

Load Scheduling and Power Trading in Systems with High Penetration of Renewable Energy Resources

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Abstract—In this paper, we focus on the problems of *load scheduling* and *power trading* in systems with high penetration of renewable energy resources (RERs). We adopt approximate dynamic programming to schedule the operation of different types of appliances including must-run and controllable appliances. We assume that users can sell their excess power generation to other users or to the utility company. Since it is more profitable for users to trade energy with other users locally, users with excess generation compete with each other to sell their respective extra power to their neighbors. A game theoretic approach is adopted to model the interaction between users with excess generation. In our system model, each user aims to obtain a larger share of the market and to maximize its revenue by appropriately selecting its offered price and generation. In addition to yielding a higher revenue, consuming the excess generation locally reduces the *reverse power flow*, which impacts the stability of the system. Simulation results show that our proposed algorithm reduces the energy expenses of the users. The proposed algorithm also facilitates the utilization of RERs by encouraging users to consume excess generation locally rather than injecting it back into the power grid.

Index Terms—Demand side management, load scheduling, power trading, approximate dynamic programming.

I. INTRODUCTION

Concerns about environmental issues and the need to reduce greenhouse gas emission have attracted considerable attention to environmentally friendly renewable energy resources (RERs). Regulations have been passed to increase the production of energy from renewable energy resources. These regulations are referred to as the renewable portfolio standard, and require the utility companies and energy providers in the United States and the United Kingdom to serve a specific minimum amount of their customers' load with RERs [1]. RERs such as solar and wind are non-dispatchable, since they are random in nature. In systems with high penetration of RERs, the power may flow from *distributed generators* (DGs) to the substation, which negatively impacts the stability of the system. If the reverse power flow exceeds a certain threshold, it causes the *voltage rise* problem, which is a major challenge in integrating a large number of DGs in the distribution network [2]–[4].

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To tackle the reverse power flow problem, it is desirable that users consume their generating power locally rather than injecting the excess power back into the grid. Storage facilities and demand side management (DSM) techniques can be adopted to shape the load pattern of the users to better match supply and demand [5]–[9]. Furthermore, users are able to trade their excess generation with other local users, if the underlying system is equipped with advanced metering infrastructure (AMI), flow control infrastructure, and communication capabilities. Such infrastructure is found in advanced networks such as microgrids. Microgrids are autonomous systems which can operate either in a grid-connected or islanded mode. These systems benefit from their own distributed generation, and to achieve a high level of reliability, they may have a grid topology different from the tree structure which is usually found in conventional distribution networks [10]–[13]. Hence, in microgrids, power can flow through different paths from one node to another. Thus, users can control the flow of power to other users in such systems. The probability that the voltage rise problem occurs increases when more users decide to inject their excess generation via the main feeder into the grid. Therefore, the ability of users to route their excess power directly to their neighbors reduces the probability that the voltage rise problem occurs. Local power trading can benefit the users by providing monetary revenue for them. Furthermore, local trading in microgrids that act autonomously and have their own market regulations facilitates the integration of RERs. Considering the conventional market regulations, there are two main barriers for the integration of RERs. First, RERs are random in nature, and it is difficult to dispatch their output generation in a day-ahead market. Second, the power generation from RERs is relatively small compared to the amounts which are traded in a day-ahead electricity market. With local trading, users can sell their excess power generation in an opportunistic manner in the current time slot, when they have a more accurate estimate of their output generation compared to the estimate they had the previous day. Furthermore, users can sell their excess generation at a smaller scale, e.g., within a microgrid.

Different DSM programs have been designed to facilitate the integration of RERs into the power grid, and the impact of different decision makers on the electricity market has been studied [14]–[23]. Most of the existing work in the literature considers the case where users can sell their excess generation back to the grid [21]–[23]. However, despite its importance,

the possibility of trading energy among local users and the related benefits that users may obtain due to competition between local generators have not been well investigated.

In this paper, we focus on modeling the interaction between users that can sell and buy locally generated electricity. In our model, users can offer their excess local generation capacity to other users with an appropriately selected price. For a given user, the selected price and the offered generation capacity depend on both the marginal cost of the user and the price offered by other users. Thus, users form a *game* in which they aim to maximize their own revenue. Due to the competition between multiple local generators, the consuming users may benefit from a lower price compared to the price advertised by the utility company. Our main contributions are as follows:

- We consider a game theoretic approach to model the interaction of users with excess power generation. Users compete to sell their extra generation to other local users. The revenue of competing users depends on their own marginal cost and the offers advertised by other competing users. Thus, each competing user chooses its offered price and output generation such that its revenue is maximized.
- We formulate the problem of selecting the offered price and output generation (i.e., the trading problem) as a linear mixed-integer program. To tackle the complexity of the trading problem, we adopt the generalized Benders' decomposition approach.
- We propose an approximate dynamic programming approach to schedule the operation of must-run, interruptible controllable, and non-interruptible controllable appliances. The linearity of the proposed approximated scheduling problem makes it possible to schedule the operation of different appliances independently. Independent scheduling of appliances significantly reduces the complexity of the scheduling algorithm and makes the real-time implementation of the algorithm possible.
- Simulation results show that our proposed algorithm reduces the energy payment of users compared to the case where trading is not applied. Our proposed scheme facilitates the integration of RERs and mitigates the reverse power flow problem by providing the users an opportunity to trade their excess generation locally.

The rest of this paper is organized as follows. The system model is introduced in Section II. In Section III, the problem formulation and algorithm description are presented. Simulation results are provided in Section IV, and Section V concludes the paper. The most important variables used in this paper are listed in Table I.

II. SYSTEM MODEL

We consider a smart power system with a single utility company and several users. Each user is equipped with a renewable DG, such as photovoltaic cells or wind turbines. The demand requirements of the users are met by their local generation, power imported from other users, and power imported from the utility company. Let $\mathcal{U}$ denote the set of users. We assume that each user $u \in \mathcal{U}$ is equipped with

TABLE I
LIST OF NOTATIONS AND VARIABLES USED IN THIS PAPER.

