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AND TELECOMMUNICATIONS

Simultaneous Wireless  
Information and Power Transfer  
(SWIPT) for IoT Devices

by

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## Abstract

The thesis considers simultaneous wireless information and power transfer (SWIPT) in multi-user multiple-input multiple-output (MIMO) systems, where multiple information receiver (IR) and multiple energy receiver (ER) are served by a transmitter and they obtain their desired resource from the same EM wave separately. Existing literatures have proven that SWIPT technology can become a key to unlock the potential of Internet-of-things (IoT), by providing a sustainable and reliable energy source to wireless devices. This thesis aims to design a resource allocation algorithm to guarantee fairness among all ERs, which maximize the minimum harvested energy ER and also providing security to all IRs. A set of non-convex optimization problem is formulated for the design. Key constants we used in the Quality-of-service (QoS) constraints are transmit power budget, the minimum acceptable achievable rate of IR, the maximum tolerable achievable rate of ER. In thesis A, maximum ratio transmission (MRT) is adopted to display suboptimal resource allocation strategy, which also shows the non-trivial trade-off between average minimum harvested power and minimum required signal-to-interference-and-noise-ratio (SINR). In thesis B, semidefinite-programming (SDP) relaxation is adopted and an optimal solution for the non-convex optimization problem was founded. Additionally, by comparing the result to the suboptimal result, it shows there is a significant performance gain by the proposed optimal scheme.

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# Chapter 1

## Introduction

Thankful to the rapid growing in internet technology, the amount of wireless communication devices have been increasing dramatically and it can be foreseen that the amount of wireless devices would reach up to 50 billion by 2020 around the world [1]. Sensors and wireless communication chips are embedded into the communication devices in order to collect and exchange information and this gives rise to the new era, namely "Internet-Of-Things (IoT)". Since the collected information are all uploaded into the internet, these smart objects which equipped with sensors are capable to provide intelligent daily life services such as sport wearables [3], smart cities [4], e-health [2], IoT in agricultural [5] and poultry [6] and etc.

However, those smart objects are powered by ordinary batteries to facilitate their daily operation. There are several disadvantages in the battery-powered sensors. Firstly, battery has a fixed life span and the lifetime of the smart object is over once the battery runs out of power, therefore this might significantly shorten the lifetime of the smart object. Secondly, as the number of smart objects will still be increasing in the future, if the batteries for each object need to be replaced after it runs out of power, this will be very costly and cumbersome, and hence it will be infeasible to continue the usage of battery-powered sensor. Due to those aforementioned disadvantages, a promising solution to provide ubiquitous and self-sustainable networks is the energy harvesting technology. Traditionally, devices harvested energy from the environment resources (e.g. solar, wind, tidal, biomass, and geothermal[7],[8]), yet this approach is usually climate-dependent and location-dependent (e.g. not all places are windy enough to generate sufficient power for the device) and therefore these green resources are not able to provide a reliable and sustainable energy source due to their uncontrollable and intermittent nature. Alternatively, both academia and industry has discovered the concept of wireless energy transfer (WET) as a solution to solve problems of traditional powered sensors [9]-[14], and hence it might be the key to unlock the potential of IoT.

### 1.1 Background

The concept of WET was proposed by Nikola Tesla back in the late nineteenth and it was implemented by a magnifying transmitter based on the Tesla coil transmitter [14]. Its main idea is to fully exploit the intrinsic energy inside electromagnetic (EM) wave and convert it into usable power and transmit that electrical energy through a wireless link. Generally, WET can be categorized into three classes: inductive coupling, magnetic resonant coupling, and RF-based WET. The first two technologies are near-field WET, which rely on near-field EM waves and there are too many limitations that hinder these technologies to achieve maximum efficiency (i.e. inductive coupling: very low efficiency in long range ;magnetic resonant coupling: low mobility due to an unique circuit alignment set-up), so they do not support the mobility of energy harvesting devices. In contrast,

RF-based WET [9]-[14] is regarded as far-field WET which exploits the far-field properties of EM waves, and this property gives rise to a new line of study, namely simultaneous wireless information and power transfer (SWIPT).

## 1.2 Communication Security

As the information receivers (IRs) and energy receivers (ERs) extract their desired resources from the same EM wave, it is possible that malicious ER can eavesdrop the intended information for IR. Although SWIPT enables the possibility of transmitting information and power concurrently, this opens up a serious security issue due to the broadcast nature of the wireless medium. Nowadays, the conventional cryptographic encryption which is employed at the application layer is used to provide communication safety [15]. These algorithms require perfect secret key management and distribution in order to work properly, yet this is unlikely to happen in the wireless IoT network [16]. Information-theoretic physical layer security provides an alternative other than the cryptographic encryption [17]-[20]. In the computer networking's Open System Interconnect (OSI) model, physical layer is the lowest and also the first layer. The physical layer security aims at providing secure communication via fully exploiting all the physical properties of a wireless communication channel [21].

On the other hand, mobility and portability of transmit antennas is a major concern in the future IoT networks, since this may cause the smart objects to be too bulky if the size of antenna is too large. Yet, severe path loss is a significant problem in transmitting information signal with a high carrier frequency in order to maintain the antenna size [22] (e.g. refer to the Friis equation, the receive power reduces by four times when the separation distance between transmit end and receive end is doubled). In order to overcome the path loss effect, the transmitter had to increase the energy of the information signal, but this also increases the information's susceptibility to eavesdropping because of the associated higher signal power in information leakage. Hence, quality-of-service (QoS) constraints are designed to protect the conflict of interest between IR and ERs. Additionally, various technologies such as energy beamforming and artificial jamming have been proposed to compensate the increase in susceptibility. In particular, artificial jamming is an effective solution which intentionally adds an artificial noise so as to degrade the quality of eavesdropper's channel and reduce the effective information received from those malicious ERs [23],[47].

