SCHOOL OF ELECTRICAL ENGINEERING
AND TELECOMMUNICATIONS

Secure Communication in Multiuser MIMO SWIPT Systems

by

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Abstract

This thesis considers simultaneous wireless information and power transfer (SWIPT) in multi-user multiple-input multiple-output (MIMO) systems where a transmitter serves an information receiver (IR) and multiple energy harvesting receivers (ERs). It is foreseen that SWIPT technology will serve as a key to unlock the potential of Internet-of-things (IoT) in the fifth-generation (5G) communication systems, via supplying wireless energy to energy limited wireless devices. This thesis aims to design a resource allocation algorithm to maximize the achievable rate of IR. The design is formulated as a non-convex optimization problem which takes into account the minimum required energy at each ER, the maximum transmit power at the transmitter, and the need of secrecy communication measure against potential eavesdropping. Due to non-convexity of the problem, in Thesis A, maximum ratio transmission (MRT) is adopted as a suboptimal resource allocation policy to depict the non-trivial trade-off between average total system data rate and average total system harvested power. In Thesis B, semidefinite programming relaxation (SDR) is applied to solve the non-convex optimization problem optimally. Simulation results show that a substantial performance gain can be achieved by the proposed optimal scheme compared to the MRT-based suboptimal scheme.

Acknowledgements

I wish to thank all the people who have made contributions to the contents of this thesis, in particular my supervisor Derrick Ng who guided me through all this.

Contents

Chapter 1

Introduction

With the rapidly growing amount of wireless communication devices and appliances such as automated control in smart cities, monitoring in e-health systems and remote security sensing [1], it is expected that the amount of wireless devices would reach up to 50 billion by 2020 around the world [2]. As a result, providing much more than faster data rate, defined as the unifying connectivity fabric, 5G also targets on enhancing mobile broadband (e.g. tactile internet), connecting massive Internet-of-Things (IoT) (e.g. wireless wearables) and enabling new mission-critical controls (e.g. autonomous vehicles). Among all 5G emerging technologies (e.g. massive multiple-input multiple-output (MIMO), millimeter wave [3] etc.) thriving to fulfill stringent quality of service (QoS) requirements, MIMO technology is used to reduce the transmit power in order to satisfy the rocketing demand of energy in both transmitters and receivers [4]. Apart from offering extra degrees of freedom to improve flexibility in resource allocation to reduce energy cost, MIMO can also enable communication security by generating energy signal for jamming the channels of potential eavesdroppers deliberately. Besides, information signal beamforming can be adopted to reduce the potential information leakage [5]. Due to high computational complexity at receivers, traditional MIMO architecture may not be suitable for portable devices. As an alternative solution, multi-user MIMO with a multiple-antenna transmitter serving multiple receivers equipped with single-antenna [6], [7], is adopted as it can shift the signal processing burden from receivers to the transmitter, and allows simple designs and cheap receiver structure.

In practice, various QoS requirements of wireless communication networks under 5G background also deserve our attention. In particular, an increasing number of mobile devices such as wireless sensors for IoT applications is gaining their popularity among the industry and becoming essential parts of wireless communication networks recently [8]. As most of these devices are battery-powered with finite battery capacity, inconvenience of battery charging or replacement are few of the major obstacles in realizing practical IoT implementation [9]. Consequently, energy harvesting technology has been proposed as a promising solution to provide ubiquitous and self-sustainable networks. Although traditional energy harvesting technology, which collects energy from natural renewable energy sources (e.g. tide and solar), enables self-sustainable networks to a certain extent, it is usually climate-dependent and location-dependent, which makes renewable energy a perpetual but intermittent energy supply [10]. Therefore, directly integrating conventional energy harvesting technology into communication devices may result in unstable communication service e.g. [10], [11]. Instead of exploiting renewable energy for energy-limited systems, a "promising" solution , radio frequency based (RF-based) energy harvesting is considered as a building block for enabling sustainable wireless sensor systems to unlock the potential of networks in IoT [12, 13, 14].

1.1 Background

Proposed by Nikola Tesla back in the late nineteenth century, wireless power transfer (WPT) was first implemented by a magnifying transmitter based on the Tesla coil transmitter [15]. Its ultimate goal was to broadcast wireless power to any location around the globe avoiding the shortcomings of conventional cabling such as being unaffordable and inconvenient to deploy [15]. Under massive impact of the industrial revolution in late 1800s, WPT was originally designed to apply on high-power machines. However, with public health concern about harmful electromagnetic radiation caused by large power emission from the transmit tower [1], progress on bringing WPT into practice was hindered in the last century. Besides, as antennas in reasonable size are required in practical design to provide mobility for portable communication devices, the wireless signal is modulated in a high carrier frequency resulting in severe path loss, which leads to a relatively small amount of collected power at the receiver side. Therefore, low power transfer efficiency is one of the challenges in implementing WPT [16]. Prevented by these two major challenges, WPT was not able to realize its further development until advancing silicon technology and wireless communication theory bring it back to life recently [1]. Therefore, collecting energy from background RF electromagnetic (EM) wave transmitted from ambient transmitters is feasible via WPT technologies. In fact, various proof-of-concepts experiments and prototypes have been developed. For example, a commercial development kit manufactured by [17], has demonstrated that a sufficient amount of harvested power (e.g. 26 μW) is enough to power up wireless IoT small sensors such as a LCD meter. Since then, RF-based energy harvesting has been a focus in wireless charging which benefits the industry with more possibilities on product design, usability, and realizability [18].

1.2 Communication Security

In the computer networking's Open System Interconnect (OSI) model, the physical layer is the lowest and also the first layer. Most of the conventional cryptographic encryption methods to ensure secure communication implemented in the upper layers such as the application layer to ensure secure communication [19]. Imperfection in physical layer such as the potential risk of noise and fading used to "hide" messages from an eavesdropper calls for security solutions at the physical layer to enhance communication security mechanisms [20]. Besides, RF-based data transmission relies on the exchange of perfect secret key information which is not practical in some of wireless communication networks [1]. Furthermore, this secret-key cryptography assumes that potential eavesdroppers have limited computational capabilities, which might pose a future threat on itself as computers with ultra-high computational capabilities would be the developing trend (e.g. quantum computers). As an alternative or a complementary solution, physical layer security aims at protecting secure communication by making use of wireless communication channels' physical natures (e.g. channel fading, etc.) [21]. The secrecy rate defining the actual data rate received by the target IR is as follows:

$$\text{SecrecyRate} = [\mathbf{A} - \mathbf{B}]^+, \tag{1.1}$$

where $\mathbf{A}$ is the capacity of the channel between the desired transmitter and receiving IR, while $\mathbf{B}$ denotes the capacity between the legitimate transmitter and the eavesdropper.

