Spectral Efficiency and Energy Efficiency in Massive MIMO Systems

by

Diwei Sun

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Abstract

5G provides users with a higher system capacity, low latency, and low system complexity. The core technology, massive multiple-input and multiple-output (MIMO), improves spectral efficiency and degrees of freedom and lower system complexity. In my project, we will analyze the relationship between signal-to-noise ratio (SNR) and capacity. In the second part, we will discuss energy efficiency. The consumed power of the system includes transmit power, circuit power consumption, and idle power consumption. The circuit power consumption dominates the system power consumption when the transmitter is equipped with massive number of antennas. Hence, to analyze this problem, we propose a power consumption model. To solve this problem, it is important to find the trade-off between system capacity and energy efficiency.

Keyword: Massive MIMO, Spectral efficiency, Energy Efficiency

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Abbreviation

$\boldsymbol{A}$ precoding matrix

y Received signal

x transmitted signal

$\boldsymbol{G}$ Precoding Matrix

$\boldsymbol{g_k}$ the kth column of $\boldsymbol{G}$

p_u the average transmitted power of each user

R Achievable capacity

I Unit Matrix

a_k the Kth column of $\boldsymbol{A}$

δ the impulse function

n the noise vector

N_t the number of activated antennas in Base station

M the number of antennas in Base Station

K The number of users in the cell

β_k the geometric attenuation and shadow fading

Γ Gamma distribution

$P_{k,total}$ the total power consumption in the kth user

P_t transmitted power

P_{cir} circuit power consumption

P_{sta} the idle power consumption

σ_{DC} the loss factors of antenna DC-DC power supply

σ_{Ms} the loss factor of antenna feeder

σ_{cool} the loss factor of active cooling system

σ_{Ms} the loss factor of main power supply

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Chapter 1

Introduction

1.1 Background of 5G

The rapid development of wireless communication technology has resulted in the explosive increases of mobile users. It has reported that there are over 1,069 million mobile users in Americas [2], while over 5 billion mobile phone users are projected to have on a global scale, which is shown in Fig. 1. The prevalence of smart devices leads to the explosive growth of requirements for digital wireless communication. As a result, the issue related to shortage of frequency resources is revealed. Although new technologies such as small cell and high order modulation can improve frequency efficiency to certain extent, it still cannot satisfy the requirements.

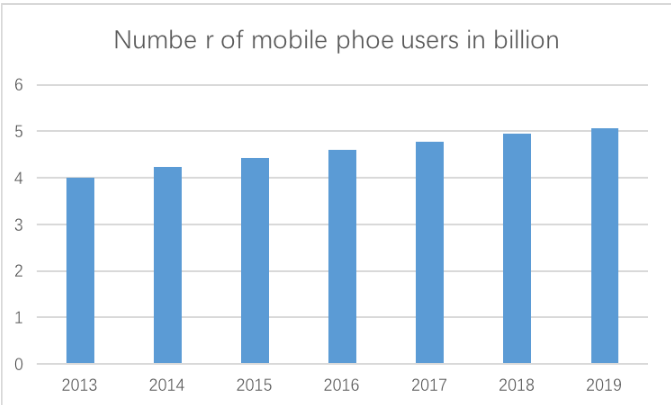

Figure 1 Total number of mobile phone users worldwide from 2013 to 2019 [1]

The fifth generation communication system (5G) is new emerging technology recent years since it enhances data speed, ultra reliable low latency, and massive machine type communication [2]. Additionally, 5G wireless communication is not only for traditional mobile devices like smart phone and laptop, but also provides services for Device to Device (D2D) communications [2].

1.2 New technology in 5G

5G is expected to provide us with high capacity, low complexity, high data rate, and ultra-low latency communication system. The advancements in 5G is shown in Fig1.2 [2].

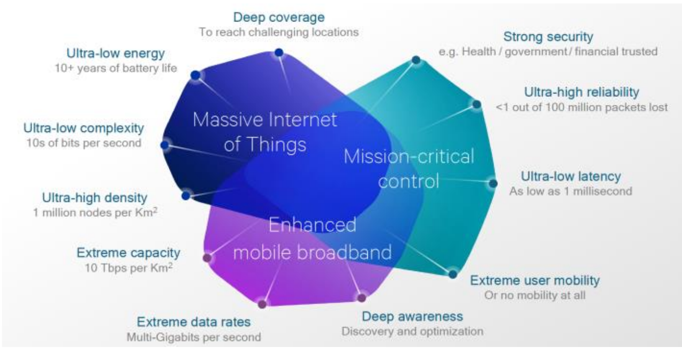

Figure 1.2 Advancement in 5G [2]

To achieve these goals, new technology including non-orthogonal multiple access (NOMA), millimeter wave communication, massive MIMO, visible light communication, will be introduced. In this project, we will mainly focus on massive MIMO systems [1,2,3].

1.3 Introduction of Massive MIMO

1.3.1 Features of Massive MIMO

Compared to long term evolution (LTE, also known as 4G) the most important feature is to allow users to enjoy much higher data speed with peak data rate of 10 Gbps. To achieve this goal, millimeter wave communication is applied. However, path loss and fading is increasingly serious as the frequency increases. Moreover, inter-user interference creates a performance bottleneck in MIMO systems.

To address this issue, massive MIMO (Multiple input and Multiple output) technology is applied. Due to application of millimeter wave communication, antennas can be designed smaller than before, while the distance among two of them will be shortened [18]. As a result, antennas array is possibly integrated in a small area [6]. When the amount of antennas increases infinitely, we can assume transmitter antennas and receiver antennas are pairwise. In this case, massive MIMO systems can be simplified as a series of SISO system, which means low interference among inter-users [7].