## 1.3 Notation

All used mathematical notations in this thesis are given in Table 1.1. Vectors and matrices are represented by boldface lower and capital case letters, respectively.  $\text{Rank}(\mathbf{V})$ ,  $\text{Tr}(\mathbf{V})$ , and  $\mathbf{V}^H$  are the rank, the trace, and Hermitian transpose of the matrix  $\mathbf{V}$ , respectively.  $\mathbf{V} \geq 0$  means  $\mathbf{V}$  is a positive semi-definite matrix.  $E\{\cdot\}$  denotes statistical expectation.  $CN(\boldsymbol{\mu}, \boldsymbol{\sigma})$ , which has a mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\sigma}$ , is used to represent the circularly symmetric complex Gaussian (CSCG) distribution.  $\mathbb{C}^{N \times M}$  represents the  $N \times M$  sets with complex entries.

Table 1.1: Mathematical Notations used in this report

Notation	Description
$\mathbf{w}_k$	Information beamforming vector
$\mathbf{v}$	Energy signal that act as artificial noise
$\mathbf{G}_j$	Channel response between the $ER_j$ and the transmitter
$\mathbf{H}_k$	Channel response between the $IR_k$ and the transmitter
$n_{IR_k}$	Noise power in the channel between $IR_k$ and the transmitter
$n_{ER_j}$	Noise power in the channel between $ER_j$ and the transmitter
$P_{max}$	Total transmit power budget
$C_k, C_k^{ER}$	IR and ER achievable rate
$R_{min}$	Minimum required achievable rate of IR
$R_{tol}$	Maximum tolerable achievable rate of ER

## Chapter 2

### System Model

Our system model is a downlink SWIPT system, c.f. Figure 2.1. We have a transmitter equipped with  $N_T$  antennas,  $K$  Information receivers and  $J$  Energy receivers. The IR is a single antenna device and the ER is a multiple-antenna device. In this thesis work, we will assume that the channel state information (CSI) for both the transmitter and receiver is known for resource allocation, which this facilitates a reliable communication with sufficiently high data rates in multiple-antenna systems. Besides, we will also assume the IRs are placed relatively further than ERs in order to avoid the hardware limitations of circuit models in IRs and ERs.

In the considered SWIPT model, IRs can only receive information while ERs can only harvest power, which are served by the same transmitter. Each kind of receiver has its own channel response. (c.f. Refer to Figure 2.1, the channel response of IR and ER are  $\mathbf{H}$  and  $\mathbf{G}$ , respectively) If the channel achievable rate for ER is sufficiently large enough, the ERs also can eavesdrop the intended information to the IRs.

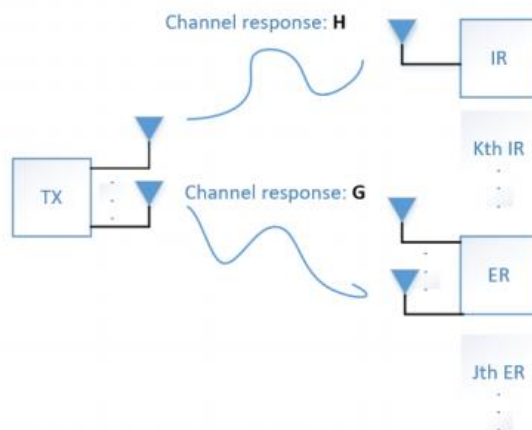


Figure 2.1: A downlink SWIPT model with  $K$  IRs and  $J$  ERs.

The following parts in this thesis is organized as follow: In section 2.1, the fundamental energy equations which are used for this thesis are covered. In section 2.2, energy beamforming and information beamforming are considered. In section 2.3, the SWIPT beamforming with both energy and information transmission is considered.



## 2.1 Channel Model

### 2.1.1 Transmitted Signal

With energy signal adopted, the transmitted signal vector  $\mathbf{y}$  is provided as below

$$\mathbf{y} = \mathbf{w}s + \mathbf{v} \quad (1)$$

where  $\mathbf{w} \in \mathbb{C}^{N_T \times 1}$  is the information beamforming vector which is a user-adjustable parameter in the transmitter side,  $s \in \mathbb{C}$  is the transmitted data symbol. Without loss of generality, we assume that  $\mathbb{E}\{|s|^2\} = 1$ .  $\mathbf{v}$  is a pseudo-random energy signal and modeled as a complex Gaussian random vector with zero mean and covariance matrix  $\mathbf{V}$ , i.e.  $\mathbf{v} \sim CN(0, \mathbf{V})$ .

### 2.1.2 Received Signal

The slow time-varying frequency flat communication channel is considered. As the transmitter transmits energy and beamforming concurrently to IRs and ERs at a single time frame, the received signals at IR and ER,  $k \in \{1, \dots, K\}$  and  $j \in \{1, \dots, J\}$  are given by

$$\mathbf{y}_{IR} = H_k^H(\mathbf{w}_k s + \mathbf{v}) + n_{IR_k}, \forall k \in \{1, \dots, K\} \quad (2)$$

$$\mathbf{y}_{ER} = G_j^H(\mathbf{w}_k s + \mathbf{v}) + n_{ER_j}, \forall j \in \{1, \dots, J\} \quad (3)$$

respectively, where  $H_k^H$  and  $G_j^H$  are channel responses between the transmitter and  $IR_k$ , and transmitter and  $ER_j$ , respectively. Both channel vectors capture the impact of small scale fading, path loss, and large scale fading of the associated channels [24].  $n_{IR_k}$  and  $n_{ER_j}$  are the additional white Gaussian noise (AWGN) of  $IR_k$  and  $ER_j$ , respectively, both have zero mean and variance  $\sigma_{IR}^2$  and  $\sigma_{ER}^2$ , respectively.

