

Quality of Sensing Aware Budget Feasible Mechanism for Mobile Crowdsensing

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Abstract—In a mobile crowdsensing system, the platform utilizes ubiquitous smartphones to perform sensing tasks. For a successful mobile crowdsensing application, the consideration of the heterogeneity of quality of sensing from different users as well as a proper incentive mechanism to motivate users to contribute to the system are essential. In this paper, we introduce quality of sensing into incentive mechanism design. Under a budget constraint, the platform aims to maximize the valuation of the performed tasks, which depends on the quality of sensing of the users. We propose ABSee, an auction-based budget feasible mechanism, which consists of a winner selection rule and a payment determination rule. ABSee is designed by adopting a greedy approach. We obtain the approximation ratio of ABSee, which significantly improves the approximation ratio of the existing budget feasible mechanisms in many cases. We further show that the approximation ratio approaches $\frac{2e}{e-1}$ when a large number of smartphone users participate in the system. ABSee also satisfies the properties of computational efficiency, truthfulness, individual rationality, and budget feasibility. Extensive simulation results show that ABSee provides a higher valuation to the platform when compared with existing budget feasible mechanisms in the literature.

Index Terms—Mobile crowdsensing, auction, budget feasible mechanism, approximation ratio.

I. INTRODUCTION

Smartphones nowadays are equipped with a variety of sensors (e.g., microphone, camera, global positioning system (GPS)) and have enhanced sensing capabilities. Mobile crowdsensing exploits the ubiquity of smartphones and utilizes their sensors to monitor the environment [1]. In mobile crowdsensing systems, the platform distributes the sensing tasks to the smartphone users to collect data. Smartphone users perform the tasks and send the results to the platform. Various mobile crowdsensing applications have been developed. They include monitoring the environment [2]–[4], creating wireless network coverage maps [5], [6], and updating the traffic condition [7].

A key factor for a successful mobile crowdsensing application is the participation of a large number of users. However, performing sensing tasks incurs a cost on the smartphone users, such as energy consumption for data sensing, packet transmission charges from the service operator, and manual

efforts. In order to motivate users to participate in mobile crowdsensing, the platform should make payments to the users to compensate their costs. The aforementioned applications in [2]–[7] are based on voluntary participation of users, where their participation cannot be guaranteed.

Another important issue in mobile crowdsensing is that users provide different quality of sensing. For example, consider a platform which monitors the noise level of a city. Each sensing task corresponds to obtaining the noise level at a sampling location in the city by utilizing the microphone of smartphones. For a specific user, the quality of sensing depends on the type of microphone on its smartphone (e.g., accuracy, calibration) and its manual effort (e.g., taking the smartphone out of its pocket to collect the sample). If the platform intends to obtain a high accuracy of the noise measurement, it would assign tasks and provide payments to the users who can provide a high quality of sensing. However, the quality of sensing of users may not be known by the platform in advance. Thus, it is required for the platform to estimate the quality of sensing of users accurately and then select the users accordingly.

Unlike most of the existing works (e.g., [8]–[22]), in this paper, we consider the quality of sensing in the incentive mechanism design. We study a practical scenario where a platform with limited budget aims to maximize the valuation of performed tasks, which depends on the quality of sensing of the users. As a practical example, in urban-scale information gathering [23]–[25], the platform with a limited budget may utilize mobile crowdsensing systems to provide its services. The platform adopts an incentive mechanism to motivate users to participate. We model the interaction between the platform and the users as an auction. The platform first announces its tasks to the users. Each user submits its bid and the subset of tasks it can perform to the platform. The platform then selects the winners according to its limited budget. The winners perform the tasks and receive payments accordingly. We design an auction-based incentive mechanism, which consists of a winner selection rule and a payment determination rule.

The budget constraint introduces inter-dependence between the winner selection rule and the payment determination rule. It makes the design of auction mechanism challenging. In this paper, we consider a practical scenario where there are many participating users in the mobile crowdsensing system. In this scenario, the existing budget feasible mechanisms, e.g., [26], [27], may not always provide a high valuation for the platform. We introduce the *crowd factor* θ , where $0 < \theta < 1$, to model the relative contribution a winner can make to the platform. When there are a few users, each user can

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significantly increase the platform's valuation and contribute to the system. In this case, the crowd factor is small. When more users participate in the auction, the relative contribution of each user becomes smaller. The relative increment in the platform valuation provided by each winner tends to zero eventually in large mobile crowdsensing systems. In this case, the crowd factor θ approaches 1. The formal definition of the crowd factor will be given in Section IV. We use the crowd factor to design a novel budget feasible mechanism. The proposed incentive mechanism is applicable not only to mobile crowdsensing systems but also to other systems where there are a large number of users.

Our main contributions are summarized as follows:

- *Novel model*: We jointly study quality of sensing and incentive mechanism design. We introduce quality of sensing into the valuation function of the platform. The valuation function is proven to be a submodular function. Since the quality of sensing of users is not known *a priori*, in our work, we accurately estimate the quality of sensing for each user. We focus on tasks with continuous values, but our model is also suitable for tasks with discrete values, including tasks with binary values.
- *Novel incentive mechanism*: We propose an Auction-based Budget feasible mechanism, which is called ABSee. ABSee consists of a winner selection rule and a payment determination rule. We first determine the crowd factor and select the winners by employing an iterative approach. We then determine the payment to the winners using the crowd factor. We rigorously prove that ABSee satisfies the properties of computational efficiency, truthfulness, individual rationality, and budget feasibility. We further analyze the crowd factor and show that it approaches 1 under some realistic scenarios.
- *Approximation ratio*: To tackle the computational complexity of the auction design, ABSee adopts a greedy approach. We first prove that the approximation ratio of ABSee, which is the ratio between the optimal valuation and the one obtained by ABSee, is $\min \left\{ \frac{2e}{\theta(e-1)}, \frac{(5-2\theta)e}{e-1} \right\}$. This always improves the approximation ratio of $\frac{5e}{e-1}$ (≈ 7.91) in the budget feasible mechanism proposed in [27]. We further show that the approximation ratio approaches $\frac{2e}{e-1}$ (≈ 3.16) in mobile crowdsensing systems with a large number of users.
- *Performance evaluation*: Through extensive numerical studies, we evaluate the performance of ABSee. Results show that a higher valuation can be obtained by adopting ABSee when compared with the budget feasible mechanisms proposed in [27] under different scenarios. The crowd factor is shown to have a significant influence on the valuation of the platform, and approaches one when a large number of users participate in the system. Moreover, our results show that the quality of sensing can be estimated accurately.

The rest of this paper is organized as follows. The related works are summarized in Section II. In Section III, we present the system model for the auction and quality of sensing. In Section IV, we propose ABSee and provide a walk-through

example. We prove that ABSee achieves all mentioned properties. We further illustrate the crowd factor and analyze the approximation ratio of ABSee in Section V. We evaluate the performance of ABSee in Section VI. Conclusion and future works are given in Section VII.

II. RELATED WORK

Different forms of incentive mechanisms have been proposed in the literature. A platform-centric model and a user-centric model are proposed in [8] to maximize the utility of the platform, which is the valuation of performed tasks minus the total payment to the users. An all-pay auction is designed in [9] to maximize the utility, where the users' population is stochastic. In [10], an auction mechanism is proposed to minimize the social cost of smartphone users. An online auction mechanism is designed in [11] for dynamic arrival and departure of smartphone users, with the goal of maximizing the valuation of performed tasks. Another online incentive mechanism is proposed in [12] to maximize the utility of the platform. Zhang *et al.* in [13] proposed different auction mechanisms for mobile crowdsourcing systems by considering the cooperation and competition among the users. They proved that the mechanisms are truthful, individually rational, budget feasible, computationally efficient, and the platform is guaranteed to obtain non-negative utility. An incentive mechanism is proposed in [14] to provide the long-term incentives to guarantee the users will participate for a long time. Ji *et al.* in [15] focused on the discretized crowdsensing where users participate in crowdsensing system in discrete time-slots. A game theoretic approach is used based on the perfect Bayesian equilibrium to maximize the platform's utility. However, quality of sensing is not considered in the aforementioned works.

The existing works on quality of sensing mainly focus on tasks with discrete values. A quality aware task assignment system is designed in [16] by considering quality measures for crowdsourcing applications. An efficient budget allocation algorithm is proposed in [17] to guarantee the accuracy of estimation of tasks with binary values. An online learning approach is adopted in [18] to maximize the quality of sensing to guarantee the robustness of the crowdsourcing system. In the data mining area, the work in [19] used expectation maximization and maximum likelihood estimation to determine the quality of sensing of tasks with binary values in social sensing applications. Davami *et al.* in [20] compared five trust prediction algorithms and calculated the quality of sensing of users for a parking application. Li *et al.* in [21] formulated an optimization problem to obtain the estimated value of the tasks and minimize the estimation error by considering tasks with both continuous and discrete values.

Besides, a few existing incentive mechanisms consider quality of sensing of tasks. Koutsopoulos in [28] proposed an incentive mechanism which considers the quality of sensing. However, the platform does not have a budget constraint and only has one task. An auction mechanism is designed in [29] to maximize the platform's valuation, where the quality of sensing of each user is assumed to be known by the platform. A sequential Bayesian approach is used in [30] to determine

TABLE I
COMPARISON OF BUDGET FEASIBLE MECHANISMS

Mechanism	Approximation ratio	Setting
[26]	117.7	General submodular functions
[27]	$\frac{5e}{e-1} \approx 7.91$	General submodular functions
[33]	31	Coverage model (a special submodular function)
ABSee	$\min\{\frac{2e}{\theta(e-1)}, \frac{(5-2\theta)e}{e-1}\}$	General submodular functions

the quality of sensing of users, but the auction mechanism is designed specifically for tasks with binary values with prior information of quality of sensing of the users. A quality-based incentive mechanism is designed in [31]. However, it does not consider the strategic behavior of users, who are interested in maximizing their own utilities and may adopt strategies to manipulate the mechanism. A quality-aware algorithm is proposed in [32], where it only considers the coverage quality. Unlike the aforementioned existing works, we consider strategic users and propose an incentive mechanism for the platform under a budget constraint, where we estimate the quality of sensing of users.

