# Optimal Access Class Barring for Stationary Machine Type Communication Devices with Timing Advance Information

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***Abstract*—The current wireless cellular networks can be used to provide machine-to-machine (M2M) communication services. However, the Long Term Evolution (LTE) networks, which are designed for human users, may not be able to handle a large number of bursty random access requests from machine-type communication (MTC) devices. In this paper, we propose a scheme that uses both access class barring (ACB) and timing advance information to prevent random access overload in M2M systems. We formulate an optimization problem to determine the optimal ACB parameter, which maximizes the expected number of MTC devices successfully served in each random access slot. Hence, the number of random access slots required to serve all MTC devices can be minimized. To reduce the computational complexity and improve the practicability of the proposed scheme, we propose a closed-form approximate solution to the optimization problem and present an algorithm to estimate the number of active MTC devices requiring access in each random access slot. The correctness of the analytical model and the accuracy of the estimation algorithm are validated via simulations. Results show that both numerical and approximate solutions provide the same performance. Our proposed scheme can reduce nearly half of the random access slots required to serve all MTC devices compared to the existing schemes, which use timing advance information only, ACB only, or cooperative ACB.**

***Index Terms*—Machine type communications, LTE random access, random access overload.**

## I. Introduction

A MACHINE-to-machine (M2M) communication system consists of a large number of machine-type communication (MTC) devices, which can communicate with the remote server or other MTC devices in a peer-to-peer manner. M2M is leading us to the Internet of Things. Its applications include smart metering, remote security sensing, health care monitoring, and fleet tracking. It is expected that more than 3.2 billion MTC devices will be deployed by 2019 [1].

Since potential M2M applications usually require seamless coverage over a large area, one approach to provide M2M services is via the existing wireless cellular networks. Meanwhile, the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) networks allow MTC devices to connect to remote servers or devices in other network domains [2]. However, the LTE networks, which are designed for human-to-human (H2H) communications, may not be optimal for M2M traffic. M2M communications differ from H2H communications in several aspects [3]. For M2M traffic, the data payload can be only several bytes, which is much smaller than the payload in H2H traffic. Bursty random access requests from many MTC devices may be sent to the same base station or evolved node B (eNB) simultaneously. Since the number of MTC devices can be much larger than the number of human users, contention among MTC devices for random access, which seldom happens in H2H communications, can occur in the M2M context. This type of contention is called *random access overload* [4].

To understand how random access overload may degrade the performance of LTE networks, we now summarize the procedures of random access for a user equipment (UE) or an MTC device. A UE first synchronizes its downlink by listening to the synchronization signals sent by eNB. Then, the master information block is received, which guides the UE to receive the system information blocks (SIBs). The SIBs help UEs to locate the reference signal, obtain valid random access preambles, and locate random access slots [5]. Random access preambles in LTE networks use Zadoff-Chu (ZC) sequences [6]. The aforementioned SIBs specify the 64 ZC sequences used for random access in the cell. ZC sequences are used as random access preambles since an eNB can distinguish each ZC sequence from the overlapping received signal and can determine its propagation delay. This is because the discrete autocorrelation of a ZC sequence creates an impulse, while two different ZC sequences are considered to be orthogonal [7].

Consider three UEs $n_1$, $n_2$, and $n_3$ in Fig. 1 (a) as an example. Without loss of generality, two propagation paths are assumed for each UE due to multipath effect. UEs $n_1$, $n_2$, and $n_3$ transmit random access preambles $A_1$, $A_2$, and $A_3$, respectively. The cyclic prefix (CP) is included for transmission of each random access preamble [8]. Due to different propagation distance from the UEs to the eNB and the multipath effect, more than one copy of each preamble with different fractions of CP are captured by the eNB in its observation interval in Fig. 1 (b). The captured signal at eNB is the summation of all signals received as shown in Fig. 1 (c). The eNB

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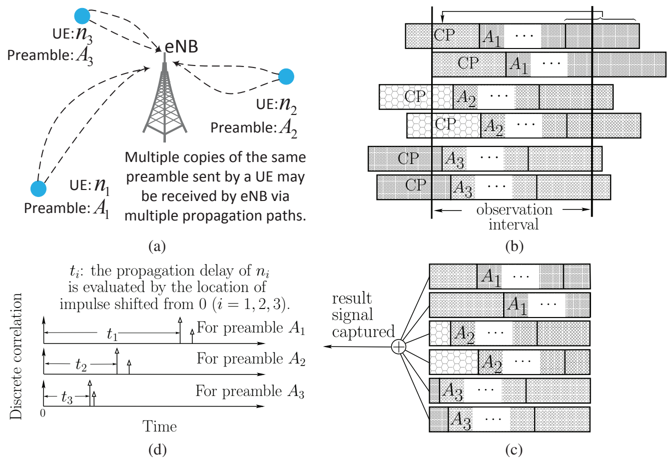


Fig. 1. Preamble detection and propagation delay evaluation in random access of LTE networks. UEs $n_1$, $n_2$, and $n_3$ are aware of the 64 random access preambles used in the cell. They select preambles $A_1$, $A_2$, and $A_3$ and transmit them with cyclic prefix (CP) to the eNB. By calculating the correlation between the overlapping cyclic shifted preamble sequences and each of the 64 preambles, the preambles $A_1$, $A_2$, and $A_3$ and their propagation delay can be determined.

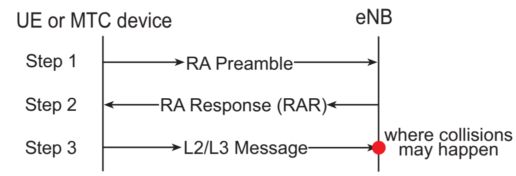


Fig. 2. The first three steps of random access in LTE networks. Multiple UEs or MTC devices may receive the same RAR if they send the same preamble in the same random access slot, so their L2/L3 messages may be transmitted on the same wireless channel and packet collisions may occur at the eNB.

determines whether the captured signal contains a specific preamble by calculating its discrete correlation with each of the 64 preambles. The signal contains a specific preamble as long as its discrete correlation contains an impulse in time domain. More than one impulse with different energy for a preamble may exist since copies of the same preamble with different fractions of CP are contained in the captured signal. The impulse with the largest amplitude determines the propagation delay as shown in Fig. 1 (d).

Fig. 2 presents the first three steps of random access in LTE networks. After receiving the random access (RA) preamble transmitted by a UE in Step 1, the preamble index and its associated propagation delay are determined. Then, the eNB sends a *random access response* (RAR) to acknowledge the UE. An RAR contains the following fields: a) a number to identify a random access slot, b) the index of the preamble received, c) the *timing advance command* [8] [9], and d) the resource allocation information. With the aforementioned fields in an RAR, a) and b) are used together to address an RAR to the UEs. The timing advance command in c) takes an index value called *timing advance* to convey the propagation delay by a multiple of 16 $T_s$, where $T_s$ denotes the *basic time unit* and is equal to $1/\left(3.072 \times 10^7\right)$ second [6]. In other words, the propagation delay determined in Fig. 1 (d) is quantized to an index value with the granularity of 16 $T_s$. Timing advance command synchronizes the uplink by informing the UE the amount of time that its data should be transmitted in advance so that the data will arrive at eNB at the anticipated time. The resource allocation information in d) is used to schedule the transmission of L2/L3 message in Step 3 for the receiver of the RAR. Some UEs may send the same preamble via the same random access slot. Thus, these UEs will receive the same RAR and send their L2/L3 messages over the same wireless channel. This may cause packet collisions at the eNB as shown in Fig. 2. Compared to H2H, the probability of this kind of packet collisions increases in the M2M random access overload scenario since the number of MTC devices requiring access to an eNB is much larger than the number of UEs. Thus, random access overload of MTC devices may degrade the performance of LTE networks.

Various works have been proposed to improve the performance of LTE networks serving MTC devices [10]. Lee *et al.* in [11] study the throughput issue and propose to split the random access preambles into two sets to serve conventional data applications of UEs and short data applications of the MTC devices separately. With the group paging approach, Wei *et al.* in [12] propose a model to estimate the number of successful and collided MTC devices in each random access slot. Liu *et al.* propose a hybrid medium access control protocol for MTC devices in [13]. The MTC devices contend for the transmission opportunities in the first period. Only successful MTC devices are assigned a time slot for their data transmissions in the second period.

MTC devices can be grouped with some MTC gateway devices [14]. Tu *et al.* in [15] and Fu *et al.* in [16] notice that those non-rechargeable MTC devices have limited energy and propose mechanisms to aggregate several short data packets at the gateway MTC device and send them together in an energy-efficient manner. Zhou *et al.* in [17] use a semi-Markov chain to determine the optimal number of short packets in an aggregated packet with a given packet collision rate.

*Access class barring* (ACB) can be used to reduce random access overload in LTE networks by broadcasting an ACB parameter $b$, where $0 \le b \le 1$, to all MTC devices via SIBs [18]. When an MTC device wants to connect to an eNB, it first generates a random number between $[0, 1]$ uniformly. It joins the random access contention only if the generated value is less than the ACB parameter $b$ broadcasted by the eNB. Lien *et al.* in [19] propose a cooperative ACB scheme to control the ACB parameters on multiple eNBs to serve MTC devices efficiently. Chou *et al.* in [20] propose to estimate the ACB parameters by predicting the number of MTC devices requiring random access. Duan *et al.* in [21] propose to dynamically update the ACB parameter based on the number of packet collisions occurred in the past.

For a stationary MTC device, since its propagation delay to the eNB is a constant, the timing advance information in RARs sent in multiple random access slots is identical. A random access protocol for stationary MTC devices is proposed in [22]. Each MTC device stores the timing advance received in a successful random access, and compares the stored value to the timing advance in subsequent random access. It sends its L2/L3 message only if the same timing advance is received. By comparing the timing advance information, the probability of packet collisions in Step 3 of random access is reduced because not all MTC devices transmit their L2/L3 messages after they receive the same RAR. However, timing advance is an index value obtained by quantizing the propagation delay in a granularity of 16 $T_s$. It may be identical for two MTC devices if the difference between their propagation distance to the eNB is less than 16 $cT_s$, *i.e.*, 156 m, where $c$ is the speed of light. Thus, only comparing the timing advance information may not be sufficient to reduce the random access overload since more MTC devices may have the same timing advance information when the density of MTC devices increases.

In this paper, we propose a scheme that jointly uses ACB and timing advance information to reduce random access overload. Our contributions are as follows:

- We formulate an optimization problem to find the optimal ACB parameter, which maximizes the expected number of MTC devices successfully served in each random access slot. Different from our previous work in [23], the *interval analysis* is used in this paper to determine the numerical solution of the optimization problem.
- To reduce the computational complexity and improve the practicability of our proposed scheme, we propose a closed-form approximate solution for the optimization problem. We further present an algorithm to estimate the number of MTC devices that require access to an eNB in each random access slot.
- Our system model is validated via simulations. Simulation results show that the approximate solution obtains the same performance as the numerical solution with the proposed scheme in reducing random access overload. Furthermore, almost 50% random access slots can be reduced by the proposed scheme when compared to the existing schemes that use timing advance information only [22], ACB only [21], or cooperative ACB [19].

The rest of the paper is organized as follows. In Section II, we introduce our system model and problem formulation. The numerical and closed-form approximate solutions for the formulated problem and the algorithm to estimate the number of MTC devices are presented in Section III. Section IV presents the simulation results. Conclusions are given in Section V.

