Cardinality Estimation in RFID Systems with Multiple Readers

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***Abstract*— Radio frequency identification (RFID) is an emerging technology for automatic object identification. An RFID system consists of a set of readers and several objects, with each object equipped with a small chip, called a tag. In this paper, we consider the anonymous cardinality estimation problem in an RFID system consisting of several readers. To achieve complete system coverage and increase the accuracy of measurement, multiple readers with overlapping interrogation zones are deployed. We study the problem under two different circumstances. First, we assume that the readers cannot perform interrogations synchronously. This models the case when the readers are not equipped with accurate clocks or synchronization imposes a high overhead. Under such condition, we propose an asynchronous exclusive estimator to estimate the number of tags that are exclusively located in the zone of a selected reader. By using this estimator, we propose an asynchronous multiple-reader cardinality estimation (A-MRCE) algorithm. In the second scenario, we assume that readers can perform interrogations synchronously. We propose a synchronous exclusive estimator and a synchronous multiple-reader cardinality estimation (S-MRCE) algorithm to estimate the total number of tags. For the exclusive estimators, we show that they are asymptotically unbiased and we derive upper bounds on the variance of error. We validate our analytical model via simulations. Results show that although the A-MRCE algorithm enjoys the asynchronous operation of the readers, it performs worse than the S-MRCE algorithm in terms of estimation error. Compared to the enhanced zero-based (EZB) and lottery frame (LoF) algorithms, the variance of the estimation error for both A-MRCE and S-MRCE algorithms increases linearly with the number of readers, while it increases exponentially for EZB and LoF algorithms.**

Keywords: RFID systems, cardinality estimation, multiple reader.

I. Introduction

Radio frequency identification (RFID) systems are increasingly being deployed as automated identification systems. These systems are expected to play an important role in various applications such as warehouse and supply chain management, object tracking, and patients' monitoring in health care facilities [1]–[3]. An RFID system consists of a set of *readers* and several objects. Each object is equipped with a small computer chip, called *tag*. Using these inexpensive tags, every object can be uniquely identified. RFID tags can be categorized into *passive* and *active* tags. A passive tag uses backscatter modulation, and its transmission power is derived from the signal of the interrogating reader [2], [4]. Passive tags can operate in different frequency bands. Low-frequency tags operate in the 124-135 kHz band and have an operating range of up to 0.5 m. Ultra high frequency tags, which operate at either 860-960 MHz or 2.45 GHz, have a range in the order of 10 m. Active tags require a power source (e.g., a battery) for data transmission and have a larger range (> 100 m).

In an RFID system, packet collisions may occur during the interrogation of a reader. This type of packet collision is called a tag-to-tag collision. Tree-walking and ALOHA-based protocols are two kinds of tag-to-tag anti-collision protocols proposed in the literature [5]–[12]. For RFID systems with multiple readers, other types of collisions (e.g., reader-to-tag and reader-to-reader collisions) may occur during the interrogations of various readers [13]. Several anti-collision interrogation techniques have been proposed for RFID systems with multiple readers in the literature [14]–[17]. A framed-slotted ALOHA-based tag anti-collision scheme has also been standardized by EPCglobal in [18]. This allows each tag to randomly select a time slot and transmit its ID. The performance of this scheme has been studied extensively recently [19], [20].

Tag estimation is widely used as a preliminary phase in ALOHA-based interrogation techniques [21]–[23]. Readers can adjust the frame size based on the estimation of tag population. Another application of tag estimation techniques, which has recently received attention, is the anonymous tracking of objects [24], [25]. In order to preserve the privacy and anonymity of the tag users, it may not be necessary to identify each individual user in some RFID applications. Instead, the goal is to estimate the total number of tags (or users) in the system. This is called the *cardinality estimation* (or tag population estimation) problem in RFID systems. The potential applications include estimating the number of attendants in large exhibitions and conferences when each attendant is equipped with an RFID tag, and urban traffic monitoring at streets and intersections when cars are equipped with RFID tags.

In [24], Kodialam *et al.* proposed the zero-based and collision-based tag estimation techniques using a framed-slotted ALOHA model with a single reader. In [25], they extended their work by introducing the enhanced-zero based (EZB) estimator, which is an asymptotically unbiased estimator. Using this technique, the mean and variance of the estimation error approach zero when the estimation process is repeated multiple times. Although the EZB algorithm can also be used for RFID systems with multiple readers, the variance of estimation error increases exponentially with the number of readers. In [22], Qian *et al.* proposed the lottery frame (LoF) scheme, which is a replicate-insensitive estimation protocol. LoF applies the hash functions with geometric distribution to tag IDs to select the time slots for transmission.