Budget feasible mechanisms for submodular functions are proposed by Singer [26] and improved by Chen *et al.* [27]. Recently, these mechanisms have been used in several applications. For example, an influence maximization problem is studied by utilizing a coverage model and a budget feasible mechanism in [33]. In this paper, we propose a novel budget feasible mechanism by introducing the crowd factor and utilizing an iterative winner selection algorithm. ABSee outperforms [27] in terms of the platform valuation. This is achieved through using the crowd factor, which is uniquely introduced in this work. ABSee can achieve a better approximation ratio comparing to the other budget feasible mechanisms as summarized in Table I. Later in Sections V and VI, we will show that θ is close to 1 and the approximation ratio of ABSee approaches $\frac{2e}{e-1} \approx 3.16$.

III. SYSTEM MODEL

A. Auction Framework

A mobile crowdsensing system, as shown in Fig. 1, is composed of a platform residing in the cloud computing center and many smartphone users. Users are connected with the cloud via wireless access networks (e.g., WiFi, LTE). The set of smartphone users is denoted by $\mathcal{N} = \{1, \dots, N\}$, where N is the number of users. We assume that $N > 1$. Note that an auction with only one user cannot guarantee truthfulness. The platform announces the sensing tasks to the smartphone users. We use $\Gamma = \{\tau_1, \dots, \tau_M\}$ to denote the set of tasks and there are M tasks in total.

We model the interactive process between the platform and the users as an auction. Each user $n \in \mathcal{N}$ submits bid b_n for the subset of tasks $\Gamma_n \subseteq \Gamma$ that it can perform to compensate its cost c_n . The platform collects the bids from all of the users and chooses a subset of users $\mathcal{S}$ to perform the tasks. These users are called the *winners*. The winners perform tasks and send the data to the platform. Then, the platform makes a

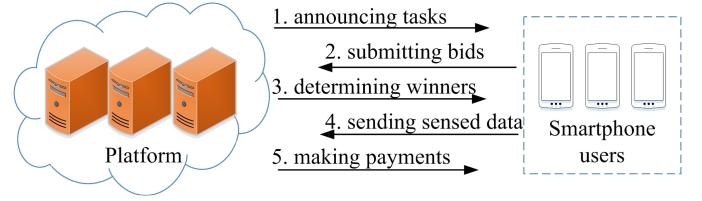


Fig. 1. A mobile crowdsensing system consists of a platform and many smartphone users. The auction mechanism is modeled as an interactive process between the platform and the users.

payment p_n to each winner $n \in \mathcal{S}$. Each user is assumed to be rational and chooses a strategy (i.e., submits bid b_n) to maximize its own utility. The utility of user $n \in \mathcal{N}$ is

$$u_n = \begin{cases} p_n - c_n, & \text{if user } n \text{ is a winner,} \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Note that the bid b_n determines whether user n can be selected as a winner or not.

B. Quality of Sensing

In a mobile crowdsensing system, the quality of sensing of a user depends on its effort and expertise to perform the tasks and the quality of the sensors of its smartphone. The quality of sensing of user $n \in \mathcal{N}$, i.e., the accuracy of the sensed data, is modeled by a *quality indicator* $q_n > 0$, which is not known *a priori* and needs to be calculated. Notice that user n may have participated in the auction multiple times if the platform conducts the auction many times¹. For example, recall the noise map application in Section I. The platform may need to determine the noise level of some locations in the city at different times for several days. It keeps a historical record of the quality indicators of different users². The platform collects all sensed data from the winners and calculates the estimated value of the tasks. It then measures the quality indicators of the winners and updates its historical record.

We now determine the quality of sensing based on the collected sensed data. Assume user n has participated in the auction and has been a winner in L auctions in the past. In the l^{th} auction, where $l \in \{1, \dots, L\}$, we use $\hat{\delta}_k^{(l)}$ and $\hat{\delta}_{k,n}^{(l)}$ to denote the estimated value of task τ_k performed by the winners and the value of task τ_k obtained from user n , respectively. The platform can estimate $\hat{\delta}_k^{(l)}$ accurately by adopting a truth discovery approach [21]. Let $\hat{q}_n^{(l)}$ denote the quality indicator of user n obtained in the l^{th} auction. Then, $\hat{q}_n^{(l)}$ can be measured as the variance of sensed data provided by user n as follows:

$$\hat{q}_n^{(l)} = \frac{1}{|\Gamma_n^{(l)}|} \sum_{\tau_k \in \Gamma_n^{(l)}} \left(\hat{\delta}_{k,n}^{(l)} - \hat{\delta}_k^{(l)} \right)^2, \quad (2)$$

¹When the platform runs the auction for multiple times, smartphone users may learn the other participants' bids and information due to privacy leakage. This issue has been studied and addressed in [34], [35], and [36], and is beyond the scope of this paper.

²Notice that a fixed pool of users is not required to estimate the quality indicators. The users can move dynamically and participate in performing different tasks in different locations. In each round of auction, the platform estimates the quality indicators and updates the historical records for those users who win the auction and perform the sensing tasks. This captures the nature of a dynamic mobile crowdsensing system.

where $\Gamma_n^{(l)}$ is the set of tasks of winner n in the l^{th} auction. Notice that $\hat{q}_n^{(l)}$ in (2) is obtained individually for user n by taking the variance of the sensed data provided by this user. If user n is able to perform a large number of sensing tasks and submits a large set $\Gamma_n^{(l)}$, then $\hat{q}_n^{(l)}$ can be obtained more accurately. Let $\bar{q}_n^{(l)}$ denote the quality indicator of user n after the l^{th} auction. Then, $\bar{q}_n^{(l)}$ can be obtained as

$$\bar{q}_n^{(l)} = \gamma \hat{q}_n^{(l)} + (1 - \gamma) \bar{q}_n^{(l-1)}, \quad l \in \{1, \dots, L\}, \quad (3)$$

where constant $0 < \gamma < 1$ is the weight for the most recent quality indicator. The platform will then store the updated quality indicator in its historical record. Notice that $\bar{q}_n^{(0)}$ represents the initial value of quality indicator, which depends on the mobile crowdsensing application. By conducting more rounds of auction, the platform can obtain a more accurate estimate of $\bar{q}_n^{(l)}$ using (3). The estimated value approaches the true value when the platform has conducted the auction for many rounds. For the sake of simplicity, we abuse the notation and remove the round of auction (i.e., (l)) and use q_n , since we focus on a specific round of auction.

We denote the quality of sensing for task τ_k given the winners $\mathcal{S}$ as $g_k(\mathcal{S})$. It represents the accuracy of the estimated value of task τ_k after aggregating all sensed data from the winners. For tasks with continuous values, the accuracy of the estimated value can be defined by the mean squared estimation error. We calculate $g_k(\mathcal{S})$ by adopting the maximum likelihood estimation [37]:

$$g_k(\mathcal{S}) = \left(\sum_{n \in \mathcal{S}: \tau_k \in \Gamma_n} \frac{1}{q_n} \right)^{-1}. \quad (4)$$

A smaller value of $g_k(\mathcal{S})$ represents a higher accuracy of the estimated value of task τ_k , i.e., better quality of sensing. We illustrate the model for quality of sensing by using tasks with continuous values, but our model is suitable for tasks with discrete values as well by updating (2) and (4). In statistics, for tasks with discrete values, $\hat{q}_n^{(l)}$ can be estimated by 0-1 loss function or squared loss function and $g_k(\cdot)$ can denote the estimation error rate [21].

C. Problem Formulation

We denote the *valuation function* obtained from the winners $\mathcal{S}$ as $V(\mathcal{S})$. We consider that each task has a different weight, which can be regarded as the importance of that task to the platform. Let $\mu_k > 0$ denote the weight of task τ_k . The valuation function is

$$V(\mathcal{S}) = \sum_{\tau_k \in \bigcup_{s \in \mathcal{S}} \Gamma_s} \mu_k \log(1 + g_k(\mathcal{S})^{-1}). \quad (5)$$

The log term in (5) reflects the diminishing marginal returns on the quality of sensing, where q_n is the quality indicator of user n in a specific round of auction. The valuation function V is defined based on q_n to reflect the valuation of the performed tasks to the platform. In general, the platform aims to have a high quality of sensing of task τ_k as denoted by $g_k(\mathcal{S})^{-1}$, which is obtained based on the quality indicators (i.e., q_n) of those users $n \in \mathcal{N}$ who can perform the task. The

higher the quality indicator (i.e., the smaller q_n), the higher valuation the platform can obtain. In addition, the weighting factor μ_k reveals the importance of task τ_k to the valuation of the platform. Similar to [9], [11], [13], we assume that the platform aims to maximize the total valuation function as given in (5) instead of considering the valuation obtained from each task individually. According to this model, any task can contribute to the valuation function if there are some winners to perform the task. By maximizing the valuation function, the platform can perform the tasks in a weighted proportionally fair manner.

We first define submodular functions. Through Lemma 1, we then prove that $V(\mathcal{S})$ is a non-negative non-decreasing submodular function. It should be noted that all results of this paper hold for any submodular valuation functions.

Definition 1. For a finite set $\mathcal{Y}$, function $f: 2^{\mathcal{Y}} \rightarrow \mathbb{R}$ is submodular if [38]

$$f(\mathcal{C} \cup \{y\}) - f(\mathcal{C}) \geq f(\mathcal{D} \cup \{y\}) - f(\mathcal{D}),$$

for any $\mathcal{C} \subseteq \mathcal{D} \subseteq \mathcal{Y}$ and $y \in \mathcal{Y} \setminus \mathcal{D}$. Moreover, a submodular function f is non-decreasing if $f(\mathcal{C}) \leq f(\mathcal{D})$ for any $\mathcal{C} \subseteq \mathcal{D}$.

Lemma 1. The valuation $V(\mathcal{S})$ in (5) is a non-negative non-decreasing submodular function.

Proof. The proof can be found in Appendix A. ■

The platform, which has a limited budget G , aims to maximize its valuation $V(\mathcal{S})$ by selecting the set of winners $\mathcal{S}$ and making payments to them properly. We aim to design a mechanism $\mathcal{M} = (\mathcal{F}, \mathcal{P})$, which consists of a winner selection rule $\mathcal{F}$ and a payment determination rule $\mathcal{P}$. Given vector $\mathbf{b} = (b_1, \dots, b_N)$ (i.e., the bids from all users) and budget G as inputs, $\mathcal{F}$ returns the set of winners $\mathcal{S}$, and $\mathcal{P}$ returns the payment vector $\mathbf{p} = (p_1, \dots, p_N)$. The mechanism $\mathcal{M}$ should satisfy the following properties:

- **Computational Efficiency:** Both the winners and payments are determined in polynomial time.
- **Truthfulness:** Each participating user submits its true cost for its bid, i.e., $b_n = c_n$, $n \in \mathcal{N}$, by which it can maximize its own utility.
- **Individual Rationality:** Each participating user has a non-negative utility, i.e., $u_n \geq 0$, $\forall n \in \mathcal{N}$.
- **Budget Feasibility:** The total payment to the users should be within the limited budget G of the platform, i.e., $\sum_{n \in \mathcal{N}} p_n \leq G$.