The gain obtained from massive number of antennas will enhance quality of service (QoS) of system, supplemented by beamforming and beam tracking. With these advantages, 5G communication systems are able to achieve high speed downlink and uplink, support 3D video and 4K videos and combine AR technology with games [2,3,4].

In massive MIMO systems, the large number of antennas can provide with a large amount of degrees of freedom (DoF) to facilitate efficient wireless communication signals, thus increasing spectral efficiency (SE) and capacity [8]. According to current research, the capacity of a point-to-point (P2P) MIMO systems based on fading channel increases linearly with the minimum number of transmitter and receiver antennas, while that of reliability improves in the order of $SNR^{N_t N_r}$ (N_t, N_r is the number of transmitters and receivers antennas, respectively). MIMO has been applied in 3GPP (third generation partnership project) and LTE (long-term evolution, also known as 4G) [9]. To satisfy users' requirements in the future, it is important for researchers to combine massive MIMO with 5G communication criteria.

Apart from capacity and spatial efficiency, energy efficiency is another essential feature for wireless communication system [5]. Massive MIMO technology not only increases capacity but improve energy efficiency (EE). For multi-user MIMO systems, EE can be optimized by algorithms such as Maximal Ratio Combining (MRC), Minimum mean square error (MMSE), and Zero-forcing (ZF) [10,11]. However, in massive MIMO systems, as the number of transmitted antennas increases over a threshold, the capacity will increase slightly. Hence, it is important for researchers to find a tradeoff between number of antennas versus energy efficiency as well as spatial efficiency.

1.3.2 Comparison between traditional MIMO and Massive MIMO

Compared to traditional MIMO, the advantages of massive MIMO include [2,3,4,5]:

- Enhancement of SE
- Massive amount of degrees of freedom in spatial domain
- Good system performance with only linear (simple) precoding scheme, e.g. Zero forcing, Maximum Ratio Transmission, Minimum Mean Square Error
- Facilitate resource allocation

1.4 Structure

This thesis will study energy efficiency and spatial efficiency in massive MIMO based on three pre-coding algorithms including MRC, MMSE and ZF in mathematical and simulation approches. A simplified massive MIMO system is built in this system which equips with adjustable number of transmitted antenna and 3 users with signal received antenna. In particular, a computational efficient resource allocation will be designed for the maximization of energy efficiency of the massive MIMO system. Besides, the impact of the amount of transmitted antenna on capacity will be assessed under different pre-coding algorithms. Based on this, energy efficiency will be then discussed. Finally, optimal and suboptimal transmission scheme will be presented in order to maximize the usage of materials and resources.

This thesis is organized as follow. In chapter 2, the background and practical model will be introduced. Chapter 3 will develop mathematical model and formulas related to pre-coding methods and energy efficiency. As well as Dinkelbach Method which are applied to solve energy efficiency model will be introduced. Based on theoretical knowledge, optimal and suboptimal programs will be given. Chapter 4 demonstrates specialized numerical results.

Chapter 2

System Model

Massive MIMO improves communication capacity significantly when total transmitted power consumption remain stable. Compared to traditional point-to-point (P2P) system [12], spectral efficiency in massive MIMO systems are enhanced significantly, which can be derived from the improvement of system achievable capacity. Massive amount of degrees of freedom in spatial domain will facilitate power allocation, contributing to energy efficiency. Hence, it is a core technology of 5G in the future. In this chapter, we will focus on the status quo of massive MIMO and issues related to EE.

2.1 System Model

In massive MIMO systems, a large number of antennas are equipped in base station. The system model is shown in Figure 2.1 [13]. The left side presents uplink process, while the right side is downlink process. From this picture, it can be seen that when users try to communicate with base station, their data stream should be pre-coded firstly according to obtained channel state information which is estimated by sending pilot sequence.

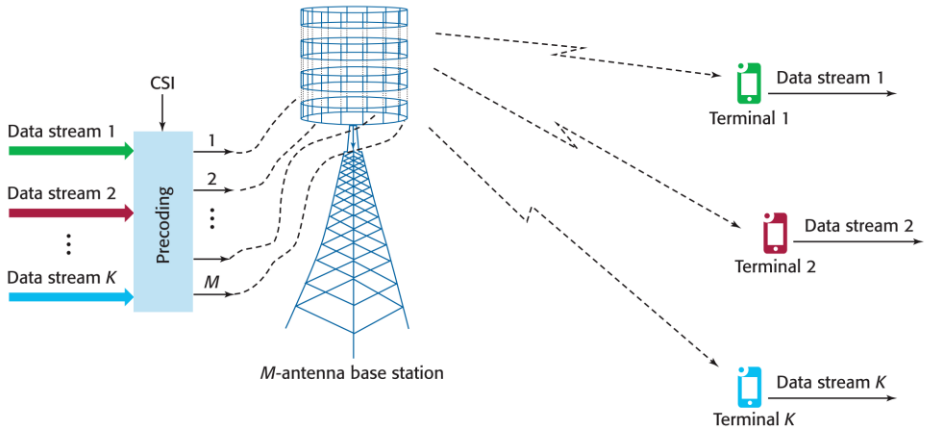

Figure 2.1 System Model of Downlink process experiences the similar process.

Based on system model, we can easily obtain mathematical model for massive MIMO systems,

$$\bar{\boldsymbol{y}} = \boldsymbol{W}^H \boldsymbol{H} x + \boldsymbol{W}^H \bar{n} \tag{2.1}$$

x is transmitted signal, $\bar{y}$ is received signal, $\vec{y} \ \epsilon\, \mathbb{C}^{N_T}$. $\boldsymbol{W} \in N_T * N_T$ is precoding matrix. $\boldsymbol{H}$ is channel matrix, $\mathbf{H}\ \epsilon\, \mathbb{C}^{N_T \times N_R}$. $\bar{n}$ is noise vector [13].