For large scale RFID systems, it is necessary to deploy multiple readers with overlapped interrogation zones to fully cover the area and achieve a high accuracy in the estimation. Consequently, a tag can be within the interrogation zone of several readers simultaneously. For tracking applications, which require privacy and anonymity of the users, each tag only transmits a portion of its ID to the reader when it is being queried. Thus, readers cannot identify uniquely the individual tags. Thus, those tags which are within the range of multiple readers may be counted multiple times. We call this problem the *multiple counting problem*. This motivates us to propose estimation algorithms which are capable of estimating the number of tags in such systems. We study the problem in two different conditions. First, we assume that readers cannot perform interrogation synchronously. We develop an asynchronous multiple-reader cardinality estimation (A-MRCE)

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algorithm under this condition. Then, we study the problem where the readers can be synchronized for interrogations, and we develop a synchronous multiple-reader cardinality estimation (S-MRCE) algorithm. The contributions of this paper are as follows:

- We propose two maximum likelihood (ML) estimators, namely asynchronous and synchronous exclusive estimators to estimate the number of tags, which are exclusively within the interrogation zone of a reader.
- We show that the error for these estimators is asymptotically normal and the estimators are asymptotically unbiased. We derive the upper bounds on the variance of the estimation error. The accuracy of these bounds is validated via simulations.
- We develop two estimation algorithms namely, A-MRCE and S-MRCE algorithms using asynchronous and synchronous exclusive estimators, respectively.
- We validate the analytical models, investigate the performance of our proposed estimators, and compare our proposed A-MRCE and S-MRCE algorithms with the EZB [25] and LoF [22] algorithms. Although all these algorithms are asymptotically unbiased, the variance of the estimation error for A-MRCE and S-MRCE algorithms increases *linearly* with the number of readers, while it increases *exponentially* for EZB and LoF algorithms.

To the best of our knowledge, there is no prior work specifically considering the problem of tag population estimation for RFID systems with multiple readers. Although EZB and LoF algorithms can be used in RFID systems with multiple readers, since they are not designed for such systems, they can have poor performance under some scenarios as shown in Section V. On the contrary, both A-MRCE and S-MRCE algorithms can be used in large scale RFID systems. Since S-MRCE algorithm needs synchronous operation of readers, this algorithm is suitable for systems where readers can operate synchronously.

In our previous work [26], we proposed a multiple-reader tag estimation (MRTE) algorithm, which is similar to the A-MRCE algorithm in this paper. However, in this paper, we present a more accurate model to determine the estimation error of the exclusive estimator. In other words, the error model for the A-MRCE algorithm is more accurate than the one in [26].

The rest of this paper is organized as follows: The system model is presented in Section II. In Section III, we first propose an asynchronous exclusive estimator. Then, we propose an A-MRCE algorithm to estimate the total number of tags in the RFID system. In Section IV, we propose a synchronous exclusive estimator and an S-MRCE algorithm. Performance evaluation and comparison are presented in Section V. Conclusions are given in Section VI.

II. System Model and Problem Statement

A. Notations and Model

Consider an RFID system with multiple readers. Readers use framed-slotted ALOHA protocol for interrogations. This is the model proposed in EPCglobal Gen2 standard [18]. We consider a fixed frame size for an interrogation process. Each reader broadcasts a query message, which includes information such as the frame size f, persistence probability p, and a random seed q at the beginning of the interrogation process. Each tag decides whether or not to transmit in the current frame based on the persistence probability p. If a tag decides to transmit, it selects a time slot based on a uniform distribution related to its ID, the persistence probability p, and the random seed q. Note that given the specific values of the frame size f, the random seed q, and persistence probability p, the tag selects exactly the same slot in a frame of size f, regardless of how many times it has received the query message. To perform different interrogations, readers can alter the seed q. The interrogation results are independent whenever a different seed q is being used. Multiple interrogations are used to improve the accuracy of the estimation process. To preserve the anonymity and privacy of the users, each tag only transmits part of its ID to the reader in the reply message. Therefore, the reader cannot individually identify the tags.

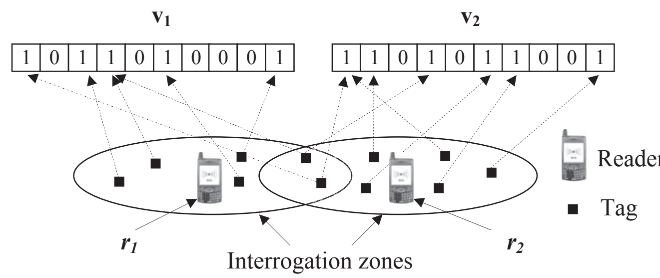


Fig. 1. An RFID system with two readers r_1 and r_2. $n_{r_1} = 6$, $n_{r_2} = 7$, $N = 11$, $f = 10$, and $p = 1$.