## II. System Model and Problem Formulation

Consider a set of stationary MTC devices $\mathcal{N}$ within the coverage of an eNB in LTE networks, where $\mathcal{N} = \{1, 2, \ldots, N\}$. MTC devices that require access to an eNB send their random access preambles in periodic random access slots. Since the MTC devices are stationary, the propagation delay for each MTC device is a constant. The timing advance is an index value (*i.e.*, $0, 1, 2, \ldots$) after quantizing the propagation delay with the granularity of 16 $T_s$ [6]. Thus, propagation delays that are close can still be quantized to two different consecutive index values when there is a multiple of 16 $T_s$ between them. Different propagation delays may be quantized to the same timing advance when they are within the same quantization granularity. To simplify our system model, we assume that propagation delays are quantized to the same timing advance if their difference is less than or equal to half of the quantization granularity. This difference is denoted by $\tau$. Thus, $\tau$ is equal to 8 $T_s$. Let $t_i$ and $T_A^i$ denote the propagation delay and timing advance of MTC device $i \in \mathcal{N}$, respectively. For an MTC device $j \in \mathcal{N} \setminus \{i\}$, we have $T_A^i = T_A^j$ if $|t_i - t_j| \leq \tau$ and $T_A^i \neq T_A^j$ if $|t_i - t_j| > \tau$. We refer to the maximum propagation distance of the MTC devices in set $\mathcal{N}$ as the *deployment range* $R$, which is determined by eNB as

$$R = \max_{i \in \mathcal{N}} \quad c\ t_i. \tag{1}$$

Each MTC device $i \in \mathcal{N}$ has stored its timing advance information $T_A^i$ in the previous successful random access. When an MTC device transmits a random access preamble in a random access slot and receives an RAR, it sends its L2/L3 message only if the timing advance in the received RAR matches $T_A^i$. Since the root mean square delay spread in microcells of urban area is $0.25\ \mu$s [24, pp. 443], which is less than $\tau = 8\ T_s$, it is feasible to compare $T_A^i$ for MTC device $i$ in different random access slots.

Consider MTC devices $n_1$, $n_2$, and $n_3$ in Fig. 1 (a) as an example and assume their propagation delays satisfy $t_1 - t_2 > \tau$, $t_1 - t_3 > \tau$ and $0 < t_2 - t_3 < \tau$. In this example, we have $\mathcal{N} = \{n_1, n_2, n_3\}$, $T_A^1 > T_A^2 = T_A^3$, and $R = ct_1$. Assume they transmit the same random access preamble in a random access slot. At least three copies of the preamble are received by eNB with similar received power [8]. Since only one RAR is transmitted for the same preamble, $n_1$, $n_2$, and $n_3$ will receive the same RAR. In this RAR, $T_A^1$, $T_A^2$, and $T_A^3$ have the same probability to be used as the timing advance. If $T_A^2$ is included in RAR, since $T_A^2$ and $T_A^3$ are the same, $n_2$ and $n_3$ will send their L2/L3 messages in the same wireless channel and packet collision will occur. If $T_A^1$ is used in RAR, since $T_A^1$ differs from both $T_A^2$ and $T_A^3$, $n_2$ and $n_3$ will not send their L2/L3 messages. The L2/L3 message sent by $n_1$ will be successfully received by eNB.

Let $r = ct$ denote the propagation distance of an MTC device with propagation delay $t$. Let $d = 8\ cT_s$ denote the minimum difference between the propagation distance of two MTC devices with different timing advance information. We consider that the MTC devices in set $\mathcal{N}$ are uniformly distributed. The probability that a randomly selected MTC device has the same timing advance information as the MTC device that has propagation distance $r$ to the eNB is

$$p(r) = \begin{cases} \frac{2}{R^2} \int_0^{r+d} \gamma \ \mathrm{d}\gamma = \left(\frac{r+d}{R}\right)^2, & \text{if } 0 \leq r < d, \\ \frac{2}{R^2} \int_{r-d}^{r+d} \gamma \ \mathrm{d}\gamma = \frac{4rd}{R^2}, & \text{if } d \leq r \leq R-d, \\ \frac{2}{R^2} \int_{r-d}^{R} \gamma \ \mathrm{d}\gamma = 1 - \left(\frac{r-d}{R}\right)^2, & \text{if } R-d < r \leq R. \end{cases} \tag{2}$$

Consider an MTC device $u \in \mathcal{N}$, we denote $I_u = 1$ as the event that $u$ passed the ACB check, and $I_u = 0$ otherwise. If the ACB parameter for the current random access slot is $b$, we have the probability $\mathbb{P}(I_u = 1) = b$. Let $Y_u$ denote a random variable, which represents the number of additional MTC devices that also passed the ACB check in the current random access slot. Then, $Y_u$ follows a binomial distribution $\mathcal{B}(N-1, b)$. We use $\Gamma_u = r$ to represent the event that an arbitrarily chosen MTC device $u$ has propagation distance $r$ from the eNB. Given $\Gamma_u = r$, the conditional probability that there are $i$ additional MTC devices which passed the ACB check and contend with $u$ in the current random access slot is

$$\begin{aligned} &\mathbb{P}\left(Y_u = i, I_u = 1 \mid \Gamma_u = r\right) \\ &= \mathbb{P}\left(Y_u = i, I_u = 1\right) \\ &= \mathbb{P}\left(Y_u = i\right)\mathbb{P}\left(I_u = 1\right) \\ &= \binom{N-1}{i}(1-b)^{N-1-i}\, b^{i+1}, \quad i = 0, 1, \ldots, N-1. \end{aligned} \tag{3}$$

Note that the random variables $I_u$ and $Y_u$ are independent of the position of MTC device $u$. Consider there are $m$ preambles in total. Let $J_u = j$ denote the event that $u$ selected preamble $j$ from $m$ preambles in a uniform manner. Let $\mathcal{L}_u \subset \mathcal{N} \setminus \{u\}$ denote the set of other MTC devices except $u$ that have passed the ACB check and have chosen preamble $j$. The cardinality of $\mathcal{L}_u$, denoted as $L_u = |\mathcal{L}_u|$, is a random variable. We have

$$\begin{aligned} &\mathbb{P}\left(L_u = k, J_u = j \mid Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &= \frac{1}{m}\binom{i}{k}\left(\frac{1}{m}\right)^k \left(1 - \frac{1}{m}\right)^{i-k}, \\ &\quad k = 0, 1, \ldots, i, \quad j = 1, \ldots, m. \end{aligned} \tag{4}$$

With $L_u = k$, $J_u = j$, $Y_u = i$, $I_u = 1$, and $\Gamma_u = r$ (*i.e.*, given the event that MTC device $u$, whose propagation distance is $r$, has passed ACB check and $i$ other MTC devices have passed ACB check as well, and meanwhile, among these $i$ MTC devices, $k$ MTC devices transmitted the same preamble $j$ as $u$), MTC device $u$ succeeds in the random access if the following two conditions are satisfied: a) $u$'s propagation delay is quantized as the timing advance information and included in RAR; b) the other $k$ MTC devices that receive the same RAR do not have the same timing advance of $u$. Let $S_u = 1$ (or $S_u = 0$) denote the event that MTC device $u$ succeeds (or fails) in the current random access. The conditional probability of $S_u = 1$ is

$$\begin{aligned} &\mathbb{P}\left(S_u = 1 \mid L_u = k, J_u = j, Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &= \frac{\binom{k}{0}\left(p(r)\right)^0 \left(1 - p(r)\right)^k}{\binom{k+1}{1}} \\ &= \frac{\left(1 - p(r)\right)^k}{k+1}. \end{aligned} \tag{5}$$

From (4) and (5), we have

$$\begin{aligned} &\mathbb{P}\left(S_u = 1, L_u = k, J_u = j \mid Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &= \mathbb{P}\left(S_u = 1 \mid L_u = k, J_u = j, Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &\quad \times \mathbb{P}\left(L_u = k, J_u = j \mid Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &= \frac{\left(1 - p(r)\right)^k}{m(k+1)}\binom{i}{k}\left(\frac{1}{m}\right)^k \left(1 - \frac{1}{m}\right)^{i-k} \\ &= \frac{1}{m(k+1)}\left(1 - \frac{1}{m}\right)^i \binom{i}{k}\left(\frac{1 - p(r)}{m-1}\right)^k, \\ &\quad k = 0, 1, \ldots, i, \quad j = 1, \ldots, m. \end{aligned} \tag{6}$$

Thus, for $i = 0, 1, \ldots, N-1$, and $0 \le r \le R$, we obtain

$$\begin{aligned} &\mathbb{P}\left(S_u = 1 \mid I_u = 1, Y_u = i, \Gamma_u = r\right) \\ &= \sum_{j=1}^{m}\sum_{k=0}^{i}\mathbb{P}\left(S_u = 1, L_u = k, J_u = j \mid Y_u = i, I_u = 1, \Gamma_u = r\right) \\ &= \left(1 - \frac{1}{m}\right)^i \sum_{k=0}^{i}\frac{1}{k+1}\binom{i}{k}\left(\frac{1 - p(r)}{m-1}\right)^k \\ &= \left(1 - \frac{1}{m}\right)^i \\ &\quad \times \left(\frac{1 - p(r)}{m-1}\right)^{-1}\sum_{k=0}^{i}\frac{1}{i+1}\binom{i+1}{k+1}\left(\frac{1 - p(r)}{m-1}\right)^{k+1} \\ &= \frac{\left(1 - \frac{1}{m}\right)^i}{i+1}\left(\frac{1 - p(r)}{m-1}\right)^{-1}\left(\left(1 + \frac{1 - p(r)}{m-1}\right)^{i+1} - 1\right) \\ &= \left(1 - \frac{1}{m}\right)^i \frac{\left(1 + \phi(r)\right)^{i+1} - 1}{\phi(r)(i+1)}, \end{aligned} \tag{7}$$

where $\phi(r) = \frac{1 - p(r)}{m-1}$ and $\frac{1}{(i+1)}\binom{i+1}{k+1} = \frac{1}{(k+1)}\binom{i}{k}$ is used in the third step. From (3) and (7), we have

$$\begin{aligned} &\mathbb{P}\left(S_u = 1, I_u = 1 \mid \Gamma_u = r\right) \\ &= \sum_{i=0}^{N-1}\mathbb{P}\left(S_u = 1, I_u = 1, Y_u = i \mid \Gamma_u = r\right) \\ &= \sum_{i=0}^{N-1}\binom{N-1}{i}(1-b)^{N-1-i}\, b^{i+1} \\ &\quad \times \left(\frac{m-1}{m}\right)^i \frac{\left(1 + \phi(r)\right)^{i+1} - 1}{\phi(r)(i+1)} \\ &= \frac{m(1-b)^N}{\phi(r)(m-1)} \\ &\quad \times \sum_{i=0}^{N-1}\frac{\left(1 + \phi(r)\right)^{i+1} - 1}{i+1}\binom{N-1}{i}\left(\frac{b(m-1)}{(1-b)m}\right)^{i+1}. \end{aligned} \tag{8}$$