The importance of computational efficiency, individual rationality, and budget feasibility is obvious while truthfulness is necessary to avoid market manipulation. Since the users are strategic, they will submit bids which maximize their own utilities. However, with truthfulness, the dominant strategy of users is to submit their true costs. These four properties together guarantee the successful practical implementation of the auction mechanism.

IV. AUCTION-BASED BUDGET FEASIBLE MECHANISM

In this section, we first propose our budget feasible mechanism ABSee by designing the winner selection rule $\mathcal{F}$ and

the payment determination rule $\mathcal{P}$. We then illustrate ABSee in details by a walk-through example.

Budget feasibility distinguishes our mechanism from other auction mechanisms and makes the mechanism design challenging. With the budget constraint, the winner selection rule $\mathcal{F}$ and the payment determination rule $\mathcal{P}$ are inter-dependent. The platform must ensure that the total payment for all selected winners is within the budget limit, while the payment should be determined carefully to ensure truthfulness. To guarantee that ABSee achieves truthfulness, according to the Myerson's characteristics [39], we have

Proposition 1. *An auction mechanism is truthful iff:*

- The winner selection rule $\mathcal{F}$ is monotone, i.e., if user n wins the auction by bidding b_n for Γ_n , it can also win the auction by bidding $b'_n \leq b_n$ for Γ_n .
- Each winner is paid the threshold payment, which refers to the highest bid the user can submit to win the auction.

Since $V(\mathcal{S})$ is a submodular function, we can adopt a greedy approach to design the monotone winner selection rule $\mathcal{F}$. Given a subset $\mathcal{A} \subseteq \mathcal{N}$, the marginal contribution of user $j \in \mathcal{N} \setminus \mathcal{A}$ is

$$V_j(\mathcal{A}) = V(\mathcal{A} \cup \{j\}) - V(\mathcal{A}). \quad (6)$$

In $\mathcal{F}$, we sort the users based on their marginal contributions per bid. The i^{th} user in the sorted list, denoted by x_i , has the largest marginal contribution per bid in subset $\mathcal{N} \setminus \mathcal{S}_{i-1}$, i.e.,

$$x_i = \arg \max_{j \in \mathcal{N} \setminus \mathcal{S}_{i-1}} \frac{V_j(\mathcal{S}_{i-1})}{b_j}, \quad (7)$$

where $\mathcal{S}_{i-1} = \{x_1, x_2, \dots, x_{i-1}\}$ and $\mathcal{S}_0 \triangleq \emptyset$. Considering the submodularity of $V(\mathcal{S})$, the sorting implies that:

$$\frac{V_{x_1}(\mathcal{S}_0)}{b_{x_1}} \geq \frac{V_{x_2}(\mathcal{S}_1)}{b_{x_2}} \geq \dots \geq \frac{V_{x_N}(\mathcal{S}_{N-1})}{b_{x_N}}. \quad (8)$$

The platform selects the winners from the above sorted list such that their marginal contribution per bid is not less than a certain threshold. The threshold determines the stopping criterion of the winner selection rule. To introduce the threshold, we first formally define θ as follows.

Definition 2. *We define the crowd factor as*

$$\theta \triangleq 1 - \frac{V^{\max}}{V(\mathcal{S})}, \quad (9)$$

where

$$V^{\max} = \max_{j \in \mathcal{N}} V(\{j\}). \quad (10)$$

To be selected as a winner, the marginal contribution per bid of user x_i must not be less than $\frac{V(\mathcal{S}_i)}{\theta G}$, where

$$\tilde{\theta} \triangleq \max \left\{ \frac{1}{2}, \theta \right\}. \quad (11)$$

We use the crowd factor θ and $\tilde{\theta}$ given in (9) and (11), respectively, to determine the portion of the budget used in the winner selection rule (i.e., $\tilde{\theta}G$) and design its stopping criterion. We will later show that using $\tilde{\theta}G$ amount of budget

for winner selection rule guarantees budget feasibility. For a fixed value of $\tilde{\theta}$, let $w \in \mathcal{N}$ be the largest index such that $\frac{V_{x_w}(\mathcal{S}_{w-1})}{b_{x_w}} \geq \frac{V(\mathcal{S}_w)}{\tilde{\theta}G}$, i.e.,

$$b_{x_w} \leq \tilde{\theta}G \frac{V_{x_w}(\mathcal{S}_{w-1})}{V(\mathcal{S}_w)}. \quad (12)$$

Then, $\mathcal{S} = \{x_1, x_2, \dots, x_w\}$ is the set of winners. However, notice that we cannot obtain θ and $\tilde{\theta}$ directly since the winners set $\mathcal{S}$ is not known *a priori*. We adopt an iterative approach to determine θ and $\mathcal{S}$ together. The value of θ in iteration t is denoted by $\theta^{(t)}$. In each iteration t , the winners set $\mathcal{S}$ is obtained by utilizing $\theta^{(t)}$, and then $\mathcal{S}$ is used to calculate $\theta^{(t+1)}$. When $\theta^{(t)}$ increases, there will be more winners added to set $\mathcal{S}$, and thus $V(\mathcal{S})$ increases. In this case, both $\theta^{(t)}$ and $V(\mathcal{S})$ increase monotonically. Since $\mathcal{S} \subseteq \mathcal{N}$ is finite, the iteration converges. When θ decreases, $V(\mathcal{S})$ will always be decreasing and similar results can be obtained. Thus, the convergence of the iteration is guaranteed. The value of $\tilde{\theta}$ can be obtained by substituting $\theta^{(t)}$ into (11) and the set of winners can be obtained accordingly.

According to the Myerson's characteristics, the goal of the payment determination rule $\mathcal{P}$ is to pay each winner the threshold payment. For each winner $x_i \in \mathcal{S}$, similar to the winner selection rule $\mathcal{F}$, we sort the users in set $\mathcal{N}' \triangleq \mathcal{N} \setminus \{x_i\}$, based on their marginal contributions per bid. We use $\mathcal{Q}_k$ to denote the first k users in this sorting, and i_k to denote the k^{th} user, i.e.,

$$i_k = \arg \max_{j \in \mathcal{N}' \setminus \mathcal{Q}_{k-1}} \frac{V_j(\mathcal{Q}_{k-1})}{b_j}.$$

Then, $V_{i_k}(\mathcal{Q}_{k-1})$ is the marginal contribution of the k^{th} user in this sorting. Similarly, we can obtain

$$\frac{V_{i_1}(\mathcal{Q}_0)}{b_{i_1}} \geq \frac{V_{i_2}(\mathcal{Q}_1)}{b_{i_2}} \geq \dots \geq \frac{V_{i_{N-1}}(\mathcal{Q}_{N-2})}{b_{i_{N-1}}}. \quad (13)$$

We use $w' \in \mathcal{N}'$ to denote the largest index such that

$$b_{i_{w'}} \leq \tilde{\theta}G \frac{V_{i_{w'}}(\mathcal{Q}_{w'-1})}{V(\mathcal{Q}_{w'})}. \quad (14)$$

Let $\beta_{i(k)}$ denote the highest bid that winner x_i can submit to replace user i_k in the k^{th} position. Furthermore, let $\rho_{i(k)}$ denote the highest bid that x_i can submit so that the marginal contribution per bid is not less than $\frac{V(\mathcal{Q}_{k-1} \cup \{x_i\})}{\tilde{\theta}G}$. We have

$$\beta_{i(k)} = \frac{V_{x_i}(\mathcal{Q}_{k-1})b_{i_k}}{V_{i_k}(\mathcal{Q}_{k-1})}, \quad \rho_{i(k)} = \tilde{\theta}G \frac{V_{x_i}(\mathcal{Q}_{k-1})}{V(\mathcal{Q}_{k-1} \cup \{x_i\})}.$$

Since x_i can be placed in any position k from 1 to $w' + 1$ to be selected as a winner, the payment p_{x_i} for winner x_i is

$$p_{x_i} = \max_{k \in \{1, \dots, w'+1\}} \{\min(\beta_{i(k)}, \rho_{i(k)})\}. \quad (15)$$

Our proposed budget feasible mechanism is shown in Algorithm 1. Through Steps 2 to 13, the crowd factor is obtained. We set $0 < \theta^{(1)} < 1$ for initialization. An iterative approach is adopted to calculate $\theta^{(t)}$. In each iteration (i.e., Steps 4 to 12), the winners are selected in a greedy manner according to their marginal contribution per bid. The value of $\tilde{\theta}$ is obtained from Step 14. Steps 15 to 20 show the winner selection rule