$\mathcal{U}$	Set of users
T	Number of time slots
$\mathcal{T}$	Set of all time slots ($\mathcal{T} \triangleq \{1, \dots, T\}$)
$\mathcal{A}_u$	Set of all appliances of user u
$T_{u,a}$	Feasible scheduling interval of appliance a for user u ($T_{u,a} \triangleq [\alpha_{u,a}, \beta_{u,a}]$)
$\alpha_{u,a}$	Earliest time at which appliance a can start operating
$\beta_{u,a}$	Deadline by which the operation of appliance a has to be finished
$\mathbf{P}_a$	Pattern of power consumption of appliance a if no scheduling is applied ($\mathbf{P}_a \triangleq (P_1^a, \dots, P_T^a)$)
P_i^a	Power consumption at i th operating cycle of appliance a
I_a	Number of operating cycles of appliance a
$\mathcal{I}_a$	Set of operating cycles of appliance a
$\chi_t^{u,a}$	State of appliance a at time slot t ($\chi_t^{u,a} \triangleq (q_t^{u,a}, w_t^{u,a})$)
$q_t^{u,a}$	Number of remaining operating cycles of appliance a
$w_t^{u,a}$	Number of time slots for which the operation of appliance a can be delayed
$x_t^{u,a}$	Indicator showing if appliance a is scheduled to operate at time slot t or not
B_b	Storage capacity of the battery
e_b	Maximum charging and discharging rates of the battery
y_t^u	Charging / discharging rate of user u 's battery
Λ_t^u	Charging state of user u 's battery at the beginning of time slot t
g_t^u	Total power exported by user u at time slot t
$\mathbb{C}_t^u(\cdot)$	Cost of providing g_t^u units of energy for user u at time slot t
a_t^u	Cost function parameter of user u
b_t^u	Cost function parameter of user u
c_t^u	Cost function parameter of user u
λ_t^h	Price value set by the utility company for time slot t
λ_t^l	Price value at which the utility company buys energy from users at time slot t
L_t^u	Load of user u at time slot t
ψ_t^u	Amount of generated power of user u at time slot t
μ_t	Market clearing price at time slot t
j_t^u	Auxiliary binary variable
h_t^u	Portion of generated power sold at price μ_t
$\mathbb{X}_t^u$	Set of actions available for each appliance a at state χ_t^u
$\mathbb{Y}_t^u$	Feasible operating set of the battery
$\mathbb{V}_t^u$	Value function of user u at time slot t
$\theta_t^u(\cdot)$	Effective market clearing price for user u at time slot t
η_t^u	Weight coefficient of user u at time slot t
δ	Observation weight
$\mathcal{G}_t$	Set of all competing users at time slot t
N_t	Number of competing users at time slot t
$\mathcal{G}_t^{-u}$	Set of all competing users other than user u
$\mathcal{O}_t^u$	Offer of competing user u at time slot t
π_t^u	Offered price of competing user u at time slot t
g_t^u	Offered generation capacity of competing user u at time slot t
G_t^u	Maximum generating capacity of competing user u at time slot t
$\mathbb{O}_t^{-u}$	Vector of offers advertised by all competing users other than user u at time slot t
z_t^n	Auxiliary binary variable
$\mathbb{S}_t^n$	Group of competing users with offered price less than or equal to π_t^n
S_t^n	Aggregate offered generation of users in $\mathbb{S}_t^n$
d_t^n, f_t	Auxiliary binary variables
K_1, K_2	Large positive constants

a smart meter which has an energy consumption controller (ECC) capable of scheduling and adjusting the household

energy consumption. The ECC units of all users are connected. We divide the intended operation cycle into $T \triangleq |\mathcal{T}|$ time slots, where $\mathcal{T} \triangleq \{1, \dots, T\}$.

Let $\mathcal{A}_u$ denote the set of all appliances of user u . We assume that based on the demand requirements of the user, each appliance can be set as either *must-run* or *controllable*. This setting is decided by the user and can vary from time to time. The ECC has no control over the operation of must-run appliances. In contrast, the operation of controllable appliances can be *delayed* or *interrupted* if necessary. Each controllable appliance can be either *interruptible* or *non-interruptible*. For interruptible appliance a , the ECC may delay or interrupt its operation. However, for non-interruptible appliance a , it is only possible to delay its operation. We assume that the mode of operation of the appliances, i.e., whether they are must-run or controllable, is not pre-determined and depends on the current preference of the user. That is, based on the preference of the user, those appliances may work either as must-run or controllable. Controllable appliances can be modeled as interruptible if the appliance's task can be completed in disjoint time intervals, and the interruption of the operation does not impact the completion of the task [24]. For example, charge of electric vehicles and pool water pumps can be modeled as interruptible controllable appliances. Since users may only require that their vehicle be fully charged or their pool be full of water by a given deadline, the charging or pumping process can be interrupted and resumed as long as it is possible to meet the deadline¹. In each time slot, appliances can be either *on* or *off*. Moreover, they can operate in a limited number of operating modes [25], [26]. Therefore, we model their operation as a discrete Markov process as will be explained in the following.

At the beginning of a time slot, each appliance of user u that is about to start operation sends an *admission request* to the ECC unit. The admission request specifies whether the appliance is must-run or controllable and its operational specifications, i.e., $T_{u,a} \triangleq [\alpha_{u,a}, \beta_{u,a}]$, where $\alpha_{u,a}$ is the earliest time at which appliance a can start operating (i.e., the time slot it becomes awake), and $\beta_{u,a}$ is the deadline by which the operation of appliance a has to be finished. The power consumption of each appliance can be different at different cycles of its operation due to changes in the amount of current being absorbed. We define vector $\mathbf{P}_a \triangleq (P_1^a, \dots, P_{I_a}^a)$ as the power consumption pattern of appliance a if *no scheduling* is applied, where P_i^a is the power consumption of appliance a at its i th operating cycle, $I_a \triangleq |\mathcal{I}_a|$ is the number of operating cycles of appliance a , and $\mathcal{I}_a \triangleq \{1, \dots, I_a\}$. We also assume that the duration of each operating cycle is one time slot. In practice, if the actual operating cycle lasts for more than one time slot, this can be modeled by introducing multiple consecutive operating cycles having identical values for the power consumption. An example for the pattern of power consumption of controllable appliance a before and after scheduling is illustrated in Fig. 1.

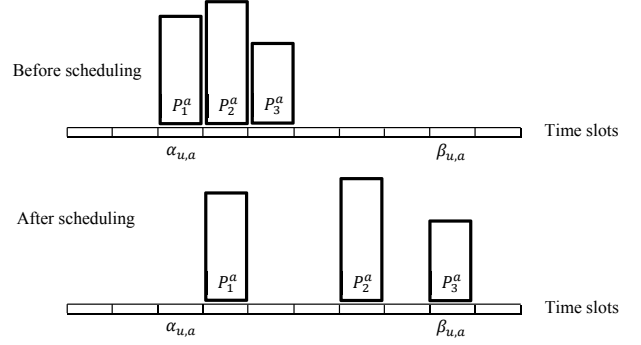


Fig. 1. An example for the pattern of power consumption of interruptible controllable appliance a before and after scheduling.

For each appliance a of user u , we define $\chi_t^{u,a} \triangleq (q_t^{u,a}, w_t^{u,a})$ as the state of appliance a at time slot t , where $q_t^{u,a}$ is the number of remaining operating cycles of appliance a , and $w_t^{u,a}$ is the number of time slots for which the operation of appliance a can be delayed. We define $x_t^{u,a} \in \{0, 1\}$ as an indicator which shows whether appliance a of user u is scheduled to operate at time slot t ($x_t^{u,a} = 1$) or not ($x_t^{u,a} = 0$). The state of appliance a at the next time slot $t+1$, $\chi_{t+1}^{u,a}$, can be inferred from its current state $\chi_t^{u,a}$, its type, and the operating decision $x_t^{u,a}$. For a must-run appliance a , the initial state is $\chi_{\alpha_{u,a}}^{u,a} = (I_a, 0)$. A must-run appliance a starts operation immediately (i.e., $x_t^{u,a} = 1$, if $q_t^{u,a} \geq 1$) and its state $\chi_{t+1}^{u,a} = (q_{t+1}^{u,a}, 0)$ evolves as

$$\chi_{t+1}^{u,a} = (q_t^{u,a} - 1, 0), \quad \text{for } q_t^{u,a} > 0. \quad (1)$$