Since $N\binom{N-1}{i} = (i+1)\binom{N}{i+1}$, equation (8) becomes

$$\begin{aligned} &\mathbb{P}\left(S_u = 1, I_u = 1 \mid \Gamma_u = r\right) \\ &= \frac{m(1-b)^N}{\phi(r)N(m-1)} \\ &= \sum_{i=0}^{N-1}\left(\left(1 + \phi(r)\right)^{i+1} - 1\right)\binom{N}{i+1}\left(\frac{b(m-1)}{(1-b)m}\right)^{i+1}. \end{aligned} \tag{9}$$

We further have

$$\begin{aligned} &\sum_{i=0}^{N-1}\left(1 + \phi(r)\right)^{i+1}\binom{N}{i+1}\left(\frac{b(m-1)}{(1-b)m}\right)^{i+1} \\ &= \left(1 + \frac{\left(1 + \phi(r)\right)b(m-1)}{(1-b)m}\right)^N - 1, \end{aligned} \tag{10}$$

and

$$\sum_{i=0}^{N-1}\binom{N}{i+1}\left(\frac{b(m-1)}{(1-b)m}\right)^{i+1} = \left(1 + \frac{b(m-1)}{(1-b)m}\right)^N - 1. \tag{11}$$

Equation (9) becomes

$$\begin{aligned}&\mathbb{P}\left(S_u=1, I_u=1 \mid \Gamma_u=r\right)\\&=\frac{m\left(1-b\right)^N}{\phi\left(r\right)N\left(m-1\right)}\\&=\left(\left(1+\frac{\left(1+\phi\left(r\right)\right)b\left(m-1\right)}{\left(1-b\right)m}\right)^N-\left(1+\frac{b\left(m-1\right)}{\left(1-b\right)m}\right)^N\right).\end{aligned} \tag{12}$$

By substituting $\phi\left(r\right)=\frac{1-p(r)}{m-1}$ into (12), we obtain

$$\begin{aligned}&\mathbb{P}\left(S_u=1, I_u=1 \mid \Gamma_u=r\right)\\&=\frac{m}{N\left(1-p\left(r\right)\right)}\left(\left(1-\frac{b}{m}p\left(r\right)\right)^N-\left(1-\frac{b}{m}\right)^N\right).\end{aligned} \tag{13}$$

An MTC device has to pass ACB check before being served in a random access slot. The probability that MTC device $u$ with propagation distance $r$ does not pass ACB check but succeeds in the random access contention is zero, *i.e.*, $\mathbb{P}\left(S_u=1, I_u=0 \mid \Gamma_u=r\right)=0$. Thus, we have

$$\begin{aligned}\mathbb{P}\left(S_u=1 \mid \Gamma_u=r\right)&=\sum_{\ell=0}^{1}\mathbb{P}\left(S_u=1, I_u=\ell \mid \Gamma_u=r\right)\\&=\mathbb{P}\left(S_u=1, I_u=1 \mid \Gamma_u=r\right).\end{aligned} \tag{14}$$

Since MTC devices in set $\mathcal{N}$ are uniformly distributed, we have

$$\begin{aligned}&\mathbb{P}\left(S_u=1\right)\\&=\frac{2m}{R^2N}\int_0^R\frac{r}{1-p\left(r\right)}\left(\left(1-\frac{b}{m}p\left(r\right)\right)^N-\left(1-\frac{b}{m}\right)^N\right)\mathrm{d}r.\end{aligned} \tag{15}$$

From (15), we can obtain the probability that an arbitrary MTC device succeeds in the current random access slot with an ACB parameter $b$. Let random variable $Z$ denote the number of MTC devices that succeed in random access. Random variable $Z$ follows a binomial distribution. That is, $Z \sim \mathcal{B}(N, \mathbb{P}\left(S_u=1\right))$. The expectation of $Z$ is given by

$$\begin{aligned}&\mathbb{E}\left[Z\right]\\&=N\mathbb{P}\left(S_u=1\right)\\&=\frac{2m}{R^2}\int_0^R\frac{r}{1-p\left(r\right)}\left(\left(1-\frac{b}{m}p\left(r\right)\right)^N-\left(1-\frac{b}{m}\right)^N\right)\mathrm{d}r.\end{aligned} \tag{16}$$

Note that equation (16) is for one random access slot. In a bursty request scenario, eNB needs to serve multiple MTC devices in a number of consecutive random access slots. To reduce the total number of random access slots required to serve all MTC devices, we need to find the optimal ACB parameter $b$ that maximizes $\mathbb{E}\left[Z\right]$ in each random access slot. Therefore, the optimization problem can be formulated as

$$\begin{aligned}\underset{b}{\text{maximize}} \quad & \mathbb{E}\left[Z\right]\\ \text{subject to} \quad & 0 \le b \le 1.\end{aligned} \tag{17}$$

From (16), we notice that the objective function in problem (17) does not have a closed-form expression and it is difficult to solve. Determining the numerical solution for the optimization problem with inequality constraints has been studied in [25, pp. 343], which provides an algorithm to find the best solution after running the *interval Newton's method* [26] over all subintervals of the parameters being optimized. However, with the given algorithm, we have to evaluate the objective function of problem (17) many times by numerical integrals. Since the number of MTC devices $N$ varies in different random access slots, the given algorithm in [25] may not be practical to be used due to its high computational complexity.

In Section III, we will use interval analysis [27] and prove that the solution to problem (17) exists on an interval where the objective function is strictly concave. With the proposed approach, not only the numerical solution can be determined efficiently, but also a closed-form approximate solution can be obtained. An algorithm to estimate the number of MTC devices requiring access to eNB in each random access slot will be given. Simulation results to be presented in Section IV show that the approximate solution obtains the same performance as the numerical solution and the proposed estimation algorithm works well with different MTC traffic models.

## III. Solutions and Proposed Algorithms

We first present our numerical and approximate solutions for problem (17). We then propose an algorithm to estimate the number of MTC devices requiring access to eNB in each random access slot, which is referred to as the *backlog* of each random access slot.

### A. Numerical Solution

Since the number of preambles $m$ is up to 64 in LTE networks and $\frac{b}{m}p\left(r\right)<\frac{b}{m}\le\frac{1}{m}<1$, the objective function in problem (17), *i.e.*, equation (16), can be approximated as

$$\begin{aligned}\mathbb{E}\left[Z\right] \approx\ & \frac{2m}{R^2}\int_0^R\frac{r}{1-p\left(r\right)}\left(e^{-\frac{Nb}{m}p(r)}-e^{-\frac{Nb}{m}}\right)\mathrm{d}r\\ \triangleq\ & \mathcal{H}\left(R,N,m,b\right).\end{aligned} \tag{18}$$

We consider the following problem in our further discussion

$$\begin{aligned}\underset{b}{\text{maximize}} \quad & \mathcal{H}\left(R,N,m,b\right)\\ \text{subject to} \quad & 0 \le b \le 1.\end{aligned} \tag{19}$$

We will show that when $N$ is large, the solution to problem (19) always exists on an interval where the objective function is strictly concave. Such a narrower interval, which contains the solution of the optimization problem, is referred as the *sharper interval* in the context of interval analysis [27, pp. 21]. To determine the sharper interval of $b$ for problem (19), we study how the value of $b$ affects the objective function $\mathcal{H}\left(R,N,m,b\right)$. We define $\rho \triangleq \frac{4(R-d)d}{R^2}$ to simplify the notation and present two propositions as follows:

*Proposition 1:* Given $R$, $N$, and $m$, the function $\mathcal{H}\left(R,N,m,b\right)$ is strictly increasing with $b$ on the interval $\left[0, \frac{m\ln\rho}{N(\rho-1)}\right]$.

*Proof:* Please refer to Appendix A.

*Proposition 2:* Given $R$, $N$, and $m$, the function $\mathcal{H}(R, N, m, b)$ is strictly concave with $b$ on the interval $\left[0, \frac{2m\ln\rho}{N(\rho-1)}\right]$.

*Proof:* Please refer to Appendix B.

Now we have the following theorem:

*Theorem 1:* Given $R$, $N$, and $m$, let $b^\star$ denote the solution of problem (19). We have

$$\begin{cases} b^\star = 1, & \text{if } N \leq \frac{m\ln\rho}{\rho-1}, \\ b^\star \in \left(\frac{m\ln\rho}{N(\rho-1)}, 1\right], & \text{if } \frac{m\ln\rho}{\rho-1} < N < \frac{2m\ln\rho}{\rho-1}, \\ b^\star \in \left(\frac{m\ln\rho}{N(\rho-1)}, \frac{2m\ln\rho}{N(\rho-1)}\right), & \text{if } N \geq \frac{2m\ln\rho}{\rho-1}. \end{cases} \tag{20}$$

*Proof:* Please refer to Appendix C.

From Theorem 1, we have $b^\star = 1$ for $N \leq \frac{m\ln\rho}{\rho-1}$. By further considering the concavity of the function $\mathcal{H}(R, N, m, b)$ with respect to $b$ on the interval $\left[0, \frac{2m\ln\rho}{N(\rho-1)}\right]$ (Proposition 2), the value of $b^\star$ for $N > \frac{m\ln\rho}{\rho-1}$ can be determined as follows. We first apply the bisection search on the interval $\left(\frac{m\ln\rho}{N(\rho-1)}, \frac{2m\ln\rho}{N(\rho-1)}\right)$ to determine the value of $b$ that maximizes the function $\mathcal{H}(R, N, m, b)$. We denote this value by $\hat{b}^\star$. Then, the solution of problem (19) is obtained by $b^\star = \min\left\{1, \hat{b}^\star\right\}$.

The procedures for an eNB to serve $N_0$ initial backlog is given in Algorithm 1. In Line 2, $N_c$ is the current backlog (*i.e.*, the number of remaining MTC devices that have not been served), which is initialized by $N_0$. $\epsilon$ is the termination threshold to search $\hat{b}^\star$. Lines 4 – 15 are the steps within one random access slot. According to above analysis, we have the optimal ACB parameter $b^\star = 1$ if $N_c \leq \frac{m\ln\rho}{\rho-1}$ (Lines 4 – 5). Otherwise, $b^\star$ is either on the interval $\left(\frac{m\ln\rho}{N_c(\rho-1)}, 1\right]$ when $\frac{2m\ln\rho}{N_c(\rho-1)} > 1$ or on the interval $\left(\frac{m\ln\rho}{N_c(\rho-1)}, \frac{2m\ln\rho}{N_c(\rho-1)}\right)$ when $\frac{2m\ln\rho}{N_c(\rho-1)} \leq 1$. Thus, the eNB first determines the value of $\hat{b}^\star$ that maximizes the objective function in problem (19) by numerical search. Then, the optimal ACB parameter for the current random access slot is $b^\star = \min\left\{1, \hat{b}^\star\right\}$ (Lines 7 – 8). After broadcasting the ACB parameter $b^\star$ in the current random access slot, eNB acknowledges each random access preamble received (Lines 10 – 12). The MTC devices being acknowledged send their L2/L3 messages to the eNB. Let $z$ denote the number of L2/L3 messages that the eNB received successfully in Line 13. Then, the eNB serves those $z$ MTC devices by allocating wireless channel to each of them. After serving those $z$ MTC devices successfully, the number of MTC devices that still need to be served is updated. The eNB continues to serve the remaining MTC devices in the subsequent random access slots until all of them are successfully served.