We now introduce some of the notations. Let N and $\mathcal{R}$ denote the number of tags and the set of readers in the system, respectively. Let $\mathcal{T}_r$ denote the set of tags within the zone of reader $r \in \mathcal{R}$. Let n_r denote the number of tags in the interrogation zone of reader $r \in \mathcal{R}$ (i.e., $n_r = |\mathcal{T}_r|$). We use $n_{\mathcal{W}}$ to denote the number of tags within the range of a set of readers $\mathcal{W}$ (i.e., $n_{\mathcal{W}} = | \cap_{w \in \mathcal{W}} \mathcal{T}_w|$). Two readers are called neighboring readers if there is a tag which is in the interrogation range of both of the readers. Let $\mathcal{H}_r$ denote the set of other readers which are neighbors of reader r. For reader r, let $\mathbf{v}_r = (v_r^1, \ldots, v_r^f)$ denote the vector created after performing an interrogation process, where v_r^l (with $l = 1, \ldots, f$) indicates whether the l^{th} time slot is empty (i.e., $v_r^l = 0$) or has at least one transmission (i.e., $v_r^l = 1$). The number of elements of vector $\mathbf{v}_r$ is equal to the frame size f. We call vector $\mathbf{v}_r$ as the *interrogation vector* of reader r. Assume reader r performs M interrogations using M different seed values. We use vectors $\mathbf{v}_r^1, \ldots, \mathbf{v}_r^M$ to denote these interrogation vectors. Fig. 1 shows a two-reader RFID system with overlapped interrogation zones and the interrogation vectors.

B. Multiple Counting Problem

For an RFID system with multiple readers, by adding up the number of tags within the zone of all readers, one can obtain an estimator for the number of tags. Since some tags may appear in the zone of several readers, they are counted multiple times by different readers. Therefore, the estimation may not be accurate especially for dense RFID systems. To obtain an accurate estimate for the number of tags, the number of tags in the overlapped areas of neighboring readers is required in addition to the number of tags within the zone of each reader. In the estimation process, once a tag is counted by a reader, other readers should exclude that tag from their estimations. The total number of tags is the summation of the number of tags estimated by all the readers, while each reader excludes the tags that have already been counted by other readers. In general, for $|\mathcal{R}|$ readers $r_1, \ldots, r_{|\mathcal{R}|} \in \mathcal{R}$ with overlapping interrogation zones, the total number of tags N in the system is

$$N = |\mathcal{T}_{r_1}| + |\mathcal{T}_{r_2} \backslash \mathcal{T}_{r_1}| + \cdots + |\mathcal{T}_{r_{|\mathcal{R}|}} \backslash \{\mathcal{T}_{r_1} \cup \cdots \cup \mathcal{T}_{r_{|\mathcal{R}|-1}}\}|. \quad (1)$$

Note that the order chosen to calculate N has no effect on the final result. Moreover, if reader $r_j \in \mathcal{R}$ shares no tags with readers $r_1, \ldots, r_{j-1}$, then $\mathcal{T}_{r_j} \backslash \{\mathcal{T}_{r_1} \cup \cdots \cup \mathcal{T}_{r_{j-1}}\} = \mathcal{T}_{r_j}$. As (1) suggests, in order to estimate N, the number of tags within the zone of a reader needs to be estimated. We call such an estimator *single reader estimator*. Moreover, we need to estimate the number of tags which are only within the zone of a reader but are not in the zone of some other readers (e.g., $|\mathcal{T}_{r_2} \backslash \mathcal{T}_{r_1}|$). To estimate the number of tags which are exclusively located within the zone of a reader, we propose two estimators in the next sections, namely asynchronous exclusive and synchronous exclusive estimators. For the single reader estimator, we use the estimator proposed in [25]. Assume that reader r has performed M interrogations using M different seed values, the we have $\mathbf{v}_r^1, \ldots, \mathbf{v}_r^M$. Let t^m denote the number of empty slots in

$\mathbf{v}_r^m,\ m=1,\dots,M$. Using the EZB algorithm, the number of tags within the zone of reader r can be estimated as [25]:

$$\tilde{n}_r = -f/p \ln\left(\sum_{m=1}^{M} t^m/Mf\right). \qquad (2)$$

It is shown in [25] and [27] that the estimation error is asymptotically normal and unbiased. The variance of the estimation error is $\sigma^2_{n_r} = f\left(\exp(pn_r/f) - (1+p^2 n_r/f)\right)/(Mp^2)$. We will use this estimator in both A-MRCE and S-MRCE algorithms.

III. Asynchronous Multiple Reader Cardinality Estimation

In some practical cases, it may not be possible for the readers to perform interrogations in a synchronous manner. This models the case when the readers are not equipped with accurate clocks or synchronization imposes a high overhead. Under such condition, we develop an asynchronous multiple reader cardinality estimation (A-MRCE) algorithm. The A-MRCE algorithm is used to estimate the total number of tags while readers perform interrogations independently and forward the information to a central controller. The A-MRCE algorithm implements (1) using asynchronous exclusive estimator described in the following subsection.

A. Asynchronous Exclusive Estimator

The asynchronous exclusive estimator is used to estimate the number of tags within the zone of a particular reader excluding the tags shared with some other readers, while readers perform interrogations independently. It facilitates implementation of equation (1). Consider reader r and a set of readers $\mathcal{W} \subseteq \mathcal{H}_r$. Let $n_{r\setminus\mathcal{W}}$ denote the number of tags within the zone of reader r that do not belong to any of the readers in set $\mathcal{W}$ (i.e., $|\mathcal{T}_r \setminus \cup_{w\in\mathcal{W}} \mathcal{T}_w|$). Given the number of tags within the zone of readers in $\mathcal{W}$, $n_\mathcal{W}$, we propose an asynchronous exclusive estimator to estimate $n_{r\setminus\mathcal{W}}$.