For a non-interruptible appliance a , it is only possible to delay its operation. Once it has started operation, it is not possible to interrupt its operation. The initial state of a non-interruptible appliance a is $\chi_{\alpha_{u,a}}^{u,a} = (I_a, \beta_{u,a} - \alpha_{u,a} - I_a + 1)$. The state evolves as

$$\chi_{t+1}^{u,a} = \begin{cases} (q_t^{u,a}, w_t^{u,a} - 1), & \text{if } x_t^{u,a} = 0, w_t^{u,a} \geq 1, \\ (q_t^{u,a} - 1, 0), & \text{if } x_t^{u,a} = 1, q_t^{u,a} \geq 1. \end{cases} \quad (2)$$

For an interruptible appliance a , it is not only possible to delay operation but also to interrupt operation if required. The initial state of an interruptible appliance a is $\chi_{\alpha_{u,a}}^{u,a} = (I_a, \beta_{u,a} - \alpha_{u,a} - I_a + 1)$, and it evolves as

$$\chi_{t+1}^{u,a} = \begin{cases} (q_t^{u,a}, w_t^{u,a} - 1), & \text{if } x_t^{u,a} = 0, w_t^{u,a} \geq 1, \\ (q_t^{u,a} - 1, w_t^{u,a}), & \text{if } x_t^{u,a} = 1, q_t^{u,a} \geq 1. \end{cases} \quad (3)$$

We also define $\chi_t^u = (\chi_t^{u,1}, \dots, \chi_t^{u,|\mathcal{A}_u|})$.

To better utilize the RERs and match demand and supply, we assume that each user is equipped with a storage device such as a battery. The charging and discharging of the battery is modeled as a continuous process. We define B_b as the storage capacity of the battery. We assume that the maximum charging and discharging rates of the battery are identical and denoted by e_b . For time slot t , we define variable $y_t^u \in [-e_b, e_b]$ as the charging / discharging rate of user u 's battery. Moreover, Λ_t^u is the charging state of user u 's battery at the beginning of time slot t . Thus, we have

$$\Lambda_t^u = \Lambda_0^u + \sum_{k=1}^{t-1} y_k^u, \quad (4)$$

¹To avoid frequent interruptions, it is possible to consider costs associated with the interruption of the operation of an appliance. However, adding this extra term makes the design more complicated and implementation in real-time settings becomes more difficult.

where Λ_0^u is the initial charging state of the battery. At any time slot t , the stored energy cannot exceed the storage capacity B_b . Moreover, it is not possible to extract more energy from the storage unit than what is stored, i.e.,

$$0 \leq \Lambda_t^u \leq B_b. \quad (5)$$

Let g_t^u denote the total power exported by user u at time slot t . Users may utilize different technologies or different types of RERs to generate power. The intermittent nature of the RERs may cause problems regarding stability, voltage regulation, and power quality. In general, the quality of the generated power needs to be enhanced before it can be transmitted to other users. Different techniques have been proposed to improve the quality of power which results in additional costs for generators [27]. Therefore, the maintenance and operation cost is different for different users. We consider a *cost function* $\mathbb{C}_t^u(g_t^u)$ indicating the cost incurred to user u for providing g_t^u units of energy at time slot t . We assume that the cost function is *increasing* and *strictly convex* in the offered energy. In this paper, we consider a quadratic cost function:

$$\mathbb{C}_t^u(g_t^u) = a_t^u (g_t^u)^2 + b_t^u g_t^u + c_t^u, \quad (6)$$

where $a_t^u > 0$ and $b_t^u, c_t^u \geq 0$ are pre-determined parameters.

III. PROBLEM FORMULATION AND ALGORITHM DESCRIPTION

We consider the problems of *load scheduling* and *power trading*. Each user schedules the operation of its appliances to reduce its energy expenses. Primarily, each user uses its local generation to meet its own demand. If the local generation is not sufficient, then the user buys energy from its neighbors and the utility company. On the other hand, users with excess generation capacity compete with each other to sell their extra generations.

We define λ_t^h as the price value set by the utility company for each time slot t . We refer to users with excess generation as *competing* users. The advertised price of competing users should be less than λ_t^h to be economically reasonable for buyers. Competing users may have different marginal production cost. Thus, their advertised prices can be different. Among the competing users, those with the smallest advertised price are selected to serve the demand. The *selected* competing users are referred to as *providing* users. Different approaches have been proposed in the literature to clear the market and pay the sellers. Providing users may sell their excess generation at their advertised price, or they may adopt a *market clearing price* (MCP) to serve the demand [28]. MCP is the highest advertised price among providing users. In our system model, we assume that the market is cleared by an MCP. Moreover, competing users who are not selected as providing users can still sell their extra generation back into the grid at a lower price $\lambda_t^l \leq \lambda_t^h$.

A. Scheduling Problem Formulation

We assume that the exact information about the list of appliances that are awake in each time slot, whether they are must-run or controllable, and the deadline by which their operation

has to be finished is revealed only gradually over time to the ECC units. The operating schedule of controllable appliances depends on the price of electricity for the current time slot (i.e., the MCP). On the other hand, the MCP depends on the tentative load schedule of each user and whether the user has excess generation or not. In general, it is difficult to formulate and solve the joint optimization problem for determining the MCP and the operating schedule of the controllable appliances. To tackle this problem, we determine the operating schedule of the appliances and the MCP in two stages, i.e., the *scheduling* stage and the *trading* stage.

At the scheduling stage, we assume that the operating schedule of the controllable appliances is determined based on the *estimated* MCP for each time slot. Each user estimates the *average* MCP for each time slot based on its observations from past operating periods. This estimate can be different for different users as the effects of both selling and buying energy are taken into account. In this stage, users determine whether they have excess generation or have to acquire energy from neighboring users. Then, at the trading stage, competing users compete and the exact MCP for the current time slot t is determined. At the trading stage, competing users prefer to sell their excess generation to their neighbors rather than to the utility company, as they can make more profit by selling at a price higher than λ_t^l . Moreover, the consuming users may also benefit from local trading because the competing users may reduce their offered price compared to λ_t^h to obtain a larger share of the market. We define

$$L_t^u = \sum_{a \in \mathcal{A}_u} P_{I_a - q_t^{u,a} + 1}^a x_t^{u,a} + y_t^u - \psi_t^u \quad (7)$$

as the load of user u at time slot t , where ψ_t^u is the amount of generated power of user u at time slot t . For the case in which the user has excess power to sell to the others, i.e., $L_t^u < 0$, we define $G_t^u \triangleq -L_t^u$ as the maximum power that can be exported to other users. For user $u \in \mathcal{U}$ at time slot $t \in \mathcal{T}$, we define $v_t(\chi_t^u, L_t^u)$ as the payment of the user for the *known* load L_t^u :

$$v_t(\chi_t^u, L_t^u) = \mu_t L_t^u j_t^u + (\mu_t h_t^u + \lambda_t^l (1 - h_t^u)) L_t^u (1 - j_t^u) \\ = \left(\mu_t j_t^u + (\mu_t h_t^u + \lambda_t^l (1 - h_t^u)) (1 - j_t^u) \right) L_t^u, \quad (8)$$

where μ_t is the MCP at time slot t , and j_t^u is a binary variable specifying whether $L_t^u \geq 0$ ($j_t^u = 1$) or not ($j_t^u = 0$). We define $h_t^u \triangleq g_t^u / G_t^u$, where g_t^u is the amount of generated power which is processed and sold to neighboring users at price μ_t .