On the other hand, severe path loss is an inevitable consequence of adopting high carrier frequency of RF wave to maintain the transmit antennas in a reasonable size [22]. Hence, when information signal transmit power is increased on purpose to improve the WPT by compensating this loss, the susceptibility of eavesdropping by ERs would also be increased due to the broadcast nature of wireless channels which arouses the security issue. In other words, communication security problem is more prominent in wireless-powered communication systems [1]. In order to fulfill these requirements, various methods such as energy beamforming or artificial jamming have been proposed. In particular, artificial jamming is an effective solution which is generated on purpose for degrading the quality of eavesdroppers' channels, which also helps facilitate an efficient energy transfer to ERs [23].

1.3 Receiver Structure

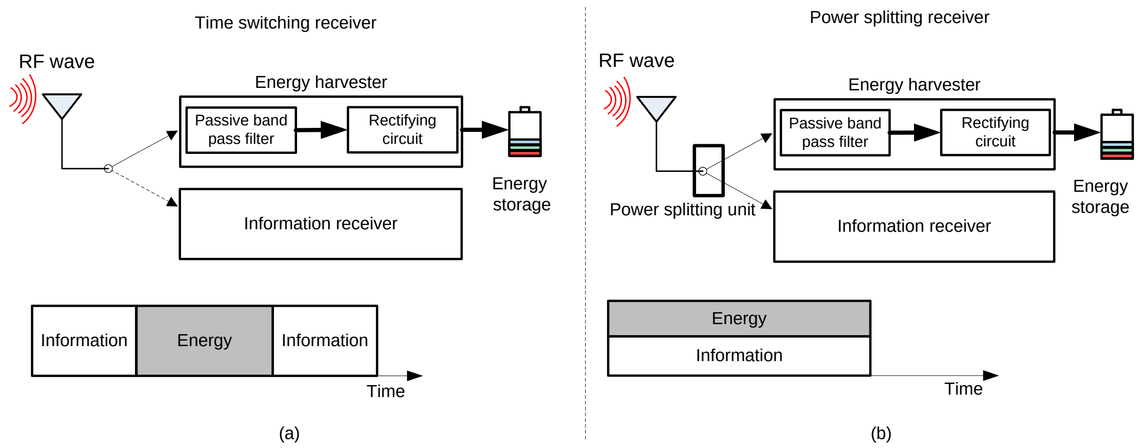

Figure 1.1: Two different receivers hardware structure; (a) Time-Switching receiver; (b) Power-Splitting receiver.

Different feasible architectures of EH receivers to enable both energy and information transmission are discussed here [5, 24, 25, 26].

- Time Switching (TS) Receiver: The working principle of TS receivers is to divide a transmission slot into two orthogonal time frames with one for pure energy harvesting and another one for pure information receiving, cf. Figure 1.1(a). Although this type of receiver is relatively simple to implement in terms of hardware circuitries, it has a stringent requirement on scheduling of information/energy and accurate timing synchronization [27].

- Power Splitting (PS) Receiver: The working principle of PS receivers is to spilt the received signal into two different power streams via the introduction of a power splitter, cf. Figure 1.1(b). Specifically, one stream is fed to an energy harvester to harvest power while the other one is connected to an IR for signal detection. Although this type of receiver provides a high flexibility in striking a balance between information decoding and energy harvesting in SWIPT systems, extra noise would be introduced into the system due to possible insertion loss or imperfect power splitting. Besides, power splitter is generally an active device requiring additional energy supply, despite an PS receiver is an energy harvester itself [28].

- Separated Receiver: Each receiver is only responsible for either harvesting energy or receiving information [29]. Compared with TS and PS receivers, the physical structure of separated receivers which requires only off-the-shell components in terms of hardware architecture makes it relatively simpler for implementation.

While advantages and disadvantages of these two types of receivers are manifest, separated receiver is adopted in this thesis. In comparison, separated receiver requires only off-the-shell components in terms of hardware architecture which makes it simple to implement in practice.

1.4 Notation

Key mathematical notations are given in Table 1.1. Boldface lower and capital case letters are used to denote vectors and matrices, respectively. Rank($\mathbf{A}$), Tr($\mathbf{A}$), and $\mathbf{A}^{\mathrm{H}}$ are the rank, the trace, and Hermitian transpose of matrix $\mathbf{A}$, respectively. $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is a positive semi-definite matrix. $\mathbb{H}^{\mathrm{N}}$ represents a set of Hermitian matrix. $\mathbb{C}^{\mathrm{N}\times\mathrm{M}}$ and $\mathbb{R}^{\mathrm{N}\times\mathrm{M}}$ represent all $N \times M$ sets with complex and real entries, respectively. $\mathbb{E}\{\cdot\}$ denotes statistical expectation. The circularly symmetric complex Gaussian (CSCG) distribution is represented by $\mathcal{CN}(\mathbf{m},\mathbf{\Sigma})$ with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$.

Table 1.1: Nomenclature adopted in this report.

Notation	**Description**
$\mathbf{h}$	Channel vector between the IR and the transmitter
$\mathbf{g}_j$	Channel vector between ER j and the transmitter
$\mathbf{w}$	Information beamforming vector
$\mathbf{w}_{\mathrm{E}}$	Energy signal beamforming vector
σ^2_{ant}, σ^2_{s}	Antenna and signal processing noise power
N_{T}	Number of transmit antennas
$R^{\mathrm{Tol}}_{\mathrm{ER}}$	Maximum tolerable data rate
$P_{\max}$	Maximum transmit power of the transmitter
$P_{\min}$	Minimum required power transfer to ERs

Chapter 2

System Model

We have one transmitter equipped with N_T antennas, one IR, and J energy harvesting receivers (ERs), with both the IR and ERs being single-antenna devices in a downlink SWIPT system [21], cf. Figure 2.1. In the following, we assume that both the transmitter and receivers know the perfect channel state information (CSI) for resource allocation.

In the considered SWIPT model [9], all receivers are separated receivers in terms of receiver architecture. Viewing from the hardware point of view, off-the-shell components for EH circuit and information decoding circuit can be used for implementation [1]. In our system model, IR can only receive information while ERs can either harvest power or eavesdrop information, which are served by the same transmitter.