Consider a time slot which is non-empty in vector $\mathbf{v}_r$. This indicates that one or more tags within the range of reader r has chosen that time slot. If this time slot is empty in $\mathbf{v}_w$ for any $w \in \mathcal{W}$, it ensures that none of the tags within the range of readers in $\mathcal{W}$ has chosen that slot. Let variable Z denote the number of time slots which are nonempty in $\mathbf{v}_r$ and empty in $\mathbf{v}_w, \forall\, w \in \mathcal{W}$. That is, given reader $r \in \mathcal{R}$ and set $\mathcal{W}$, we have $Z = \sum_{l=1}^{f} v_r^l \prod_{w\in\mathcal{W}}(1-v_w^l)$. The time slots, which are non-empty in $\mathbf{v}_r$ and empty in $\mathbf{v}_w$, are chosen by a subset of tags in the set $\{\mathcal{T}_r \setminus \{\cup_{w\in\mathcal{W}} \mathcal{T}_w\}\}$. This suggests that the variable Z can be used to estimate $n_{r\setminus\mathcal{W}}$. The following theorem characterizes the distribution of variable Z.

Theorem 1: The random variable Z has a normal distribution with mean μ_z and variance σ_z^2 for large values of f, $n_{r\setminus\mathcal{W}}$ and $n_\mathcal{W}$:

$$\mu_z = f\left(1-\exp\left(-\tfrac{pn_{r\setminus\mathcal{W}}}{f}\right)\right)\exp\left(-\tfrac{pn_\mathcal{W}}{f}\right). \qquad (3)$$

$$\begin{aligned}\sigma_z^2 \;&=\; \mu_z\left(1-\mu_z/f\left(1+\frac{p^2 n_\mathcal{W}}{f}\right)\right)\\ &\quad - p^2 n_{r\setminus\mathcal{W}} \exp\left(-\frac{2p}{f}(n_{r\setminus\mathcal{W}}+n_\mathcal{W})\right). \end{aligned} \qquad (4)$$

The proof of Theorem 1 is given in Appendix A. By using the interrogation vectors $\mathbf{v}_r^1,\dots,\mathbf{v}_r^M$, and $\mathbf{v}_w^1,\dots,\mathbf{v}_w^M,\ \forall\ w \in \mathcal{W}$, one can obtain M samples of the random variable Z, namely $z^1,\dots,z^M$. Let $\mathbf{z}$ denote a vector composed of all the samples (i.e., $\mathbf{z} = (z^1,\dots,z^M)$). From Theorem 1, the likelihood function of $\mathbf{z}$ given $n_{r\setminus\mathcal{W}}$ and $n_\mathcal{W}$ is

$$L\left(\mathbf{z}; n_{r\setminus\mathcal{W}}, n_\mathcal{W}\right) = \prod_{m=1}^{M} \frac{1}{\sqrt{2\pi\sigma_z^2}}\exp\left(-\frac{(z^m-\mu_z)^2}{2\sigma_z^2}\right). \qquad (5)$$

Algorithm 1 A-MRCE Algorithm executed at reader $r \in \mathcal{R}$.

1: Upon receiving parameters $p, \mathbf{q}$, and f from the controller, reader r performs M interrogation processes using seed values $\mathbf{q}$.
2: Reader r sends its location info and the interrogation vectors $\mathbf{v}_r^1,\dots,\mathbf{v}_r^M$ to the controller.

The maximum likelihood (ML) estimate of $n_{r\setminus\mathcal{W}}$ is the value that maximizes the log-likelihood function as follows:

$$\begin{aligned}\tilde{n}^M_{r\setminus\mathcal{W}} \;&=\; \underset{n_{r\setminus\mathcal{W}}}{\arg\max} \ln L\left(\mathbf{z}; n_{r\setminus\mathcal{W}}, n_\mathcal{W}\right)\\ &=\; \underset{n_{r\setminus\mathcal{W}}}{\arg\max}\left\{-\frac{M}{2}\ln\left(\sigma_z^2\right) - \sum_{m=1}^{M}\frac{(z^m-\mu_z)^2}{2\sigma_z^2}\right\}. \end{aligned} \qquad (6)$$