For each user u , the power scheduling is done by its ECC unit at current time slot t by solving the following optimization problem, which aims to minimize the *expected* energy cost in the upcoming time slots:

$$\mathbb{V}_t^u(\chi_t^u) = \underset{\substack{\mathbf{x}_t^u \in \mathbb{X}_t^u, \\ y_t^u \in \mathbb{Y}_t^u}}{\text{minimize}} \quad v_t(\chi_t^u, L_t^u) + \mathbb{E} \left\{ \mathbb{V}_{t+1}^u(\chi_{t+1}^u) \mid \chi_t^u \right\}, \quad (9)$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation with respect to the demand and generation uncertainties in upcoming time slots, $\mathbf{x}_t^u \triangleq (x_t^{u,1}, \dots, x_t^{u,|\mathcal{A}_u|})$, $\mathbb{X}_t^u$ is the set of actions available for each

appliance a at state χ_t^u , and $\mathbb{Y}_t^u$, which is the feasible operating set of the battery, is defined as

$$\mathbb{Y}_t^u = \left\{ y_t^u \mid y_t^u \in [-e_b, e_b], 0 \leq \Lambda_t^u \leq B_b \right\}. \quad (10)$$

$v_t(\chi_t^u, L_t^u)$ is as (8), while the second term on the right hand side of (9) is the expected cost of energy in the upcoming time slots, which we will refer to it as the *cost-to-go*. We refer to $\mathbb{V}_t^u(\cdot)$ as the value function of user u at time slot t , and $\mathbb{V}_{T+1}^u(\cdot) \triangleq 0$. Considering (8), we define

$$\theta_t^u(\chi_t) \triangleq \mu_t j_t^u + (\mu_t h_t^u + \lambda_t^l(1 - h_t^u))(1 - j_t^u) \quad (11)$$

as the *effective* MCP for user u at time slot t . $\theta_t^u(\cdot)$ incorporates the effects of both electricity payment and electricity revenue for user u . $\theta_t^u(\cdot)$ depends on the scheduling and trading decisions of user u , the decisions of other users, and the realizations of random events (i.e., the power generation from RERs and the demand requirements of the users). Thus, it is very difficult to calculate $\theta_t^u(\cdot)$ directly. To tackle this problem, we propose an *approximate dynamic programming* approach to estimate the solution of problem (9). One approach to approximate the value function is to adopt *parametric models* [29]. The value function is replaced with a *linear regression*. Problem (9) can be approximated as

$$\hat{\mathbb{V}}_t^u(\chi_t^u) = \underset{\substack{\mathbf{x}_t^u \in \mathbb{X}_t^u, \\ y_t^u \in \mathbb{Y}_t^u}}{\text{minimize}} \quad \eta_t^u L_t^u + \mathbb{E} \left\{ \hat{\mathbb{V}}_{t+1}^u(\chi_{t+1}^u) \mid \chi_t^u \right\}, \quad (12)$$

where η_t^u is the weight coefficient of user u at time slot t . A comparison of (9) and (12) reveals that the coefficient η_t^u approximates the function $\theta_t^u(\chi_t)$ in (8). However, as new observations about the true value function for each time slot are revealed, the weight coefficients η_t^u are updated accordingly, as will be explained later in this section.

The linearity of (12) makes it possible to calculate the optimal values of $\mathbf{x}_t^{u,a}$ and y_t^u independently, and thus, to separate the scheduling process of individual appliances. That is, for each individual appliance a , we need to solve the following *dynamic program*:

$$\hat{\mathbb{V}}_t^{u,a}(\chi_t^{u,a}) = \underset{\substack{\mathbf{x}_t^{u,a} \in \mathbb{X}_t^{u,a}}}{\text{minimize}} \quad \eta_t^u P_{I_a - q_t^{u,a} + 1}^a x_t^{u,a} + \hat{\mathbb{V}}_{t+1}^{u,a}(\chi_{t+1}^{u,a}), \quad (13)$$

where $\mathbb{X}_t^{u,a}$ is the set of actions available for appliance a at state $\chi_t^{u,a}$, and $\hat{\mathbb{V}}_t^{u,a}(\chi_t^{u,a})$ is the approximate value function for appliance a in state $\chi_t^{u,a}$. When state $\chi_t^{u,a}$ is equal to $(0, w_t^{u,a})$, $\hat{\mathbb{V}}_t^{u,a}(\chi_t^{u,a}) = 0$ since the operation of the appliance is finished. Problem (13) can be solved by backward induction.

For operation of the battery, due to the continuity of the decision variables, it is more convenient to represent its scheduling process in the form of an optimization problem

$$\hat{\mathbb{V}}_t^{u,b}(\Lambda_t^u) = \underset{\substack{y_k^u \in \mathbb{Y}_k^u, \\ k \in \mathcal{T}_t}}{\text{minimize}} \quad \sum_{k \in \mathcal{T}_t} \eta_k^u y_k^u, \quad (14)$$

where $\hat{\mathbb{V}}_t^{u,b}(\Lambda_t^u)$ is the approximate value function for the battery in state Λ_t^u , and $\mathcal{T}_t \triangleq \{t, \dots, T\}$.

We assume that at the end of the operation period, the true value function for the whole operation period can be

observed (i.e., the total electricity cost for all T time slots). We denote the true value function for user u as $\mathbb{V}_u$. However, based on the column vector of value function parameters, $\boldsymbol{\eta}_u = (\eta_1^u, \dots, \eta_T^u)$, and the column vector of total load in each time slot, $\mathbf{L}_u = (L_1^u, \dots, L_T^u)$, the approximated value function is

$$\hat{\mathbb{V}}_u = \mathbf{L}_u^T \boldsymbol{\eta}_u, \quad (15)$$

where T is the transpose operator. After the true value function has been observed, this *new* information is used to adjust the *old* estimate of parameter $\boldsymbol{\eta}_u$. Let m denote the number of observations obtained so far. We define $\mathbb{V}_{u,m}$, $\hat{\mathbb{V}}_{u,m}$, $\boldsymbol{\eta}_{u,m}$, and $\mathbf{L}_{u,m}$ as the values of $\mathbb{V}_u$, $\hat{\mathbb{V}}_u$, $\boldsymbol{\eta}_u$, and $\mathbf{L}_u$ corresponding to the m th observation, respectively. As the new $(m+1)$ th observation arrives, we update the value function parameters based on the *recursive least square* method, i.e.,

$$\begin{aligned} \boldsymbol{\eta}_{u,m+1} &= \boldsymbol{\eta}_{u,m} \\ &+ \frac{\mathbf{H}_m \mathbf{L}_{u,m+1}}{\frac{1}{\delta} + \mathbf{L}_{u,m+1}^T \mathbf{H}_m \mathbf{L}_{u,m+1}} \left(\mathbb{V}_{u,m+1} - \hat{\mathbb{V}}_{u,m+1} \right), \end{aligned}$$

where

$$\hat{\mathbb{V}}_{u,m+1} = \mathbf{L}_{u,m+1}^T \boldsymbol{\eta}_{u,m},$$

and

$$\mathbf{H}_{m+1} = \mathbf{H}_m - \frac{\mathbf{H}_m \mathbf{L}_{u,m+1} \mathbf{L}_{u,m+1}^T \mathbf{H}_m}{\frac{1}{\delta} + \mathbf{L}_{u,m+1}^T \mathbf{H}_m \mathbf{L}_{u,m+1}}.$$

Here, $\mathbf{H}_0$ is a positive definite matrix, and $0 < \delta < 1$ is the observation weight. We note that the influence of the recent observations decreases more rapidly for smaller values of δ .