### B. Closed-form Approximate Solution

Note that random access overload usually happens when a large number of MTC devices require access in the LTE networks. When $N_c > \frac{m\ln\rho}{\rho-1}$, eNB still needs to evaluate the numerical integral many times to determine $\hat{b}^\star$ according to Line 7 in Algorithm 1. Thus, Algorithm 1 may not be practical due to its high computational complexity. We propose a closed-form solution $\tilde{b}^\star$ to approximate $\hat{b}^\star$. Then, the approximate solution to problem (19) is given by

$$b^\star_{appr} = \min\left\{1, \tilde{b}^\star\right\}. \tag{21}$$

---

**Algorithm 1** Procedures for an eNB to serve MTC devices with ACB parameter $b^\star$ in random access slots.

1: Initialize $R$, $m$, $d$, $N_0$.
2: Set $N_c := N_0$, $\rho := \frac{4(R-d)d}{R^2}$, $\epsilon := 10^{-3}$.
3: **Repeat**
4:     **if** $N_c \leq \frac{m\ln\rho}{\rho-1}$ **then**
5:         Set $b^\star := 1$.
6:     **else**
7:         Determine $\hat{b}^\star$ by using bisection search on the concave interval $\left(\frac{m\ln\rho}{N_c(\rho-1)}, \frac{2m\ln\rho}{N_c(\rho-1)}\right)$ until $\left|\frac{\partial}{\partial b}\mathcal{H}(R, N_c, m, b)\,|_{b=\hat{b}^\star}\right| < \epsilon$.
8:         Set $b^\star := \min\left\{1, \hat{b}^\star\right\}$.
9:     **end if**
10:     Broadcast $b^\star$ for the next available random access slot via system information blocks.
11:     Listen and receive the random access preambles.
12:     Send RARs for the received preambles.
13:     Set $z :=$ number of L2/L3 messages which are successfully received.
14:     Serve these $z$ MTC devices by allocating a wireless channel to each of them.
15:     Set $N_c := N_c - z$.
16: **Until** $N_c = 0$.

---

We now describe how to determine $\tilde{b}^\star$. We first define a set of network scenarios $\mathcal{S} = \left\{\zeta_1, \ldots, \zeta_{|\mathcal{S}|}\right\}$, where the three-tuple $\zeta_i = (R_i, N_i, m_i)$, for $i = 1, \ldots, |\mathcal{S}|$, denotes a network scenario. For each network scenario $\zeta_i$, we determine $\hat{b}^\star_i = \arg\max_{b\in(\beta_i, 2\beta_i)} \mathcal{H}(R_i, N_i, m_i, b)$, where $\beta_i = \frac{m_i \ln\rho_i}{N_i(\rho_i-1)}$ and $\rho_i = \frac{4(R_i-d)d}{R_i^2}$. Note that $(\beta_i, 2\beta_i)$ is a local concave interval of $\mathcal{H}(R_i, N_i, m_i, b)$ with respect to $b$ (by Proposition 2), and the length of the interval is inversely proportional to $N_i$. When $N_i$ is large, *i.e.*, $N_i > \frac{m_i\ln\rho_i}{\rho_i-1}$, the length of the interval $(\beta_i, 2\beta_i)$ is small. Therefore, we introduce a variable $\gamma_{\mathcal{S}}$ $(0 < \gamma_{\mathcal{S}} < 1)$ for the set $\mathcal{S}$. For each network scenario $\zeta_i \in \mathcal{S}$, we use the interior point $\tilde{b}_i(\gamma_{\mathcal{S}}) = \gamma_{\mathcal{S}}\beta_i + (1-\gamma_{\mathcal{S}})\,2\beta_i$ on the interval $(\beta_i, 2\beta_i)$ to approach to $\hat{b}^\star_i$. For network scenario $\zeta_i$, let $\delta_i(\gamma_{\mathcal{S}})$ denote the relative error of $\tilde{b}_i(\gamma_{\mathcal{S}})$ from the optimal value $\hat{b}^\star_i$, which is determined by the value of $\gamma_{\mathcal{S}}$. We have

$$\begin{aligned} \delta_i(\gamma_{\mathcal{S}}) &= \frac{\tilde{b}_i(\gamma_{\mathcal{S}}) - \hat{b}^\star_i}{\hat{b}^\star_i} \\ &= \gamma_{\mathcal{S}}\frac{\beta_i - \hat{b}^\star_i}{\hat{b}^\star_i} + (1-\gamma_{\mathcal{S}})\frac{2\beta_i - \hat{b}^\star_i}{\hat{b}^\star_i}, \\ &\quad 0 < \gamma_{\mathcal{S}} < 1, \quad i = 1, \ldots, |\mathcal{S}|. \end{aligned} \tag{22}$$

We now determine the optimal value of $\gamma_{\mathcal{S}}$, *i.e.*, the value of $\gamma_{\mathcal{S}}$ that minimizes the overall relative errors of all network scenarios in set $\mathcal{S}$. The sign of relative error $\delta_i(\gamma_{\mathcal{S}})$ given by (22) can be positive or negative depending on the network scenario $\zeta_i \in \mathcal{S}$. To evaluate the overall relative errors for all network scenarios in $\mathcal{S}$, we determine the square root of the sum of squares of the relative errors for all network scenarios in set $\mathcal{S}$. Thus, with the given set $\mathcal{S}$, we need to solve the following problem to determine the optimal value of $\gamma_{\mathcal{S}}$

$$\begin{aligned} &\underset{\gamma_{\mathcal{S}}}{\text{minimize}} \quad \left( \sum_{i=1}^{|\mathcal{S}|} \delta_i^2 \left( \gamma_{\mathcal{S}} \right) \right)^{\frac{1}{2}} \\ &\text{subject to} \quad 0 < \gamma_{\mathcal{S}} < 1. \end{aligned} \tag{23}$$

Problem (23) can be solved by transforming it to a geometry problem as follows. We denote two vectors $\boldsymbol{a} = \left( \frac{\beta_1 - \hat{b}_1^\star}{\hat{b}_1^\star}, \ldots, \frac{\beta_{|\mathcal{S}|} - \hat{b}_{|\mathcal{S}|}^\star}{\hat{b}_{|\mathcal{S}|}^\star} \right)$ and $\boldsymbol{b} = \left( \frac{2\beta_1 - \hat{b}_1^\star}{\hat{b}_1^\star}, \ldots, \frac{2\beta_{|\mathcal{S}|} - \hat{b}_{|\mathcal{S}|}^\star}{\hat{b}_{|\mathcal{S}|}^\star} \right)$. Note that $\boldsymbol{a}$ and $\boldsymbol{b}$ can be taken as two points in a hyperspace with $|\mathcal{S}|$ dimensions. For any $\gamma_{\mathcal{S}}$ on $(0, 1)$, we have the third point in the $|\mathcal{S}|$-dimensional space given by $\boldsymbol{c} = \gamma_{\mathcal{S}} \boldsymbol{a} + (1 - \gamma_{\mathcal{S}}) \boldsymbol{b}$. Since $\frac{\beta_i - \hat{b}_i^\star}{\hat{b}_i^\star} < 0$ and $\frac{2\beta_i - \hat{b}_i^\star}{\hat{b}_i^\star} > 0$ hold for any network scenario $\zeta_i \in \mathcal{S}$, the point $\boldsymbol{a}$ is in the $\left(2^{|\mathcal{S}|} - 1\right)^{\text{th}}$ quadrant of the $|\mathcal{S}|$-dimensional space and $\boldsymbol{b}$ lies in the first quadrant. That is, problem (23) aims to determine a point on the open line segment from $\boldsymbol{a}$ to $\boldsymbol{b}$ in the $|\mathcal{S}|$-dimensional space, which obtains the minimum distance to the origin. Such a geometry problem is ready to be solved, and the solution to problem (23) is given by

$$\gamma_{\mathcal{S}}^\star = -\frac{(\boldsymbol{a} - \boldsymbol{0})^T (\boldsymbol{b} - \boldsymbol{a})}{\|\boldsymbol{b} - \boldsymbol{a}\|_2^2}, \tag{24}$$

where $(\cdot)^T$ denotes the transpose.

To better evaluate $\gamma_{\mathcal{S}}^\star$ for typical LTE networks, we compose the set $\mathcal{S}$ by enumerating network scenarios with the parameters given as follows: $R_i$ from 200 m to 2 km with increment of 5 m, $N_i$ from 40 devices to 3040 devices with increment of 5 devices, and $m_i$ from 10 preambles to 64 preambles with increment of 2 preambles. Therefore, 6,074,908 network scenarios are included in the set $\mathcal{S}$. We evaluate $\gamma_{\mathcal{S}}^\star$ with the set $\mathcal{S}$ by equation (24) and we obtain $\gamma_{\mathcal{S}}^\star = 0.83$.

Note that $\tilde{b}^\star$ in (21) is used to approximate $\hat{b}^\star$ in Theorem 1 on the interval $\left( \frac{m \ln \rho}{N(\rho-1)}, \frac{2m \ln \rho}{N(\rho-1)} \right)$. When random access overload occurs, the number of MTC devices $N$ is large and the length of interval $\left( \frac{m \ln \rho}{N(\rho-1)}, \frac{2m \ln \rho}{N(\rho-1)} \right)$ is small. Thus, we apply $\gamma_{\mathcal{S}}^\star = 0.83$ given by the set of network scenarios $\mathcal{S}$ to obtain a value of $\tilde{b}^\star$ on the interval $\left( \frac{m \ln \rho}{N(\rho-1)}, \frac{2m \ln \rho}{N(\rho-1)} \right)$. We have

$$\tilde{b}^\star = \gamma_{\mathcal{S}}^\star \frac{m \ln \rho}{N(\rho - 1)} + (1 - \gamma_{\mathcal{S}}^\star) \frac{2m \ln \rho}{N(\rho - 1)} = \frac{1.17 m \ln \rho}{N(\rho - 1)}. \tag{25}$$

According to (21), the closed-form approximate solution of problem (19) is given by

$$b_{appr}^\star = \min \left\{ 1, \frac{1.17 m \ln \rho}{N(\rho - 1)} \right\}. \tag{26}$$

We study the relative error between the numerical solution $b^\star$ and the closed-form approximate solution $b_{appr}^\star$ of problem (19). With all network scenarios in set $\mathcal{S}$, the cumulative distribution function is shown in Fig. 3. We find that the relative error from $b_{appr}^\star$ to $b^\star$ is bounded by $\pm 2.6\%$.

The procedures for an eNB to serve all $N_0$ MTC devices are updated by Algorithm 2, where the eNB does not need to search for $\hat{b}^\star$ numerically as Line 7 in Algorithm 1. Instead, the ACB parameter $b_{appr}^\star$ is given by a closed-form expression as Line 4 in Algorithm 2. Simulation results to be presented in the next section show that $b_{appr}^\star$ obtains the same performance as $b^\star$ with our proposed scheme.

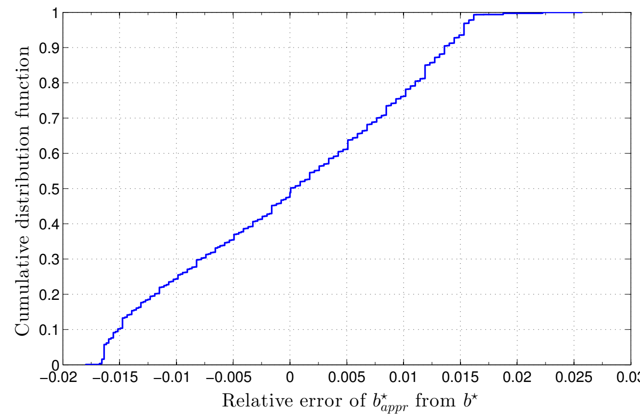


Fig. 3. Cumulative distribution function of the relative error between $b_{appr}^\star$ and $b^\star$.