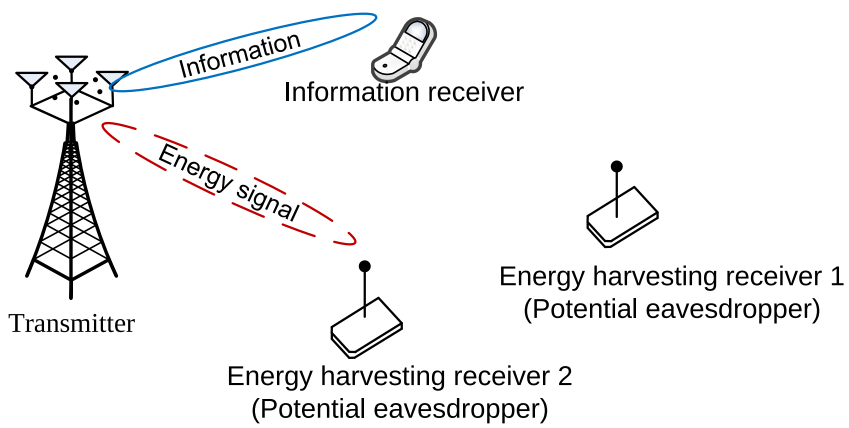

Figure 2.1: A downlink SWIPT model with one IR and $J = 2$ ERs. The ERs also act as potential eavesdroppers eavesdropping information transmitted from the transmitter to the IR.

The following parts in this thesis are presented as follows. In Section 2.2.1, pure energy beamforming is considered which is followed by Section 2.2.2, where we only consider information beamforming. With communication security taken into account, Section 2.2.3 considers SWIPT beamforming with both energy and information transmission.

2.1 Channel Model

2.1.1 Transmitted Signal

With energy signal adopted, the transmit signal vector $\mathbf{x}$ is given here as

$$\mathbf{x} = \mathbf{w}s + \mathbf{w}_{\mathrm{E}}, \tag{2.1}$$

where $\mathbf{w}_{\mathrm{E}}$ is a pseudo-random energy signal and modeled as a complex Gaussian random vector with zero-mean and covariance matrix $\mathbf{W}_{\mathrm{E}}$, i.e., $\mathbf{w}_{\mathrm{E}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_{\mathrm{E}})$, $\mathbf{w}$ is the beamforming vector of the information signal, and s is the information signal.

2.1.2 Received Signals

We consider slow time-varying frequency flat communication channels. In a single time frame, the transmitter transmits energy and information to receivers concurrently. With the transmitted signal $\mathbf{x}$ applied, the signals received at IR and ER $j \in \{1, \ldots, J\}$ are given by

$$y = \mathbf{h}^H(\mathbf{w}s + \mathbf{w}_{\mathrm{E}}) + n \text{ and} \tag{2.2}$$

$$y_{\mathrm{ER}_j} = \mathbf{g}_j^H(\mathbf{w}s + \mathbf{w}_{\mathrm{E}}) + n_{\mathrm{ER}_j}, \forall j \in \{1, \ldots, J\}, \tag{2.3}$$

respectively, where $\mathbf{h}^H$, and $\mathbf{g}_j^H$ are channel vectors between the transmitter and IR, transmitter and ER j, respectively. Both vector variables capture the impact of small scale fading, path loss, and large scale fading of the associated channels [30]. n, n_{ER_j} are additive white Gaussian noise (AWGN) of IR and ER j from the receiving antenna, respectively, with zero-mean and variance σ^2_{ant} and $\sigma^2_{\mathrm{ant}_j}$, respectively.

2.2 Problem Formulation

2.2.1 Energy Beamforming

In this section, we only consider energy transmission with the generated energy signal to facilitate WPT from the transmitter to the ERs. Figure 2.2 shows the block diagram of an ER. Since RF-based EH circuits can be implemented via various hardware architectures [31], we do not assume a specific hardware design. Instead, we adopt an information theoretic approach which can isolate the proposed beamforming design from any particular hardware implementation. We also assume without loss of generality that $\mathbb{E}\{|\mathbf{s}|^2\} = 1$. Without considering information transmission, the transmitted signal is given as $\mathbf{x} = \mathbf{w}_{\mathrm{E}}$, the harvested energy at ER j, $\Phi_{\mathrm{ER}_j}^{\mathrm{Linear}}$, is typically modelled by the following linear model [32], [10]:

$$
\begin{aligned}
\Phi_{\mathrm{ER}_j}^{\mathrm{Linear}} &= \eta_j P_{\mathrm{ER}_j}, && (2.4)\\
P_{\mathrm{ER}_j} &= \mathbb{E}\{|\mathbf{g}_j^H \mathbf{x}|^2\} && (2.5)\\
&= \mathrm{Tr}\Big(\mathbb{E}\{\mathbf{w}_{\mathrm{E}}\mathbf{w}_{\mathrm{E}}^H\}\mathbf{g}_j\mathbf{g}_j^H\Big) && (2.6)\\
&= \mathrm{Tr}\Big(\mathbf{W}_{\mathrm{E}}\mathbf{g}_j\mathbf{g}_j^H\Big), && (2.7)
\end{aligned}
$$

cf. equation (2.4), where $\eta_j \in [0,1]$ is the RF-to-electrical energy conversion efficiency, $\mathbf{W}_{\mathrm{E}}$ is the transmit covariance matrix of the energy signal and P_{ER_j} denotes the received power from the channel.

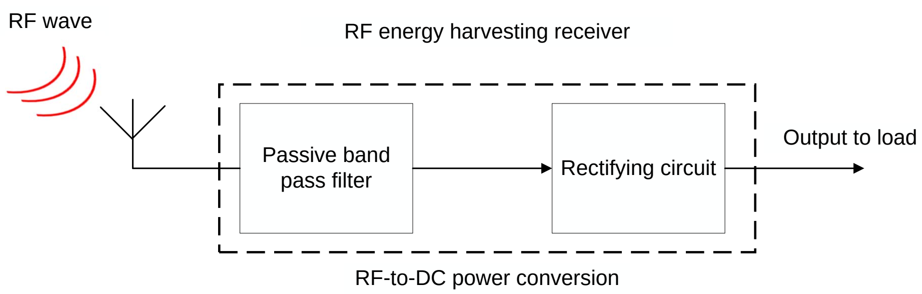

Figure 2.2: Block diagram of an ER.

From ERs perspectives, we can formulate Problem 1:

Problem 1. Total Harvested Power Maximization

$$\begin{aligned} \underset{\mathbf{W}_{\mathrm{E}} \in \mathbb{H}^{N_{\mathrm{T}}}}{\text{maximize}} \quad & \mathrm{Tr}(\mathbf{W}_{\mathrm{E}}\mathbf{G}) \\ \text{s.t.} \quad & \mathrm{C1}: \ \mathrm{Tr}(\mathbf{W}_{\mathrm{E}}) \le P_{\max}, \\ & \mathrm{C2}: \ \mathbf{W}_{\mathrm{E}} \succeq \mathbf{0}. \end{aligned} \tag{2.8}$$

where $P_{\max}$ is the maximum transmit power budget offered by the transmitter and $\mathbf{G} = \sum_{j=1}^{J} \eta_j \mathbf{g}_j \mathbf{g}_j^H$ represents the equivalent channel between the transmitter and J ERs.

In Problem 1, we target at maximizing the total harvested power of ERs while guaranteeing the total transmit power is no larger than the maximum transmit power budget.