B. Trading Problem Formulation

In this section, we consider the trading stage and focus on the competing users and how they interact. If the aggregate excess generation of the competing users is more than the demand, the competing users choose their offered price and generation such that they will be selected as *providing users*, and their payoff will be maximized. The competing users also take into account the offers advertised by other competing users. In our system model, users do consider the effect of their actions on the MCP. Thus, we need to analyze the Nash equilibrium of the *game* played among multiple competing users who compete to have some share of the market. In this game theoretic model, the strategy of each user consists of its offered price and generation for sale to other users. Let $\mathcal{G}_t = \{1, \dots, N_t\}$ be the set of all competing users at time slot t , where $N_t \triangleq |\mathcal{G}_t|$. Moreover, $\mathcal{G}_t^{-u} \triangleq \mathcal{G}_t \setminus \{u\}$ is defined as the set of all competing users other than user u . We denote the *offer* of competing user u as $\mathcal{O}_t^u \triangleq (\pi_t^u, g_t^u)$, where π_t^u is the offered price and g_t^u is the offered generation capacity. We define $\mathbb{O}_t^{-u} \triangleq (\mathcal{O}_t^n \mid n \in \mathcal{G}_t^{-u})$ as the vector of offers advertised by all competing users other than user u . The information about the offers of the other users, i.e., $\mathbb{O}_t^{-u}$ is received by user u . Without loss of generality, we assume that the elements of vector $\mathbb{O}_t^{-u}$ are *sorted in an ascending order* based on the entries π_t^n , $n \in \mathcal{G}_t^{-u}$.

User u chooses its offer $\mathcal{O}_t^u$ such that it will be selected as a providing user and its payoff is maximized. To identify

whether offer $\mathcal{O}_t^u$ will place user u among the providing users or not, the rank of offer $\mathcal{O}_t^u$ among other offers $\mathbb{O}_t^{-u}$ has to be evaluated. We adopt the binary auxiliary variable $z_t^n, n \in \mathcal{G}_t^{-u}$, to indicate whether the offered price π_t^n is lower or equal to the selected price π_t^u ($z_t^n = 1$) or not ($z_t^n = 0$). Thus,

$$r_t^u = \sum_{n \in \mathcal{G}_t^{-u}} z_t^n \quad (16)$$

indicates the number of users with lower offered prices than user u (i.e., the rank of offer $\mathcal{O}_t^u$ among other offers $\mathbb{O}_t^{-u}$). To calculate binary variable z_t^n , the following constraint is added to the trading optimization problem

$$\frac{\pi_t^u}{\pi_t^n} > z_t^n, \quad \forall n \in \mathcal{G}_t^{-u}. \quad (17)$$

If $\pi_t^n > \pi_t^u$, constraint (17) ensures that $z_t^n = 0$, i.e., the constraint is active. If the constraint is inactive, i.e., $\pi_t^n \leq \pi_t^u$, z_t^n can be either 0 or 1. To enforce $z_t^n = 1$ while the constraint is inactive, an auxiliary term is added to the objective of the optimization problem such that, z_t^n is set to 1. To this end, a term $-K_1 z_t^n$ is added to the objective of the user's minimization problem, where K_1 is a large positive constant. That is, while constraint (17) is inactive, z_t^n is set to 1 for minimization of the objective function.

Starting from the user with the smallest offered price, competing users will be added to the set of providing users until there is enough generation to serve the demand. In this case, the MCP will be equal to the highest offered price among the group of providing users. We define $\mathbb{S}_t^n, n \in \mathcal{G}_t$, as a group of competing users with offered price less than or equal to π_t^n . The MCP will be equal to $\pi_t^n, n \in \mathcal{G}_t$, if the following two conditions are satisfied:

- The aggregate offered generation of the users within $\mathbb{S}_t^n, n \in \mathcal{G}_t$, is sufficient to meet the demand.
- Among all groups that satisfy condition (a), $\mathbb{S}_t^n$ has the smallest number of members.

Condition (b) ensures that the process of adding competing users to the set of providing users will be stopped if we have enough generation to meet the demand. If the aggregate offered generation of all competing users is less than the total demand, all competing users will be selected as providing users and the MCP will be equal to the price advertised by the utility company.

We define $S_t^n, n \in \mathcal{G}_t$, as

$$S_t^n = \sum_{i \in \mathbb{S}_t^n} g_t^i = \begin{cases} \sum_{i=1}^n g_t^i + (1 - z_t^n)g_t^u, & \text{if } n \in \mathcal{G}_t^{-u}, \\ \sum_{i \in \mathcal{G}_t^{-u}} z_t^i g_t^i + g_t^u, & \text{if } n = u \end{cases} \quad (18)$$

to denote the aggregate offered generation of the users within $\mathbb{S}_t^n$. For each group $\mathbb{S}_t^n, n \in \mathcal{G}_t$, we assign a binary auxiliary variable d_t^n to indicate whether the MCP is set to the offered price π_t^n ($d_t^n = 1$) or not ($d_t^n = 0$). We also define auxiliary variable d_t^0 which plays the same role as d_t^n for the utility company's price λ_t^h . The MCP is finally selected from one of the advertised prices π_t^n or λ_t^h . Thus, we have

$$\sum_{n \in \mathcal{G}_t} d_t^n + d_t^0 = 1. \quad (19)$$

To enforce condition (a) for evaluating the MCP, the constraint

$$\frac{S_t^n}{D_t} \geq d_t^n, \quad \forall n \in \mathcal{G}_t \quad (20)$$

is added to the trading optimization problem, where D_t is the total demand at time slot t . If $S_t^n < D_t$, then constraint (20) is *active* and $d_t^n = 0$. If constraint (20) is *inactive*, i.e., $S_t^n \geq D_t$, d_t^n can be either 0 or 1. To satisfy constraint (19) and to enforce condition (b) for evaluating the MCP, one of the variables $d_t^n, n \in \mathcal{G}_t$, or d_t^0 has to be set to 1 if its corresponding constraint (20) is *inactive* and the associated set $\mathbb{S}_t^n$ has the smallest number of members. To this end, an auxiliary term $K_2(\sum_{n \in \mathcal{G}_t^{-u}} n d_t^n + (N_t + 1)d_t^0 + r_t^u d_t^u)$ is added to the objective function of the trading optimization problem, where K_2 is a large positive constant. For the added term, the coefficients of the binary variables $d_t^n, n \in \mathcal{G}_t$, are equal to the size of the groups $\mathbb{S}_t^n$. Thus, to minimize the objective function, among the variables $d_t^n, n \in \mathcal{G}_t$, for which the corresponding constraint (20) is inactive, the one with the smallest weight is set to 1. However, if the corresponding constraint (20) is active for all variables $d_t^n, n \in \mathcal{G}_t$, variable d_t^0 will be set to 1. Finally, to evaluate the payoff of user u , we define the auxiliary variable f_t which indicates whether the offer $\mathcal{O}_t^u$ will place user u in the set of providing users ($f_t = 1$) or not ($f_t = 0$).