In practice, $n_\mathcal{W}$ is not given and an estimation should be used instead. Let $\tilde{n}^M_\mathcal{W}$ denote the estimation of $n_\mathcal{W}$ using M interrogation vectors. By taking the derivative of (6) and equating it to zero, we can find the closed form expression for the ML estimator. For large values of M (e.g., $M > 10$) and f (e.g., $f > 100$), the exclusive estimator in (6) can be approximated as

$$\begin{aligned}\tilde{n}^M_{r\setminus\mathcal{W}} \;&\approx\; \left\{n_{r\setminus\mathcal{W}} \;\middle|\; \mu_z - \frac{1}{M}\sum_{m=1}^{M} z^m = 0\right\}\\ &=\; -\frac{f}{p}\ln\left(1-\frac{\sum_{m=1}^{M} z^m}{Mf}\exp\left(\frac{p\tilde{n}^M_\mathcal{W}}{f}\right)\right). \end{aligned} \qquad (7)$$

Eq. (7) is the asynchronous exclusive estimator. In the next subsection, we explain how $n_\mathcal{W}$ can be estimated. The inaccuracy in estimation of $n_{r\setminus\mathcal{W}}$ comes from the random nature of samples taken from the system and also the error in estimation of $n_\mathcal{W}$. The following theorem characterizes the error of the estimator in (7).

Theorem 2: Given the error in estimation of $n_\mathcal{W}$ follows a normal distribution, for large values of M and f, the error in estimation of $n_{r\setminus\mathcal{W}}$ using (7) has a normal distribution. Moreover, this estimator is asymptotically unbiased if the mean of error in estimation of $n_\mathcal{W}$ approaches zero for large values of M. The variance of error is bounded by

$$\begin{aligned}\sigma^2_{n_{r\setminus\mathcal{W}}}(M) \;&\le\; \frac{1}{p^2}\exp\left(\frac{2p\left(n_{r\setminus\mathcal{W}}+n_\mathcal{W}\right)}{f}\right)\frac{\sigma_z^2}{M}\\ &\quad + \left(\exp\left(\frac{pn_{r\setminus\mathcal{W}}}{f}\right)-1\right)^2\sigma^2_{n_\mathcal{W}}(M), \end{aligned} \qquad (8)$$

where $\sigma^2_{n_{r\setminus\mathcal{W}}}(M)$ and $\sigma^2_{n_\mathcal{W}}(M)$ are the variance of errors in estimation of $n_{r\setminus\mathcal{W}}$ and $n_\mathcal{W}$ using M independent set of interrogation vectors, respectively.

The proof of Theorem 2 is given in Appendix B. We will compare $\sigma^2_{n_{r\setminus\mathcal{W}}}(M)$ and its upper bound in Section V.

B. A-MRCE Algorithm

We now present the A-MRCE algorithm to estimate the total number of tags in an RFID system with multiple readers. The A-MRCE algorithm is a centralized algorithm. The controller is a centralized unit which is responsible to estimate the total number of tags in the system. Each reader performs interrogations individually and transmits its interrogation vectors to the controller. The controller uses these vectors to estimate the number of tags. The A-MRCE algorithm, which is shown in Algorithms 1 and 2, implements equation (1) using the single reader and the asynchronous exclusive estimators. Algorithm 1 shows part of the A-MRCE algorithm performed by a reader $r \in \mathcal{R}$. Algorithm 2 shows part of the A-MRCE algorithm performed by the controller. When the algorithm is invoked, the controller informs the persistence probability p, frame size f, and a set of M random seeds $\mathbf{q}$ (i.e., $|\mathbf{q}| = M$) to all the readers.

Algorithm 2 A-MRCE Algorithm executed at the controller.

1: Input: Set of readers $\mathcal{R}$, neighboring set of reader r, $\mathcal{H}_r$, and the interrogation vectors of reader r, $\forall r \in \mathcal{R}$.
2: *Initialization*: Set $\tilde{N} := 0$, and $\Gamma := \{\}$
3: **while** $\Gamma \neq \mathcal{R}$
4: Select a reader r randomly from the set $\mathcal{R} \backslash \Gamma$.
5: Set $\mathcal{W} := \Gamma \cap \mathcal{H}_r$.
6: **if** $\mathcal{W} = \{\}$
7: Calculate $\tilde{n}_r$ using (2).
8: Set $\tilde{N} := \tilde{N} + \tilde{n}_r$.
9: **else**
10: Calculate $\tilde{n}_{\mathcal{W}}$ using (2) if $|\mathcal{W}| = 1$ or using A-MRCE algorithm if $|\mathcal{W}| > 1$.
11: Calculate $\tilde{n}_{r \backslash \mathcal{W}}$ using asynchronous exclusive estimator (7).
12: Set $\tilde{N} := \tilde{N} + \tilde{n}_{r \backslash \mathcal{W}}$.
13: **end if**
14: Set $\Gamma := \Gamma \cup \{r\}$.
15: **end while**

Each reader $r \in \mathcal{R}$ then performs M interrogation processes and sends the interrogation vectors $(\mathbf{v}_r^1, \ldots, \mathbf{v}_r^M)$ to the controller. The readers also inform the controller about their location by sending the location information to the controller. $\mathcal{H}_r$ is the set of readers whose interrogation vectors overlap with the interrogation zone of reader r. Based on the interrogation range of each reader and its location, $\mathcal{H}_r$, which is the set of neighboring readers of reader r can also be determined. The readers may encounter reader-to-tag and reader-to-reader collision during their interrogations. To avoid the collision, several techniques have been proposed in the literature [13]–[17] for RFID systems with multiple readers. However, it is not the focus of this paper and we assume that the readers employ one of these techniques to perform interrogations.