Algorithm 1: ABSee Budget Feasible Mechanism

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1 Input  $\Gamma, V(\cdot), G, \mathcal{N}, \mathbf{b}$ 
  /* Winner selection  $\mathcal{F}$  */
2  $0 < \theta^{(1)} < 1, t \leftarrow 0, V^{\max} \leftarrow \max_{j \in \mathcal{N}} V(\{j\})$ 
3 do
4    $t \leftarrow t + 1, i \leftarrow 1, \mathcal{S} \leftarrow \emptyset$ 
5    $x_i \leftarrow \arg \max_{j \in \mathcal{N}} \frac{V_j(\emptyset)}{b_j}$ 
6   while  $b_{x_i} \leq \theta^{(t)} G \frac{V_{x_i}(\mathcal{S})}{V(\mathcal{S} \cup \{x_i\})}$  do
7      $\mathcal{S} \leftarrow \mathcal{S} \cup \{x_i\}$ 
8      $i \leftarrow i + 1$ 
9      $x_i \leftarrow \arg \max_{j \in \mathcal{N} \setminus \mathcal{S}} \frac{V_j(\mathcal{S})}{b_j}$ 
10  end
11  if  $\mathcal{S} = \emptyset$  then break;
12   $\theta^{(t+1)} \leftarrow 1 - \frac{V^{\max}}{V(\mathcal{S})}$ 
13 while  $\theta^{(t+1)} \neq \theta^{(t)}$ ;
14  $\tilde{\theta} \leftarrow \max\{\frac{1}{2}, \theta^{(t)}\}$ 
15  $i \leftarrow 1, \mathcal{S} \leftarrow \emptyset, x_i \leftarrow \arg \max_{j \in \mathcal{N}} \frac{V_j(\emptyset)}{b_j}$ 
16 while  $b_{x_i} \leq \tilde{\theta} G \frac{V_{x_i}(\mathcal{S})}{V(\mathcal{S} \cup \{x_i\})}$  do
17    $\mathcal{S} \leftarrow \mathcal{S} \cup \{x_i\}$ 
18    $i \leftarrow i + 1$ 
19    $x_i \leftarrow \arg \max_{j \in \mathcal{N} \setminus \mathcal{S}} \frac{V_j(\mathcal{S})}{b_j}$ 
20 end
  /* Payment determination  $\mathcal{P}$  */
21  $p_n \leftarrow 0, \forall n \in \mathcal{N}$ 
22 for  $x_i \in \mathcal{S}$  do
23    $\mathcal{N}' \leftarrow \mathcal{N} \setminus \{x_i\}, \mathcal{Q} \leftarrow \emptyset, k \leftarrow 0$ 
24   do
25      $k \leftarrow k + 1$ 
26      $\rho_{i(k)} \leftarrow \frac{\tilde{\theta} G V_{x_i}(\mathcal{Q})}{V(\mathcal{Q} \cup \{x_i\})}$ 
27     if  $\mathcal{N}' \setminus \mathcal{Q} = \emptyset$  then
28        $p_{x_i} \leftarrow \max\{p_{x_i}, \rho_{i(k)}\}$ 
29       break
30     else
31        $i_k \leftarrow \arg \max_{j \in \mathcal{N}' \setminus \mathcal{Q}} \frac{V_j(\mathcal{Q})}{b_j}$ 
32        $\beta_{i(k)} \leftarrow \frac{V_{x_i}(\mathcal{Q}) b_{i_k}}{V_{i_k}(\mathcal{Q})}$ 
33        $p_{x_i} \leftarrow \max\{p_{x_i}, \min\{\beta_{i(k)}, \rho_{i(k)}\}\}$ 
34        $\mathcal{Q} \leftarrow \mathcal{Q} \cup \{i_k\}$ 
35     end
36   while  $b_{i_k} > \tilde{\theta} G \frac{V_{i_k}(\mathcal{Q} \setminus \{i_k\})}{V(\mathcal{Q})}$ ;
37 end
38 return  $(\mathcal{S}, \mathbf{p})$ 

```

$\mathcal{F}$. The set of winners will be updated when $\theta^{(t)}$ is replaced by $\tilde{\theta}$. Steps 21 to 37 show the payment determination rule. We finally obtain the set of winners $\mathcal{S}$ and the payment vector $\mathbf{p}$.

We use the example in Fig. 2 to provide a walk-through example. In this figure, the squares represent the users and the circles represent the tasks. The bid for the tasks each user can perform and the quality indicator of the user are given above the squares. The weight of each task is also given in this figure. Assume the platform has a budget $G = 30$.

Winner selection:

- 1) According to Step 2 of Algorithm 1, suppose we choose $\theta^{(1)} = \frac{1}{2}$ and calculate $V^{\max} = V(\{2\}) = 7 \times \log(1 + \frac{1}{0.1}) = 16.79$. Then, according to Steps 3 to 13, winners are selected and θ is updated in each iteration as follows.
- 2) $t = 1, \theta^{(1)} = \frac{1}{2}$.

- $\mathcal{S} = \emptyset: \frac{V_1(\emptyset)}{b_1} = \frac{V(\{1\})}{b_1} = \frac{8 \times \log(1 + \frac{1}{0.2})}{4} = 3.58,$

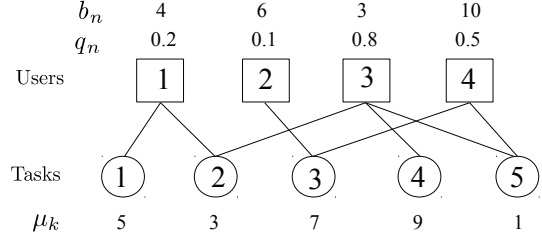


Fig. 2. An instance of the system with four users and five tasks. $\mathcal{N} = \{1, \dots\}$, $\Gamma = \{\tau_1, \dots\}$, $q_1 = 0.2, q_2 = 0.1, q_3 = 0.8, q_4 = 0.5, b_1 = 4, b_2 = 6, b_3 = 3, b_4 = 10, \mu_1 = 5, \mu_2 = 3, \mu_3 = 7, \mu_4 = 9, \mu_5 = 1, \Gamma_1 = \{\tau_1, \tau_2\}, \Gamma_2 = \{\tau_3\}, \Gamma_3 = \{\tau_2, \tau_4, \tau_5\},$ and $\Gamma_4 = \{\tau_3, \tau_5\}$.

$$\frac{V_2(\emptyset)}{b_2} = 2.8, \frac{V_3(\emptyset)}{b_3} = 3.51, \frac{V_4(\emptyset)}{b_4} = 0.88.$$

$$x_1 = 1, 3.58 = \frac{V_1(\emptyset)}{b_1} \geq \frac{V(\emptyset \cup \{1\})}{\theta^{(1)} G} = \frac{14.33}{15} = 0.96.$$

- $\mathcal{S} = \{1\}: \frac{V_2(\{1\})}{b_2} = 2.8, \frac{V_3(\{1\})}{b_3} = 2.89, \frac{V_4(\{1\})}{b_4} = 0.88.$

$$x_2 = 3, 2.89 = \frac{V_3(\{1\})}{b_3} \geq \frac{V(\{1,3\})}{\theta^{(1)} G} = \frac{23.01}{15} = 1.53.$$

- $\mathcal{S} = \{1, 3\}: \frac{V_2(\{1,3\})}{b_2} = 2.8, \frac{V_4(\{1,3\})}{b_4} = 0.83.$

$$x_3 = 2, 2.8 = \frac{V_2(\{1,3\})}{b_2} \geq \frac{V(\{1,2,3\})}{\theta^{(1)} G} = \frac{39.8}{15} = 2.65.$$

- $\mathcal{S} = \{1, 2, 3\}: \frac{V_4(\{1,2,3\})}{b_4} = 0.18.$

$$x_4 = 4, 0.18 = \frac{V_4(\{1,2,3\})}{b_4} < \frac{V(\{1,2,3,4\})}{\theta^{(1)} G} = 2.8.$$

Thus, $\mathcal{S} = \{1, 2, 3\}, \theta^{(2)} = 1 - \frac{V^{\max}}{V(\mathcal{S})} = 0.58,$ and $\theta^{(2)} \neq \theta^{(1)}.$

3) $t = 2, \theta^{(2)} = 0.58.$

- $\mathcal{S} = \emptyset: x_1 = 1, 3.58 = \frac{V_1(\emptyset)}{b_1} \geq \frac{V(\emptyset \cup \{1\})}{\theta^{(2)} G} = 0.82.$

- $\mathcal{S} = \{1\}: x_2 = 3, 2.9 = \frac{V_3(\{1\})}{b_3} \geq \frac{V(\{1,3\})}{\theta^{(2)} G} = 1.32.$

- $\mathcal{S} = \{1, 3\}: x_3 = 2, 2.8 = \frac{V_2(\{1,3\})}{b_2} \geq \frac{V(\{1,2,3\})}{\theta^{(2)} G} = \frac{39.8}{17.4} = 2.29.$

- $\mathcal{S} = \{1, 2, 3\}: x_4 = 4, 0.18 = \frac{V_4(\{1,2,3\})}{b_4} < \frac{V(\{1,2,3,4\})}{\theta^{(2)} G} = \frac{42.07}{17.4} = 2.42.$

Thus, we still have $\mathcal{S} = \{1, 2, 3\}$ with $\theta^{(3)} = 1 - \frac{V^{\max}}{V(\mathcal{S})} = 0.58$ and $\theta^{(2)} = \theta^{(3)}.$

4) According to Step 14 of Algorithm 1, since $\theta^{(3)} > 0.5,$ we obtain $\tilde{\theta} = \theta^{(3)} = 0.58.$ Thus, the set of winners $\mathcal{S} = \{1, 2, 3\}.$

Payment determination:

- 1) $x_1 = 1$: Winners and the next user after the selected winners are $1_1 = 3, 1_2 = 2,$ and $1_3 = 4.$

$$\beta_{1(1)} = \frac{V_{x_1}(\mathcal{Q}_0) b_{1_1}}{V_{1_1}(\mathcal{Q}_0)} = \frac{V_1(\emptyset) b_3}{V_3(\emptyset)} = 4.08, \rho_{1(1)} = \frac{\tilde{\theta} G V_{x_1}(\mathcal{Q}_0)}{V(\mathcal{Q}_0 \cup \{x_1\})} = \tilde{\theta} G = 17.4, \beta_{1(2)} = 4.45, \rho_{1(2)} = 9.43, \beta_{1(3)} = 69.3, \rho_{1(3)} = 5.45. \text{ Thus, } p_{x_1} = 5.45.$$

- 2) $x_2 = 3$: Winners and the next user after the selected winners are $2_1 = 1, 2_2 = 2,$ and $2_3 = 4.$

$$\beta_{2(1)} = 2.94, \rho_{2(1)} = 17.4, \beta_{2(2)} = 3.1, \rho_{2(2)} = 6.56, \beta_{2(3)} = 37.74, \rho_{2(3)} = 3.79. \text{ Thus, } p_{x_2} = 3.79.$$

- 3) $x_3 = 2$: Winners and the next user after the selected winners are $3_1 = 1, 3_2 = 3,$ and $3_3 = 4.$

$$\beta_{3(1)} = 4.69, \rho_{3(1)} = 17.4, \beta_{3(2)} = 5.81, \rho_{3(2)} = 9.39, \beta_{3(3)} = 20.23, \rho_{3(3)} = 7.34. \text{ Thus, } p_{x_3} = 7.34.$$

Hence, $p_1 = p_{x_1} = 5.45, p_2 = p_{x_3} = 7.34, p_3 = p_{x_2} = 3.79,$ and $p_4 = 0.$ Notice that $p_1 + p_2 + p_3 + p_4 < G.$

V. MECHANISM ANALYSIS

In this section, we first prove that ABSee satisfies all of the properties introduced in Section III. Then, we calculate the approximation ratio of ABSee.

A. Properties of ABSee

Theorem 1. *ABSee is computationally efficient.*