2.2 Precoding

Precoding provides two fundamental advantages, including eliminate interference and performing beamforming to the desired users. In general, there are two types of pre-coding, non-linear precoding schemes and linear pre-coding schemes. Non-linear pre-coding can achieve both of these two function, while the linear one can only reduce inter-users interference [14].

In wireless communication system, due to the geographic effect, received signal cannot be obtained simultaneously [15]. Inter-user interference cannot be eliminated by multi-user detection as well. Under this circumstance, precoding will play a significant role in improving system performance.

Compared to nonlinear precoding schemes, the complexity of linear precoding schemes the complexity is remarkably lower. Moreover, due to a massive amount of DoF in massive MIMO, linear precoding schemes are enough to satisfy communication requirements [13,14].

2.2.1 Linear Precoding Schemes

In massive MIMO systems, when the amount of transmitted antennas approaches infinity, the system can be simplified as a Single-input-to-Single-output (SISO) systems [11]. Therefore, to optimize spectral resources in massive MIMO systems, pre-coding is used at the transmit side in order to reduce the complexity of system, diminish noise effect and optimize stream data transmission based on channel state information (CSI) [12,13,14]. There are three common linear pre-coding schemes, including MRC, ZF and MMSE.

(1) Maximum-ratio Transmission

This scheme is to maximize SNR by seeking to maximize the power at the receiver combiner. MRC is considered as a viable linear reception scheme for massive MIMO systems since it can be applied in a distributed manner. The mathematical model for MRC is shown as below,

$$A = \frac{\boldsymbol{g_k}}{\|\boldsymbol{g_k}\|} \quad (2.2)$$

Moreover, MRC has a satisfactory performance in the low-power regime, even approaching to optimal performance as the amount of antennas grows infinitely. However, as the power increase, systems based on MRC scheme suffer from serious inter-user interference [1].

(2) Zero-forcing

The ZF scheme is to eliminate inter-user interference by projecting the received signals into the orthogonal elements. It can be written as

$$\boldsymbol{A} = \boldsymbol{G}(\boldsymbol{G}^H\boldsymbol{G})^{-1} \quad (2\text{-}3)$$

Since ZF scheme does not take noise into consideration, system based on ZF precoding scheme has a poor performance in low power regime. The performance in high-power regime approaches to optimal [1].

(3) MMSE

MMSE scheme seeks to eliminate inter-user interference as well as noise. Compared to MRC and ZF, system complexity of MMSE is relatively higher.

$$\boldsymbol{A} = \boldsymbol{G}(\boldsymbol{G}^H\boldsymbol{G} + \frac{1}{P_u}I_k)^{-1} \quad (2.4)$$

From mathematical perspective, MRC has the lowest complexity among these three precoding schemes. MMSE requires perfect channel state information [1].

2.2.2 Spectral Efficiency

Spectral Efficiency refers to achievable data rate over a given bandwidth in a specific wireless communication system [16].

Based on Shannon Theorem [14], wireless communication system capacity is given by

$$R = \log_2\left(I_{N_R} + P\, \boldsymbol{A}^H \boldsymbol{G}\right) \tag{2.5}$$

R is achievable data rate, where I_{N_R} is an $N_R * N_R$ identity matrix. P represents transmission power consumption. **H** is channel matrix [16].

By introducing pre-coding schemes in, the average system capacity is written as,

$$R^{\mathcal{U}} = \sum_{k=1}^{k} \mathcal{E}\{\log_2(1 + SINR^{\mathcal{U}}\} \tag{2.6}$$

$\mathcal{U}$ represents pre-coding schemes, i.e. MMSE, ZF, MRC respectively. Signal-to-interference-noise-ratio is given by

$$SINR_k{}^{\mathcal{U}} = \frac{\left|\vec{a}^H \, \overrightarrow{g_k}^{\mathcal{U}}\right|^2}{\sum_{j\neq k}^{k}\left|\overrightarrow{a_k}^H \, \overrightarrow{g_k}^{\mathcal{U}}\right|^2 + \sigma^2} \tag{2.7}$$

σ^2 is noise power variance, while a_k is the kth column of A.

2.3 Energy Efficiency in Massive MIMO

The energy efficiency (EE) is defined as the ratio of capacity versus transmitted power consumption, presenting the number of bits transmitted per Joule [17]. In massive MIMO system, when allocating transmitted power equally to antennas, the spatial efficiency will be improved by M and $\sqrt{M}$ for perfect CSI and imperfect CSI system, respectively (where M represents the number of transmitted antennas).

Moreover, with the increase amount of antennas, noise and small scale fading will decrease correspondingly; correlation among channels will be reduced by expending distance among antennas. According to statistics, channels between antennas and users will approach to orthogonal when the number of antennas is overwhelmingly large than that of users [18]. Traditional MIMO systems pay more attention on transmitted power consumption instead of EE [20].

2.3.1 Power Consumption Model

Since a large number of antennas are involved in, circuit power consumption which is mainly yielded by radio frequency (RF) chain cannot be neglected [11,19]. In communication system, each antenna is equipped with one RF chain. RF chain in downlink consists of mixer, filter, digital-to-analog converter (DAC) as well as synchronizers, which is shown in Fig.2.1

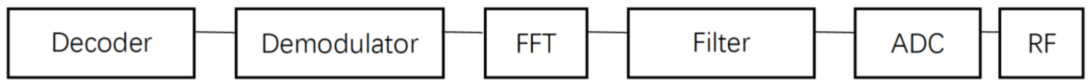

Figure 2.2 Structure of RF chain [12]

Received signal x is firstly fed into band-selective filter, power amplifier, synchronization, low-pass filer and auto gain control module (AGC). Then, an A/D converter is adapted to converts analogue signal to digital signal by quantization.