Given $\mathbb{O}_t^{-u}$, each competing user u solves the following optimization problem to calculate its offered price and generation capacity

$$\text{minimize } \mathbb{F}(\mathbf{I}_t, \mathbf{C}_t) \quad (21a)$$

$$\text{subject to } f_t, d_t^0, d_t^n, z_t^n, d_t^u \in \{0, 1\}, \quad \forall n \in \mathcal{G}_t^{-u}, \quad (21b)$$

$$\text{constraints (17), (19), (20),} \quad (21c)$$

$$\frac{\sum_{n \in \mathcal{G}_t} d_t^n \pi_t^n + d_t^0 \lambda_t^h}{\pi_t^u} \geq f_t, \quad (21d)$$

$$\lambda_t^l \leq \pi_t^u \leq \lambda_t^h, \quad (21e)$$

$$0 \leq g_t^u \leq G_t^u, \quad (21f)$$

where $\mathbf{I}_t = (\mathbf{d}_t, \mathbf{z}_t, f_t)$, $\mathbf{C}_t = (g_t^u, \pi_t^u)$, $\mathbf{d}_t \triangleq (d_t^0, d_t^n | n \in \mathcal{G}_t)$, $\mathbf{z}_t \triangleq (z_t^n | n \in \mathcal{G}_t^{-u})$, and

$$\begin{aligned} \mathbb{F}(\mathbf{I}_t, \mathbf{C}_t) = & \mathbb{C}_t^u(g_t^u) - \left(\sum_{n \in \mathcal{G}_t} d_t^n \pi_t^n + d_t^0 \lambda_t^h \right) g_t^u f_t - \lambda_t^l (G_t^u - g_t^u) \\ & - K_1 \sum_{n \in \mathcal{G}_t^{-u}} z_t^n - K_2 f_t \\ & + K_2 \left(\sum_{n \in \mathcal{G}_t^{-u}} n d_t^n + (N_t + 1)d_t^0 + r_t^u d_t^u \right), \end{aligned} \quad (22)$$

where $K_1 \gg K_2$, and G_t^u is the maximum power that can be provided to other users by user u at time slot t . The first term in (22) is the cost of providing g_t^u units of energy for the intended user u , c.f. (6). The second and the third terms reflect the revenue of user u for the offer (π_t^u, g_t^u) . The term

$$\mu_t = \sum_{n \in \mathcal{G}_t} d_t^n \pi_t^n + d_t^0 \lambda_t^h \quad (23)$$

is equal to the MCP. Constraint (21d) checks whether or not the considered user u is selected as a providing user. If the

offered price π_t^u is greater than the MCP, then user u is not a providing user ($f_t = 0$). Otherwise, f_t can be either 0 or 1. In this case, as f_t appears in the objective function with a negative sign, it will be set to $f_t = 1$. The offered price cannot exceed the price advertised by the utility company and the amount of generation cannot exceed the total generation capacity as reflected by (21e) and (21f), respectively.

Problem (21) is a mixed-integer program. By adopting the generalized Benders' decomposition approach [30], problem (21) can be re-written as

$$\text{minimize}_{\mathbf{I}_t} \mathbb{V}(\mathbf{I}_t) \quad (24a)$$

$$\text{subject to (19), (21b),} \quad (24b)$$

$$\frac{\pi_t^{u*}}{\pi_t^n} \geq z_t^n, \quad \forall n \in \mathcal{G}_t^{-u}, \quad (24c)$$

$$\frac{S_t^{n*}}{D_t} \geq d_t^n, \quad \forall n \in \mathcal{G}_t, \quad (24d)$$

$$\frac{\sum_{n \in \mathcal{G}_t^{-u}} d_t^n \pi_t^n + d_t^u \pi_t^{u*} + d_t^0 \lambda_t^h}{\pi_t^{u*}} \geq f_t, \quad (24e)$$

where

$$\mathbb{V}(\mathbf{I}_t) = \text{minimize}_{\mathbf{C}_t} \mathbb{F}(\mathbf{I}_t, \mathbf{C}_t) \quad (25a)$$

$$\text{subject to (21c)-(21f),} \quad (25b)$$

and problem (25) should be feasible for the set of *complicating* variables $\mathbf{I}_t$ that are held fixed. Here, $\mathbf{C}_t^* = (\pi_t^{u*}, g_t^{u*})$ is the solution of (25), and S_t^{n*} is as in (18), where $\mathbf{C}_t$ is set to $\mathbf{C}_t^*$.

The procedure for solving problem (21) is as follows:

Step 1: Let $\mathbf{I}_t^0$ be an initial vector of complicating variable $\mathbf{I}_t$ for which problem (25) is feasible. Solve subproblem (25) to obtain optimal vector $\mathbf{C}_t^*$. $BFS = \mathbb{F}(\mathbf{I}_t^0, \mathbf{C}_t^*)$ is the best value of problem (21) found so far, and $\mathbb{V}(\mathbf{I}_t) = \mathbb{F}(\mathbf{I}_t, \mathbf{C}_t^*)$ forms the objective function of problem (24). Select the convergence tolerance parameter $\epsilon > 0$.

Step 2: Solve problem (24) to obtain $\mathbf{I}_t^*$ and $\mathbb{V}(\mathbf{I}_t^*)$. If $|\mathbb{V}(\mathbf{I}_t^*) - BFS| < \epsilon$, terminate.

Step 3: Solve subproblem (25) to obtain $\mathbf{C}_t^*$ for the complicating variables found in Step 2, $\mathbf{I}_t^*$. If $|\mathbb{F}(\mathbf{I}_t^*, \mathbf{C}_t^*) - BFS| < \epsilon$, terminate. If $\mathbb{F}(\mathbf{I}_t^*, \mathbf{C}_t^*) \leq BFS$, update $BFS = \mathbb{F}(\mathbf{I}_t^*, \mathbf{C}_t^*)$. Return to Step 2.

We note that for a fixed value of $\mathbf{I}_t$, problem (25) is a convex quadratic optimization problem and can be solved efficiently by convex optimization techniques. Moreover, $\mathbb{V}(\mathbf{I}_t)$ is bounded. Furthermore, problem (24) is a quadratic integer program, which can be solved efficiently with optimization software such as MOSEK [31]. Following the discussion in [30], the adopted generalized Benders' decomposition procedure converges to the optimum value. The proof of the following theorem can be found in [30].

Theorem 1: For a finite discrete vector $\mathbf{I}_t$, the generalized Benders' decomposition procedure to solve (21) terminates in a finite number of steps for any given $\epsilon > 0$.

C. Market Clearing Game

From problem (21), the payoff of each user depends on its offer (π_t^u, g_t^u) as well as the offers of the other users. Hence, we have the following game among the users:

- **Players:** Competing users.
- **Strategies:** Each user selects its offered price and generating capacity (π_t^u, g_t^u) to minimize its cost.
- **Costs:** The cost of each user is determined based on optimization problem (21).

In the proposed market clearing game, first, each competing user assumes random offers for other competing users $\mathcal{O}_t^{-u}$. This assumption is required since at the beginning, competing user u has no prior information about the other users. Next, each competing user u solves its own local problem (21). That is, each user plays its best response to the offers advertised by other users. Each competing user broadcasts its new offer through the communication infrastructure. We note that each user broadcasts its offer only if it has been changed compared to its previous offer. Moreover, it will update its local information about the offers of the other users whenever it receives a broadcast message. This procedure continues until no competing user changes its offer.