2.2.2 Information Beamforming

In this section, we only consider information transmission between the transmitter and the IR. Without considering energy transfer, the transmitted signal is given by $\mathbf{x} = \mathbf{w}s$. Similar to the case of pure energy transfer, it is assumed that without loss of generality which means $\mathbb{E}\{|\mathbf{s}|^2\} = 1$. Indicating how much information is successfully transmitted to the IR from the transmitter, the achievable rate (bit/s/Hz) between the transmitter and the IR is given by [33]

$$R = \log_2\left(1 + \frac{\mathrm{Tr}(\mathbf{WH})}{\sigma_{\mathrm{ant}}^2 + \sigma_{\mathrm{s}}^2}\right), \tag{2.9}$$

where $\mathbf{W}$ is the covariance matrix of the information signal and the term $\frac{\mathrm{Tr}(\mathbf{WH})}{\sigma_{\mathrm{ant}}^2 + \sigma_{\mathrm{s}}^2}$ represents the signal-to-noise ratio (SNR) of the IR.

From the IR point of view, the achievable rate maximization design can be formulated as problem below:

Problem 2. Achievable Rate Maximization

$$\begin{aligned} \underset{\mathbf{W}\in\mathbb{H}^{N_\mathrm{T}}}{\text{maximize}} \quad & R=\log_2\left(1+\frac{\mathrm{Tr}(\mathbf{W}\mathbf{H})}{\sigma_{\mathrm{ant}}^2+\sigma_{\mathrm{s}}^2}\right) \\ \text{s.t.} \quad & \mathrm{C1}:\ \mathrm{Tr}(\mathbf{W})\le P_{\max}, \\ & \mathrm{C2}:\ \mathbf{W}\succeq\mathbf{0}, \\ & \mathrm{C3}:\ \mathrm{Rank}(\mathbf{W})=1. \end{aligned} \tag{2.10}$$

In Problem 2, we try to maximize the achievable data rate of IR while guaranteeing the transmit power from the transmitter is no larger than the transmit power budget.

2.2.3 SWIPT Beamforming

In this section, we consider the SWIPT system with both information and energy transfer. In particular, the transmit signal is given as $\mathbf{x}=\mathbf{w}s+\mathbf{w}_\mathrm{E}$ when information and energy transmission are considered. For ERs, the total harvested power is to be maximized while for the IR, the achievable data rate is to be maximized. However, it is possible for ERs to eavesdrop information transmitted from the transmitter to the IR. As a result, to guarantee secure communication, the achievable data rate of ER j should be below a tolerable level.

Problem 3. Achievable Rate Maximization [Generalization]

$$\begin{aligned} \underset{\mathbf{W},\mathbf{W}_\mathrm{E}\in\mathbb{H}^{N_\mathrm{T}}}{\text{maximize}} \quad & R=\log_2\left(1+\frac{\mathrm{Tr}(\mathbf{W}\mathbf{H})}{\sigma_{\mathrm{ant}}^2+\sigma_{\mathrm{s}}^2}\right) \\ \text{s.t.} \quad & \mathrm{C1}:\ \mathrm{Tr}((\mathbf{W}+\mathbf{W}_\mathrm{E})\mathbf{G})\ge P_{\min}, \\ & \mathrm{C2}:\ \mathrm{Tr}(\mathbf{W}+\mathbf{W}_\mathrm{E})\le P_{\max}, \\ & \mathrm{C3}:\ R_j^{\mathrm{ER}}\le R_{\mathrm{tol}}^{\mathrm{ER}}, \forall j\in\{1,\dots,J\}, \\ & \mathrm{C4}:\ \mathbf{W}\succeq\mathbf{0}, \\ & \mathrm{C5}:\ \mathbf{W}_\mathrm{E}\succeq\mathbf{0}, \\ & \mathrm{C6}:\ \mathrm{Rank}(\mathbf{W})=1. \end{aligned} \tag{2.11}$$

where $R_j^{\mathrm{ER}} = \log_2\left(1 + \frac{\mathrm{Tr}(\mathbf{W}\mathbf{G}_j)}{\mathrm{Tr}(\mathbf{G}_{\mathrm{j}}\mathbf{W}_{\mathrm{E}})+\sigma_{\mathrm{ant}}^2+\sigma_{\mathrm{s}}^2}\right)$ is the achievable rate of ER j, C1 means the total system harvested power should be no less than $P_{\min}$, the minimum required total system power transfer, C2 denotes the total system transmit power should be no larger than $P_{\max}$, the maximum transmit power budget offered by transmitter, C3 constrains the the achievable rate of ER j such that it is smaller than $R_{\mathrm{tol}}^{\mathrm{ER}}$, the maximum tolerable achievable rate of ER j, and the highlighted terms are non-convex functions.

Chapter 3

Resource Allocation Design

3.1 Suboptimal Solution

As Problem 3 is a generalization of both Problems 1 and 2 with security issue taken into account. In fact, once the structure of the optimal beamforming for Problem 3 is derived, it is able to be used to obtain the optimal solution of Problems 1 and 2. However, with two non-convex terms in Problem 3, it is difficult to directly derive the optimal solutions for Problem 3. As in non-convex functions, there might be multiple local maximum points [34]. In general one can the apply exhaustive search method to find the globally optimal solution. However, the computational complexity grows exponentially with respect to the number of antennas and ERs which is not suitable for medium and large size of systems. In the following, we first consider some simple suboptimal designs by using MRT [35]. Then we design the optimal solution based on SDP-relaxation.

MRT-based Suboptimal Solution:
In fact, the key to solve the problem suboptimally is to transform the non-convex problem into a convex one and then solve the transformed problem via existing convex optimization techniques. The suboptimal algorithm is presented in Table 3.1 on the top of next page. After the first two steps, we already use MRT to convert the non-convex problem into a convex one, which facilitate the use of CVX, a convex problem solver, for solving the problem [36]. It is note-worthy that the first two steps of adopting MRT to solve Problem 3 are tantamount to solving Problem 2 solitarily, as we fix the beamforming direction pointing right the IR to get a maximum achievable data rate for IR.

Table 3.1: Suboptimal Resource Allocation Algorithm

Algorithm Suboptimal Optimization

Initialize the minimum required total system power transfer $P_{\min} = 0$ and δ is a small positive constant

repeat {Loop}

    Adopt a fixed beamforming direction pointing at $\frac{\mathbf{h}^*}{\|\mathbf{h}\|}$

    Optimize the transmit power of the beamforming vector

    Solve Problem 3 numerically with convex problem matlab solver, e.g. (CVX), [36], [37]

    if Problem 3 is still feasible **then**

        $P_{\min} = P_{\min} + \delta$

    end if

until Problem 3 becomes infeasible

3.2 Optimal Solution

Problem 3 is non-convex because of constraint C3 and rank-one matrix constraint C6. SDP relaxation is applied to derive a tractable problem formulation by relaxing constraint 6 $\text{Rank}(\mathbf{W}) = 1$ from Problem 3 [38]. Therefore, the studied problem changes into a convex SDP problem and we can apply convex problem solver CVX to solve it [39]. Before we can directly apply CVX for problem solving, we need to study the tightness of the SDP relaxation in Theorem 1.