In this case, more antennas contribute to more circuit power consumption. As a result, massive MIMO comprises energy efficiency for spatial efficiency. Additionally, system loss associated to hardware for RF seriously threats on system performance. Hardware loss includes quantization error, phase error, phase shift of carrier frequency and sampling frequency, nonlinear power amplifier [30].

In conclusion, a large amount of antennas in massive MIMO systems does improve spatial efficiency and capacity but lead to extra power consumption based on analysis. Therefore, it is important to find a balanced trade-off among different aspects of system performance.

2.3.2 Mathematical Model of Power consumption

Based on analysis above, we can get the mathematical model for total power consumption,

$$P_{k,total} = \frac{\frac{P_t}{\eta\left(1-\sigma_{feed}\right)}+P_{cir}+P_{sta}}{(1-\sigma_{DC})(1-\sigma_{Ms})(1-\sigma_{cool})} \tag{2-8}$$

where$P_{k,total}$ is kth use's total power consumption. P_{cir} is circuit power consumption which can be calculated by $P_{cir} = N\left(P_{dac} + P_{mix} + P_{filt}\right) + P_{syn}$, P_{dac} is DAC power consumption. P_{mix} is mixer power consumption. P_{filt} is filter power consumption. N is activated transmitted antennas. P_{sta} is idle power consumption. σ_{DC}, σ_{feed},σ_{cool}, σ_{Ms} are the loss factors of antenna DC-DC power supply, antenna feeder, active cooling system and main power supply respectively [23].

In this project, we ignore loss factors [11].

Therefore, EE can be given in mathematical way as below

$$EE = \frac{\sum Capacity}{\sum_{i=1}^{k} P_{i,total}} \tag{2.9}$$

2.4 Summary

In this chapter, we discuss system model of massive MIMO. By analyzing system capacity and power consumption, we build mathematical model for energy efficiency. According to the mathematical model of the system, we obtain that the energy efficiency is closely associated to the transmitted power and the number of activated antennas. We will discuss further in the next chapters.

Chapter 3

Problem Formulation

In the previous chapter, the basic massive MIMO system model has been discussed. In massive MIMO systems, since a large amount of antennas arrays are implemented, we can achieve a desired capacity with a lower power consumption. In this chapter, we will concentrate on the achievable capacity of massive MIMO systems.

In theory, the base station will apply the maximum likelihood detector in order to achieve optimal capacity. Under this circumstance, however, the complexity in received side will be exponentially raised as increase of the number of users. As we discussed previously, in massive MIMO systems, linear coding schemes can also meet our requirements. Therefore, in this chapter, we will theoretically analyze the system performance of massive MIMO based on ZF, MRC and MMSE.

3.1 Achievable capacity under perfect CSI.

We first consider the wireless communication system under perfect CSI, which is known as $\boldsymbol{G}$. Mathematically, assuming a detector matrix $\boldsymbol{A}$ whose dimensions are $\boldsymbol{N} \times \boldsymbol{K}$, where $\boldsymbol{N}$ and $\boldsymbol{K}$ is the number of antenna in BS and the number of users in a cell, respectively. $\boldsymbol{A}$ is dependent on $\boldsymbol{G}$, which is defined as below [1,5],

$$\boldsymbol{A} = \begin{cases} G & for\ MRC \\ G(G^H G)^{-1} & for\ ZF \\ G(G^H G + \frac{1}{Pu} I_k)^{-1} & for\ MMSE \end{cases} \tag{3.1}$$

For a receiver with linear detector, the received signal can be written as:

$$\boldsymbol{r} = \boldsymbol{A}^H * \boldsymbol{y} \tag{3.2}$$

Based on the equation in Equation (2.1) and (3.2), received vector can be rewritten as:

$$\boldsymbol{r} = \sqrt{p_u}\boldsymbol{A}^H \boldsymbol{G} x + \boldsymbol{A}^H n. \tag{3.3}$$

For specific user, the received signal of user K is given by:

$$r_k = \sqrt{p_u} a_k^H g_k x_k + \sqrt{P_u} \sum_{i=1, i\neq k}^{K} a_k^H g_i x_i + a_k^H n \tag{3.4}$$

where a_k is the kth column of A, while g_k is the kth column of channel realization G. In the received signal model, we regarded system noise and interference term as a random variable with zero-mean and variance $p_u \sum_{i=1,i\neq k}^{K} |a_k^H g_i|^2 + ||a_k||^2$ [25, 26,28]. When we consider this channel as an additive white Gaussian noise, the lower bound of capacity can be approached.

Assuming the channel is ergodic, the achievable capacity in MU-MIMO uplink can be given by [22]

$$R_{P,K} = E\left\{log_2\left(1 + \frac{p_u |a_k^H g_k|^2}{p_u \sum_{i=1,i\neq k}^{K} |a_k^H g_i|^2 + ||a_k||^2}\right)\right\} \tag{3.5}$$

In theory, we can encode information over realizations in baseband to obtain this capacity. In practice, however, we can encode source information in wideband domain, such as Orthogonal Frequency-division Multiplexing (OFDM).

When we consider that received side has perfect CSI, the receiver can reconstruct signal perfectly. Therefore, the influence of noise and interference caused by channel can be cancelled. The SINR of MIMO system is determined by the transmitted power and small scale fading β_k.

3.1.1 Maximum-Ratio Combing

In Equation3.1, with MRC detector, $A = G$ [49]. We can get $a_k = g_k$. Therefore, Equation 3.5 can be rewritten as

$$R_{P,K}^{MRC} = E\left\{log_2\left(1 + \frac{p_u|g_k|^4}{p_u \sum_{i=1,i\neq k}^{K} |g_k^H g_i|^2 + ||a_k||^2}\right)\right\} \tag{3.6}$$

The total transmitted power in transmitted side is E_u. With M antennas in BS, p_u is given by

$$p_u = \frac{E_u}{M}, \tag{3.7}$$

Thus, as the number of antenna goes infinitely, the achievable capacity of receiver with MRC detector can be written as [41],

$$R_{P,K}^{MRC} \rightarrow log_2(1 + \beta_k E_u). \tag{3.8}$$

Based on the above analysis, when the number of antenna goes infinitely, the massive MIMO system can be simplified as a number of SISO system. Under this condition, the intra-cell interference and fast-fading will be significantly alleviated. In other words, when the same transmitted power, the system capacity will be improved by M times. Moreover, the spectral efficiency is also increased by K times as the system can serve K users simultaneously [26, 27, 28].