## 2.2 Problem Formulation

### 2.2.1 Energy Beamforming

In this section, only the energy transmission and conversion process between the transmitter to the ER module is considered. Figure 2.2 shows the block diagram of an ER. Existing literatures describe different kind of RF-based EH circuits via various hardware architectures [25]-[27], therefore we do not assume a specific hardware design. In this thesis, we assume the harvested energy and transmitted power is linearly proportional to each other, and this can avoid all the effects from any particular hardware implementation that might complicate our problem. The transmitted signal is given as  $\mathbf{x} = \mathbf{v}$ , the

harvested energy at  $ER_j$ ,  $\phi_{ER_j}^{Linear}$ , can be modelled in a linear relationship (c.f. Equation (2)) [28],[29]:

$$\phi_{ER_j}^{Linear} = \eta_j P_{ER_j} \quad (2)$$

$$P_{ER_j} = E \left\{ |\mathbf{G}_j^H \mathbf{x}|^2 \right\} \quad (3)$$

$$= Tr(E\{\mathbf{v}\mathbf{v}^H\} \mathbf{G}_j \mathbf{G}_j^H) \quad (4)$$

$$= Tr(\mathbf{V} \mathbf{G}_j \mathbf{G}_j^H) \quad (5)$$

Refer to (4),  $\eta_j \in [0,1]$  is the radio-frequency(RF)-to-electrical energy conversion efficiency and  $\mathbf{V}$  is the transmit covariance matrix of the energy signal and  $P_{ER_j}$  is the received power from the RF generator.

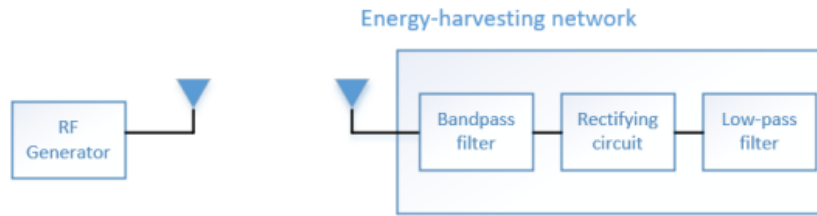


Figure 2.2: Energy transfer flow from the transmitter to the ER

From ER perspective and without consideration on the information secrecy, we can formulate Problem 1:

Problem 1: Minimum Harvested Power Maximization

$$\text{maximize}_{\mathbf{w}, \mathbf{V}} \min_{j \in \{1, J\}} \mu_j \left[ Tr \left( \mathbf{G}_j \mathbf{G}_j^H \left( \sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H + \mathbf{v}\mathbf{v}^H \right) \right) \right] \quad (6)$$

subject to

$$C1: Tr(\mathbf{V}) + \sum_{k=1}^K \|\mathbf{w}_k\|^2 \leq P_{max}$$

$$C2: \mathbf{W} \geq \mathbf{0}$$

where  $P_{max}$  is the maximum transmit power budget constructed by the artificial noise and the transmitter and the term equipped with trace in the objective optimization equation denotes the total received power at the ER side. In Problem 1, we aim to maximize the minimum harvested power among  $J$  ERs while guaranteeing the total transmit power would not exceed the maximum transmit power budget.

## 2.2.2 Information Beamforming

In this section, only the information transmission between the transmitter and the IR is considered. As mentioned earlier, one way to increase the harvested power efficiently is by increasing the transmit power, but the trade-off of this method is the increase in the susceptibility to eavesdropping because of the associated higher signal power in information leakage. Therefore, it is essential to maintain the harvested energy level and also protect the information secrecy for IR users.

One key parameter to measure security is the secrecy rate and this measures the difference between the achievable rate of  $IR_k$  and  $ER_j$ , c.f. Equation (9), where  $C_k$  and  $C_k^{ER}$  are the achievable rate of IRs and ERs, respectively.

$$Secrecy\ rate = [C_k - C_k^{ER}]^+ \quad (7)$$

In general, if the secrecy rate is sufficiently large enough, this provides a certain extent of security to the IRs. To guarantee the security of IR, it is preferential to maximize the secrecy rate by enlarging former term and minimizing latter term.

From the IR perspective, the minimum harvested power maximization design with concern of secrecy can be formulated as Problem 2:

Problem 2: Minimum harvested power maximization [Secure]

$$\underset{\mathbf{W}, \mathbf{V}}{\text{maximize}} \min_{j \in \{1, j\}} \mu_j \left[ \text{Tr} \left( \mathbf{G}_j \mathbf{G}_j^H \left( \sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H + \mathbf{v} \mathbf{v}^H \right) \right) \right] \quad (8)$$

subject to

C1:  $\text{Tr}(\mathbf{V}) + \sum_{k=1}^K \|\mathbf{w}_k\|^2 \leq P_{max}$

C2:  $C_k \geq R_{min}, \forall k$

C3:  $C_k^{ER} \leq R_{tol}$

C4:  $\mathbf{V} \geq \mathbf{0}$

C5:  $\mathbf{W} \geq \mathbf{0}$

In Problem 2, we try to maximize the secrecy rate by ensuring the achievable rate of IR to be larger than a pre-determined value (i.e.  $R_{min}$ ) and limiting the achievable rate of ER to be smaller than a maximum tolerable value (i.e.  $R_{tol}$ ), while guaranteeing transmit power must not exceed the transmit power budget.