After receiving the required information (interrogation vectors and location information) from all the readers, the controller estimates the total number of tags using asynchronous exclusive estimator by invoking Algorithm 2. Algorithm 2 is equivalent to applying equation (1) iteratively to estimate the total number of tags. Steps $4-14$ denote one iteration of the algorithm. At each iteration, the algorithm selects a reader r randomly from the readers which have not been selected yet (i.e., $\mathcal{R} \backslash \Gamma$). Then, the controller calculates the number of tags within the range of the selected reader which have not already been counted and adds it to $\tilde{N}$. The set $\mathcal{W}$ denotes the set of neighbors of reader r which has been selected by the algorithm in the previous iterations (i.e., the tags within the range of the readers in the set $\mathcal{W}$ have been counted till now). If this set is empty, the algorithm uses the single reader estimator in (2) to calculate $\tilde{n}_r$ and adds this number to the current estimation of the total number of tags $\tilde{N}$ (Steps $6-9$). If the set $\mathcal{W}$ is non-empty, then the algorithm uses the asynchronous exclusive estimator to calculate $\tilde{n}_{r \backslash \mathcal{W}}$. To do so, the asynchronous exclusive estimator requires the value of $\tilde{n}_{\mathcal{W}}$. If the set $\mathcal{W}$ has only one member, then $\tilde{n}_{\mathcal{W}}$ can be calculated using single reader estimator in (2). Otherwise, the controller invokes the A-MRCE algorithm (Algorithm 2) again to estimate the number of tags within the range of readers in the set $\mathcal{W}$. This shows a *recursive* operation of the A-MRCE algorithm. We notice that since the controller already has the interrogation vectors of all the readers including those in set $\mathcal{W}$, it does not need to ask the readers to perform interrogations again.

C. Estimation Error and Discussion

To estimate $\tilde{n}_{r \backslash \mathcal{W}}$, the asynchronous exclusive estimator uses the estimation of $n_{\mathcal{W}}$. Therefore, the A-MRCE algorithm needs to calculate $\tilde{n}_{\mathcal{W}}$. If the set $\mathcal{W}$ has one element, then the single reader estimator can be used and the estimation error in estimating $n_{\mathcal{W}}$ has a normal distribution with zero mean. If the set $\mathcal{W}$ has more than one reader, the algorithm is invoked again to estimate $n_{\mathcal{W}}$. This gives a recursive operation of the A-MRCE algorithm. In the recursive procedure, the first level of the estimation is obtained by using the asynchronous exclusive estimator which has an error with normal distribution and zero mean. Based on Theorem 2, the next levels have also an estimation error with normal distribution and zero mean. Consequently, the estimation of $n_{\mathcal{W}}$ has a normal distribution with zero mean.

We now describe how to determine the estimation error for the A-MRCE algorithm. The value of $\tilde{N}$ obtained by the A-MRCE algorithm is composed of several estimations from the single reader and the asynchronous exclusive estimators. All the readers contribute to the estimation of $\tilde{N}$. Since different terms for $\tilde{N}$ are from either single reader or asynchronous exclusive estimators, they have asymptotically normal distributions with zero mean. Therefore, $\tilde{N}$ has asymptotically normal distribution with zero mean. In general, different terms of $\tilde{N}$ are not independent. However, the summation of the variance of error of these terms can give an upper bound for the variance of the error for A-MRCE algorithm. The summation of the variances depends on the order that readers are selected in the algorithm (Step 4 of algorithm 2).

For pure random selection of the readers (Step 4 of Algorithm 2), we calculate the expected value of the summation of variances over different runs of the algorithm. For reader $r \in \mathcal{R}$ with $|\mathcal{H}_r|$ neighbors, let $\mathcal{Q}_r$ denote the power set of $\mathcal{H}_r$. The power set of a set is the set of all subsets of that set. In different runs of the A-MRCE algorithm, the number of tags within the zone of reader r may appear in various forms in $\tilde{N}$. For example, it can be either $\tilde{N} := \tilde{N} + \tilde{n}_r$ or $\tilde{N} := \tilde{N} + \tilde{n}_{r \backslash \mathcal{W}}$ for any $\mathcal{W} \in \mathcal{Q}_r$. For set $\mathcal{W} \in \mathcal{Q}_r$, the probability that $\tilde{N}$ contains $\tilde{n}_{r \backslash \mathcal{W}}$ is equal to the probability that only readers in set $\mathcal{W}$ from the neighbors of r are selected before reader r within the algorithm run. Assuming that the reader selection is purely random, this probability is equal to the probability that among reader r and its neighbors (i.e., $\mathcal{H}_r$), readers in set $\mathcal{W}$ are selected before reader r. The probability that any subset of neighbors of r with cardinality $|\mathcal{W}|$ are selected by the algorithm before reader r is $1/(|\mathcal{H}_1| + 1)$ and the number of such subsets is $\binom{|\mathcal{H}_r|}{|\mathcal{W}|}$. Let $P_{n_{r \backslash \mathcal{W}}}$ denote the probability that $n_{r \backslash \mathcal{W}}$ appears in $\tilde{N}$. This probability can be written as

$$P_{n_{r \backslash \mathcal{W}}} = \frac{1}{(|\mathcal{H}_r| + 1)\binom{|\mathcal{H}_r|}{|\mathcal{W}|}}. \tag{9}$$