3.1.2 Zero-Forcing Receiver

According to Equation 3.5, for ZF receiver, the detector matrix A is $G(G^H G)^{-1}$ [38]. Thus,

$$A^H = (G^H G)^{-1} G^H \tag{3-9}$$

Therefore,

$$a_k^H g_i = \delta_{ki} \tag{3.10}$$

where δ_{ki} is the impulse function.

By applying Jensen's inequality, the lower bound of capacity for ZF detector can be given by

$$R_{p,k}^{ZF} \geq log_2\left(1 + \frac{p_u}{E\{[(G^H G)^{-1}]_{kk}\}}\right) \tag{3.11}$$

When we consider the wireless communication system under Rayleigh Channel, the noise and interference can be rewritten as [37],

$$E\{[(G^H G)^{-1}]_{kk}\} = \frac{1}{\beta_k} E\{[(H^H H)^{-1}]_{kk}\} \qquad (3.12)$$

$$= \frac{1}{K\beta_k} E\{tr(H^H H)^{-1}]\}$$

$$= \frac{1}{(M-K)\beta_k}, for\ M \geq K+1$$

The lower bound then can be simplified as

$$\widetilde{R_{P,K}^{zf}} = log_2(1 + p_{\mathrm{u}}(M-K)\beta_k) \qquad (3.13)$$

With the condition $p_u = \frac{E_u}{M}$ [36], the equation (3.13) can be rewritten as

$$\widetilde{R_{P,K}^{zf}} = log_2\left(1 + \frac{E_u}{M}(M-K)\beta_k\right) \rightarrow log_2(1+\beta_k E_u)\ M\ goes\ infinitely. \quad (3.14)$$

According to Equation (3.8) and (3.14), we can see that the lower bound becomes exact when M goes infinitely.

3.1.3 Minimum Mean-Squared Error Receiver

We have obtained the detector matrix A in Equation (3.1), which is given by $A = G(G^H G + \frac{1}{Pu} I_k)^{-1}$ [35]. Thus,

$$A^H = (G^H G + \frac{1}{Pu} I_k)^{-1} G^H = G^H (G G^H + \frac{1}{Pu} I_M)^{-1} \qquad (3.15)$$

$$a_k = (G G^H + \frac{1}{Pu} I_M)^{-1} g_k = \frac{\Lambda_k^{-1} g_k}{g_k^H \Lambda_k^{-1} g_k + 1} \qquad (3.16)$$

Where $\Lambda_k \triangleq \sum_{i=1, i\neq k}^{K} g_i g_i^H + \frac{1}{p_u} I_M$.

Then, the lower bound capacity for MMSE detector can be simplified as,

$$R_{P,k}^{mmse} = E\left\{log_2(\frac{1}{[I_K+p_u G^H G)^{-1}]_{kk}})\right\} \quad (3.17)$$

$$\geq \widetilde{R_{P,K}^{mmse}} = log_2(1+\frac{1}{E\{1/\gamma_k\}})$$

Where $\gamma_k \triangleq \frac{1}{[I_K+p_u G^H G)^{-1}]_{kk}}$. γ_k follows Gamma distribution, with PDF

$$p_{\gamma_k}(\gamma) = \frac{\gamma^{\alpha_k-1}e^{-\gamma/\theta_k}}{\Gamma(\alpha_k)\theta_k^{\alpha_k}} \quad (3.18)$$

Where

$$\alpha_k = \frac{(M-K+1+(K-1)\mu)^2}{M-K+1+(K-1)\kappa}$$

$$\theta_k = \frac{M-K+1+(K-1)\kappa}{M-K+1+(K-1)\mu} p_u \beta_k$$

Considering that the channel is under Rayleigh Fading, the lower bound of system capacity can be given by,

$$\tilde{R}_{P,k}^{mmse} = log_2(1+\frac{\Gamma(\alpha_k)}{\Gamma(\alpha_k-1)}\theta_k) \quad (3.19)$$

Since Γ follows Gamma distribution, we have $\Gamma(x+1) = x\Gamma(x)$. Thus, the Equation (3.19) can be rewritten as

$$\tilde{R}_{P,k}^{mmse} = log_2(1+(\alpha_k-1)\theta_k) \quad (3.20)$$

Table 3.1 Achievable Capacity with Perfect CSI

MRC	$R_{P,K}^{MRC} \rightarrow log_2(1+\beta_k E_u)$
ZF	$\widetilde{R_{P,K}^{zf}} = log_2(1+p_{\mathrm{u}}(M-K)\beta_k)$
MMSE	$\tilde{R}_{P,k}^{mmse} = log_2(1+(\alpha_k-1)\theta_k)$

3.2 Problem Formulation

In this section, we will demonstrate the problem formulations for optimizing energy efficiency in massive MIMO systems.

3.2.1 Optimization for EE

As we have analyzed the achievable data rate of these three linear detectors in the last sub session, we will combine this result and the energy consumption model in massive MIMO given in Chapter 2.3. We can obtain the EE model in massive MIMO systems with downlink communication as:

$$EE = \frac{\sum Capacity}{\sum_{i=1}^{k} P_{i,total}} \tag{3.21}$$

where $P_{k,total} = \frac{\frac{P_t}{\eta(1-\sigma_{feed})}+P_{cir}+P_{sta}}{(1-\sigma_{DC})(1-\sigma_{Ms})(1-\sigma_{cool})}$.