## 2.2.3 SWIPT Beamforming

In this section, both information and energy transfer are all considered. For ERs, the minimum harvested energy is to be maximized while for the IR, the secrecy rate is to be maximized so as to prevent malicious ERs to eavesdrop any information that is for IRs. In order to meet the aforementioned requirements for the problem, we can formulate a generalized form for the minimum harvested power maximization as Problem 3:

Problem 3: Minimum harvested power maximization

$$\underset{\mathbf{W}, \mathbf{V}}{\text{maximize}} \min_{j \in \{1, j\}} \mu_j \left[ \text{Tr} \left( \mathbf{G}_j \mathbf{G}_j^H \left( \sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H + \mathbf{v} \mathbf{v}^H \right) \right) \right] \quad (11)$$

subject to

C1:  $\text{Tr}(\mathbf{V}) + \sum_{k=1}^K \|\mathbf{w}_k\|^2 \leq P_{max}$

C2:  $C_k \geq R_{min}, \forall k$

C3:  $C_k^{ER} \leq R_{tol}$

C4:  $\mathbf{V} \succeq \mathbf{0}$

C5:  $\mathbf{W} \succeq \mathbf{0}$

C6 :  $\text{Rank}(\mathbf{W}) \leq 1$

In Problem 3, C1 limits the total power from the transmitter and the artificial noise must not exceed the transmit power budget; C2 ensures the achievable rate of  $IR_k$  is larger than a pre-determined minimum value, while C3 limits the achievable rate of any ERs to be smaller than a maximum tolerable value.

However, we need to derive the problem in a generalized form, as the constraint C2 and C3 are not in the most generalized form. As mentioned earlier, the achievable rate of either IR or ER are both the channel capacity between the transmitter and themselves, and therefore we can apply the Shannon-Hartley theorem and the substituted equations for C2 and C3 are given by, c.f. (12) and (13).

$$\text{C2: } \log_2 \left( 1 + \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k^H \mathbf{w}_j|^2 + \text{Tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{V}_k) + \sigma_k^2} \right) \geq R_{min} \quad (9)$$

$$C3: \log_2 \left( 1 + \frac{|g_j^H w_k|^2}{\sum_{j \neq k} |g_j^H w_k|^2 + \text{Tr}(g_j g_j^H V_k) + \sigma_k^2} \right) \geq R_{tol} \quad (10)$$

Proposition 1.1. For  $R_{min} > 0, \forall j$ , the following implication holds for constraint C2 and the new C2\* is given by:

$$|h_k^H w_k|^2 \geq \mu_{req} \left[ \sum_{j \neq k} |h_k^H w_k|^2 + \text{Tr}(h_k h_k^H V) + \sigma_k^2 \right]$$

Where  $\mu_{req} = 2^{R_{min}} - 1$  is an auxiliary constant

Now, we apply Proposition 1.1 to (12) and (13) and replacing the original constraint C2 with constraint C2\* and also C3 with constraint C3\*. After that, we rewrite the optimization problem as Problem 4:

Problem 4: Minimum harvested power maximization

$$\underset{\mathbf{W}, \mathbf{V}}{\text{maximize}} \min_{j \in \{1, j\}} \mu_j \left[ \text{Tr} \left( G_j G_j^H \left( \sum_{k=1}^K w_k w_k^H + v v^H \right) \right) \right] \quad (11)$$

subject to

$$C1: \text{Tr}(\mathbf{V}) + \sum_{k=1}^K \|w_k\|^2 \leq P_{max}$$

$$C2^*: |h_k^H w_k|^2 \geq \mu_{req} \left[ \sum_{j \neq k} |h_k^H w_k|^2 + \text{Tr}(h_k h_k^H V) + \sigma_k^2 \right], \forall k$$

$$C3^*: |g_j^H w_k|^2 \geq \mu_{req} \left[ \sum_{j \neq k} |g_j^H w_k|^2 + \text{Tr}(g_j g_j^H V) + \sigma_k^2 \right]$$

$$C4: \mathbf{V} \succeq \mathbf{0}$$

$$C5: \mathbf{W} \succeq \mathbf{0}$$

$$C6: \text{Rank}(\mathbf{W}) \leq 1$$

In the last step, we simplify the objective optimization problem by replacing it by  $\tau$  and introducing a new constraint C7 and rewrite the problem as Problem 5, which is the generalized form. Besides, for simplicity,  $\mathbf{W} = \mathbf{w} \mathbf{w}^H$  and  $\mathbf{V} = \mathbf{v} \mathbf{v}^H$  are new optimization variable matrix and adopted in Problem 5. The highlighted constraints are non-convex functions, which requires additional work to solve and will be discussed in the next chapter.

Problem 5: Minimum harvested power maximization [generalized]

$$\underset{\mathbf{W}, \mathbf{V}}{\text{Maximize}} \quad \tau \quad (15)$$

Subject to C1, C4, C5,

$$\text{C2*}: \text{Tr}(\mathbf{W}\mathbf{H}_k) \geq \mu_{req} \left[ \sum_{j \neq k} \text{Tr}(\mathbf{W}\mathbf{H}_k) + \text{Tr}(\mathbf{H}_k \mathbf{V}) + \sigma_k^2 \right], \forall k$$

$$\text{C3*}: \text{Tr}(\mathbf{W}\mathbf{G}_j) \geq \mu_{max}^{tot} \left[ \sum_{j \neq k} \text{Tr}(\mathbf{W}\mathbf{G}_j) + \text{Tr}(\mathbf{G}_j \mathbf{V}) + \sigma_k^2 \right]$$

$$\text{C6}: \text{Rank}(\mathbf{W}) \leq 1$$

$$\text{C7}: \tau \leq \mu_j \left[ \text{Tr} \left( \mathbf{G}_j \mathbf{G}_j^H \left( \sum_{k=1}^K \mathbf{W} + \mathbf{V} \right) \right) \right], \forall j \in \{1, \dots, J\}$$

## Chapter 3

### Resource Allocation Design

#### 3.1 Suboptimal Solution

In the previous chapter, a generalized optimal beamforming problem (i.e. Problem 5) is derived, which takes account in the strict requirement on energy transmission and information security. Yet, it is very difficult to directly solve the problem due to the presence of two non-convex constraints (i.e. C2\* and C3\*), because there are no systematic way to solve them. In a non-convex function, it might have multiple local maximum and minimum points [30]. In general, exhaustive search method may be adopted to find the globally optimal solution by calculating the associated values at a number of equally spaced points [56], however the computational complexity grows exponentially with respect to number of antennas and ERs, and hence this search method is not feasible when considering medium and large size of systems (i.e. MIMO). In the following work, we first consider some suboptimal designs by using Maximum Ratio Transmission (MRT) [15]. After that, we formulate the optimal solution based on the concept of Semidefinite Programming (SDP) Relaxation.

