of SWIPT.

5.2 Dinkelbach Method

Literatures [13], [14] and [15] explain that a nonlinear fractional programming problem with concave numerator and convex denominator can be represented by a parametric program which is a convex programming.

In particular, Dinkelbach's method is a branch of fractional programming which adopts iterative procedure to solve the equivalent parametric program of a problem. This iterative approach to fractional problems effectively reduces the computational complexity that cannot be avoided in a conventional fractional programming. [16], [17]

The iterative algorithm employed in Dinkelbach's method allows to find the maximum feasible q by solving the optimization problem $(F(q_k) = max\{N(x) - q_kD(x)|x \in S\})$ repetitively with the updated q until $F(q_k)$ approaches a pre-set tolerance value (usually set close to zero).

Chapter 6

Simulation Results

In this section the simulated results of the proposed optimal solution is demonstrated. The trade-offs and factors which contribute to the results are identified and the trend of the results is analyzed with theoretical reasoning.

6.1 Energy Efficiency and Total Transmission Power

Figure 6.1: Simulation result of Energy efficiency versus Total Transmission Power.

The figure demonstrates the average energy efficiency versus the total transmit power Pmax. It can be seen that the energy efficiency of the optimal solution obtained increases monotonically with respect to the total transmit power.

The acceleration of the escalation slows down as it reaches the upper limit, because the effect of Pmax on the gain of energy efficiency becomes less significant due to the maximum error free data rate a channel can support.

The significance of this result is that once the upper bound limit of the energy efficiency is achieved by employing adequate amount of transmission power, further increase in transmission power will introduce loss in energy efficiency.

As can be seen in Figure 6.1, the energy efficiency increases with the number of transmission antennas as a result of a improved accuracy in beamforming with larger number of transmission antennas. However, the effect of increasing number of transmit antennas Nt loses significance as the number of transmit antennas exceeds a sufficient value. This result is because the channel capacity scales logarithmically with respect to Nt.

Since the circuit power consumption increases linearly with respect to the number of transmit antennas and the gain obtained with additional transmit antennas is limited, excessively increasing the number of Nt becomes insufficient to compensate for the increase in consumption of circuit power ie. The cost invested in providing circuit power.

Therefore, implementing excessively large amount of transmission power or large number of transmission antenna will not necessarily achieve higher energy efficiency or cost effectiveness.

6.2 Energy efficiency and Number of Iterations

Figure 6.2: Simulation result of Energy efficiency versus Number of Iterations.

In Fig. 6.2 the trend in average energy efficiency is demonstrated against the number of iterations of Dinkelbach algorithm. The average energy efficiency increases monotonically with increasing number of iterations as the value of q approaches the optimal value in each iteration until it finally converges.

The similar increasing trend in energy efficiency is exhibited throughout the different values of transmission power. The overall average energy efficiency increases with increasing transmission power. However, it can be observed that the effect of increasing the number of transmission power on average energy efficiency becomes less significant as the value of transmission power becomes bigger. This is because the acceleration of the increase in energy efficiency becomes significantly slower with increasing transmission power, as can be observed

in figure 6.1.

The convergence of all three considered scenarios with different value of transmission power occurred at the same number of iterations, meaning the value of transmission power has little significance in the rate of conversion employing Dinkelbach method.

Therefore, it can be deduced that while increasing the transmission power can help increase the overall energy efficiency, blindly increasing the transmission power is not an ultimate solution. Also, the reliability of Dinkelbach algorithm in finding the optimal value of beamforming vector for maximum energy efficiency is supported with all considering scenarios achieving convergence before 10 iterations.

6.3 Energy Efficiency and Number of Eavesdroppers

Figure 6.3: Simulation result of Energy efficiency and Number of Eavesdroppers.

The Fig. 6.3 depicts the energy efficiency of the system for different number of energy harvesting receivers, and the effect of employing different number of transmission antennas. The overall average energy efficiency increases with increasing number of antennas as expected similarly to the case discussed in section 6.1, however the average energy efficiency monotonically decreases with increasing number of energy harvesting receivers.

The energy efficiency of the system decreases with increasing number of energy harvesters, because the power invested in operating the increased number of energy harvester also increases. Therefore even though the increase in number of energy harvesters may contribute to increase in energy efficiency in terms of energy harvested, it is not a sufficient way of improving the energy efficiency especially while guaranteeing quality of service.