Problem 3. Achievable Rate Maximization [Generalization]

$$
\begin{aligned}
\underset{\mathbf{W},\mathbf{W}_\mathrm{E}\in\mathbb{H}^{N_\mathrm{T}}}{\text{minimize}} \quad & -\text{Tr}(\mathbf{W}\mathbf{H}) \qquad (3.1)\\
\text{s.t.} \quad & \text{C1}: \; P_{\min} - \text{Tr}((\mathbf{W}+\mathbf{W}_\mathrm{E})\mathbf{G}) \le \mathbf{0},\\
& \text{C2}: \; \text{Tr}(\mathbf{W}+\mathbf{W}_\mathrm{E}) - P_{\max} \le \mathbf{0},\\
& \text{C3}: \; \text{Tr}(\mathbf{W}\mathbf{G}_j) - (2^{R_\mathrm{tol}^\mathrm{ER}} - 1)\,\text{Tr}(\mathbf{G}_j\mathbf{W}_\mathrm{E} + \sigma_\mathrm{tol}^2) \le \mathbf{0}, \forall j \in \{1,\dots,J\},\\
& \text{C4}: \; -\mathbf{W} \preceq \mathbf{0},\\
& \text{C5}: \; -\mathbf{W}_\mathrm{E} \preceq \mathbf{0},\\
& \cancel{\text{C6}: \; \text{Rank}(\mathbf{W}) = 1.}
\end{aligned}
$$

Theorem 1: We assume that channels $\mathbf{H}$ and $\mathbf{G}_j$ are statistically independent and Equation 3.1 is feasible. The optimal information beamforming matrix $\mathbf{W}$ is rank-one with probability one, i.e. $\text{Rank}(\mathbf{W}) = 1$.

Proof 1: Please refer to Appendix B. $\square$

Thus, the applied SDP relaxation is proven tight as long as the channel state assumptions in Theorem 1 are fulfilled. Therefore, the considered information beamforming is optimal for the maximization of achievable data rate under the considered framework [40].

Chapter 4

Simulation

4.1 Simulation Parameters

In this section, numerical examples are used to present us the non-trivial trade-off between the total system data rate and the total system harvested power for the considered SWIPT model. Unless further specified, Table 4.1 below lists some important parameters adopted in the simulation. In the following scenario, the IR is placed 100 meters away from the transmitter while all the ERs are 10 meters away from the transmitter.

Table 4.1: Parameters in simulation.

Centre frequency of carrier signal	915 MHz
Bandwidth	200 kHz
Gain of transceiver antenna	10 dBi
Transmit antenna number N_T	$3, 6, 9$
Noise power σ^2	-95 dBm
Maximum transmit power P_{max}	1 W
$\mathbf{g}_j$ fading distribution	Ricean with Ricean factor 3 dB
$\mathbf{h}$ fading distribution	Rayleigh

4.2 Simulation Results

Figure 4.1 shows the non-trivial trade-off between the total system data rate and the total system harvested power for the SWIPT model under the proposed suboptimal beamforming scheme. In general, the area enclosed by the curve of a certain transmit antenna number is the achievable region which means that all points lie inside or on the curve can be achieved by tuning the relevant system parameters. By comparing intersecting points of both y and x-axis for different transmit antenna numbers, it can be observed that with increasing number of transmitter antennas, the maximum total system data rate of IR and the maximum total system harvested power of ERs increase due to extra spatial degrees of freedom supplied by multiple transmit antennas which improve the accuracy in beamforming. On the other hand, by making comparison between two enlarged regions (i.e. region between $N_T = 3$ and $N_T = 6$, region between $N_T = 6$ and $N_T = 9$), it is manifest that with increasing number of transmitter antennas, the increasing rate of the enlarged region decreases due to channel hardening [33].

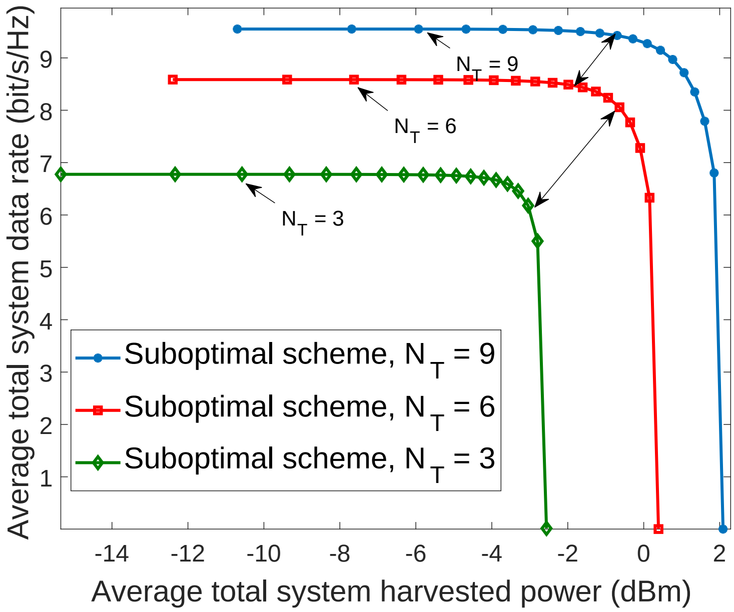

Figure 4.1: Average total system data rate (bit/s/Hz) versus average total system harvested power (dBm).

Figure 4.2 below displays a comparison of the non-trivial trade-off between the total system data rate and the total system harvested power for the SWIPT model under proposed suboptimal and optimal beamforming scheme. It can be verified from the graph that with the optimal beamforming design the achievable region enclosed by the optimal curve of a certain N_T is further enlarged due to its more flexibility in beamforming. It can also be seen that the suboptimal scheme cannot achieve the maximum system data rate as communication security is taken into account.