##### MRT-based Suboptimal Solution

Generally, one efficient way to solve the non-convex problem is to start by transforming the non-convex problems into convex and then solve the transformed problem by the use of some convex optimization techniques. The algorithm we will use is the MRT and its suboptimal algorithm is presented in Table 3.1 below, and this algorithm only interests in maximizing the gain in the signal of interest direction while disregarding the potential interferences. After transforming the problem to convex, we use a convex problem solver, namely CVX [31] to solve the convex problem in a systematic approach.

Table 3.1 Suboptimal Resource Allocation Algorithm

Algorithm Suboptimal Optimization
1. Initialize the maximum transmit power budget( $P_{max}$ ), the minimum IR achievable rate( $R_{min}$ ) and maximum tolerable ER achievable rate ( $R_{tol}$ ), and the minimum required harvested power for the target ER( $P_{min}$ )
2. <b>Repeat</b> {Loop}
3. Adopt a fixed beamforming direction pointing at $\frac{\mathbf{g}^*}{\ \mathbf{g}\ }$
4. Optimize the transmit power of the beamforming vector
5. Solve Problem 5 with convex problem matlab solver (e.g. CVX)

6. **If** Problem 5 is feasible **then**
7.      $P_{min} = P_{min} - \varphi$ , ( $\varphi$  is a small positive constant)
8.     **end if**
9. **Until** Problem 5 becomes infeasible

## 3.2 Optimal Solution

In Problem (6), the non-convexity arises due to the constraint C2\*, C3\* and the rank-one matrix constraint C6. The SDP relaxation is an alternative to obtain a tractable and optimal solution. In general, SDP relaxation is a strategy that relaxes non-convex constraints and transform the problem to a convex SDP problem [32], so it can be solved optimally and efficiently by CVX [33].



Problem 6: SDP Transformation

$$\text{Maximize}_{\mathbf{W}, \mathbf{V}} \quad \tau$$

Subject to

$$\text{C1: } \text{Tr}(\mathbf{V}) + \sum_{k=1}^K \|\mathbf{w}_k\|^2 - P_{max} \leq 0$$

$$\text{C2*}: -\text{Tr}(\mathbf{W}\mathbf{H}_k) + \mu_{req} \left[ \sum_{j \neq k} \text{Tr}(\mathbf{W}\mathbf{H}_k) + \text{Tr}(\mathbf{H}_k \mathbf{V}) + \sigma_k^2 \right] \leq 0, \forall k$$

$$\text{C3*}: \text{Tr}(\mathbf{W}\mathbf{G}_j) - \mu_{max}^{tot} \left[ \sum_{j \neq k} \text{Tr}(\mathbf{W}\mathbf{G}_j) + \text{Tr}(\mathbf{G}_j \mathbf{V}) + \sigma_k^2 \right] \leq 0$$

$$\text{C4: } -\mathbf{V} \preceq \mathbf{0}$$

$$\text{C5: } -\mathbf{W} \preceq \mathbf{0}$$

~~$$\text{C6: } \text{Rank}(\mathbf{W}) \leq 1$$~~

$$\text{C7: } \tau - \mu_j \left[ \text{Tr} \left( \mathbf{G}_j \mathbf{G}_j^H \left( \sum_{k=1}^K \mathbf{W} + \mathbf{V} \right) \right) \right] \leq 0, \forall j \in \{1, \dots, J\}$$

Note that from problem (6), the constraint C6 is relaxed due to SDP relaxation. As a result, the whole optimization problem is convex and solvable. However if  $\text{Rank}(\mathbf{W}) > 1$  occurs, the relaxation may not be tight and thus further proof is required to reveal the tightness of the adopted SDP relaxation in (6) by the following theorem:

**Theorem 1.** *Assume the channels  $\mathbf{H}$  and  $\mathbf{G}$  are statistically independent and the transformed problem in (6) is feasible, it is always possible to construct an at-most rank-one beamforming matrix  $\mathbf{W}$ , i.e.  $\text{Rank}(\mathbf{W}) \leq 1$*

*Proof:* Please refer to the Appendix

Since rank of the beamforming matrix is limited to 1, the implemented SDP relaxation is always tight as long as the conditions in Theorem 1 are fulfilled. Hence, there is an optimal solution that satisfies the transformation problem and it is also feasible to maximize the minimum amount of harvested power by  $ER_j$  while providing security to  $IR_k$ .

## Chapter 4

### Simulation

#### 4.1 Simulation Parameters

In this section, all the important simulation parameters being used throughout the simulation process are specified in Table 4.1. Besides, as mentioned earlier, we assume the IR is placed relatively further than the ERs in order to avoid circuit saturation, (i.e. In the simulation environment, the IR is placed 100 meters away from the transmitter and all ERs are 10 meters away from the transmitter).

Table 4.1: Parameters used in simulation

Carrier center frequency	915 MHz
Small-scale fading distribution	Rician fading with Rician factor 3 dB
Transmit power budget, $P_{max}$	46 dBm
Total noise variance, $\sigma_s^2$	-23 dBm
Number of receive antennas at each ER	2
Receive antenna gain	6 dB
Maximum tolerable channel capacity at ERs, $R_{ER_j}$	1 bit/s/Hz
RF energy to electrical energy conversion efficiency for $ER_j, \mu_j$	0.5

#### 4.2 Simulation Results

Figure 4.2.1 below shows the non-trivial trade-off between average minimum harvested energy and minimum required signal-to-interference-and-noise-ratio (SINR) for the SWIPT model under **suboptimal beamforming scheme**. Generally, all the points lie inside or on the curve can be achieved by changing the relevant system parameters. By

comparing different results for different transmit antenna numbers, it can be concluded that increase in number of transmit antennas results in a substantial increase in harvested energy when the minimum required SINR is the same. This is because more transmit antennas improve the accuracy in beamforming and thus the associated performance (i.e. a sharper and narrower main lobe which focuses most of its energy in the signal of interest direction), and therefore this provides an extra spatial degrees of freedom. Besides, the minimum harvested power decreases with increasing minimum required SINR, and this can be explained by the resource allocation between the ERs and IRs, respectively. Since the overall resource is scarce and limited, the enlargement in minimum required SINR introduces a more stringent requirement (i.e. C2 becomes more stringent), this forces the transmitter to allocate more resources to IR and hence less resource is allocated to ERs.