Proof. Finding the user with the maximum marginal contribution per bid takes $O(N^2M)$ time since calculating $V(\mathcal{S})$ takes $O(NM)$ time. Both the winner selection rule $\mathcal{F}$ and the payment determination rule $\mathcal{P}$ have nested loops. The while loop in $\mathcal{F}$ takes $O(N^3M)$ time because the maximum number of the winners is N . Moreover, the do-while loop runs at most N times to update $\theta^{(t)}$. Thus, the running time of $\mathcal{F}$ is $O(N^4M)$. Similar to $\mathcal{F}$, the running time of the payment determination rule $\mathcal{P}$ is also $O(N^4M)$. ■

Note that the running time shown above is conservative. In practice, the number of winners is much less than N . The above complexity analysis is provided for one round of auction as the platform does not need to run multiple rounds at the same time. However, the number of auctions will increase in a long run to update the quality indicator of users.

Theorem 2. *ABSee is truthful.*

Proof. It is required to show that ABSee satisfies the Myerson's characteristics. In the greedy approach in $\mathcal{F}$, since a lower bid can only put the user in the same or a prior position, the monotonicity is guaranteed. Notice that the monotonicity is not influenced by the value of $\tilde{\theta}$. The iteration in the winner selection rule $\mathcal{F}$ cannot change the monotonicity of the greedy approach. Thus, we only need to show that the winners receive the threshold payments. The proof follows the approach presented in [26]. Consider winner x_i . From (15), let $r \leq w' + 1$ be the index such that

$$p_{x_i} = \max_{k \in \{1, \dots, w' + 1\}} \{\min(\beta_{i(k)}, \rho_{i(k)})\} = \min\{\beta_{i(r)}, \rho_{i(r)}\}.$$

Recall w' from (14). We show that user x_i wins the auction by bidding $b_{x_i} \leq p_{x_i}$, and loses the auction if $b_{x_i} > p_{x_i}$.

If $b_{x_i} \leq p_{x_i}$, we have $b_{x_i} \leq \beta_{i(r)}$. User x_i can be placed in the first $w' + 1$ positions in the sorted list given by (13). We also have $b_{x_i} \leq \rho_{i(r)}$. Thus, it will be chosen as a winner.

If $b_{x_i} > p_{x_i}$, we have two cases.

Case 1: $\beta_{i(r)} \leq \rho_{i(r)}$, i.e., $p_{x_i} = \beta_{i(r)}$. In this case, x_i will be placed after the r^{th} position in the sorted list. If $\beta_{i(r)} = \max_{k \in \{1, \dots, w' + 1\}} \beta_{i(k)}$, x_i cannot be a winner. Otherwise, if $\beta_{i(r)} < \beta_{i(k)}$ for some $k \in \{1, \dots, w' + 1\}$, we have $\rho_{i(k)} < \beta_{i(r)} = p_{x_i} < b_{x_i}$. Thus, x_i cannot be placed at the k^{th} position to be selected as a winner.

Case 2: $\beta_{i(r)} > \rho_{i(r)}$, i.e., $p_{x_i} = \rho_{i(r)}$. If $\rho_{i(r)} = \max_{k \in \{1, \dots, w' + 1\}} \rho_{i(k)}$, x_i cannot be a winner. Otherwise, if $\rho_{i(r)} < \rho_{i(k)}$ for some $k \in \{1, \dots, w' + 1\}$, we have $\beta_{i(k)} < \rho_{i(r)} = p_{x_i} < b_{x_i}$. Thus, x_i cannot be a winner.

Since the winner selection rule $\mathcal{F}$ is monotone and each winner receives the threshold payment, we conclude that ABSee is truthful. ■

Theorem 3. *ABSee is individually rational.*

Proof. If a user is not a winner, its utility is zero as shown in (1). Consider a winner x_i , whose payment p_{x_i} is obtained from (15). Since ABSee is truthful, we have $b_{x_i} = c_{x_i}$ for a winner x_i . If there exists k from 1 to $w' + 1$ such that $b_{x_i} \leq \min\{\beta_{i(k)}, \rho_{i(k)}\}$, we have $b_{x_i} \leq p_{x_i}$. Then, we can obtain $u_{x_i} = p_{x_i} - c_{x_i} \geq 0$. For winner x_i , the payment determination rule $\mathcal{P}$ implies that $\mathcal{Q}_k = \mathcal{S}_k$, for $k < i$, thus $V_{x_i}(\mathcal{S}_{i-1}) = V_{x_i}(\mathcal{Q}_{i-1})$. Recall that user x_i is sorted in the i^{th} position in (8) among users $\mathcal{N}$ and user i_k is sorted in the k^{th} position in (13) among users $\mathcal{N} \setminus \{x_i\}$. Consider the case of $k = i$, we have

$$b_{x_i} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{S}_{i-1} \cup \{x_i\})} = \tilde{\theta}G \frac{V_{x_i}(\mathcal{Q}_{k-1})}{V(\mathcal{Q}_{k-1} \cup \{x_i\})} = \rho_{i(k)}. \quad (16)$$

From (7), we have $\frac{V_{x_i}(\mathcal{S}_{i-1})}{b_{x_i}} \geq \frac{V_{i_k}(\mathcal{S}_{i-1})}{b_{i_k}}$. We can obtain that

$$b_{x_i} \leq \frac{V_{x_i}(\mathcal{S}_{i-1})b_{i_k}}{V_{i_k}(\mathcal{S}_{i-1})} = \frac{V_{x_i}(\mathcal{Q}_{k-1})b_{i_k}}{V_{i_k}(\mathcal{Q}_{k-1})} = \beta_{i(k)}. \quad (17)$$

From (16) and (17), we have $b_{x_i} \leq \min\{\beta_{i(k)}, \rho_{i(k)}\}$ when $k = i$. This results in $b_{x_i} \leq p_{x_i}$ and completes the proof. ■

To prove the budget feasibility of ABSee, we use the following lemma.

Lemma 2. *The payment p_{x_i} for each winner x_i satisfies $p_{x_i} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{Q}_{w'})}$.*

Proof. The proof can be found in Appendix B. ■

By utilizing Lemma 2, we now prove the budget feasibility.

Theorem 4. *ABSee is budget feasible.*

Proof. Since the proof of budget feasibility is similar to [27] when $\tilde{\theta} = \frac{1}{2}$ (i.e., $\theta < \frac{1}{2}$), we only focus on the case that $\tilde{\theta} = \theta \geq \frac{1}{2}$. To prove $\sum_{n \in \mathcal{N}} p_n \leq G$, it is equivalent to prove that $\sum_{x_i \in \mathcal{S}} p_{x_i} \leq G$. According to the payment determination rule $\mathcal{P}$, we have $V(\mathcal{S}) - V(\mathcal{Q}_{w'}) \leq \max_{j \in \mathcal{N}} V(\{j\})$. Thus,

$$1 - \frac{V(\mathcal{Q}_{w'})}{V(\mathcal{S})} = \frac{V(\mathcal{S}) - V(\mathcal{Q}_{w'})}{V(\mathcal{S})} \leq \frac{\max_{j \in \mathcal{N}} V(\{j\})}{V(\mathcal{S})}.$$

Then,

$$\frac{V(\mathcal{Q}_{w'})}{V(\mathcal{S})} \geq 1 - \frac{\max_{j \in \mathcal{N}} V(\{j\})}{V(\mathcal{S})} = \theta.$$

Thus, $V(\mathcal{Q}_{w'}) \geq \theta V(\mathcal{S})$. From Lemma 2, for each winner x_i ,

$$p_{x_i} \leq \theta G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{Q}_{w'})} \leq \theta G \frac{V_{x_i}(\mathcal{S}_{i-1})}{\theta V(\mathcal{S})} = \frac{V_{x_i}(\mathcal{S}_{i-1})G}{V(\mathcal{S})}.$$

According to (6) and $\mathcal{S} = \{x_1, x_2, \dots, x_w\}$, we have

$$\begin{aligned} \sum_{x_i \in \mathcal{S}} V_{x_i}(\mathcal{S}_{i-1}) &= V_{x_1}(\emptyset) + V_{x_2}(\mathcal{S}_1) + \dots + V_{x_w}(\mathcal{S}_{w-1}) \\ &= V(\mathcal{S}). \end{aligned}$$

Thus, $\sum_{x_i \in \mathcal{S}} p_{x_i} \leq \sum_{x_i \in \mathcal{S}} V_{x_i}(\mathcal{S}_{i-1}) \frac{G}{V(\mathcal{S})} = G$. ■

B. Crowd Factor

We now analyze the crowd factor θ . For a successful mobile crowdsensing application, there are usually a large number

of participating users and the platform selects many winners. Thus, $V(\mathcal{S})$ is much larger than $V^{\max}$ (i.e., $V(\mathcal{S}) \gg V^{\max}$) and θ approaches 1. We study the following scenario as an example to analyze the behavior of θ . Consider each user has the same quality indicator, i.e., $q_n = q$, $n \in \mathcal{N}$. The number of tasks each user can perform is also the same, i.e., $|\Gamma_n| = \Upsilon$, $n \in \mathcal{N}$. The subsets of tasks each user can perform are pairwise disjoint, i.e., $\Gamma_n \cap \Gamma_{n'} = \emptyset$, $n, n' \in \mathcal{N}$. All tasks have the same weight, i.e., $\mu_k = \mu$, $\tau_k \in \Gamma$. The bid b_n is equal to b . Then, for winner x_i , we have