We note that for any practical trading scenario, users are not allowed to apply arbitrarily small changes to their offered prices and generations. That is, offered prices and generations are selected from a countable finite set. Therefore, the existence of a Nash equilibrium for the considered game directly follows from [32, Theorem 3.3.22]. For the Nash equilibrium to be unique, the best response mapping of the users is required to be concave [33]. The best response mapping is a function which maps the offer profile of all the competing users to a new offer profile. Based on this mapping, the new offer of each user is the best response of the user to the previous offer profile of all the other users. For our proposed trading algorithm, the best response of the user is determined as the solution to optimization problem (21). Therefore, it does not have a closed-form solution, and it is difficult to show its concavity. However, we have simulated different scenarios and observed the uniqueness of the Nash equilibrium in all instances. Nevertheless, since we are not able to simulate all possible scenarios, this is not a proof for the uniqueness of the Nash equilibrium for the proposed trading algorithm.

D. Algorithm Description

In this subsection, we explain the steps of the proposed Algorithm 1 for load scheduling and power trading. At the beginning, the list of the appliances that should be scheduled is updated, c.f. Line 1. Based on the estimated MCP, the operating schedule of each appliance is determined as in (13). Moreover, the charging and discharging rate of the battery is calculated as in (14), c.f. Line 2. At the end of the scheduling stage, the user determines whether it has excess generation or not. Users with excess generation receive the information about the offers of other competing users. Each competing user solves (21) by adopting the generalized Benders' decomposition approach to obtain $\mathcal{O}_t^u$. Users update their offered price and generation in response to the advertised prices by other competing users. This process continues until the users reach a Nash equilibrium of the trading game as described in Section III-C, c.f. Lines 4 to 7. Users that are selected as providing users sell g_t^u units of their extra generation to consuming users

Algorithm 1: Load scheduling and power trading algorithm executed at each time slot $t \in \mathcal{T}$ for user $u \in \mathcal{U}$.

- 1: Update the list of the appliances that are to be scheduled.
- 2: Schedule the appliances as in (13) and (14).
- 3: **if** there is excess generation
- 4: **repeat**
- 5: Receive offers of other users $\mathcal{O}_t^{-u}$.
- 6: Solve (21) to obtain the offer $\mathcal{O}_t^u$.
- 7: **until** a Nash equilibrium of the trading game is reached
- 8: **if** selected as a providing user
- 9: Sell g_t^u units of excess generation to other users at the MCP.
- 10: **end if**
- 11: Sell the remaining generation to the grid at price λ_t^l .
- 12: **end if**

at the MCP, c.f. Line 9. Competing users that are not selected as providing users can sell their excess generation back to the grid at the lower price λ_t^l , c.f. Line 11.

IV. PERFORMANCE EVALUATION

In this section, we present simulation results and assess the performance of our proposed DSM program. In our simulation setting, we assume that the operation period is divided into 24 one-hour time slots. The scheduling and trading stages take place at the beginning of each time slot. For our problem formulation, since the trading algorithm converges in a few iterations, this stage lasts only for a few seconds. However, we note that the time granularity depends on how accurate we can estimate the power generation from the RERs, and how fast the trading stage is finished. In any case, the duration of the trading stage should be kept small compared to the duration of each time slot. To this end, a practical scenario, the trading algorithm may be confined to a limited number of iterations. We consider a system with $|\mathcal{U}| = 50$ users. Each user possesses various must-run and controllable appliances. We assume that the information about the energy requirements of the users is not known at the beginning of the operation period. That is, the list of appliances that are awake in each time slot, whether they are must-run or controllable, and the deadline by which the operation of each appliance has to be finished are not known *a priori*. We run the simulation multiple times with different patterns for the times at which the appliances become awake. We then present the average results. For a typical user, we consider on average 16 appliances. We consider different power consumptions for different operation cycles of the appliances. The pattern of power consumption of the appliances is known to the ECC. Some of the appliances and their operating specifications are summarized in Table II. In our simulation settings, we assume that λ_t^h varies between 12 cents/kWh and 24 cents/kWh for off-peak and on-peak time slots, respectively. The parameter λ_t^l is set to 4 cents/kWh. The cost function parameters a_t^u and b_t^u are different for different users varying between 0.1 and 0.6, and c_t^u is set to 0, c.f. (6).

To have a baseline to compare with, we consider a system without ECC deployment and trading opportunity, where each

TABLE II
OPERATING SPECIFICATIONS OF DIFFERENT APPLIANCES.

	average P_i^a (kW)	arrival interval
Electric stove	1.5	[06:00, 14:00]
Clothes dryer	0.5	[14:00, 22:00]
Vacuum cleaner	1	[06:00, 15:00]
Refrigerator	0.125	[06:00, 09:00]
Air conditioner	1	[12:00, 22:00]
Dishwasher	1	[15:00, 24:00]
Heater	1.5	[15:00, 03:00]
Water heater	1.5	[06:00, 23:00]
Pool pump	2	[12:00, 21:00]
Electric vehicle	2.5	[16:00, 24:00]
Lighting	0.5	[16:00, 24:00]
TV	0.25	[16:00, 01:00]
PC	0.25	[08:00, 24:00]
Ironing appliance	1	[06:00, 16:00]
Hairdryer	1	[06:00, 13:00]
Other	1.5	[06:00, 24:00]

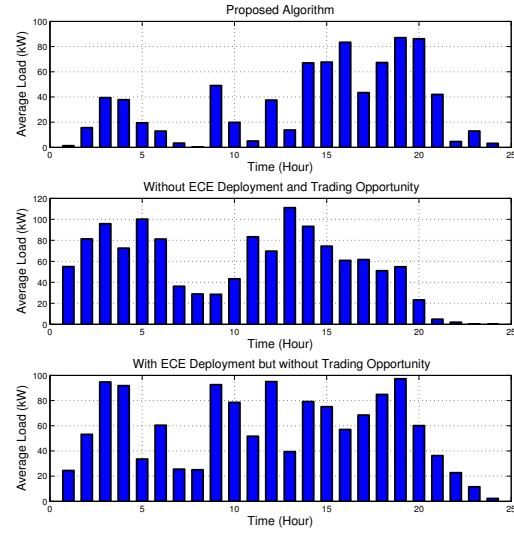


Fig. 2. Average imported power from utility company for different scenarios.

appliance a starts operation with its power pattern $\mathbf{P}_a$ directly after it has sent an admission request to the ECC unit. In this model, the excess generation of each user is sold to the utility company if it is not consumed or stored. For the system without ECC deployment, users are *not* responding to the variations of the price parameters. In order to be able to better evaluate the effect of trading, we also consider a system in which each users is equipped with an ECC unit to schedule the operation of its controllable appliances, but is not able to trade its excess generation with other users. Our simulations were executed on a computer system with Intel(R) Core(TM) i7 CPU 3.07 GHz processor, 12 GB RAM, and Windows 7 operating system.