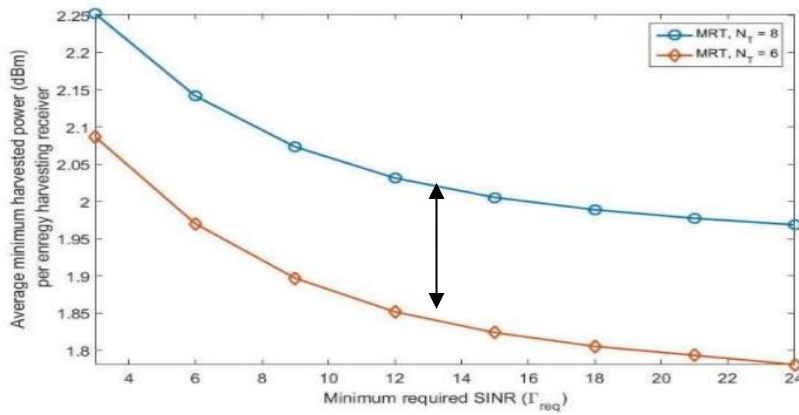


Figure 4.2.1: Average minimum harvested power (dBm) versus minimum required SINR  $\Gamma_{req}$  (dB)

Figure 4.2.2 below shows the non-trivial trade-off between average minimum harvested energy and minimum required SINR for the SWIPT model under the **optimal beamforming scheme**. By comparing the results under the optimal and suboptimal beamforming scheme, it can be seen from the graph that the achievable region enclosed by the optimal beamforming scheme is larger than the suboptimal beamforming scheme. Additionally, the maximum minimum harvested energy under the optimal scheme is larger than the suboptimal scheme (c.f. Figure 4.2.1 and Figure 4.2.2, when 8 transmit antennas are used the maximum minimum harvested energy of suboptimal and optimal is 2.25 dBm and 22.5 dBm, respectively).

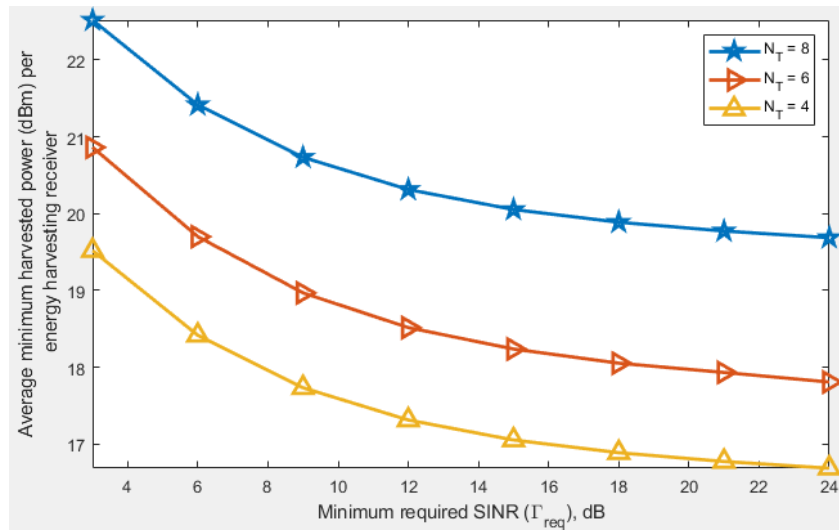


Figure 4.2.2: Average minimum harvested power (dBm) versus minimum required SINR  $\Gamma_{req}$  (dB)

Figure 4.2.3 below shows the relationship between average minimum harvested energy and transmit power budget for the SWIPT model under the optimal beamforming scheme. It can also be seen that increase in number of transmit antennas results in a performance gain in the minimum harvested power. Moreover, the minimum harvested power increases with increasing transmit power budget (i.e.  $P_{max}$ ). This is because of the relaxation in the QoS constraint (i.e. C1), and there is more radiation power available in the system for ERs to utilize [21], which also means there are additional resources for them to harvest.

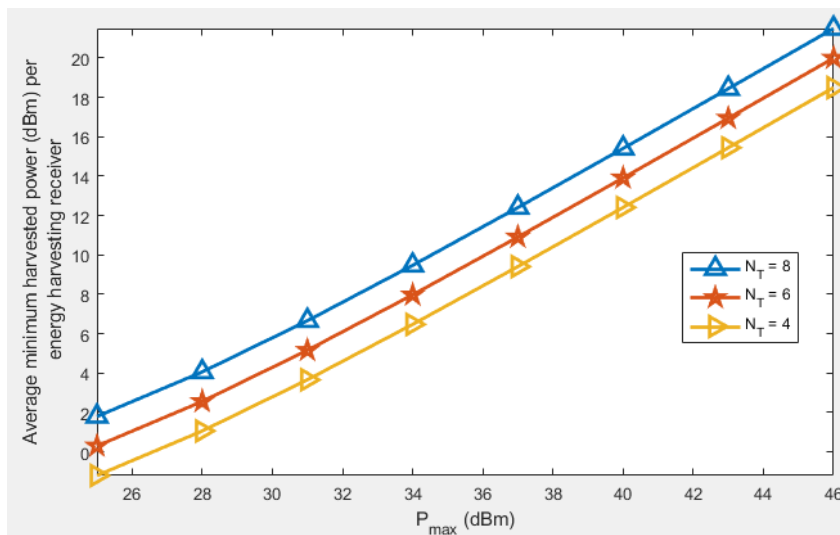


Figure 4.2.3: Average minimum harvested power (dBm) versus transmit power budget (dBm)