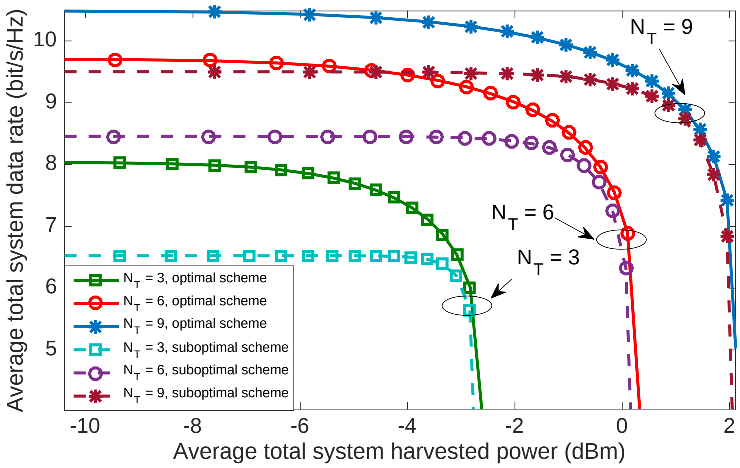

Figure 4.2: Average total system data rate (bit/s/Hz) versus average total system harvested power (dBm).

Figure 4.3 and Figure 4.4 below show the relationships between the average data rate, the average total harvested power of the system, and the average total transmit power, respectively. Both the average data rate of the system and the average total system harvested power increase when there is more transmit power available in the system. This is because a larger amount of radiated power is available in the system for the IR to receive and ERs to harvest, which means more resources for IR and ERs to utilize [21]. In particular, in Figure 4.4, by making comparison between two performance gains, the enlarged regions (i.e. region between $N_T = 8$ and $N_T = 10$, region between $N_T = 10$ and $N_T = 12$), it is manifest that with increasing number of transmitter antennas, the increasing rate of the enlarged region decreases due to channel hardening [33].

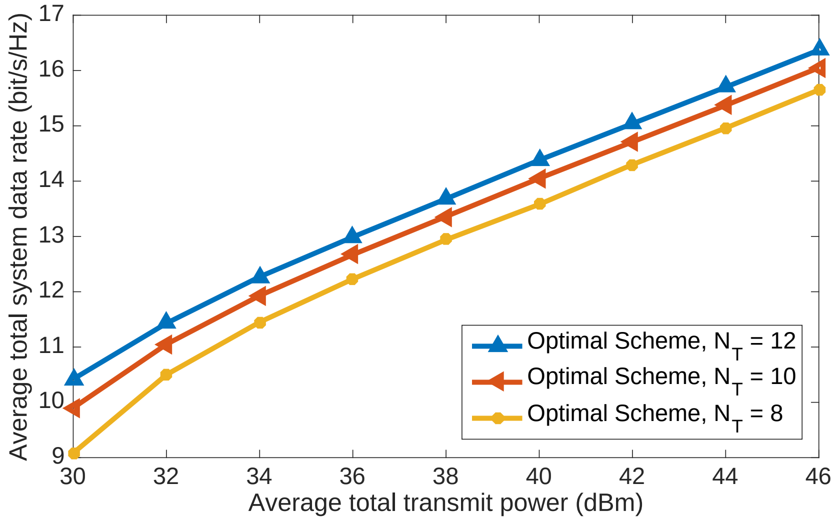

Figure 4.3: Average total system data rate (bit/s/Hz) versus average total system transmit power (dBm).

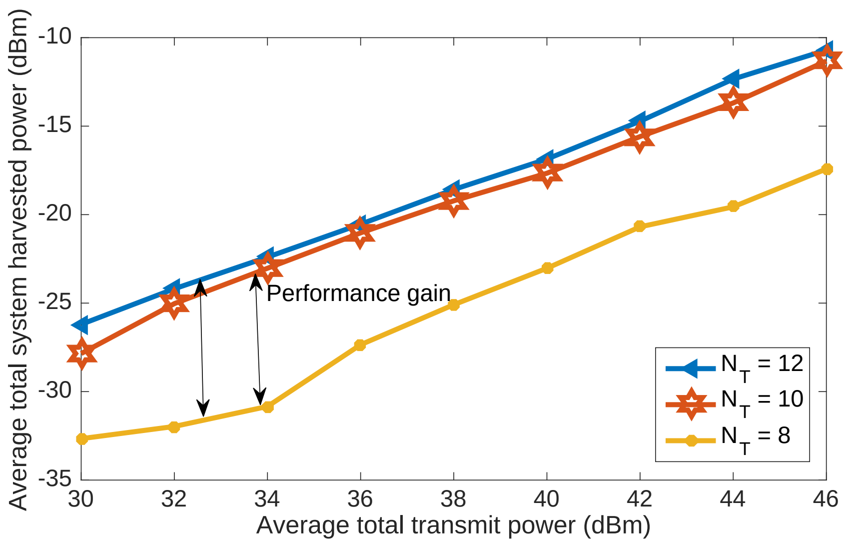

Figure 4.4: Average total system harvested power (dBm) versus average total system transmit power (dBm).

Figure 4.5 illustrates a relationship between the average data rate of the system and the average total transmit power with different numbers of ERs present in the system. Under a given amount of total transmit power, the average total system data rate decreases when there are more ERs in the system. In fact, when there are more ERs in the system, both the QoS requirements on communication security and minimum required harvested power become more stringent. Particularly, the transmitter is forced to steer the direction of information signal towards the ERs. This will decrease the received signal strength of the desired signal at the IR. Besides, the transmitter would also increase the transmit power of energy signal to neutralize the higher potential of information leakage, which leads to a further reduction in the data rate. Furthermore, the proposed suboptimal scheme is able to guarantee both the QoS requirements of min. required harvested power and communication security.

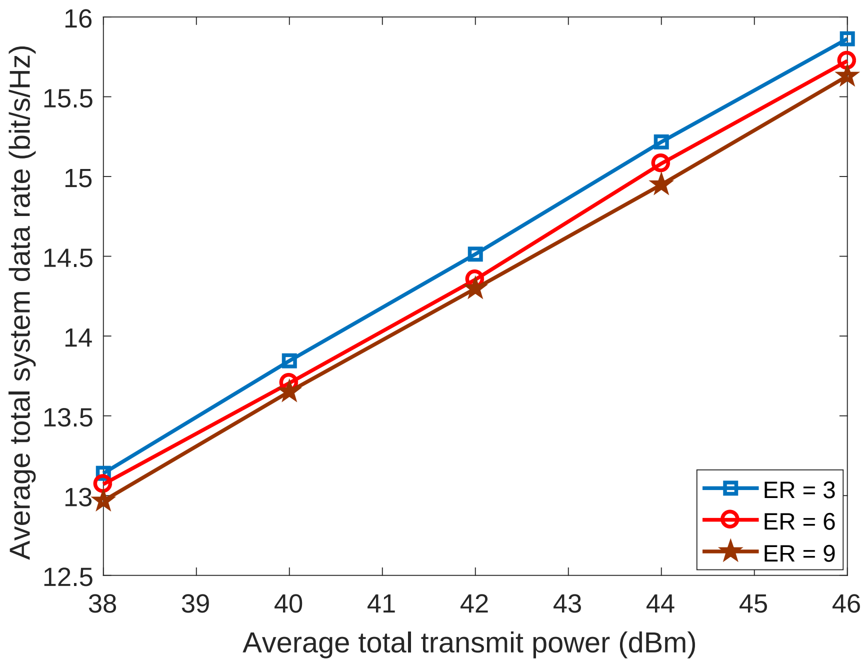

Figure 4.5: Average total system data rate (bit/s/Hz) versus average total system transmit power (dBm).