Therefore, when the reader r is selected within the algorithm run, with probability $P_{n_{r \backslash \mathcal{W}}}$ we have $\mathcal{W} = \Gamma \cap \mathcal{H}_r, \ \forall \ \mathcal{W} \in \mathcal{Q}_r$. The expected value for the summation of variances of errors in $\tilde{N}$ gives an upper bound on the variance of estimation error σ^2_{asyn}:

$$\sigma^2_{\text{asyn}} \leq \sum_{r \in \mathcal{R}} \sum_{\mathcal{W} \in \mathcal{Q}_r} P_{n_{r \backslash \mathcal{W}}} \sigma^2_{n_{r \backslash \mathcal{W}}}. \tag{10}$$

IV. Synchronous Multiple Reader Cardinality Estimation (S-MRCE)

We proposed the A-MRCE algorithm in the previous section based on the assumption that readers cannot be synchronized for interrogations. In this section, we consider the case that readers have the ability to be synchronized for interrogations (e.g., they are equipped with accurate clocks). We develop a synchronous multiple reader cardinality estimation (S-MRCE) algorithm, which is suitable for RFID systems with multiple synchronized readers. The readers operate synchronously in a sense that they start interrogation at certain times and perform interrogations periodically one after another. We first propose a synchronous exclusive estimator, which is a building block of the S-MRCE algorithm.

A. Synchronous Exclusive Estimator

The synchronous exclusive estimator is developed to estimate $n_{r \backslash \mathcal{W}}$ for $\mathcal{W} \subseteq \mathcal{H}_r$ using interrogation vectors obtained from synchronous operation of the readers. The main idea behind the operation of the synchronous exclusive estimator is the use of different seed values for different tags in one interrogation. In fact, the reader performs interrogation while the shared tags (i.e., the tags in $\mathcal{T}_r \cap \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$) and the tags which are exclusively located within the range of the reader have different seed values. Consider reader r, the set of readers $\mathcal{W} \subseteq \mathcal{H}_r$, and the vector $\mathbf{v}_r$ obtained from the interrogation of tags within the range of r using seed value q. Also, consider vector $\mathbf{u}_r$ obtained from the interrogation of tags in $\mathcal{T}_r$ while the tags in $\mathcal{T}_r \backslash \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$) use seed value q and the tags in $\mathcal{T}_r \cap \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$ use seed value q'. Since the tags select the same slot whenever they are interrogated with the same seed value, vectors $\mathbf{v}_r$ and $\mathbf{u}_r$ have common information about $n_{r\backslash\mathcal{W}}$. The difference between these two vectors comes from the shared tags, which have been interrogated with different seed values. The number of slots with at least one transmission in either $\mathbf{v}_r$ or $\mathbf{u}_r$ represents the existence of a tag in set $\mathcal{T}_r$ while the tags in $\mathcal{T}_r \cap \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$ are counted twice. Let Y denote the number of time slots which are nonempty either in $\mathbf{v}_r$ or $\mathbf{u}_r$. That is, given reader $r \in \mathcal{R}$ and set $\mathcal{W}$, we have $Y = \sum_{l=1}^{f}\left(v_r^l + u_r^l - v_r^l u_r^l\right)$. The samples of random variable Y can be used to estimate the value of $n_{r\backslash\mathcal{W}}$. The following theorem characterizes the distribution of variable Y.

Theorem 3: The random variable Y has a normal distribution with mean μ_y and variance σ_y^2 for large values of f, n_r, and $n_{r\backslash\mathcal{W}}$ as:

$$\mu_y = f\left(1 - \exp\left(-p\nu/f\right)\right), \tag{11}$$

$$\sigma_y^2 = f\exp\left(-p\nu/f\right)\left(1 - \left(1 + p^2\nu\right)\exp\left(-p\nu/f\right)\right), \tag{12}$$

where $\nu = (2n_r - n_{r\backslash\mathcal{W}})$.