Therefore, in order to optimize system performance in terms of EE, the problem formulation can be written as

$$\max_{\boldsymbol{A},\boldsymbol{N}} \frac{R_{P,k}^{mmse,zf,mrc}(M)}{Kp_u + M*P_c} \tag{3.22a}$$

$$s.t\ C1:\ M \le N_{max}, \tag{3.22b}$$

$$C2:\ R_{P,k}^{mmse,zf,mrc}(M) \ge\ R_{min}, \tag{3.22c}$$

$$\mathrm{C3}:\ \boldsymbol{A} \ge 0. \tag{3.22d}$$

$C1, C2, C3$ are constraints, which are related to total power consumption, required system capacity, and precoding matrix, respectively. $C1$ limits the maximum number of antennas in BS. $C2$ is the equation of achievable data rate, which requires the minimum system capacity. M is the number of antennas in BS, while K is the number of users in one cell. A in $C3$ is the optimal precoding matrix, which is expected to be positive. $\boldsymbol{H}$ is channel state information [32, 33, 35].

3.2.2 Analysis for problem formulation

Although the number of antennas can achieve full degree of freedom, which in other words, intra-cell interference can be effectively alleviated, the path loss and large-scale fading such as shadowing cannot be neglected. So different users in the same cell may have different SNR. Under this circumstance, an effective power control should be implemented.

In theory, to design an effective power control scheme, transmitted power p_u and the number of users K should be taken into account. The power control factor should only be associated to large-scale fading. However, in practice, such a power control system may introduce new trade off. Furthermore, the tradeoff of spectral versus energy efficiency depends on the optimization of the number of active users. If the channel coefficients of different users are largely different, we should also consider whether the coefficient should be flexible for different users.

In principle, both ways can perform as power control though they have different operation meanings. In our analysis, we will ignore the effect of large-scale fading, while mathematically is shown as Equation

For energy consumption, since not only transmitted power consumption but the circuit power consumption increases by the increment of the number of transmitted antennas, the tendency of EE can be regarded as a concave curve. Therefore, although the absolute value of EE is decreasing with the increase of p_u, M is increased to maximize EE.

3.3 Convex Programming-based Dinkelbach Method

As the problem formulation in Equation (3.22) includes a non-convex objective, we cannot solve it with CVX method directly. Under this condition, Dinkelbach Method can be applied [40, 41, 42].

The maximum energy efficient that the system can achieve is given by q. Therefore, the Equation (3.22a) can be rewritten as

$$\max_{N_t} R_{P,k}^{mmse,zf,mrc} - q * \left(p_u * \mathrm{K} + M * P_c\right) = 0 \tag{3.23}$$

The pseudo code is shown in Table (3.2)

Table 3.2 Pseudo-code of Dinkelbach Method for Imperfect CSI

1). Initialize q = 0, and $\delta = 0$ as the stopping criterion
2). Repeat
3). For a given q, solve problem (3.22) to obtain the maximum EE
4). If $R_{P,k}^{mmse,zf,mrc} - q * \left(p_u * \mathrm{K} + M * P_c\right) \le \delta$
5). Convergence = TRUE
6). RETURN $\{N_t^*\}$=$\{N_t\}$ and $q^* = q$
7). ELSE
8). Set $q = \frac{R_{P,k}^{mmse,zf,mrc}(N_t)}{p_u + M*P_c}$, Convergence = FALSE
9). END IF
10). UNTIL Convergence = TRUE

3.4 Summary

In this chapter we have discussed the mathematic model for the whole system in details. Based on these models, we formulated an optimization problem which depicts the trade-off between spectral efficiency and energy efficiency. Since the optimization consists of non-linear variables, we implement Dinkelbach Method to solve it.

Chapter 4

Simulation Results

We have analyzed mathematical model for spectral efficiency and energy efficiency in Chapter. In this chapter, we present numerical results to verify the theoretic analyses and simulation results. We will first demonstrate EE with the Dinkelbach results and then the simulation results will be given.

4.1 EE DM-CVX

Based on pseudo codes in Table 3.1, the simulation results are given in Fig 4.1,

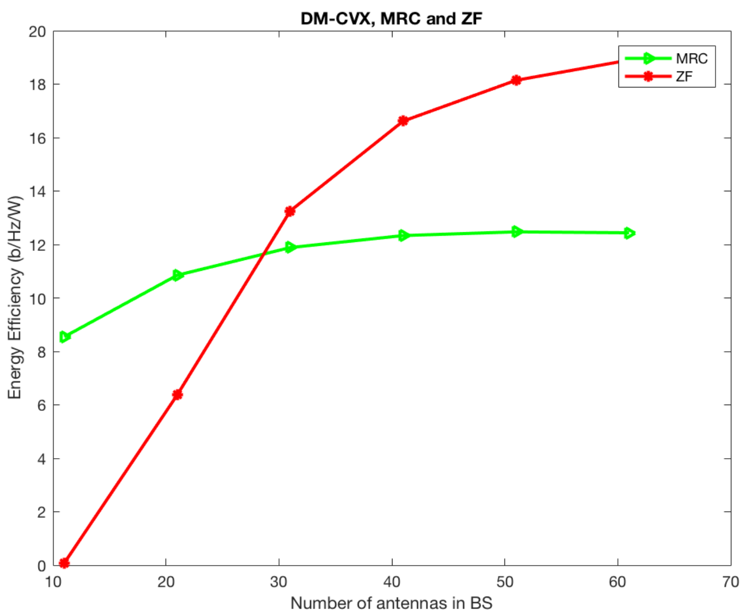

Fig. 4.1 Convergence behavior of the Energy Efficiency

Proposition: Channel is under Rayleigh distribution and Receiver side has imperfect CSI.