Chapter 7

Future Works

7.1 Future Research

Figure 7.1: A comparison between linear and non-linear energy harvesting model.

The current system model developed in thesis A assumes linear harvested energy with perfect CSI, which is not the behaviour a practical SWIPT convey. Figure 5.2 is a graph showing the trend of traditional linear model compared to the measured data from [12], [21]-[23], demonstrating the significance of non-linear model in generating an optimal system model in practice.

Acknowledging the implication of non-linearity and imperfect channel state information in harvested energy generated by different factors, they can be examined in depth through the commencing research as well as the security of wireless information transmission which is is crucial [33]-[37].

Conclusion

The ultimate goal of this thesis is to design a computational efficient beamformer which maximises the energy efficiency of SWIPT systems, while guaranteeing the quality of service. Thorough research on the different design factors which contribute to the performance of SWIPT system were conducted to develop a system model. The focus in designing process was to make sure the model is efficient and feasible with high quality of service – which is why the separated receiver technique was selected for its feasibility, despite the drawback in performance, then compensating using the beamforming technique.

Through literature review and simulation of the system model, the significance of non-trivial trade-off between energy efficiency and transmission power on the feasibility and practicality of SWIPT system was identified. It was evident that due to the constraint imposed by Shannon's capacity, the increase in signal to noise ratio (or the intensity of received signal) will cause the energy efficiency to elevate abruptly at start and quickly decrease at a certain point.

By analyzing the properties of non-convex optimization and different fractional programming, the formulated problems of the system are converted into its equivalent convex optimization. The iterative algorithm of the Dinkelbach method is employed for the maximization of energy efficiency by obtaining the Lgrangian dual function of the objective function. The simulation results demonstrate the effect of factors such as the total transmission power and number of antennas to the overall energy efficiency of the system, and that the proposed optimization algorithm will successfully converge to the optimal solution at a small number of iterations.

Appendix

As established in the in the optimal solution section, the optimization problem is convex and strictly feasible (there exists a matrix W which satisfies all conditions with strict inequality) - therefore satisfying the Slater's constraint and guaranteeing that strong duality holds. Having established that the strong duality holds, the lagrange dual function has significant implication, as by definition of strong duality, the optimal value of the primal problem equals to the optimal value of its dual problem.

To prove that the SDP relaxation is tight, Karush-Kuhn-Tucker (KKT) condition [26] of the optimization problem is analyzed. First we derive the Lagrangian function:

$$
L(\boldsymbol{W}, \lambda_1, \lambda_2, \lambda_3) = -\log_2\left(1 + \frac{\tau}{\delta^2}\right) + q\left(\text{Tr}(\boldsymbol{W}) + P_c\right) + \lambda_1(Tr\boldsymbol{W} - P_{\text{max}})
$$

+
$$
\sum_{j=1}^J \lambda_{2j} (P_{\text{min}_j} - \zeta \text{ Tr}(\boldsymbol{g}_j \boldsymbol{g}_j^H \boldsymbol{W}))
$$

+
$$
\lambda_3 \left(\tau - \text{Tr}(\boldsymbol{W} \boldsymbol{h} \boldsymbol{h}^H)\right) - Tr(\boldsymbol{Y} \boldsymbol{W})
$$
 (7.1)

Where Y and $\lambda_1, \lambda_{2_j}, \lambda_3$ are the dual variables corresponding to the constraints C4 and C1, C2, C3 of the optimization problem respectively.The lagrangian derived above is used in finding the dual problem of the primal problem. The dual problem can be written as:

$$
\max_{\mathbf{Y}, \lambda_1, \lambda_2, \lambda_3} \inf_{\mathbf{W}, \tau \in \mathbf{H}^T} L
$$
\n(7.2)