## Chapter 5

### Future Development

As I completed my thesis B work, there are still some future developments that should be done in order to optimize my thesis topic. The first thing is to extend the current study of perfect CSI to imperfect CSI [34]-[37], as it is very difficult to have perfect CSI in real-life, and hence the validity of the results is higher, but this might complicate the existing problem due to more factors are taken into consideration (e.g. channel effects to the signal need to be considered). The second thing to do is to use the non-linear energy harvesting (EH) model inside the ER module [38]-[40], as it is optimistic in assuming the harvested energy and the RF received energy is linearly proportional to one another, so this can more accurately characterize the non-linearity of practical EH circuits. The last thing to do is to extend the scope of the work from multi-user MIMO to the massive MIMO system, which there are a massive number of IRs and ERs to serve

by the transmitter, and also future research should focus on the advancement in computational efficiency of precoding design [41].

## Chapter 6

### Conclusion

WET has been foreseen that they have the potential to become the key and unlock potential of IoT. With the advancement in wireless technologies (e.g. massive MIMO), these technologies provide an extra spatial degrees of freedom to optimize the beamforming performance, and hence they can be adopted to provide a better self-sustainable and secure communication in the SWIPT model. In this thesis, our aim is to design a resource allocation algorithm to ensure ER user's fairness and IR user's security. We successfully formulated the beamforming design for the SWIPT systems as a generalized non-convex optimization problem (c.f. Problem 5). At first, we adopt the MRT to determine a suboptimal solution for the problem. By exploiting SDP relaxation, we were able to solve the non-convex optimization problem optimally. After solving the problem, we display the optimal solution of the proposed optimal algorithm via Matlab simulation. Results reveal the non-trivial trade-off between the minimum harvested energy and the minimum required SINR due to the stringent requirements on different QoS constraints. Besides, we observe that the maximum minimum harvested energy was enlarged by equipping more transmit antenna and enlargement of transmit power budget. Nonetheless, possible future developments on extending the scope of our current work is discussed in the previous chapter.

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## Appendix A

### Appendix A-Optimization in Communication

The use of optimization methods is ubiquitous in communications and signal processing [42]. Optimization is a filtration process that selects the best decision from few sets of available alternatives that are subject to some kind of criterions. In general, an optimization problem can be to minimize an objective function that several constraints are given and the outcome solution must be selected on the basis of strictly satisfying the given constraints [43],[50].

From the communication perspective, optimization is a powerful analytic tool to convert real-life complicated communication systems into discrete mathematical models for problem solving, for example it is useful to use mathematical variables to represent throughput and system's secrecy rate [45],[46], and this allows us to visualize those intangible quantities and guarantee certain level of performance or security in the design of system model. An optimization problem can usually represent in the form of:

$$\begin{aligned} & \text{minimize } f(\mathbf{x}) && \text{(A.1)} \\ & \text{subject to } g(\mathbf{x}) \leq 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$

where  $\mathbf{x}$  is the desired variable to optimize and  $f$  is the objective function

After formulating a set or optimization problem to represent a real-life problem, the next thing is to know how to solve an optimization problem. The major issue to encounter when solving this kind of problem is the problem's convexity. The convexity of problem will determine the difficulty on solving this problem, and also it can tell us on how to solve this problem in a robust, efficient and distributed way [30].

#### A.1 Convexity

In general, convexity can be categorized into two main types which are convex and non-convex. There is a systematic way to solve convex problems optimally due to the fact that the local minimum is also the global minimum in a convex problem [44], and hence it is a easy task to optimise a convex problem. On the other hand, as a non-convex problem may have multiple local optimum points, the locally optimal solutions may not be the globally optimal ones [48], and therefore it is difficult to determine the optimal solution. Moreover, we may encounter some non-convex problems in real-life, so one powerful technique to handle non-convex problem is the SDP relaxation [49].

## A.2 KKT

In general, many optimization algorithms can be understood as methods for numerically solving the Karush-Kuhn-Tucker (KKT) conditions [51]. Let  $\mathbf{x}$ ,  $\mathbf{y}$  be the dual and  $\mathbf{z}$  be the primary variables for the Lagrangian function. Equations below are considered as the KKT conditions on Lagrangian function A.1.

$$\mathbf{x} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0} \quad (\text{A.6})$$

$$\mathbf{y}g(\mathbf{z}) = 0 \quad (\text{A.7})$$

$$\nabla f(\mathbf{z}) + \mathbf{x}\nabla g(\mathbf{z}) + \mathbf{u}\nabla h(\mathbf{z}) = 0 \quad (\text{A.8})$$

where (A.7) is relevant to dual feasibility, and (A.8) refers to complementary slackness.

## A.3 SDP Relaxation

SDP relaxation is a technique that relaxes non-convex constraint in rank-constrained optimization problem [52],[53]. By applying SDP in optimization problems, we obtain an upper bound of the optimal value in the considered problem, since the rank constraint is removed. Generally, the obtained solution from this may not satisfy the original rank constraint, and hence we need to examine the tightness of the SDP relaxation so as to assure the solution's feasibility. This means that we need to prove the rank of the primal optimization matrix variable is one, and we can do it by utilizing the Lagrangian function and KKT conditions [54]. In particular, it is possible to have cases that the SDP relaxation is proven tight and also the solution is the globally optimal solution for the original problem [55].