**Algorithm 2** Procedures for an eNB to serve MTC devices with ACB parameter $b_{appr}^\star$ in random access slots.

1: Initialize $R$, $m$, $d$, $N_0$.
2: Set $N_c := N_0$, $\rho := \frac{4(R-d)d}{R^2}$.
3: **Repeat**
4: Set $b_{appr}^\star := \min \left\{ 1, \frac{1.17 m \ln \rho}{N_c(\rho - 1)} \right\}$.
5: Broadcast $b_{appr}^\star$ for the next available random access slot via system information blocks.
6: Listen and receive the random access preambles.
7: Send RARs for the received preambles.
8: Set $z :=$ number of L2/L3 messages which are successfully received.
9: Serve these $z$ MTC devices by allocating a wireless channel to each of them.
10: Set $N_c := N_c - z$.
11: **Until** $N_c = 0$.

## C. Backlog Estimation Algorithm for Proposed Scheme

The initial backlog $N_0$ in Algorithms 1 and 2 may not be available to eNB. Thus, the actual backlog $N_c$ in each random access slot may not be used to determine $b^\star$ or $b_{appr}^\star$. Meanwhile, MTC devices may not require access to eNB at the same time since they may not be activated simultaneously. Furthermore, it is difficult to determine the duration required to activate all MTC devices and determine the probability that an MTC device is activated in a specific random access slot. We propose to estimate the actual backlog $N_c$ in the current random access slot based on the *preamble collision ratio* in the previous random access slot. The preamble collision ratio is defined as the ratio of the number of different preambles that are transmitted by MTC devices in a random access slot but those fail to serve any MTC device over the total number of preambles available. Let $\hat{N}$ denote an estimation of the $N_c$ backlog in the current random access slot. When $N_c$ MTC devices perform random access, the sub-optimal ACB parameter is $b_{\hat{N}} = \min\{1, \frac{1.17 m \ln \rho}{\hat{N}(\rho - 1)}\}$, which is determined by (26) based on the backlog estimation $\hat{N}$. We consider

**Algorithm 3** Procedures for an eNB to serve MTC devices with ACB parameter $b_{\hat{N}}$ based on backlog estimation $\hat{N}$.

1: Initialize $R$, $m$, $d$.
2: Set $\rho := \frac{4(R-d)d}{R^2}$.
3: Initialize $k := 0$, $\hat{N}^{(1)} := \frac{1.17m\ln\rho}{\rho-1}$.
4: **Repeat**
5: Set $k := k+1$, $b_{\hat{N}}^{(k)} := \frac{1.17m\ln\rho}{\hat{N}^{(k)}(\rho-1)}$.
6: Broadcast $b_{\hat{N}}^{(k)}$ for the next available random access slot via system information blocks.
7: Listen and receive the random access preambles.
8: Set $x_{\hat{N}}^{(k)} :=$ number of preambles which are not selected by any MTC device.
9: Send RARs for the received preambles.
10: Set $z_{\hat{N}}^{(k)} :=$ number of L2/L3 messages which are successfully received.
11: Serve these $z_{\hat{N}}^{(k)}$ MTC devices by allocating a wireless channel to each of them.
12: Set $\hat{N}^{(k+1)} :=$ $\max\left\{\frac{1.17m\ln\rho}{\rho-1}, \hat{N}^{(k)} f_R^{-1}\left(1-\frac{x_{\hat{N}}^{(k)}+z_{\hat{N}}^{(k)}}{m}\right)\right\}$.
13: **Until** $x_{\hat{N}}^{(k)} = m$ and $b_{\hat{N}}^{(k)} = 1$.

$\hat{N} \geq \frac{1.17m\ln\rho}{(\rho-1)}$ since we have $b_{\hat{N}} = 1$ for any backlog estimation $\hat{N} \in [0, \frac{1.17m\ln\rho}{(\rho-1)}]$. Let random variable $\Psi_{\hat{N}}$ denote the preamble collision ratio when the sub-optimal ACB parameter $b_{\hat{N}} = \frac{1.17m\ln\rho}{\hat{N}(\rho-1)}$ is utilized. We have the following theorem:

*Theorem 2:* Given $R$ and the ratio of actual backlog and its estimation $\frac{N_c}{\hat{N}}$, the expected preamble collision ratio $\mathbb{E}[\Psi_{\hat{N}}]$ is approximated by

$$\begin{aligned} &\mathbb{E}\left[\Psi_{\hat{N}}\right] \\ \approx& 1-e^{-\frac{N_c}{\hat{N}}\left(\frac{1.17\ln\rho}{\rho-1}\right)} \\ &-\frac{2}{R^2}\int_0^R \frac{r}{1-p\left(r\right)}\left(e^{-\frac{N_c}{\hat{N}}\left(\frac{1.17\ln\rho}{\rho-1}p(r)\right)}-e^{-\frac{N_c}{\hat{N}}\left(\frac{1.17\ln\rho}{\rho-1}\right)}\right)\mathrm{d}r \\ \triangleq& f_R\left(\frac{N_c}{\hat{N}}\right), \end{aligned} \tag{27}$$

which is a strictly increasing function of $\frac{N_c}{\hat{N}}$.

*Proof:* Please refer to Appendix D.

Thus, the inverse function of $f_R$ in (27) exists. Since the deployment range $R$ of the MTC devices is known by eNB, the inverse function $f_R^{-1}$ can be stored on the eNB by a lookup table to reduce computational complexity. $N_c$ is equal to $\hat{N} f_R^{-1}(\mathbb{E}[\Psi_{\hat{N}}])$. However, only one instance $\psi_{\hat{N}}$ of random variable $\Psi_{\hat{N}}$ can be obtained for the actual backlog $N_c$. This is because $N_c$ may change over random access slots since some MTC devices may have been served and an unknown number of MTC devices may be activated. For two consecutive random access slots, $N_c$ does not change significantly. We propose to estimate $N_c$ in a random access slot based on the value of $\psi_{\hat{N}}$ in the previous random access slot with the backlog estimation $\hat{N}$. We denote by $\hat{N}^{(k)}$ the backlog estimation in the $k^{\text{th}}$ random access slot. The backlog estimation for the $(k+1)^{\text{th}}$ random access slot is obtained by $\hat{N}^{(k+1)} = \hat{N}^{(k)} f_R^{-1}(\psi_{\hat{N}}^{(k)})$. The procedures are given in Algorithm 3. The backlog estimation for the first random access slot is initialized by $\hat{N}^{(1)} = \frac{1.17m\ln\rho}{\rho-1}$ in Line 3. With the ACB parameter $b_{\hat{N}}^{(k)}$ determined for the $k^{\text{th}}$ random access slot in Line 5, the number of preambles not used and the number of MTC devices successfully served are denoted by $x_{\hat{N}}^{(k)}$ and $z_{\hat{N}}^{(k)}$, which are determined in Line 8 and Line 10, respectively. The backlog estimation for the next random access slot is obtained with the current backlog estimation $\hat{N}^{(k)}$ and the preamble collision ratio $1-\frac{x_{\hat{N}}^{(k)}+z_{\hat{N}}^{(k)}}{m}$ (Line 12). The loop is terminated by Line 13 when no preamble is selected (*i.e.*, $x_{\hat{N}}^{(k)}=m$) while no MTC device is blocked (*i.e.*, $b_{\hat{N}}^{(k)}=1$), which means all MTC devices have been served.

## IV. Performance Evaluation

In this section, we first validate our system model by comparing the analytical and simulation results of the number of successfully served MTC devices in a random access slot. Then, we present that using the optimal ACB parameter $b^\star$ takes the least number of random access slots to serve all MTC devices comparing with using sub-optimal ACB parameters. We show that the closed-form approximate solution $b^\star_{appr}$ in Algorithm 2 achieves the same performance as the numerical solution $b^\star$ in Algorithm 1. With the same network settings, we also present the performance of Algorithm 3, which uses ACB parameter $b_{\hat{N}}$ determined by the backlog estimation $\hat{N}$ in each random access slot. By applying the MTC traffic models from [28], we further compare $b_{\hat{N}}$ with $b^\star$ in each random access slot of a simulation run to show its accuracy.

### *A. Model Validation*

To present the correctness of our system model, we compare the average number of MTC devices served in a random access slot in simulations to its expectation calculated analytically. We consider $N$ MTC devices require access to eNB in a random access slot together. They are deployed within the deployment range $R$ of $1.5$ km. For the granularity of quantizing the propagation delay to timing advance, we have $\tau = 8T_s = 0.26$ $\mu$s, and $d = c\tau = 78$ m [8], where $T_s = 1/\left(3.072\times10^7\right)$ sec is the basic time unit [6] and $c = 3\times10^8$ m/sec. We vary the number of MTC devices $N$ from 150 to 1050 in simulations. In each simulation run, we first consider the ACB check on each MTC device with parameter $b$, and then let those MTC devices which passed the ACB check contend for $m=64$ preambles [6]. We check each preamble and increase the number of successfully served MTC devices by 1 when one of the following two cases happens: 1) the preamble is selected by exactly one MTC device; 2) the preamble is chosen by multiple MTC devices, but the timing advance of the selected one is different from the others. The number of MTC devices successfully served in the random access slot with a given ACB parameter $b$ is determined after each simulation run. We plot the average result of $5\times10^3$ simulation run and the corresponding analytical result given by (16) in Fig. 4. We find that the analytical result given by (16) closely matches with the average of the simulation results. For a small ACB parameter, more MTC devices can be successfully served in a random access slot if a relatively larger ACB parameter is used because MTC devices are excessively blocked. When the value of ACB parameter is larger than the

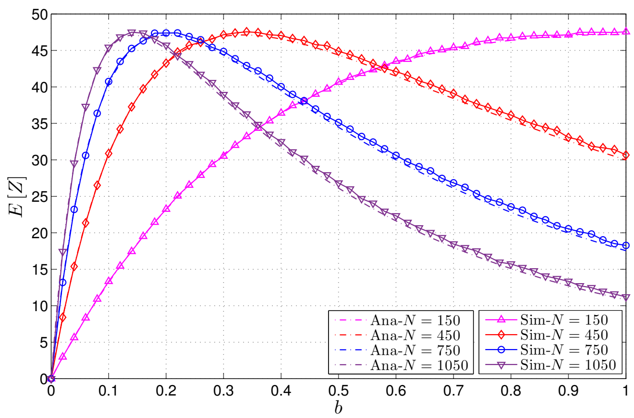


Fig. 4. Analytical and simulation results of the expected number of MTC devices served $\mathbb{E}[Z]$ in a random access slot with different ACB parameters.