Fig. 2 depicts simulation results for the average total power imported from the utility company for the proposed load scheduling algorithm, the system without ECC deployment and trading, and the system with ECC deployment but without trading. The average imported energy for the system without ECC deployment and trading is 1360.9 kWh, while this value is 1313.6 kWh for the system in which users are equipped with

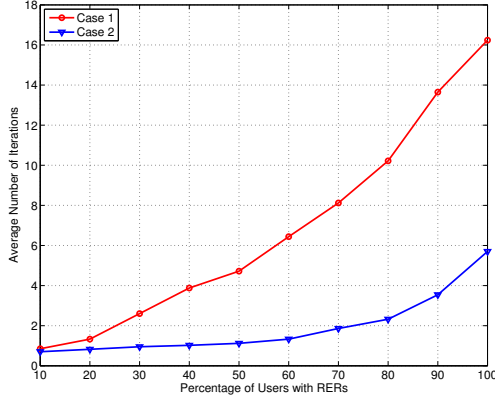


Fig. 3. Number of iterations required to converge vs. the percentage of users equipped with RERs (Case 1: $G_t^u = 6$ kW, Case 2: $G_t^u = 3$ kW).

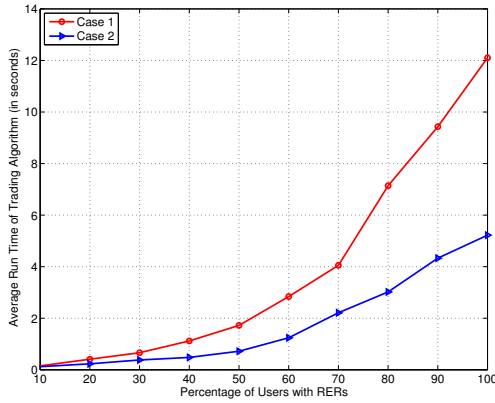


Fig. 4. Run time of trading algorithm vs. the percentage of users equipped with RERs (Case 1: $G_t^u = 6$ kW, Case 2: $G_t^u = 3$ kW).

ECCs but cannot trade. The patterns of power consumption of the latter two systems are different since the system with ECC deployment shifts most of the load to low price time slots. Our proposed load scheduling algorithm reduces the average imported energy to 820.2 kWh because of the trading among the users. The average electricity cost of the users for the system without ECC deployment and trading is \$62.91. For the system with ECC deployment but without trading, this value is reduced to \$54.73. Our proposed algorithm further reduces the electricity cost of the users to \$40.37. The advantages of the proposed algorithm are twofold. First, users are able to decrease their energy expenses by selling their excess generation to other users with a price higher than λ_t^l . Second, buyers may also benefit from the price reduction due to the competition between multiple sellers.

To assess the speed of convergence of our proposed trading algorithm, the number of iterations required to find a Nash equilibrium for different percentage of users that are equipped with RERs is illustrated in Fig. 3. Moreover, Fig. 4 depicts the corresponding average run time of the trading algorithm for different percentage of users that are equipped with RERs. We consider two different cases for the average generating capacity of the users equipped with RERs. In the first case, we assume on average $G_t^u = 6$ kW, while for the second case, we have $G_t^u = 3$ kW. Simulations are repeated multiple times and only the average results are presented. We note that as

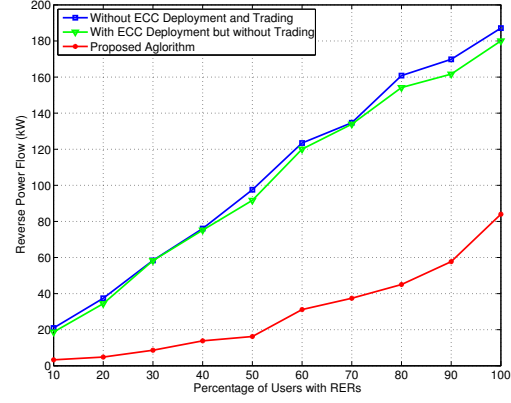


Fig. 5. Reverse power flow vs. the percentage of users equipped with RERs.

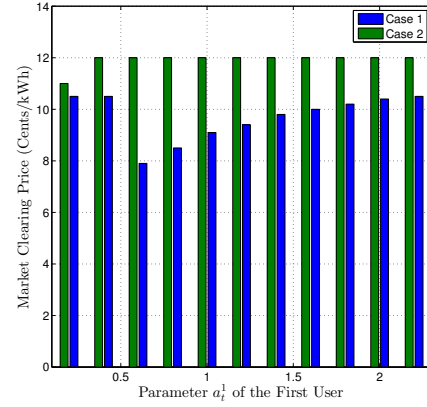


Fig. 6. MCP for $G_t^u = 12$ kW (Case 1) and $G_t^u = 6$ kW (Case 2).

the percentage of users equipped with RERs increases, more users are likely to participate in the trading game to sell their excess generation. Therefore, the number of iterations required by the trading algorithm to converge increases. However, even for a high percentage of users equipped with RERs, the trading algorithm converges in only a few iterations which makes the implementation of the algorithm in practical applications possible.

Our proposed DSM program encourages the utilization of RERs by providing a trading opportunity for the users. To evaluate the effect of the proposed algorithm on the amount of power being injected back into the grid (i.e., the reverse power flow), due to the mismatch between supply and demand, we show in Fig. 5 the reverse power flow as a function of the percentage of users that are equipped with RERs. Fig. 5 reveals that the amount of reverse power flow is significantly reduced for the proposed algorithm.

Due to the competition between the users, the electricity may be offered at a price lower than the price advertised by the utility company. To better understand the effect of the competition between the users on the MCP, we focus on a simplified model in which a single time slot is considered, and three competing users try to sell their excess generation to serve a demand of $D_t = 10$ kW. We consider two different cases. In the first case, each competing user has enough generation to meet the demand (i.e., $G_t^u = 12$ kW), whereas in the second case, the generation capacity of each user is

not sufficient to meet the total demand (i.e., $G_t^u = 6$ kW). We assume $\lambda_t^h = 12$ cents/kWh, and the cost parameters of the last two users are fixed. Simulation results for the MCP for different values of the first user's parameter a_t^u in (6) are depicted in Fig. 6. The parameter a_t^u for the last two users is set to 0.6. For the first case, if the production cost of the first user is low enough, the user prefers to reduce its production and shares a small portion of the market with other competing users with higher offered prices to keep the price as high as possible to maximize its revenue. On the other hand, if the production costs of the users are comparable, the users will also share the market. In this case, the MCP will drop due to the competition between the users. When the generating capacity of each individual user is not sufficient to meet the total demand, the users will share the market and try to keep the price high to maximize their revenue.

V. CONCLUSIONS

In this paper, we proposed a load control algorithm for DSM. We considered the problem of joint *load scheduling* and *power trading*. An approximate dynamic program was proposed to schedule the operation of different types of appliances, and a game theoretic approach was adopted to model the interaction of the users with excess power generation. Users with excess power generation choose their offered price and output generation such that they obtain a larger share of the market and their revenue is maximized. Simulation results showed that our proposed algorithm reduces the energy costs of the users. That is, competing users may sell their extra generation to local users at a price higher than the buying price of the utility company, and consuming users may buy the electricity from neighboring users at a price lower than the selling price of the utility company. Moreover, the possibility to trade facilitates the integration of RERs by encouraging the users to consume their excess generation locally which mitigates the reverse power flow problem.

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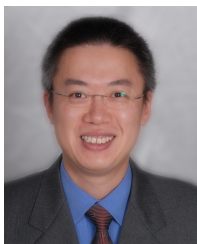


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