## Appendix B

### Appendix B - Proof of Theorem 1

Strong duality of the transformed optimization problem can be shown by showing the problem is convex and satisfies the Slater's constraint qualification. By proving the strong duality, the solution of its dual problem is also applicable in its primary problem. In this sub-section, our aim is to prove the Problem 5 has a rank-one beamforming matrix. To achieve this, we need to define the Lagrangian function of the problem (c.f. Equation (B.1)):

$$\begin{aligned}
L = & \tau + \lambda_1 \left[ \sum_{k=1}^K \|w_k\|^2 \right] \\
& + \sum_{k=1}^K \lambda_k \left[ -Tr(\mathbf{W}_k \mathbf{H}_k) + \mu_{\text{req}} \left( \sum_{j \neq k} Tr(\mathbf{W}_k \mathbf{H}_j) \right) \right] \\
& + \lambda_3 \left[ Tr(\mathbf{W}_k \mathbf{G}_j) - \mu_{\text{max}}^{tol} \left[ \sum_{j \neq k} Tr(\mathbf{W}_k \mathbf{G}_j) \right] \right] \\
& + \lambda_4 \left[ -\mu_j * Tr \left( \mathbf{G}_j \left( \sum_{k=1}^K w_k \right) \right) \right] - \lambda_6 \left[ \sum_{k=1}^K Tr(\mathbf{W}_k \mathbf{Y}_k) \right] + \Delta
\end{aligned} \tag{B.1}$$

where  $\Delta$  denotes the variables and the constants that are not dependent of  $\mathbf{W}$  and hence they are irrelevant in the proof.  $\mathbf{Y}$  and  $\lambda_1, \lambda_k, \forall k \in \{1, \dots, K\}, \lambda_3, \lambda_4, \lambda_6$  are Lagrangian multiplier associated with constraints C7 and C1, C2, C3, C4, respectively.

In the next step, we explore the structure details of  $\mathbf{W}$  by studying the Karush-Kuhn-Tucker (KKT) conditions. The required conditions that we needed for the proof are given by:

$\mathbf{Y} \geq 0, \lambda_1, \lambda_k, \lambda_3, \lambda_4, \lambda_6 \geq 0,$	(B.2)
$\mathbf{Y}_k \mathbf{W}_k = 0,$	(B.3)
$\mathbf{Y} = -\mathbf{H} + \mathbf{A}$	(B.4)
$\mathbf{A} = \lambda_1 [\mathbf{I}] + \mu_{\text{req}} \left( \sum_{j \neq k} \mathbf{H}_j \lambda_j \right) + \lambda_3 \left[ \mathbf{G}_j - \mu_{\text{max}}^{tol} \left[ \sum_{j \neq k} \mathbf{G}_j \right] \right] + \lambda_4 [-\mu_j \mathbf{G}_j]$	(B.5)
$\mathbf{H} = -\lambda_k \mathbf{H}_k$	(B.6)

where (B.4) is obtained by taking the first derivative of (B.1) with respect to  $\mathbf{W}$  and the details of  $\mathbf{A}$  is also shown in (B.5). The conditions proposed in (B.2) are the complementary slackness property which implies the columns of matrix  $\mathbf{W}$  fall into the null-space spanned by  $\mathbf{Y}$  for  $\mathbf{W} \neq 0$ . Therefore, we can prove rank of the optimal beamforming matrix  $\mathbf{W}$  is either one or zero, if  $\text{Rank}(\mathbf{Y}) \geq N_T - 1$  can be proved. Hence, we can study the structure of  $\mathbf{Y}$  by examing (B.4).

As indicated in (B.4),  $\mathbf{Y}$  is formed by the addition of  $\mathbf{H}$  and  $\mathbf{A}$ , and then this implies we need to understand each matrices separately in order to gain an understanding of  $\mathbf{Y}$ . Firstly, we need to prove by contradiction that  $\mathbf{A}$  is a positive definite matrix with probability one. If this is false, then  $\mathbf{A}$  is a positive semi-definite matrix. In general, it has

at least one eigenvalue and the associated eigenvector is named as  $\mathbf{v}$ . Without loss of generality, a matrix  $\mathbf{V} = \mathbf{v}\mathbf{v}^H$  is formed from its eigenvectors. After that, if we multiply both sides of (B.4) with  $\mathbf{V}$  and apply the trace operator, we can get

$$\text{Tr}(\mathbf{YV}) = -\text{Tr}(\mathbf{HV}) + \text{Tr}(\mathbf{AV}) = -\text{Tr}(\mathbf{HV}) \quad (\text{B.7})$$

An assumption in Theorem 1 is made that H and G are statistically independent, so we confirm  $\text{Tr}(\mathbf{HV}) > 0$ , yet this creates a contradiction because  $\text{Tr}(\mathbf{YV}) > 0$ . Since the relationship in (B.7) is not satisfied, matrix A is a positive definite matrix and its rank is a full matrix, i.e.  $\text{Rank}(\mathbf{A}) = N_T$ .

Besides, it can be proved by the KKT conditions that constraint C2 in Problem 5 is satisfied with equality for the optimal solution and hence  $\lambda_k > 0$ . By examining (B.4), (B.5), (B.6) and some basic inequality for the rank of matrices, we obtain

$$\begin{aligned} \text{Rank}(\mathbf{Y}) + \text{Rank}(\mathbf{W}) &\geq \text{Rank}(\mathbf{Y} + \lambda_k \mathbf{H}_k) && (\text{B.8}) \\ &= \text{Rank}(\mathbf{A}) = N_T \\ &\Rightarrow \text{Rank}(\mathbf{Y}) \geq N_T - 1 \end{aligned}$$

As a result,  $\text{Rank}(\mathbf{Y})$  can only be either  $N_T$  or  $N_T - 1$ , this implies that  $\text{Rank}(\mathbf{W})$  also can only be 0 or 1 respectively. However it is impossible for it to become 0 as this does not satisfy the minimum SINR requirement in C2. Therefore,  $\text{Rank}(\mathbf{W}) = 1$  is proved and the rank-one solution can be constructed.