In this simulation, we assume that the circuit power consumption is 0.1W for each set, while transmitted power is equal to 1W. Shadowing factor $\sigma = 8\ dB$ and path loss factor is $n = 3.8$. The distance between users and BS are all larger than 100 meter.

From this Picture we can see, for MRC detectors, when the number of antennas in BS is around 60-70, the EE can be maximized and the maximum value is approximately 12.85 b/Hz/W.

For ZF detector, when the number of antennas in BS is 70, the EE is maximized and the maximum number is around 19.11 b/Hz/W.

When the number of antennas is smaller than 30, the EE of MRC is better than that of ZF. However, when the number of antennas larger than 30, the performance of ZF will exceed that of MRC.

Since MRC algorithm ignores the effect of interference, which in other words, MRC only takes noise effect into consideration, the system performance will suffer from critical intra-cell interference when the number of antennas is larger than 30. On the other hand, for ZF algorithm, which only consider noise effects, when the number of antenna approaches infinity, interference among users will dominant the system performance. Therefore, when the number of antennas is larger than 30, the capacity of ZF algorithm is better than that of MRC algorithm.

4.2 Simulation result for EE and Spectral Efficiency

System capacity is determined by transmitted power and activated number of antennas. Thus, we analyze the relationship between transmitted power and capacity as well as number of antennas versus capacity separately.

4.2.1 Simulation for Spectral Efficiency in terms of SNR

To show how power consumption influences on capacity, we firstly simulate system capacity under different SNR values.

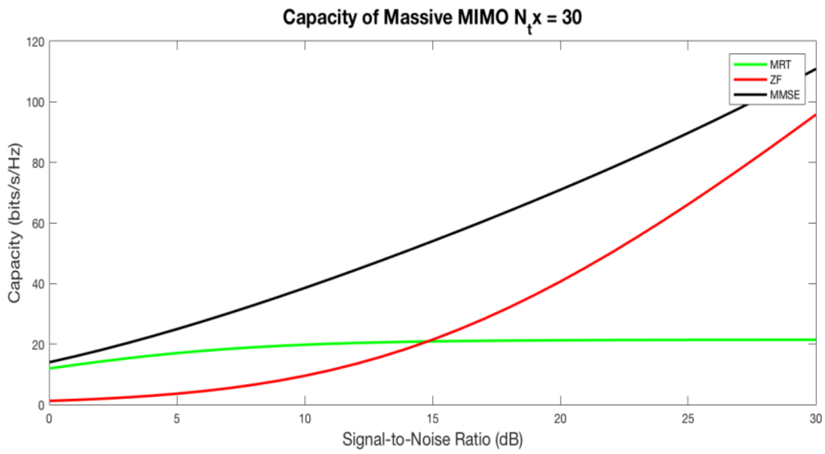

Figure 4.2 SNR – Capacity

In Fig 4.2, we assume channel is Rayleigh fading, and the number of antennas are set to 30. Path loss has been ignored, while circuit power consumption is 0.1 Watt per RF chain. Channel state information is perfectly known in both the transmit side and the receive side.

From this figure, it can be clearly seen that MMSE system has the best performance in terms of capacity among the whole power regime. When SNR is equal to 0dB, the achievable data rate is 21 bits/s/Hz. And as the transmitted power increases, the capacity increases correspondingly.

That of ZF experiences a similar process. However, since it does not take noise into account, it has a poor performance in low power regime. Once SNR is larger than 10dB, the capacity of ZF increases significantly. It even approaches to that of MMSE in high power regime. For system based on MRC precoding scheme, it has a relatively good performance, which is approaching to MMSE in low power regime due to less inter-users interference. However, when SNR is larger than 10 dB, the capacity almost remains stable, which mainly results from high interference among users.

From this figure, we can conclude that MMSE has the best system performance in terms of capacity when channel state information is perfectly known. For the other two precoding schemes, that of ZF has a suboptimal performance in high power regime, while MRC has a good performance in low power regime. As we mentioned before, ZF do not requires perfect CSI and MRC has the lowest complexity. Under this circumstance, it is important to find a tradeoff between spectral efficiency and other system requirements.

4.2.2 Spectral Efficiency versus Number of antennas

Based on simulation results shown in Fig3.1, we can know that the capacity will increase as the transmitted power increases. We will analyze the relationship between capacity and the number of antennas which is shown in Fig 4.3.

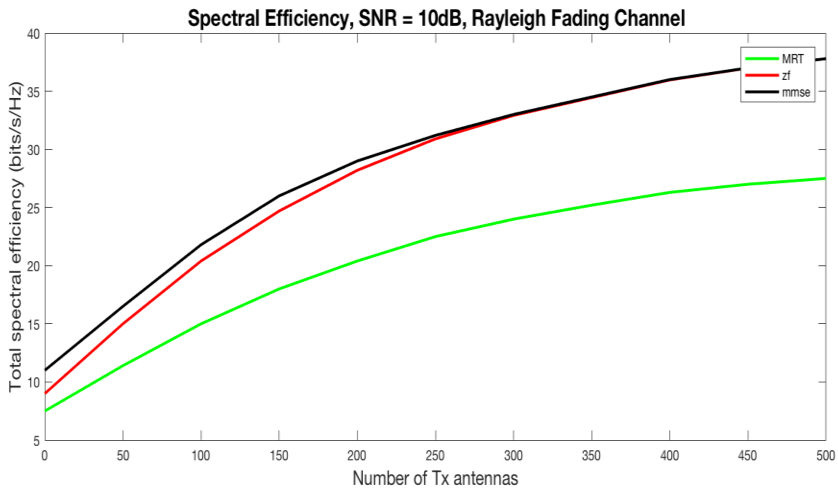

Figure 4.3 Number of Antennas - Spectral Efficiency

The simulation is done with SNR equal to 10dB, while channel is assumed as Rayleigh fading and both transmit side and receive side have perfect channel state information.