$$V_{x_i}(\mathcal{S}_{i-1}) = \sum_{\tau_k \in \Gamma_{x_i}} \mu_k \log \left(1 + \frac{1}{q} \right) = \Upsilon \mu \log \left(1 + \frac{1}{q} \right),$$

which is denoted by V . Thus, we obtain

$$V(\mathcal{S}) = \sum_{x_i \in \mathcal{S}} V_{x_i}(\mathcal{S}_{i-1}) = |\mathcal{S}| V, \quad (18)$$

and $V^{\max} = V$. From the inequalities (8) and (12), the largest index of winners (i.e., w) satisfies $\frac{V}{b} \geq \frac{wV}{\theta G}$. When $\theta \geq \frac{1}{2}$, we have $\tilde{\theta} = \theta$. Thus, we obtain $w = \lfloor \frac{\theta G}{b} \rfloor$. When $\frac{G}{b}$ is large, the number of winners $|\mathcal{S}| = w \approx \frac{\theta G}{b}$. From (9), we have

$$\theta = 1 - \frac{V^{\max}}{V(\mathcal{S})} = 1 - \frac{V}{wV} = 1 - \frac{1}{w} \approx 1 - \frac{b}{\theta G}.$$

Notice that according to (18), we have $V(\mathcal{S}) = |\mathcal{S}| V = wV$. We now solve the equation $\theta = 1 - \frac{b}{\theta G}$ to obtain the value of θ . When $\frac{4b}{G} < 1$, we have $\theta = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4b}{G}} \right)$. For $w \approx \frac{\theta G}{b}$, the number of winners should be at least $\frac{\theta G}{b}$, i.e., the number of users should be at least $\frac{\theta G}{b}$. Thus, when $\frac{4b}{G} \ll 1$ and $N > \frac{G}{b}$, i.e., $N \gg 4$, the crowd factor θ approaches 1. Since the conditions $\frac{4b}{G} \ll 1$ and $N \gg 4$ can easily be satisfied in a practical mobile crowdsensing system, θ is close to 1 with high probability. Note that the bid of users (i.e., b) is usually much less than the total budget G .

In general, according to the definition of crowd factor θ , i.e., $\theta \triangleq 1 - \frac{V^{\max}}{V(\mathcal{S})}$, it depends on $V^{\max}$ and $V(\mathcal{S})$. Since $V^{\max}$ is fixed given a set of users, θ is mainly determined by $V(\mathcal{S})$. The value of $V(\mathcal{S})$ becomes larger, when many users win the auction and perform the tasks. In this case, θ will be close to 1. Moreover, a higher budget of the platform increases the number of winners, which consequently increases $V(\mathcal{S})$ and θ . Although we analyzed the crowd factor for the above example, in Section VI, we will show that θ is close to 1 under different scenarios in a mobile crowdsensing system.

C. Approximation Ratio Analysis

A budget feasible mechanism is called α -approximate [26] if it determines a subset $\mathcal{S} \subseteq \mathcal{N}$ such that $\text{opt}(\mathcal{N}) \leq \alpha V(\mathcal{S})$, where $\text{opt}(\mathcal{N})$ is the optimal value of the following problem:

$$\begin{aligned} & \underset{\mathcal{S} \subseteq \mathcal{N}}{\text{maximize}} && V(\mathcal{S}) \\ & \text{subject to} && \sum_{x_i \in \mathcal{S}} c_{x_i} \leq G. \end{aligned} \quad (19)$$

Problem (19) is a budgeted submodular function maximization problem [40]. Similar to the Knapsack problem, problem (19)

is also an NP-hard problem.

To determine the approximation ratio of our proposed mechanism, we first introduce a fractional greedy algorithm [27] to solve problem (19). Similar to our proposed winner selection rule $\mathcal{F}$, we select winners with the highest marginal contribution per bid until we cannot add more winners due to budget limit. We assume that the contribution of a user can be fractional. Let H be the largest index such that $\sum_{i=1}^H c_{x_i} \leq G$. We define $C_{x_{H+1}} \triangleq G - \sum_{i=1}^H c_{x_i}$ and $V'_{x_{H+1}}(\mathcal{S}_H) \triangleq V_{x_{H+1}}(\mathcal{S}_H) \frac{C_{x_{H+1}}}{c_{x_{H+1}}}$. The valuation obtained by adopting the fractional greedy algorithm can be defined as:

$$\tilde{V}(\mathcal{S}) \triangleq \sum_{i=1}^H V_{x_i}(\mathcal{S}_{i-1}) + V'_{x_{H+1}}(\mathcal{S}_H). \quad (20)$$

We have the following lemma from [27]:

Lemma 3. *The fractional greedy solution has an approximation ratio of $\frac{e}{e-1}$ for problem (19). That is,*

$$\text{opt}(\mathcal{N}) \leq \left(\frac{e}{e-1} \right) \tilde{V}(\mathcal{S}),$$

where $\text{opt}(\mathcal{N})$ is the optimal value given user set $\mathcal{N}$.

By utilizing Lemma 3, we have the following theorem:

Theorem 5. *ABSee achieves an approximation ratio of $\min \left\{ \frac{2e}{\theta(e-1)}, \frac{(5-2\theta)e}{e-1} \right\}$.*

Proof. Recall that the winners $\mathcal{S} = \{x_1, x_2, \dots, x_w\}$. For any $i \in \{w+1, \dots, H\}$, we have

$$\frac{c_{x_i}}{V_{x_i}(\mathcal{S}_{i-1})} \geq \frac{c_{x_{w+1}}}{V_{x_{w+1}}(\mathcal{S}_{w+1})} > \frac{\tilde{\theta}G}{V(\mathcal{S}_{w+1})}.$$

Notice that x_i is no longer a winner when $i > w$. Thus, according to the winner determination rule, we obtain

$$c_{x_i} > \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{S}_{w+1})}, \quad C_{x_{H+1}} > \tilde{\theta}G \frac{V'_{x_{H+1}}(\mathcal{S}_H)}{V(\mathcal{S}_{w+1})}.$$

Then, we have

$$\tilde{\theta}G \frac{\sum_{i=w+1}^H V_{x_i}(\mathcal{S}_{i-1}) + V'_{x_{H+1}}(\mathcal{S}_H)}{\sum_{i=1}^{w+1} V_{x_i}(\mathcal{S}_{i-1})} < \sum_{i=w+1}^H c_{x_i} + C_{x_{H+1}} \leq G,$$

and

$$\sum_{i=w+1}^H V_{x_i}(\mathcal{S}_{i-1}) + V'_{x_{H+1}}(\mathcal{S}_H) < \frac{\sum_{i=1}^{w+1} V_{x_i}(\mathcal{S}_{i-1})}{\tilde{\theta}}. \quad (21)$$

From (20) and (21), we have

$$\begin{aligned} \tilde{V}(\mathcal{S}) &= \sum_{i=1}^H V_{x_i}(\mathcal{S}_{i-1}) + V'_{x_{H+1}}(\mathcal{S}_H) \\ &= \sum_{i=1}^w V_{x_i}(\mathcal{S}_{i-1}) + \sum_{i=w+1}^H V_{x_i}(\mathcal{S}_{i-1}) + V'_{x_{H+1}}(\mathcal{S}_H) \\ &< \sum_{i=1}^w V_{x_i}(\mathcal{S}_{i-1}) + \frac{\sum_{i=1}^w V_{x_i}(\mathcal{S}_{i-1}) + V_{x_{w+1}}(\mathcal{S}_w)}{\tilde{\theta}} \end{aligned}$$

$$\leq (1 + \frac{1}{\theta}) \sum_{i=1}^w V_{x_i}(\mathcal{S}_{i-1}) + \frac{V^{\max}}{\theta}. \quad (22)$$

Recall that $V(\mathcal{S}) = \sum_{i=1}^w V_{x_i}(\mathcal{S}_{i-1})$. If $\theta \geq \frac{1}{2}$, we have $\tilde{\theta} = \theta$. Thus,

$$\begin{aligned} \tilde{V}(\mathcal{S}) &< \frac{V(\mathcal{S}) + V(\mathcal{S}) - V^{\max}}{V(\mathcal{S}) - V^{\max}} V(\mathcal{S}) + \frac{V(\mathcal{S})}{V(\mathcal{S}) - V^{\max}} V^{\max} \\ &= \frac{2V(\mathcal{S})}{V(\mathcal{S}) - V^{\max}} V(\mathcal{S}) = \frac{2}{\theta} V(\mathcal{S}). \end{aligned}$$

From Lemma 3, we have $\text{opt}(\mathcal{N}) \leq \frac{e}{e-1} \tilde{V}(\mathcal{S})$. Then,

$$\frac{\text{opt}(\mathcal{N})}{V(\mathcal{S})} \leq \frac{e}{e-1} \frac{\tilde{V}(\mathcal{S})}{V(\mathcal{S})} < \frac{2e}{\theta(e-1)}.$$

If $\theta < \frac{1}{2}$, we set $\tilde{\theta} = \frac{1}{2}$ in (22) and we have

$$\begin{aligned} \tilde{V}(\mathcal{S}) &\leq 3V(\mathcal{S}) + 2V^{\max} = V(\mathcal{S}) \left(3 + 2 \frac{V^{\max}}{V(\mathcal{S})} \right) \\ &= V(\mathcal{S}) (5 - 2\theta). \end{aligned}$$

Therefore,

$$\text{opt}(\mathcal{N}) < \frac{(5-2\theta)e}{(e-1)} V(\mathcal{S}).$$

Thus, ABSee achieves an approximation ratio of $\min \left\{ \frac{2e}{\theta(e-1)}, \frac{(5-2\theta)e}{e-1} \right\}$. When θ is close to 1, the approximation ratio approaches $\frac{2e}{e-1}$. ■

VI. PERFORMANCE EVALUATION