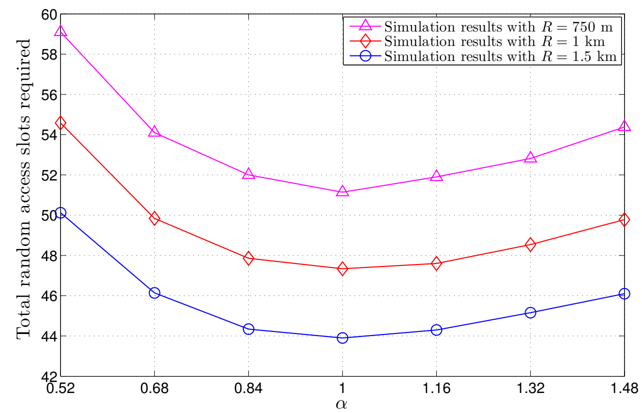


Fig. 5. Total random access slots required versus $\alpha$ and different deployment range $R$. The optimal ACB parameter $b^\star$ (*i.e.*, when $\alpha = 1$) requires the minimum number of random access slots to serve all MTC devices. ($N_0 = 2000$)

optimal value, the number of successfully served MTC devices reduces since letting more MTC devices participate in random access will increase the packet collision rate at the eNB.

### *B. Effect of Optimal ACB Parameters*

With the proposed scheme that uses both timing advance information and ACB to reduce the random access overload, we consider there are $N_0$ MTC devices at the beginning of a simulation run. Each MTC device within the initial backlog $N_0$ needs to be served exactly once. An MTC device which has not been served will keep on trying to pass the ACB check and request access to eNB until it is served in a random access slot. We run simulations over consecutive random access slots and count the number of random access slots required to serve all MTC devices. We show that using the optimal ACB parameter $b^\star$ can reduce the number of random access slots required to serve all $N_0$ backlog MTC devices when compared with using sub-optimal ACB parameters. We introduce a positive multiplier $\alpha$ and use the ACB parameter $b = \min\{1, \alpha b^\star\}$ in simulations. That is, the optimal ACB parameter $b^\star$ is utilized when $\alpha = 1$. We change the value of $\alpha$ from $0.52$ to $1.48$ with step size $0.16$ for each simulation run. Thus, random access slots required to serve $N_0$ MTC devices with the optimal or sub-optimal ACB parameters are compared. We run simulations 100 times with various $R$ and the initial backlog $N_0$ is equal to $2000$. The average results are presented in Fig. 5. We observe that using the optimal ACB parameter $b^\star$ (*i.e.*, $\alpha = 1$) in our proposed scheme requires the minimum number of random access slots to serve all MTC devices.

### *C. Performance Comparison with Other Schemes*

We compare our proposed scheme that uses both ACB and timing advance information with the following schemes in terms of total random access slots required to serve all MTC devices: (a) the scheme that uses only timing advance information in [22], (b) the scheme that uses only ACB in [21], (c) the cooperative ACB scheme that coordinates multiple eNBs in [19]. We first consider the traffic model that all $N_0$ MTC devices are activated simultaneously. For our proposed scheme, we run simulations with Algorithms 1 – 3, respectively. That is, we not only compare the performance of the proposed scheme with the numerical solution $b^\star$ and its closed-form approximation $b^\star_{appr}$ when the actual backlog $N_c$ in each random access slot is available to the eNB (Algorithms 1 and 2), but also study the performance of the proposed scheme by using the ACB parameter $b_{\hat{N}} = \frac{1.17m \ln \rho}{\hat{N}(\rho-1)}$ determined by the backlog estimation $\hat{N}$ in each random access slot (Algorithm 3). To obtain the best performance of the scheme that uses ACB only, we refer to the work in [21] and use its optimal ACB parameter $m/N_c$ in each random access slot with the actual backlog $N_c$. To simulate the cooperative ACB scheme, we use four eNBs to serve backlog MTC devices and allocate a number of preambles to each of them randomly in each simulation run. To compare cooperative ACB with other schemes in a fair manner, the total number of preambles used by the eNBs in cooperative ACB is the same as the number of preambles used by the eNB in other schemes. To achieve the best performance for cooperative ACB, we consider that each of the four eNBs is aware of the actual number of MTC devices requiring access to it.

We first compare the total random access slots required by the aforementioned schemes when $N_0 = 800$ MTC devices are activated together. The average results of $500$ simulation run with varying deployment range $R$ are shown in Fig. 6. Our proposed scheme consumes the least number of random access slots to serve all MTC devices. Both schemes that use timing advance information have better performance when the deployment range $R$ increases because fewer MTC devices have the same timing advance in sparse networks and the collision probability decreases accordingly. Results also show that the cooperative ACB scheme that uses four eNBs with total $m = 64$ preambles obtains the same performance as the scheme that uses only ACB with one eNB of $64$ preambles. This is because each eNB in the cooperative ACB scheme determines its optimal ACB parameter based on the actual number of MTC devices requiring access to it. Results show that for schemes only using ACB, the maximum number of MTC devices that can be served in a random access slot is

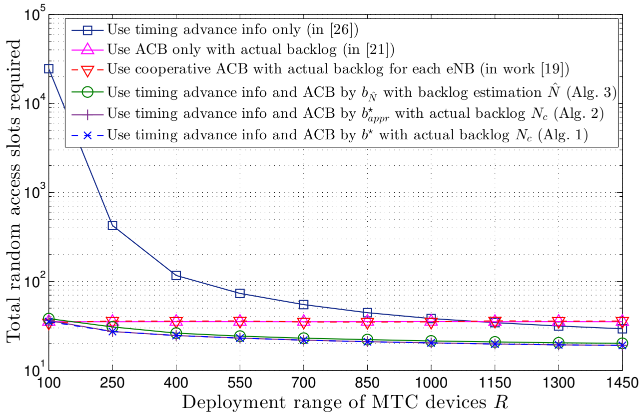


Fig. 6. Total random access slots required versus the deployment range of MTC devices $R$. ($N_0 = 800$, $m = 64$)

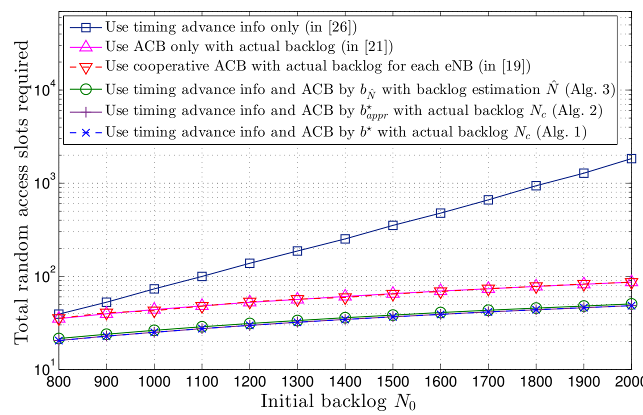


Fig. 7. Total random access slots required versus initial backlog $N_0$. ($R = 1$ km, $m = 64$)

determined by the total number of preambles in the network. This also explains the reason why using both timing advance information and ACB obtains much better performance than using either timing advance information or ACB only. We further notice that in sparse networks, using only timing advance information may require fewer random access slots than using ACB only. The reason is that the number of MTC devices $N_0 = 800$ is not very large and the optimal ACB parameter is equal to one in sparse networks. The effect of reducing random access overload with ACB vanishes. However, comparing timing advance in the received RAR before transmitting the L2/L3 message is still helpful to avoid packet collisions. We also find that the proposed scheme requires more random access slots to serve all MTC devices if these MTC devices are deployed in a smaller area. Eventually, when the deployment range $R$ is equal to $100$ m, the proposed scheme requires the same number of random access slots to serve all MTC devices as the scheme that uses ACB only. This phenomenon can be explained as follows. When $N_0$ MTC devices are located in a smaller area, the density of MTC devices increases and more MTC devices have the same timing advance information. Thus, the scheme that uses ACB only with one eNB is a special case of our proposed scheme when $R$ is small enough such that all MTC devices have identical timing advance information.

We compare Algorithms 1 – 3 for our proposed scheme. Results in Fig. 6 show that using $b^{\star}_{appr}$ (Algorithm 2) requires the same number of random access slots to serve $N_0 = 800$ initial backlog as using $b^{\star}$ (Algorithm 1) when the actual backlog $N_c$ is available to the eNB. Without the actual backlog $N_c$, almost the same performance is obtained by using the ACB parameter $b_{\hat{N}}$ determined by the backlog estimation $\hat{N}$ for each random access slot (Algorithm 3).

We now present how the number of initial backlog $N_0$ affects the number of random access slots required. Simulation results are presented in Fig. 7. For the schemes using ACB, the number of random access slots required to serve $N_0$ backlog MTC devices increases with $N_0$ linearly. The cooperative ACB scheme obtains the same performance as the scheme that uses only ACB. This follows the same reasons that we have explained for Fig. 6. For the scheme which uses timing advance information only, the required number of random

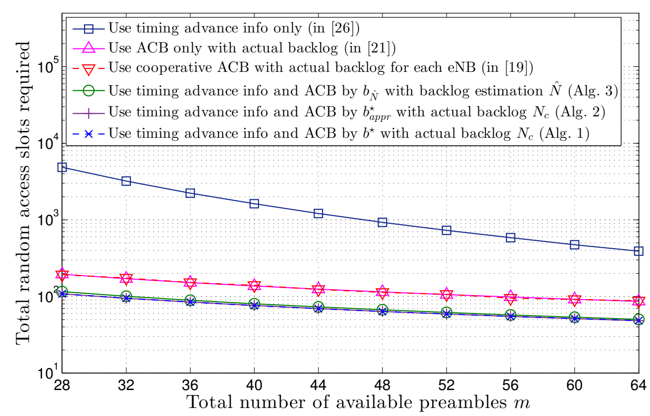


Fig. 8. Total random access slots required versus number of preambles $m$. ($N_0 = 2000$, $R = 1$ km)

access slots increases exponentially. This is because when $N_0$ increases, more packet collisions occur at the beginning of each simulation run. Thus, using only timing advance information requires more slots to serve all MTC devices. Our proposed scheme that uses both timing advance information and ACB requires the least random access slots in all scenarios, which reduces half of the number of random access slots compared to the other two schemes. Fig. 7 also compares the performance of the proposed scheme by using ACB parameters $b^{\star}$ in Algorithm 1 and $b^{\star}_{appr}$ in Algorithm 2 when actual backlog $N_c$ is available. The simulation results obtained by Algorithms 1 and 2 coincide with each other. Compared with Algorithms 1 and 2, Algorithm 3 obtains nearly the same performance by using the ACB parameter $b_{\hat{N}}$ determined by backlog estimation $\hat{N}$ in each random access slot.