The spectral efficiency of all of these three precoding schemes increases as the number of transmitted antennas increases. MMSE has the best performance, while ZF is not far behind. In particular, when the number of transmitted antennas goes to infinite, performance of ZF approaches to that of MMSE. The capacity of MRC is relatively lower than others. Moreover, when the number of antennas is smaller than 250, the slopes of these three curves are relatively large. However, the spectral efficiency increases slightly when the number is larger than 250.

From this figure, we can see that the spectral efficiency cannot increase infinitely with the limitation of power consumption. A large amount of antennas contributes to high power consumption as well as cost for hardware devices.

When the number of antennas goes infinity, the interference among users will dominant the system performance. Under this condition, the more users in cells, the stronger inter-cell inference will produce. The system cannot linear increase as the number of antennas increases.

These two simulation figures illustrate how the capacity is influenced by transmitted power and number of antennas. It can be concluded that the system capacity cannot be enhanced infinitely. Therefore, it is important to find out the optimal number of activated antennas for maximizing the system energy efficiency.

4.3 Energy Efficiency.

Theoretical analysis in Chapter 2 reveals that the energy efficiency is determined by precoding scheme, the number of antennas as well as transmitted power consumption. We mainly focus on how the transmitted power influences on energy efficiency. The relationship is shown in Fig 4.3.

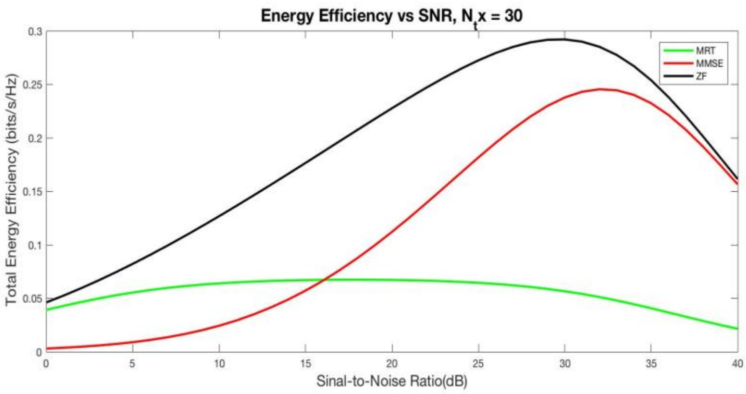

Figure 4.4 Energy Efficiency - Transmitted Power consumption

The number of antennas is set to 30. Communication channel is under Rayleigh fading and channel state information is perfectly known in both transmit and receive side.

From the numerical results, we can see that MMSE still has the best performance among the whole power regime, from 0dB to 40dB. It increases from 0.05 bits/Joule with SNR equal to 0dB to 0.3 bits/Joule with SNR equal to 28dB. And it decreases significantly when SNR exceeds 28dB. When SNR is equal to 40dB, energy efficiency is only 0.16 bits/Joule.

The similar process happens to ZF despite of poor performance in low power regime. When SNR is larger than 10dB, the energy efficiency is enhanced dramatically and reached its highest point when SNR is equal to 32dB. It goes back to 0.13 when SNR increases to 40dB.

On the other hand, MRC has totally different process from the others. The change among the whole power regime is not noticeable. In low power regime, its performance approaches to that of MMSE. And as SNR goes to large, it reaches its highest point, as many as 0.08 bits/Joule, when SNR is equal to 10dB. After that, it remains stable until SNR exceeds 30dB. Then, the EE will decrease as the number of antennas increases.

Although EE of ZF approaches to that of MMSE, it consumes more power. On the contrary, MRC obtains the highest EE with the lowest transmitted power consumption.

As we mentioned in the last session, the spectral efficiency of Massive MIMO systems cannot increase linearly as the number of antennas increases. Therefore, the relationship between EE and transmitted power (which is determined by SNR and the number of antennas) is convex. There is a optimization formulation between the achieved data rate and EE of massive MIMO system.

4.4 Summary

Combining theoretical knowledge, Dinkelbach Method and numerical results, if we do not consider system complexity, MMSE has the best performance in terms of spectral efficiency and energy efficiency (perfect CSI is assumed). Compared to ZF, MRC performs better in low power regime (with SNR ranging from 0dB to 15dB). On the contrary, the performance of ZF in high power regime (with SNR larger 10dB) is better and even approaches to that of MMSE. Moreover, the simulation results demonstrate that massive MIMO systems cannot enhance spectral efficiency and energy efficiency infinitely due to the existence of inference among users. To optimize system performance and maximize economic benefit, it is significant to find out the optimal power allocation.

Chapter 5

Conclusion

Since the appearance of Massive MIMO, the system performance of wireless communication system has been improved significantly in terms of capacity, latency, reliability and e.t.

In this thesis,

(1) we firstly analyze the history of wireless communication system.

(2) Introduce the importance of massive MIMO, system model, mathematic model and problem formulation.

(3) And to make the problem formulation solvable, we apply Dinkelbach Method to address this optimization problem with CVX.

(4) Finally, combining numerical results, we present that massive MIMO remarkably improves the energy efficiency and spectral efficiency.

We mainly focus on energy efficiency in massive MIMO systems. Although we have studied power allocation in uplink transmission, there are still some aspects that should be done in the future.

5.1 Future Works

(1) The energy efficiency model in this thesis only suits to single cell massive MIMO system. In the future, it can be developed to multi-cells Massive MIMO system.

(2) We consider that both transmit and receive sides have perfect CSI. It is expected to develop to imperfect CSI.

(3) Distances between users and BS is not be taken into account in this thesis. This should be considered in the future for more accurate simulation results.

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