In this section, we first evaluate the performance of the approach for estimating the quality of sensing of each user. We then compare the performance of ABSee with GREEDY-SM and RANDOM-SM mechanisms proposed in [27], which are budget feasible mechanisms and satisfy all the desirable properties. We assume that tasks and users are randomly distributed within a 1 km $\times$ 1 km region. A user can perform a task if the distance between the user and the task is less than 50 m. The cost of user n (i.e., c_n) is $\eta_n |\Gamma_n|$, in which η_n is uniformly distributed over $[1, 5]$. The weight of task τ_k (i.e., μ_k) in (5) is uniformly distributed over $[1, 10]$. The results are obtained by averaging over 100 instances.

The platform calculates and keeps a record of the quality indicators of the participating users. To simulate this process, consider the noise map application. We assume each task corresponds to the noise level of a specific position. The real noise level of task τ_k in the l^{th} auction, denoted by $\delta_k^{(l)}$, is uniformly distributed over $[0, 5]$. The sensing data $\hat{\delta}_{k,n}^{(l)}$ provided by user n is generated from a Gaussian distribution $N(\delta_k^{(l)}, q_n)$, where the quality indicator q_n of each user is uniformly distributed over $[0, 1]$. After collecting sensing data for task τ_k , the platform calculates the estimated value $\hat{\delta}_k^{(l)}$ and updates the estimated quality indicator $\bar{q}_n^{(l)}$. At the beginning, we assume that the estimated quality indicator $\bar{q}_n^{(0)}$ is uniformly distributed in $[0, 1]$ for all $n \in \mathcal{N}$. In the next rounds of auction, we set the estimated quality indicator of those users who have not won the auction in previous rounds as the average of quality indicators of other users. The weight γ in (3) is set to 0.5.

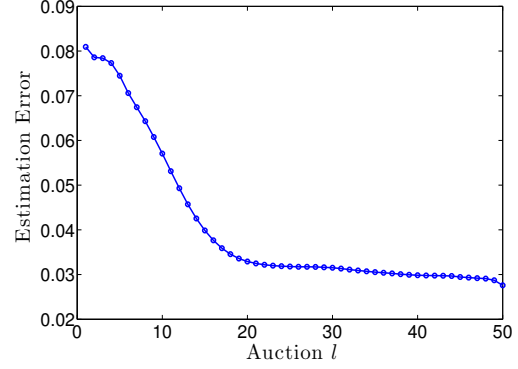


Fig. 3. Estimation error versus the number of auctions l ($M = 500, N = 1000, G = 5000$). A more accurate estimation is obtained when the platform runs more rounds of auction.

Through Fig. 3, we first evaluate the accuracy of estimation. We use $V(\mathcal{S})$ to denote the valuation that the platform can obtain when the quality indicator q_n of the users is known by the platform. Similarly, the valuation that the platform obtains according to the estimated quality indicator $\bar{q}_n^{(l)}$ is denoted by $\bar{V}(\mathcal{S}^{(l)})$. Then, the estimation error of the quality of sensing can be calculated as $\frac{|V(\mathcal{S}) - \bar{V}(\mathcal{S}^{(l)})|}{V(\mathcal{S})}$. Fig. 3 shows the estimation error of the quality of sensing. Results show that when the platform conducts more auctions, it obtains more accurate estimation of the quality indicators and selects the users more properly to perform the tasks. When the quality estimation has errors, the set of winners may be different from what obtained when the quality indicators are accurately estimated. Thus, the valuation obtained by the platform is affected by the estimation error. Since the platform aims to achieve a high valuation, the estimation error is measured by the relative deviation of the valuation, i.e., $\frac{|V(\mathcal{S}) - \bar{V}(\mathcal{S}^{(l)})|}{V(\mathcal{S})}$.

We now evaluate our proposed budget feasible mechanism. Figs. 4–6 show the valuation function $V(\mathcal{S})$ obtained from ABSee in comparison with GREEDY-SM and RANDOM-SM. We can see that ABSee significantly improves the valuation obtained by GREEDY-SM and RANDOM-SM. Notice that from (12), when $\tilde{\theta}$ increases, the number of winners will also increase. Since more winners are selected, ABSee can provide a higher valuation. In Fig. 4, the value of $V(\mathcal{S})$ increases when there are more tasks. According to (5), an increase in the number of tasks can provide a higher valuation to the platform. The platform can also obtain a higher valuation when the number of users increases as shown in Fig. 5. In this case, the number of users who can perform the tasks becomes larger. Thus, those users with lower bids but higher quality of sensing can be chosen. Fig. 6 confirms that the more budget the platform has, the higher valuation it can obtain. This is because it can select more winners to have the tasks performed.

Fig. 7 shows the number of winners $|\mathcal{S}|$ versus the number of users N in ABSee in comparison with GREEDY-SM and RANDOM-SM. Our proposed mechanism slightly selects more winners than GREEDY-SM and RANDOM-SM to perform the tasks, while it still satisfies budget feasibility. This is because we use the crowd factor and spend θG portion of the budget to determine the winners, while GREEDY-SM and RANDOM-SM use a fixed budget. Fig. 7 also shows that

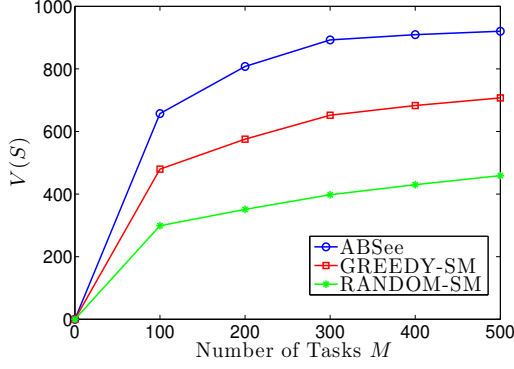


Fig. 4. Valuation function $V(S)$ versus M ($N = 1000$, $G = 100$).

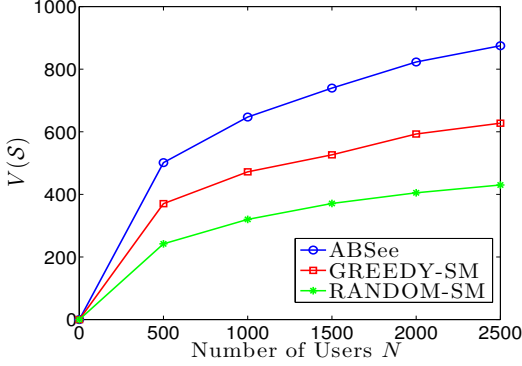


Fig. 5. Valuation function $V(S)$ versus N ($M = 100$, $G = 100$).

the number of winners increases when more users participate in the system. However, when the total number of users is very large, there will not be significant increase on the number of winners any more. When a large number of users participate, the platform selects the users with high quality of sensing and low bids to perform the tasks. However, due to the limited budget of the platform, the number of winners grows slowly.

Through Fig. 8, we investigate whether ABSee is budget feasible. Fig. 8 shows the total payment versus the number of users for different amount of budget where there are $M = 100$ tasks. The total payment increases when more users participate in the system. This is because the platform selects more winners to perform the tasks. However, when the number of users becomes very large, the number of winners does not increase anymore. From Fig. 8, we can see that the total payment is always less than the budget, which shows that ABSee satisfies budget feasibility.

Fig. 9 shows the crowd factor θ versus the number of users for different amount of budget. According to the definition of crowd factor θ , when there are more users or more budget, the number of winners increases, which result in a higher valuation for the platform. Thus, the crowd factor becomes larger, which captures a practical mobile crowdsensing system. Fig. 10 further shows the cumulative distribution function (CDF) of θ . Among 100 experiments, we can see that θ is close to 1 with a high probability.

We now verify the approximation ratio of ABSee through Fig. 11. Since problem (19) is an NP-hard problem, it is time consuming to obtain the optimal value. Thus, we cannot compare the optimal value and the value obtained from ABSee

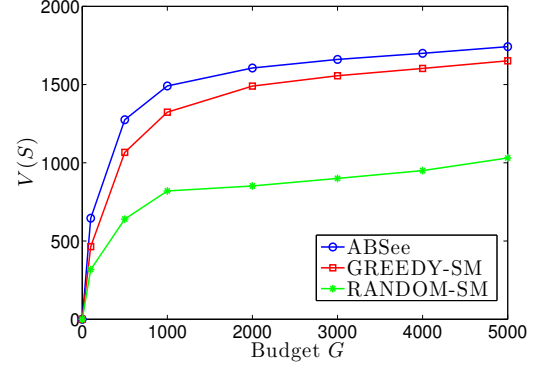


Fig. 6. Valuation function $V(S)$ versus budget G ($M = 100$, $N = 1000$).

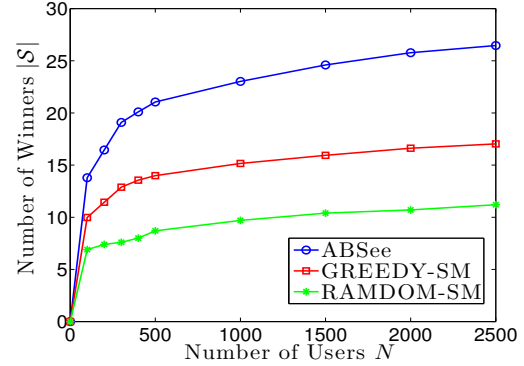


Fig. 7. The number of winners $|S|$ versus N ($M = 100$, $G = 100$).

directly. We circumvent this issue by comparing $\tilde{V}(S)$ and $V(S)$. Recall that $\tilde{V}(S)$ is the valuation obtained from a fractional greedy algorithm. We know $opt(\mathcal{B}) \leq \frac{e}{e-1} \tilde{V}(S) < \frac{2e}{\theta(e-1)} V(S)$ from Section V. Notice that θ is always greater than $\frac{1}{2}$ according to the simulation results and we have $\hat{\theta} = \theta$. Thus, we can validate the approximation ratio by showing $\theta \frac{\tilde{V}(S)}{V(S)} < 2$. Fig. 11 confirms that $\theta \frac{\tilde{V}(S)}{V(S)}$ is always smaller than 2. Note that θ is close to 1 when the number of users and budget are large. Thus, the approximation ratio $\frac{opt(\mathcal{B})}{V(S)}$ is always less than $\frac{2e}{\theta(e-1)}$, which approaches to $\frac{2e}{e-1}$.