We present simulation results for different number of preambles in Fig. 8. For all schemes, the number of random access slots required increases exponentially when fewer preambles are available. For the scheme that uses timing advance information only, the number of slots required to serve $N_0$ backlog MTC devices is an order of magnitude higher than those required by other schemes. Compared with schemes that use timing advance information only, ACB only, or cooperative ACB, our proposed scheme only requires half of random ac-

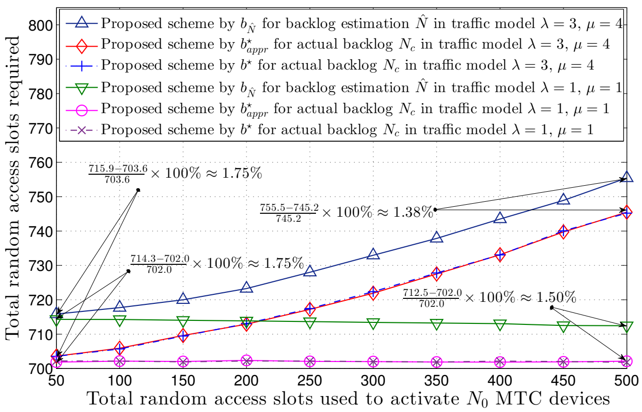


Fig. 9. Total random access slots required to serve $N_0$ MTC devices versus the number of random access slots used to activate $N_0$ MTC devices. ($N_0 = 30000$, $R = 1$ km, $m = 64$)

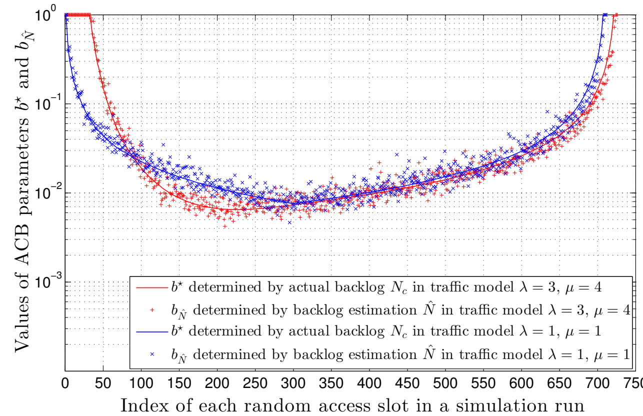


Fig. 10. Comparison of $b^{\star}$ and $b_{\hat{N}}$ determined by the actual backlog $N_c$ and its estimation $\hat{N}$ in each random access slot, respectively. ($N_0 = 30000$, $R = 1$ km, $m = 64$)

cess slots to serve all MTC devices because comparing timing advance information reduces the packet collision probability at eNB. The performance of the proposed scheme by using ACB parameter $b^{\star}$ and its approximation $b^{\star}_{appr}$ coincide with each other when the actual backlog $N_c$ is available to the eNB (Algorithms 1 and 2). Moreover, using the ACB parameter $b_{\hat{N}}$ determined by backlog estimation $\hat{N}$ in each random access slot (Algorithm 3) obtains almost the same performance as using $b^{\star}$ or $b^{\star}_{appr}$ for the actual backlog $N_c$.

### D. Performance with Different Traffic Models

The MTC devices may not be activated simultaneously but may be activated within a period of time. Let $V$ denote the length of time that all $N_0$ MTC devices are activated. According to the work in [28], the probability density function that a given MTC device is activated at time $v$ ($0 \leq v \leq V$) is given by $q(v; \lambda, \mu, V) = \frac{v^{\lambda-1}(V-v)^{\mu-1}}{V^{\lambda+\mu-2}\mathrm{Beta}(\lambda,\mu)}$, where $\mathrm{Beta}(\lambda, \mu)$ is the beta function. Two traffic models are suggested in [28] by changing $\lambda$ and $\mu$. We have $\lambda = 1$ and $\mu = 1$ when MTC devices are uniformly activated within the activation period. Otherwise, we have $\lambda = 3$ and $\mu = 4$. We increase $V$ from 50 to 500 random access slots to activate $N_0 = 30000$ MTC devices. For each pair of $\lambda$ and $\mu$, we compare the performance of our proposed scheme by using the ACB parameters $b^{\star}$ and $b^{\star}_{appr}$ when the actual backlog $N_c$ in each random access slot is available to the eNB. We also simulate our proposed scheme with ACB parameter $b_{\hat{N}}$ for the backlog estimation $\hat{N}$ in each random access slot. The average results of 200 simulations are given in Fig. 9. We find that the same performance is obtained with ACB parameters $b^{\star}$ and $b^{\star}_{appr}$ when the actual backlog $N_c$ is available. Without the actual backlog $N_c$, our proposed Algorithm 3, which applies ACB parameter $b_{\hat{N}}$ determined by the backlog estimation $\hat{N}$, takes only $1.38\%$ – $1.75\%$ more random access slots to served $N_0$ MTC devices compared with using ACB parameters $b^{\star}$ and $b^{\star}_{appr}$.

With the same values of $N_0$, $R$, and $m$ given above, Fig. 10 compares the ACB parameters $b^{\star}$ determined by the actual backlog $N_c$ and $b_{\hat{N}}$ determined by the backlog estimation $\hat{N}$ in each random access slot of a simulation run. We conduct two simulation run with two traffic models $\lambda = 3$, $\mu = 4$ and $\lambda = 1$, $\mu = 1$, respectively. We find that the ACB parameter $b_{\hat{N}} = \frac{1.17m \ln \rho}{\hat{N}(\rho-1)}$ determined by the backlog estimation $\hat{N}$ in each random access slot is close to the $b^{\star}$ determined by the actual backlog $N_c$ in the corresponding random access slot. Moreover, both $b^{\star}$ and $b_{\hat{N}}$ are equal to 1 in the first 33 random access slots with the traffic model $\lambda = 3$, $\mu = 4$. This is because the backlog increases slowly at the beginning of the simulation. This also explains the reason why more random access slots are required to serve all MTC devices when they are activated within a longer activation duration $V$ with the traffic model $\lambda = 3$, $\mu = 4$ in Fig. 9.

## V. Conclusions

In this paper, we proposed to use both ACB and the timing advance information to relieve the random access overload in M2M systems. We determined the optimal ACB parameter $b^{\star}$ which maximizes the expected number of MTC devices successfully served in each random access slot. To reduce the computational complexity in determining the optimal ACB parameter $b^{\star}$, we proposed a closed-form solution $b^{\star}_{appr}$ to approximate $b^{\star}$. We also presented an algorithm to estimate the number of MTC devices that require access to eNB in each random access slot. Through simulations, we validated our analytical results and showed that the closed-form approximate solution $b^{\star}_{appr}$ obtains the same performance as the numerical solution $b^{\star}$. We further showed that our scheme works well with the proposed backlog estimation algorithm in various traffic models. We found that almost $50\%$ of the random access slots can be saved to serve all MTC devices when compared with other schemes that use either timing advance information or ACB only. For future work, our proposed scheme can be extended by using different ACB parameters for MTC devices with different priorities or QoS requirements. Moreover, for uniformly distributed MTC devices, the MTC devices that are closer to the eNB may have a higher chance to be served. This is because the number of MTC devices with the same timing advance information decreases when their distance to the eNB is reduced. This fairness issue in our proposed scheme is expected to be addressed in the future work. We expect that our work will motivate more efficient mechanisms that can better support M2M applications in LTE networks.

## Appendix A
## Proof of Proposition 1

The first-order partial derivative of $\mathcal{H}(R,N,m,b)$ with respect to (*w.r.t.*) $b$ is given by

$$\frac{\partial}{\partial b}\ \mathcal{H}(R,N,m,b) = \frac{2N}{R^2}\int_0^R \frac{r}{1-p(r)}\left(e^{-\frac{Nb}{m}} - p(r)\, e^{-\frac{Nb}{m}p(r)}\right)\mathrm{d}r. \quad (28)$$

Let function $g(b,r) = e^{-\frac{Nb}{m}} - p(r)\, e^{-\frac{Nb}{m}p(r)}$. The first-order partial derivative of $g(b,r)$ *w.r.t.* $r$ is given by

$$\frac{\partial}{\partial r}\ g(b,r) = \frac{\partial g(b,r)}{\partial p(r)}\frac{\partial p(r)}{\partial r} = \frac{\partial p(r)}{\partial r}\left(\frac{Nb}{m}p(r)-1\right)e^{-\frac{Nb}{m}p(r)}. \quad (29)$$

Note that $e^{-\frac{Nb}{m}p(r)} > 0$. The sign of (29) is determined by $\frac{\partial p(r)}{\partial r}\left(\frac{Nb}{m}p(r)-1\right)$. It can be shown that $p(r)$ increases with $r$ on $[0, R-d)$ and decreases with $r$ on $(R-d, R]$. Let $p_u$ denote the maximum value that $p(r)$ can obtain. We have $p_u = \frac{4(R-d)d}{R^2}$. When $\frac{Nb}{m}p(r) - 1 < 0$, we have $b < \frac{m}{Np(r)}$. To make the inequality hold for $r \in [0,R]$, we have $b < \min_{r\in[0,R]} \frac{m}{Np(r)}$. That is,

$$b < \frac{m}{Np_u}. \quad (30)$$

For $b \in \left[0, \frac{m}{Np_u}\right)$, we have $\frac{\partial}{\partial r}g(b,r) < 0$ for $r \in [0, R-d)$ and $\frac{\partial}{\partial r}g(b,r) > 0$ for $r \in (R-d, R]$. Thus, $g(b,r)$ obtains the minimum value at $r = R-d$ on the interval $[0,R]$. When $g(b, R-d) \geq 0$, *i.e.*, $e^{-\frac{Nb}{m}} - p_u e^{-\frac{Nb}{m}p_u} \geq 0$, we obtain another interval of $b$ as follows:

$$b \leq \frac{m \ln p_u}{N(p_u - 1)}. \quad (31)$$

Note that $0 < p_u < 1$, we introduce another variable $\vartheta = 1 - p_u$, where $0 < \vartheta < 1$. We have

$$\frac{\ln p_u}{p_u - 1} = \frac{\ln(1-\vartheta)}{-\vartheta} = \frac{-\sum_{k=1}^{\infty}\frac{\vartheta^k}{k}}{-\vartheta} = \sum_{k=0}^{\infty}\frac{\vartheta^k}{k+1}, \quad (32)$$

and

$$\frac{1}{p_u} = \frac{1}{1-\vartheta} = \sum_{k=0}^{\infty}\vartheta^k. \quad (33)$$

Since $\sum_{k=0}^{\infty}\frac{\vartheta^k}{k+1} < \sum_{k=0}^{\infty}\vartheta^k$, the interval of $b$ given by (30) contains the interval of $b$ given by (31). Thus, for $b \in \left[0, \frac{m\ln p_u}{N(p_u-1)}\right]$ and $r \in [0,R]$, we have $g(b,r) \geq 0$ and the equality holds when $b = \frac{m\ln p_u}{N(p_u-1)}$ and $r = R-d$.