Chapter 5

Future Development

Upon the completion of my thesis B, here are some possible future developments. The first one is to consider the use of non-linear EH model. Recently, it has been shown on simulation results that non-linear EH model is closer to real-life measured data than linear EH model is, as non-linear EH model can accurately characterize the non-linearity of practical EH circuits [41]. Secondly, an extension can be done on my thesis by considering different types of receivers (e.g. time-switching receivers and power-splitting receivers) as the receiver type focused in the thesis is separated receiver. Furthermore, we can extend the current study of perfect CSI to the case of imperfect CSI [42, 43, 44]. In practice, it is over optimistic in assuming perfect CSI for resource allocation design. However, the consideration of imperfect CSI would complicate the problem on hand. Besides, the current multi-user MIMO system can be extended to the massive MIMO system where further research could focus on improving computational efficiency of precoding design [45].

Chapter 6

Conclusion

WPT has been shown to be a feasible solution to enable sustainability of wireless low-power devices. With extra spatial degrees of freedom and a designed energy signal generation offered by multiuser MIMO, SWIPT is enabled to provide self-sustainable and secure communication. In this thesis, we aimed at designing a resource allocation algorithm to ensure secure SWIPT by considering the linear EH model. We formulated the beamforming design for secure SWIPT systems as a non-convex optimization problem. Realizing the non-convexity of Problem 3, we adopted the MRT-based scheme as a suboptimal solution to the optimization problem. By exploiting SDP-relaxation, we solved the non-convex optimization problem optimally. Then, we illustrated the performance of the proposed suboptimal algorithm via Matlab simulation. Simulation results depicted a non-trivial trade-off between achievable data rate of the IR and total harvested power of the ERs. Besides, advantages offered by equipping more transmit antennas at the transmitter in enlarging achievable region was demonstrated by our simulation results. We also observed that with more ERs present in the system, the achieved information rate by the IR decreases under a fixed amount of transmit power as the constraints on energy harvesting and security become stringent. Additionally, possible future developments on extending our current work is discussed.

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Appendix A

Appendix A–Optimization in Communication

Mathematical optimization is a process to select a best decision variable subject to some constraints or criterion from few sets of available alternatives [46]. An optimization problem normally includes maximizing or minimizing an objective function by opting given sets of allowed inputs systematically and compute the regarding output values of that function [47].

In communication, optimization is often linked to physical meaning, which is a powerful analytic tool to turn real-life complicated communication systems into discrete mathematical models for problem solving [48]. For example, it is useful in representing and using system information, guaranteeing the requirements from the applications of the information such as accuracy and throughput are met, regulating competition among users and handling communication between multiple transmitters and multiple receivers [49]. Optimization can often be seen in this form:

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g(\mathbf{x}) \leq 0, \\ & h(\mathbf{x}) = 0, \end{aligned} \tag{A.1}$$

where $\mathbf{x}$ is the optimization variable and f is the objective function.
After optimization problem formulation based on the real-life scenario of the physical system, we now look at how to solve an optimization problem. It is widely known that linear programming is easy to solve. However, linearity is not the distinguished line between a hard and

an easy optimization problem, but convexity, which sometimes indicates if this optimization problem can be solved uniquely and globally, and how we can solve it in a robust, efficient and distributed way [34].

A.1 Convexity

In the research area of communication system, if the optimization problem is recognized and proved to be a convex optimization problem which is to minimize a convex function over a convex set, the local minimum of the objective function f is the global minimum of f over subjected constraints [50]. In other words, the solution of solving a convex optimization problem is optimal. That's why it is important to know how to recognize and utilize convex functions. However, in real life after problem formulation we often encounter some problems which do not preserve convexity, i.e. non-convex optimization problem, whose locally optimal solutions might not be the globally optimal ones [51]. In these cases, some techniques are required and very useful in finding the globally optimal solutions such as SDP-relaxation [52].

A.2 KKT

Under particular circumstances [53], Karush-Kuhn-Tucker (KKT) conditions are sufficient and necessary for optimality in convex optimization problems. However, KKT conditions become only necessary when it comes to non-convex optimization problems [54]. In general, plenty of optimization algorithms can be understood as methods for numerically solving the KKT equations and inequalities [55]. Let $\mathbf{y}$, $\mathbf{u}$ be the dual and $\mathbf{x}$ be the primal variables for the Lagrangian function. Consider applying general KKT conditions on Lagrangian function A.1:

$$\mathbf{y} \succeq \mathbf{0}, \mathbf{u} \succeq \mathbf{0}, \tag{A.2}$$

$$\mathbf{y}g(\mathbf{x}) = 0, \tag{A.3}$$

$$\nabla f(\mathbf{x}) + \mathbf{y}\nabla g(\mathbf{x}) + \mathbf{u}\nabla h(\mathbf{x}) = 0, \tag{A.4}$$

where equation A.3 relates to dual feasibility, and equation A.4 refers to complementary slackness.

A.3 SDP Relaxation

SDP relaxation is a technique which can be applied to relax the non-convex rank constraint in rank-constrained optimization problem [56]. However, by applying SDP in optimization problems, one can obtain an upper bound of the optimal value of the considered problem as the rank constraint is removed. In general, the obtained solution by SDP relaxation may not satisfy the original rank constraint. Hence, there is a need to study the tightness of the SDP relaxation, which means, in this thesis, we aim at proving that the rank of the primal optimization matrix variable is one, by exploiting the Lagrangian function of the optimization problem and KKT conditions [57]. In some cases, SDP relaxation can be proven to be tight and the obtained solution from the rank-constraint relaxed problem is also the globally optimal solution for the original problem [58].