In Fig. 12, we compare the running time of ABSee with GREEDY-SM. ABSee is slightly slower than GREEDY-SM since it needs to calculate the crowd factor θ using an iterative algorithm. However, as shown in Figs. 4–6, it significantly improves the platform's valuation comparing to GREEDY-SM.

VII. CONCLUSION

In this paper, we considered quality of sensing of the smartphone users in a mobile crowdsensing system. The platform estimates the quality of sensing of the users and keeps a historical record. It aims to maximize the valuation of the performed tasks with a limited budget. We introduced crowd factor and designed an auction-based budget feasible mechanism called ABSee, which selects the winners and determines the payment to the them. In addition to budget feasibility, we proved that ABSee satisfies computational efficiency, truthfulness, and individual rationality. We also showed that the approximation ratio of ABSee approaches $\frac{2e}{e-1}$ in mobile crowdsensing systems. Simulation results showed that the quality of sensing of the users can be estimated accurately.

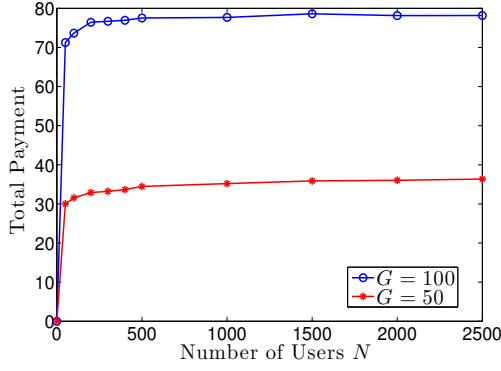


Fig. 8. Total payment versus the number of users N ($M = 100$).

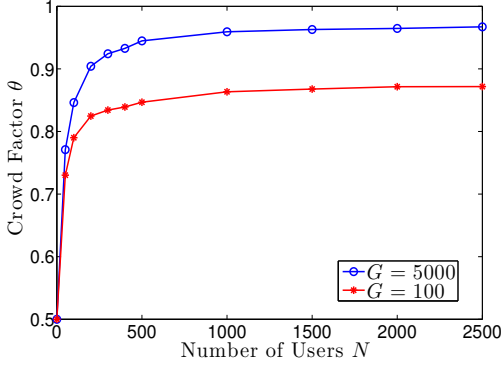


Fig. 9. Crowd factor θ versus the number of users N ($M = 100$).

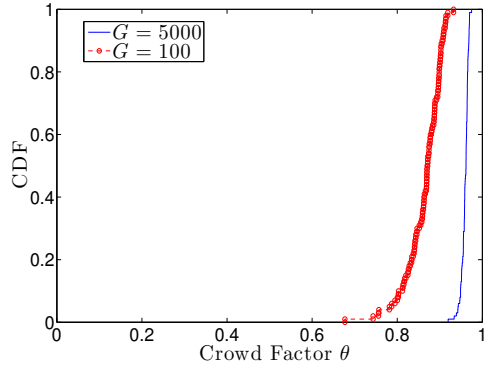


Fig. 10. CDF of θ ($M = 100$, $N = 1000$).

Furthermore, the platform can obtain a higher valuation when it implements ABSee in comparison with GREEDY-SM and RANDOM-SM proposed in [27].

In terms of future work, possible extensions are as follows. First, there may exist several platforms providing sensing services, all of which aim to maximize their valuations. We will propose a double auction mechanism to manage such sensing market. Second, we will further explore the mechanism design when the platform aims to minimize the total payment to the users. Third, privacy leakage is interesting to investigate especially when the number of auctions is large.

APPENDIX

A. Proof of Lemma 1

From Definition 1, we need to show that

$$V(\mathcal{S} \cup \{j\}) - V(\mathcal{S}) \geq V(\mathcal{Z} \cup \{j\}) - V(\mathcal{Z}),$$

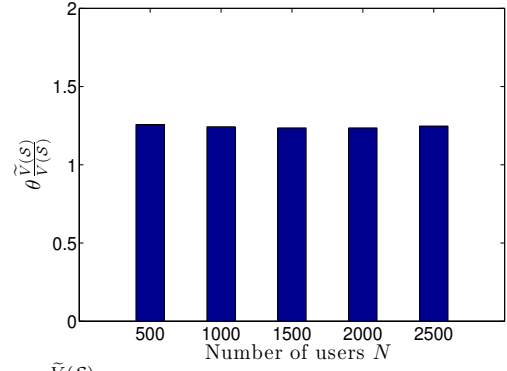


Fig. 11. $\theta \frac{\tilde{V}(\mathcal{S})}{V(\mathcal{S})}$ versus the number of users N ($M = 100$, $G = 100$). We have $\theta \frac{\tilde{V}(\mathcal{S})}{V(\mathcal{S})} \leq 2$, which validates the approximation ratio.

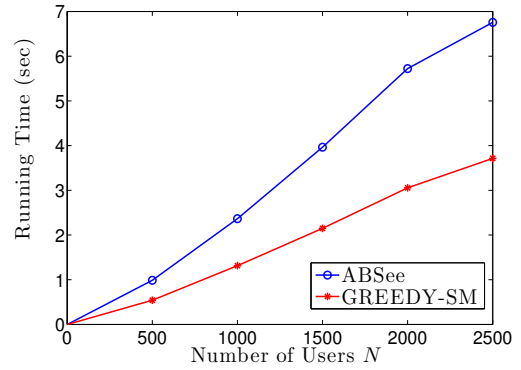


Fig. 12. Running time versus the number of users N ($M = 200$, $G = 500$).

for any $\mathcal{S} \subseteq \mathcal{Z} \subseteq \mathcal{N}$ and $j \in \mathcal{N} \setminus \mathcal{Z}$, where $\mathcal{N}$ is the set of smartphone users. According to (4) and (5), we have

$$V(\mathcal{S}) = \sum_{\tau_k \in \cup_{s \in \mathcal{S}} \Gamma_s} \mu_k \log \left(1 + \sum_{n \in \mathcal{S}: \tau_k \in \Gamma_n} l_n \right),$$

where $l_n = \frac{1}{q_n}$ to simplify the expression.

Given sets $\mathcal{S}$ and $\mathcal{Z}$, let $s_k = \sum_{n \in \mathcal{S}: \tau_k \in \Gamma_n} l_n$ and $z_k = \sum_{n \in \mathcal{Z}: \tau_k \in \Gamma_n} l_n$. We define

$$\begin{aligned} \Phi_{j,1} &\triangleq \Gamma_j \cap (\cup_{n \in \mathcal{S}} \Gamma_n) \cap (\cup_{n \in \mathcal{Z}} \Gamma_n), \\ \Phi_{j,2} &\triangleq (\Gamma_j \cap (\cup_{n \in \mathcal{Z}} \Gamma_n)) \setminus (\cup_{n \in \mathcal{S}} \Gamma_n), \\ \Phi_{j,3} &\triangleq \Gamma_j \setminus ((\cup_{n \in \mathcal{Z}} \Gamma_n) \cup (\cup_{n \in \mathcal{S}} \Gamma_n)). \end{aligned}$$

We have $\Phi_{j,1} \cup \Phi_{j,2} \cup \Phi_{j,3} = \Gamma_j$ while $\Phi_{j,1} \cap \Phi_{j,2} = \Phi_{j,2} \cap \Phi_{j,3} = \Phi_{j,1} \cap \Phi_{j,3} = \emptyset$. Then,

$$\begin{aligned} V(\mathcal{S} \cup \{j\}) - V(\mathcal{S}) &= \sum_{\tau_k \in \Phi_{j,1}} \mu_k \log \left(1 + \frac{l_n}{1 + s_k} \right) \\ &\quad + \sum_{\tau_k \in \Phi_{j,2}} \mu_k \log(1 + l_n) + \sum_{\tau_k \in \Phi_{j,3}} \mu_k \log(1 + l_n), \\ V(\mathcal{Z} \cup \{j\}) - V(\mathcal{Z}) &= \sum_{\tau_k \in \Phi_{j,1}} \mu_k \log \left(1 + \frac{l_n}{1 + z_k} \right) \\ &\quad + \sum_{\tau_k \in \Phi_{j,2}} \mu_k \log \left(1 + \frac{l_n}{1 + z_k} \right) + \sum_{\tau_k \in \Phi_{j,3}} \mu_k \log(1 + l_n). \end{aligned}$$

The reason for the difference between the second terms in the above equations is that for any task $\tau_k \in \Phi_{j,2}$, it can

be performed by user j and the users in $\mathcal{Z}$ but it cannot be performed by the users in $\mathcal{S}$. Since $\mathcal{S} \subseteq \mathcal{Z}$ and $l_n > 0$ for $\tau_k \in \Gamma_n$, $n \in \mathcal{N}$, we have $0 \leq s_k \leq z_k$. Therefore,

$$V(\mathcal{S} \cup \{j\}) - V(\mathcal{S}) \geq V(\mathcal{Z} \cup \{j\}) - V(\mathcal{Z}).$$

Since μ_k and q_n are non-negative, $V(\mathcal{S})$ is non-negative. It can be observed that $V(\mathcal{S})$ is also non-decreasing, which completes the proof. ■

B. Proof of Lemma 2

For a winner x_i , from the submodularity of the valuation function, we have

$$V_{x_i}(\mathcal{S}_{i-1}) \geq V_{x_i}(\mathcal{Q}_{k-1}), \forall k \geq i. \quad (23)$$

Let r be the index for which $p_{x_i} = \min\{\beta_{i(r)}, \rho_{i(r)}\}$. We now prove that $r \geq i$. By contradiction, assume $r < i$, we have $x_r = \arg \max_{j \in \mathcal{N} \setminus \mathcal{S}_{r-1}} \frac{V_j(\mathcal{S}_{r-1})}{b_j}$ from (7). Thus, $\frac{V_{x_r}(\mathcal{S}_{r-1})}{b_{x_r}} > \frac{V_{x_i}(\mathcal{S}_{r-1})}{b_{x_i}}$. We assume the strict inequality holds to simplify the proof. The same results can be obtained if the equality is considered. From the definition of $\beta_{i(r)}$, we have $\frac{V_{i_r}(\mathcal{Q}_{r-1})}{b_{i_r}} = \frac{V_{x_i}(\mathcal{Q}_{r-1})}{\beta_{i(r)}}$. Since $\mathcal{Q}_k = \mathcal{S}_k$ and $x_k = i_k$ for $k < i$, we have $V_{x_i}(\mathcal{S}_{r-1}) = V_{x_i}(\mathcal{Q}_{r-1})$, $V_{x_r}(\mathcal{S}_{r-1}) = V_{i_r}(\mathcal{Q}_{r-1})$, and $b_{x_r} = b_{i_r}$. According to these equalities, we conclude that $p_{x_i} \leq \beta_{i(r)} < b_{x_i}$, which is in contradiction with Theorem 3. Hence, we have $r \geq i$.

If $r \leq w'$, then $\frac{V_{i_r}(\mathcal{Q}_{r-1})}{b_{i_r}} \geq \frac{V_{i_{w'}}(\mathcal{Q}_{w'-1})}{b_{i_{w'}}$. From (23), we have

$$\begin{aligned} p_{x_i} \leq \beta_{i(r)} &= \frac{V_{x_i}(\mathcal{Q}_{r-1})b_{i_r}}{V_{i_r}(\mathcal{Q}_{r-1})} \leq \frac{V_{x_i}(\mathcal{Q}_{r-1})b_{i_{w'}}}{V_{i_{w'}}(\mathcal{Q}_{w'-1})} \\ &\leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{Q}_{r-1})}{V(\mathcal{Q}_{w'})} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{Q}_{w'})}. \end{aligned} \quad (24)$$

If $r = w' + 1$, we have

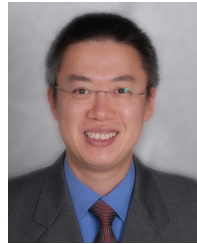
$$p_{x_i} \leq \rho_{i(r)} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{Q}_{w'})}{V(\mathcal{Q}_{w'} \cup \{x_i\})} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{Q}_{w'})}. \quad (25)$$

From (24) and (25), we conclude that $p_{x_i} \leq \tilde{\theta}G \frac{V_{x_i}(\mathcal{S}_{i-1})}{V(\mathcal{Q}_{w'})}$, which completes the proof. ■

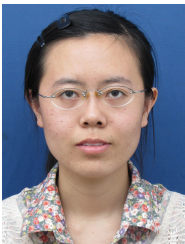
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