Consider (28) and note that $\frac{2N}{R^2} > 0$. For $b \in \left[0, \frac{m\ln p_u}{N(p_u-1)}\right]$, the integrand of the integral is positive for $r \in (0, R-d)$ and $r \in (R-d, R]$, and is nonnegative when $r = 0$ or $r = R-d$. Thus, $\frac{\partial}{\partial b}\mathcal{H}(R,N,m,b) > 0$ for $b \in \left[0, \frac{m\ln p_u}{N(p_u-1)}\right]$. That is, $\mathcal{H}(R,N,m,b)$ is strictly increasing with $b$ on the interval $\left[0, \frac{m\ln\rho}{N(\rho-1)}\right]$ by noting that $\rho = p_u$, which completes the proof. ■

## Appendix B
## Proof of Proposition 2

The second-order partial derivative of $\mathcal{H}(R,N,m,b)$ *w.r.t.* $b$ is given by

$$\frac{\partial^2}{\partial b^2}\ \mathcal{H}(R,N,m,b) = \frac{2N^2}{mR^2}\int_0^R \frac{r}{1-p(r)}\left(p^2(r)\, e^{-\frac{Nb}{m}p(r)} - e^{-\frac{Nb}{m}}\right)\mathrm{d}r. \quad (34)$$

Let function $f(b,r) = p^2(r)\, e^{-\frac{Nb}{m}p(r)} - e^{-\frac{Nb}{m}}$. The first-order partial derivative of $f(b,r)$ *w.r.t.* $r$ is given by

$$\frac{\partial}{\partial r}\ f(b,r) = \frac{\partial f(b,r)}{\partial p(r)}\frac{\partial p(r)}{\partial r} = -\frac{\partial p(r)}{\partial r}\left(\frac{Nb}{m}p(r)-2\right)p(r)\, e^{-\frac{Nb}{m}p(r)}. \quad (35)$$

Since $-p(r)\, e^{-\frac{Nb}{m}p(r)} < 0$, the sign of (35) is determined by $\frac{\partial p(r)}{\partial r}\left(\frac{Nb}{m}p(r)-2\right)$. Denote $p_u$ as the maximum value that $p(r)$ can obtain, (*i.e.*, $p_u = \frac{4(R-d)d}{R^2}$). When $\frac{Nb}{m}p(r) - 2 < 0$, we have $b < \frac{2m}{Np(r)}$. To make the inequality hold for $r \in [0,R]$, we have $b < \min_{r\in[0,R]}\frac{2m}{Np(r)}$. That is,

$$b < \frac{2m}{Np_u}. \quad (36)$$

Since $p(r)$ increases with $r$ on $[0, R-d)$ and decreases with $r$ on $(R-d, R]$. For $b \in \left[0, \frac{2m}{Np_u}\right)$, we have $\frac{\partial}{\partial r}f(b,r) > 0$ for $r \in [0, R-d)$ and $\frac{\partial}{\partial r}f(b,r) < 0$ for $r \in (R-d, R]$. Thus, $f(b,r)$ obtains its maximum value at $r = R-d$ on the interval $[0,R]$. By making $f(b, R-d) \leq 0$, *i.e.*, $p_u^2 e^{-\frac{Nb}{m}p_u} - e^{-\frac{Nb}{m}} \leq 0$, we obtain another interval of $b$ which is given by

$$b \leq \frac{2m\ln p_u}{N(p_u - 1)}. \quad (37)$$

Since $\frac{\ln p_u}{p_u - 1} < \frac{1}{p_u}$, the interval of $b$ in (37) is contained by the interval of $b$ in (36). Thus, for $b \in \left[0, \frac{2m\ln p_u}{N(p_u-1)}\right]$ and $r \in [0,R]$, we have $f(b,r) \leq 0$ and the equality holds when $b = \frac{2m\ln p_u}{N(p_u-1)}$ and $r = R-d$.

Consider equation (34) and note that $\frac{2N^2}{mR^2} > 0$. For $b \in \left[0, \frac{2m\ln p_u}{N(p_u-1)}\right]$, the integrand of the integral is negative for $r \in (0, R-d)$ and $r \in (R-d, R]$, and is nonpositive when $r = 0$ or $r = R-d$. Thus, $\frac{\partial^2}{\partial b^2}\mathcal{H}(R,N,m,b) < 0$. That is, $\mathcal{H}(R,N,m,b)$ is strictly concave with $b$ on the interval $\left[0, \frac{2m\ln\rho}{N(\rho-1)}\right]$ by noting that $\rho = p_u$, which completes the proof. ■

## Appendix C
## Proof of Theorem 1

First of all, we introduce the following lemma:

*Lemma 1:* There exists exactly one value of $b$ that makes $\frac{\partial}{\partial b}\ \mathcal{H}(R,N,m,b) = 0$.

*Proof:* We assume both $b = b'$ and $b = b' + \sigma$ can make (28) equal to 0. We have

$$\int_0^R \frac{r}{1-p(r)}e^{-\frac{Nb'}{m}}\ \mathrm{d}r = \int_0^R \frac{rp(r)}{1-p(r)}e^{-\frac{Nb'}{m}p(r)}\ \mathrm{d}r, \quad (38)$$

and

$$\int_0^R \frac{r}{1-p(r)} e^{-\frac{N(b'+\sigma)}{m}} \, \mathrm{d}r = \int_0^R \frac{rp(r)}{1-p(r)} e^{-\frac{N(b'+\sigma)}{m} p(r)} \, \mathrm{d}r. \tag{39}$$

We multiply $e^{-\frac{N\sigma}{m}}$ on both sides of (38) and note that the left hand side of the result is identical with the left hand side of (39). Thus, the difference of their right hand sides is 0, *i.e.*,

$$\int_0^R \frac{rp(r)}{1-p(r)} e^{-\frac{Nb'}{m} p(r)} \left( e^{-\frac{N\sigma}{m} p(r)} - e^{-\frac{N\sigma}{m}} \right) \, \mathrm{d}r = 0. \tag{40}$$

Note that $\frac{rp(r)}{1-p(r)} e^{-\frac{Nb'}{m} p(r)} \geq 0$ with the equality holds at $r = 0$. Moreover, $e^{-\frac{N\sigma}{m} p(r)} - e^{-\frac{N\sigma}{m}} > 0$ when $\sigma > 0$. In addition, $e^{-\frac{N\sigma}{m} p(r)} - e^{-\frac{N\sigma}{m}} < 0$ when $\sigma < 0$. Thus, $\sigma = 0$ is the only passibility, which completes the proof. □

By substituting $b = \frac{2m \ln \rho}{N(\rho-1)}$ into (28), which is the right boundary of interval $\left[0, \frac{2m \ln \rho}{N(\rho-1)}\right]$ in Proposition 2, we have

$$\begin{aligned} &\frac{\partial}{\partial b} \mathcal{H}(R, N, m, b) \mid_{b=\frac{2m \ln \rho}{N(\rho-1)}} \\ &= \frac{2N}{R^2} \int_0^R \frac{r}{1-p(r)} \left( \rho^{\frac{2}{1-\rho}} - p(r) \rho^{\frac{2}{1-\rho} p(r)} \right) \mathrm{d}r. \end{aligned} \tag{41}$$

Note that the sign of (41) depends on $R$ only, and it is always negative for $R > 2d$. Based on Lemma 1 and Propositions 1 and 2, the value of $b$ that makes $\frac{\partial}{\partial b} \mathcal{H}(R, N, m, b)$ equal to 0 must be on the interval $\left( \frac{m \ln \rho}{N(\rho-1)}, \frac{2m \ln \rho}{N(\rho-1)} \right)$. By taking the constraint in problem (19) (*i.e.*, $0 \leq b \leq 1$) into account, we have $b^\star = 1$ when $\frac{m \ln \rho}{N(\rho-1)} \geq 1$ (*i.e.*, $N \leq \frac{m \ln \rho}{(\rho-1)}$). By letting $\frac{2m \ln \rho}{N(\rho-1)} \leq 1$, we have $b^\star \in \left( \frac{m \ln \rho}{N(\rho-1)}, \frac{2m \ln \rho}{N(\rho-1)} \right)$ for $N \geq \frac{2m \ln \rho}{(\rho-1)}$. Otherwise, for $\frac{m \ln \rho}{(\rho-1)} < N < \frac{2m \ln \rho}{(\rho-1)}$, we have $b^\star \in \left( \frac{m \ln \rho}{N(\rho-1)}, 1 \right]$, which completes the proof. ■

## APPENDIX D
## PROOF OF THEOREM 2

We consider $N_c$ MTC devices are requiring access to the eNB in the current random access slot. With the backlog estimation $\hat{N}$, the ACB parameter $b_{\hat{N}} = \frac{1.17m \ln \rho}{\hat{N}(\rho-1)}$ is used in the random access slot. By using ACB parameter $b_{\hat{N}}$, let random variables $X_{\hat{N}}$ and $Z_{\hat{N}}$ denote the number of preambles not used by any MTC device and the number of MTC devices successfully served in the current random access slot, respectively. Thus, $Z_{\hat{N}}$ is equal to the number of selected preambles that succeed to serve MTC devices. Since the number of available preambles is $m$, the expected preamble collision ratio is given by

$$\begin{aligned} \mathbb{E}\left[\Psi_{\hat{N}}\right] &= \mathbb{E}\left[\frac{m - X_{\hat{N}} - Z_{\hat{N}}}{m}\right] \\ &= 1 - \frac{\mathbb{E}\left[X_{\hat{N}}\right]}{m} - \frac{\mathbb{E}\left[Z_{\hat{N}}\right]}{m}. \end{aligned} \tag{42}$$

Recall that $I_u = 1$ (or $I_u = 0$) denotes the event that an arbitrary MTC device $u$ passes the ACB check (or not) and $J_u = j$ denotes the event that MTC device $u$ selects preamble $j$ uniformly from $m$ available preambles. The probability that $u$ selects preamble $j$ is given by

$$\begin{aligned} \mathbb{P}(I_u = 1, J_u = j) &= \mathbb{P}(I_u = 1)\, \mathbb{P}(J_u = j \mid I_u = 1) \\ &= \frac{1.17 \ln \rho}{\hat{N}(\rho-1)}. \end{aligned} \tag{43}$$

For an arbitrary preamble $j$, let $K_j = 0$ denote the event that preamble $j$ is not selected by any MTC device. The probability of $K_j = 0$ is given by

$$\begin{aligned} \mathbb{P}(K_j = 0) &= \binom{N}{0} \left( \frac{1.17 \ln \rho}{\hat{N}(\rho-1)} \right)^0 \left( 1 - \frac{1.17 \ln \rho}{\hat{N}(\rho-1)} \right)^N \\ &= \left( 1 - \frac{1.17 \ln \rho}{\hat{N}(\rho-1)} \right)^N. \end{aligned} \tag{44}$$

For large $\hat{N}$ and $N_c$, we have the following approximation

$$\frac{\mathbb{E}\left[X_{\hat{N}}\right]}{m} = \frac{m\mathbb{P}(K_j = 0)}{m} \approx e^{-\frac{N_c}{\hat{N}} \left( \frac{1.17 \ln \rho}{\rho-1} \right)}. \tag{45}$$

By substituting ACB parameter $b_{\hat{N}} = \frac{1.17m \ln \rho}{\hat{N}(\rho-1)}$ into (18) and rearranging the result, we have

$$\begin{aligned} &\frac{\mathbb{E}\left[Z_{\hat{N}}\right]}{m} \\ &\approx \frac{2}{R^2} \int_0^R \frac{r}{1-p(r)} \left( e^{-\frac{N_c}{\hat{N}} \left( \frac{1.17 \ln \rho}{\rho-1} p(r) \right)} - e^{-\frac{N_c}{\hat{N}} \left( \frac{1.17 \ln \rho}{\rho-1} \right)} \right) \mathrm{d}r. \end{aligned} \tag{46}$$

Equation (27) is obtained by substituting (45) and (46) into (42). We now consider another estimation $\hat{N}'$ of the $N_c$ MTC devices, we have $\frac{N_c}{\hat{N}'} > \frac{N_c}{\hat{N}}$ if and only if $\hat{N}' < \hat{N}$. Thus, another sub-optimal ACB parameter $b_{\hat{N}'} = \frac{1.17m \ln \rho}{\hat{N}'(\rho-1)}$ for $\hat{N}'$ must be greater than the sub-optimal ACB parameter $b_{\hat{N}} = \frac{1.17m \ln \rho}{\hat{N}(\rho-1)}$ for $\hat{N}$. That is, increasing $\frac{N_c}{\hat{N}}$ actually increases the ACB parameter used in a random access slot. Thus, the expected preamble collision ratio in (27) strictly increases with the value of $\frac{N_c}{\hat{N}}$, which completes the proof. ■

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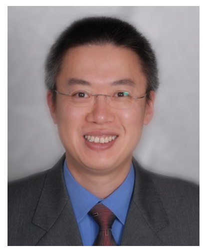

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