Appendix B

Appendix B–Proof of Theorem 1

Since in Problem B.1 we intend to maximize the achievable data rate of the IR i.e. $R = \log_2\left(1+\frac{\mathrm{Tr}(\mathbf{WH})}{\sigma_{\mathrm{ant}}^2+\sigma_{\mathrm{s}}^2}\right)$ which is a logarithmic function, it is equivalent to maximizing $\mathrm{Tr}(\mathbf{WH})$ as other variables are constants unrelated to $\mathbf{W}$. Therefore, by applying SDP relaxation, we can rewrite Problem B.1 as following:

Problem B.1. Achievable Rate Maximization [Generalization]

$$\begin{aligned}
\underset{\mathbf{W},\mathbf{W}_{\mathrm{E}}\in\mathbb{H}^{N_{\mathrm{T}}}}{\text{minimize}} \quad & -\mathrm{Tr}(\mathbf{WH}) \qquad \text{(B.1)}\\
\text{s.t.} \quad & \mathrm{C1}:\ P_{\min}-\mathrm{Tr}((\mathbf{W}+\mathbf{W}_{\mathrm{E}})\mathbf{G})\leq\mathbf{0},\\
& \mathrm{C2}:\ \mathrm{Tr}(\mathbf{W}+\mathbf{W}_{\mathrm{E}})-P_{\max}\leq\mathbf{0},\\
& \mathrm{C3}:\ \mathrm{Tr}(\mathbf{WG}_j)-(2^{R_{\mathrm{tol}}^{\mathrm{ER}}}-1)\,\mathrm{Tr}(\mathbf{G}_j\mathbf{W}_{\mathrm{E}}+\sigma_{\mathrm{tol}}^2)\leq\mathbf{0},\forall j\in\{1,\dots,J\},\\
& \mathrm{C4}:\ -\mathbf{W}\preceq\mathbf{0},\\
& \mathrm{C5}:\ -\mathbf{W}_{\mathrm{E}}\preceq\mathbf{0},\\
& \cancel{\mathrm{C6}:\ \mathrm{Rank}(\mathbf{W})=1}.
\end{aligned}$$

It can be verified that B.1 is convex and satisfied the Slater's constraint qualification, the strong duality holds for B.1 with SDP relaxation applied, solving the dual problem is tantamount to solving the primal problem. In this section, we study the tightness of the SDP relaxation. To this end, we prove this by first defining the Lagrangian function:

$$\begin{aligned}
L \;=\;& -\mathrm{Tr}(\mathbf{WH})-\lambda_{\mathrm{C1}}\,\mathrm{Tr}((\mathbf{W}+\mathbf{W}_{\mathrm{E}})\mathbf{G})+\lambda_{\mathrm{C2}}\,\mathrm{Tr}(\mathbf{W}+\mathbf{W}_{\mathrm{E}})\\
&+\lambda_{\mathrm{C3}}\,\mathrm{Tr}(\mathbf{WG}_j)-\mathrm{Tr}(\mathbf{YW})+\Delta, \qquad \text{(B.2)}
\end{aligned}$$

where Δ represents the variables and the constants that are independent of $\mathbf{W}$ and therefore irrelevant in the proof. $\mathbf{Y}$ and λ_{C1}, λ_{C2}, λ_{C3} are dual variables related to the constraints C4 and C1, C2, C3, respectively. Now, we can express the dual problem of B.1 with SDP relaxation applied:

$$\max_{\mathbf{Y},\lambda_{\mathrm{C1}},\lambda_{\mathrm{C2}},\lambda_{\mathrm{C3}}} \ \min_{\mathbf{W},\mathbf{W}_{\mathrm{E}}\in\mathbb{H}^{N_{\mathrm{T}}}} \ L \tag{B.3}$$

Then, we apply KKT conditions:

$$\mathbf{Y} \succeq \mathbf{0}, \lambda_{\mathrm{C1}}, \lambda_{\mathrm{C2}}, \lambda_{\mathrm{C3}} \geq 0, \tag{B.4}$$

$$\mathbf{Y}\mathbf{W} = \mathbf{0}, \tag{B.5}$$

$$\mathbf{Y} = -\mathbf{H} + \mathbf{B}, \tag{B.6}$$

where equation B.6 is derived by taking the derivative of the Lagrangian function with respect to $\mathbf{W}$ and $\mathbf{B} = -\lambda_{\mathrm{C1}}\mathbf{G} + \lambda_{\mathrm{C2}}\mathbf{I} + \lambda_{\mathrm{C3}}\mathbf{G}_{\mathrm{j}}$. Equation B.4 is the complementary slackness property which implies that the columns of matrix $\mathbf{W}$ fall into the null space of $\mathbf{Y}$ for $\mathbf{W} \neq \mathbf{0}$. Hence, if we can prove that $\mathrm{Rank}(\mathbf{Y}) = N_{\mathrm{T}} - 1$, the optimal beamforming matrix $\mathbf{W}$ is a rank-one matrix. To obtain the structure of $\mathbf{Y}$, we prove by contradiction that $\mathbf{B}$ is positive definite but not positive semi-definite with probability one which means that $\mathbf{B}$ has no null space in the following. Since $\mathbf{Y}\mathbf{W} = \mathbf{0}$, $\mathbf{Y}\mathbf{W} = -\mathbf{H}\mathbf{W} + \mathbf{B}\mathbf{W} = \mathbf{0}$. Besides, as by physical meaning the achievable data rate of IR should be greater than 0, i.e., $-\mathbf{H}\mathbf{W} \prec \mathbf{0}$ and by KKT $\mathbf{Y} \succeq \mathbf{0}$, we know that $\mathbf{B}\mathbf{W} \neq \mathbf{0}$. Since we know that $\mathbf{W}$ is positive definite, $\mathbf{B}$ can only be positive definite i.e. $\mathrm{Rank}(\mathbf{B}) = N_{\mathrm{T}}$ but not positive semi-definite which enables $\mathbf{B}$ to have null space to make $\mathbf{B}\mathbf{W} = \mathbf{0}$.

Before we further proceed, we introduce the following rank inequalities

Lemma 1: Let $\mathbf{A}$ and $\mathbf{B}$ be two matrices with same dimension. The inequality of matrix $\mathrm{Rank}(\mathbf{A}+\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A}) - \mathrm{Rank}(\mathbf{B})$ holds.

Proof: By basic rule of inequality for the rank of matrix, $\mathrm{Rank}(\mathbf{A}) + \mathrm{Rank}(\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A} + \mathbf{B})$ with both matrices of same dimension. Thus we have $\mathrm{Rank}(\mathbf{A}+\mathbf{B}) + \mathrm{Rank}(-\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A})$. Since $\mathrm{Rank}(\mathbf{B}) = \mathrm{Rank}(-\mathbf{B})$, we can now prove Lemma 1.

Now, we can exploit equation B.6 and a transformation from a basic rule of inequality for the rank of matrix (see Lemma1):

$$\mathrm{Rank}(\mathbf{Y}) = \mathrm{Rank}(-\mathbf{Y}) = \mathrm{Rank}(-\mathbf{B}+\mathbf{H}) \geq \mathrm{Rank}(-\mathbf{B}) - \mathrm{Rank}(\mathbf{H}) = N_{\mathrm{T}} - 1 \tag{B.7}$$

To satisfy the minimum SINR requirement of IR, $\mathbf{W} \neq \mathbf{0}$ is required. Thus, $\mathrm{Rank}(\mathbf{Y}) = N_{\mathrm{T}} - 1$ and $\mathrm{Rank}(\mathbf{W}) = 1$.