Now we concentrate on the Karush-Kuhn-Tucker (KKT) conditions which are related to the structure of the matrix W and proving the theory. One condition is that the gradient of Lagrangian with respect to the control variable vanishes. By taking the derivative of the Lagrangian function with respect to W , the value of Y at which the gradient of the Lagrangian function equals to zero can be found:

$$
\frac{\partial L}{\partial \mathbf{W}} = q\mathbf{I} + \lambda_1 \mathbf{I} - \sum_{j=1}^{J} \lambda_{2j} \zeta \operatorname{Tr} \left(\mathbf{g}_j \mathbf{g}_j^{\mathbf{H}} \right) - \lambda_3 \mathbf{h} \mathbf{h}^{\mathbf{H}} - \mathbf{Y} \tag{7.3}
$$

$$
\mathbf{Y} = q\mathbf{I} + \lambda_1 \mathbf{I} - \sum_{j=1}^{J} \lambda_{2j} \zeta g_j g_j^H - \lambda_3 \mathbf{h} \mathbf{h}^H
$$
 (7.4)

where \boldsymbol{I} is an identity matrix and satisfies the constraint:

$$
\mathbf{Y} \succcurlyeq \mathbf{0}, \lambda_1, \lambda_{2_j}, \lambda_3 \ge 0 \tag{7.5}
$$

Now we consider complementary slackness to further analyze the KKT conditions. Complementary slackness explains that if a dual variable is greater than zero then the corresponding primal constraint must be an equality, and that if the primal constraint is greater than zero then the corresponding dual variable is equality. This means that the columns of matrix W fall into the null-space spanned by Y for $W \neq 0$.

$$
YW = 0 \tag{7.6}
$$

In other words, the rank of the optimal beamforming matrix W will less than or equal to one if the rank of \boldsymbol{Y} is greater than or equal to the number of transmission antennas (dimension of W) minus one.

To find the rank of the matrix we analyze the structure of Y by re-writing it in the form:

$$
Y = B - H
$$

\n
$$
Y = qI + \lambda_1 I - \sum_{j=1}^{J} \lambda_{2_j} \zeta \text{ Tr}(g_j g_j^H) - \lambda_3 h h^H
$$
 (7.7)

Where \bm{B} and \bm{H} are matrices with same dimensions. For matrix \bm{B} to have a full-rank, (ie. $Rank(\mathbf{B}) = N_T$), **B** must be a positive definite matrix with probability one.

To prove matrix B is a positive definite matrix, let us consider the case where \bm{B} is a positive semi-definite matrix. If \boldsymbol{B} is a positive semi-definite matrix, there must exist at least one zero eigenvalue v. Now by multiplying the equation (9.7) on both sides by a matrix $V = v v^H$ and applying the trace operation on both sides we get:

$$
\operatorname{Tr}(\boldsymbol{Y}\boldsymbol{V}) = -\operatorname{Tr}(\boldsymbol{H}\boldsymbol{V}) + \operatorname{Tr}(\boldsymbol{B}\boldsymbol{V})
$$

=
$$
-\operatorname{Tr}(\boldsymbol{H}\boldsymbol{V})
$$
 (7.8)

Matrices H and G_i are statistically independent, therefore $Tr(HV)$ is larger than zero. However, $Tr(YV)$ is greater than or equal to zero hence the two inequalities contradict each other. This proves matrix **B** is a positive definite matrix with full-rank, $Rank(B) = N_T$.

Now to find the rank of matrix \boldsymbol{Y} , we apply basic rule of inequality. Consider two matrices \bm{A} and \bm{B} with same dimensions. By basic rule of inequality and rank of matrices, the summation of rank of individual matrices \bm{A} and \bm{B} is larger than the rank of the matrix $A + B$ ie. $Rank(A) + Rank(B) \geq Rank(A + B)$.

Therefore, it can said that $Rank(\mathbf{A} + \mathbf{B}) + Rank(-\mathbf{B}) \geq Rank(\mathbf{A})$, and rearranging the inequality we get $Rank(\mathbf{A} + \mathbf{B}) \geq Rank(\mathbf{A}) - Rank(\mathbf{B})$. Now applying this inequality, we can compute the rank of matrix Y :

$$
Rank(Y) = Rank(-Y)
$$

= Rank(-B + H) (7.9)

$$
\geq Rank(-B) - Rank(H) \geq N_t - 1
$$

Since the columns of matrix W fall into the null-space spanned by Y and we have established that $Rank(Y) \ge N_t - 1$, we can conclude that $Rank(W) \le 1$ for, completing the proof for theorem 1.

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