The proof is similar to the proof given in [28] for the occupancy problem and we omit it due to the lack of space. Using M different pairs of seed values for the tags in $\mathcal{T}_r \backslash \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$ and $\mathcal{T}_r \cap \{\cup_{w\in\mathcal{W}}\mathcal{T}_w\}$, M different samples of random variable Y can be obtained, namely $y^1, \ldots, y^M$. Let $\mathbf{y}$ denote the vector of all samples (i.e., $\mathbf{y} = \left(y^1, \ldots, y^M\right)$). The likelihood function of $n_{r\backslash\mathcal{W}}$ using $\mathbf{y}$ is similar to that obtained in (5) if the mean and variance of variable Z is replaced by variable Y. The ML estimate of $n_{r\backslash\mathcal{W}}$ with M different samples can be obtained by taking derivative from the likelihood function and equating it to zero similar to the approach used for (6). For large values of M (e.g., $M > 10$) and f (e.g., $f > 100$), the synchronous exclusive estimator can be approximated as

$$\begin{aligned}\hat{n}_{r\backslash\mathcal{W}}^M &\approx \left\{ n_{r\backslash\mathcal{W}} \;\middle|\; \mu_y - \frac{1}{M}\sum_{m=1}^{M} y^m = 0 \right\} \\ &= 2n_r + \frac{f}{p}\ln\left(1 - \frac{\sum_{m=1}^{M} y^m}{Mf}\right),\end{aligned} \tag{13}$$

where $\hat{n}_{r\backslash\mathcal{W}}^M$ denotes the estimation of $n_{r\backslash\mathcal{W}}$ using the synchronous exclusive estimator with M samples. When using the synchronous exclusive estimator, we use an estimation of n_r. This estimation is obtained using the single reader estimator which is asymptotically unbiased. For large values of M, $\sum_m y^m/M$ converges to μ_y and $\ln\left(1 - \sum_m y^m/(Mf)\right)$ converges to $-p/f\nu$. Since the error in the estimation of n_r approaches zero, the synchronous exclusive estimator is asymptotically unbiased. Since it is an ML estimator, the error distribution is asymptotically normal [29]. The error in the estimation of n_r is also asymptotically normal and is added to the error generated from the random samples taken from the system. Assume that there is no error in estimation of n_r. In this case, the error in $\hat{n}_{r\backslash\mathcal{W}}^M$ originates from the second term in (13). Since the synchronous exclusive estimator is an ML estimator, the variance of error can be approximated using the Cramer-Rao bound [29]. The Fisher information for this exclusive estimator can be written as

$$\begin{aligned}&\mathcal{I}_{\text{syn}}(n_{r\backslash\mathcal{W}}) = E_{y|n_{r\backslash\mathcal{W}}}\left\{\left(\frac{\partial}{\partial n_{r\backslash\mathcal{W}}}\ln L(y; n_{r\backslash\mathcal{W}})\right)^2\right\} \\ &= E_{y|n_{r\backslash\mathcal{W}}}\left\{\left(\frac{\partial}{\partial n_{r\backslash\mathcal{W}}}\ln\prod_{m=1}^{M}\frac{1}{\sqrt{2\pi\sigma_y^2}}\exp\left(-\frac{\left(y^m - \mu_y\right)^2}{2\sigma_y^2}\right)\right)^2\right\}.\end{aligned} \tag{14}$$

Algorithm 3 S-MRCE Algorithm to calculate the total number of tags in the system.

1: The controller sends the parameters p, f, vector $\mathbf{q}$, and random slot numbers $\{s_1, \ldots, s_{|\mathcal{R}|}\}$ to all readers.
2: **for** $k = 0, \ldots, |\mathcal{R}| - 1$
3:     **if** $s_r = k$,
4:         Reader r starts periodic interrogation process using seed vector $\mathbf{q}$.
5:     **end if**
6: **end for**
7: **for** $r \in \mathcal{R}$
8:     Reader r calculates $\tilde{n}_r$ using (2) and calculates $\tilde{n}_{r\backslash\mathcal{W}}$ using synchronous exclusive estimator (13).
9:     Reader r sends the estimated value $\tilde{n}_{r\backslash\mathcal{W}}$ to the controller.
10: **end for**
11: The controller sums up the received value from all readers to find the estimation of N.

Equation (14) can be approximated as

$$\mathcal{I}_{\text{syn}}(n_{r\backslash\mathcal{W}}) \approx \frac{\left(\mu_y'\right)^2}{\sigma_z^2} = \frac{p^2}{f\left(1 - \left(1 + p^2\nu\right)\exp\left(-p\nu/f\right)\right)}, \tag{15}$$

where μ_y' denotes the derivative of μ_y with respect to $n_{r\backslash\mathcal{W}}$. Now, we consider the error in estimation of n_r. Similar to the estimation error of asynchronous exclusive estimator, the error generated from inaccurate estimation of n_r and the error from the random samples have different signs. Therefore, the summation of the variance of these two errors gives an upper bound on the variance of estimation error for synchronous exclusive estimator. Since the term n_r in (13) has coefficient two, its variance has coefficient four in the summation of variances:

$$\sigma_{n_{r\backslash\mathcal{W}}}^2(M) \le \left(M\mathcal{I}_{\text{syn}}(n_{r\backslash\mathcal{W}})\right)^{-1} + 4\sigma_{n_r}^2. \tag{16}$$

B. S-MRCE Algorithm

We now present the S-MRCE algorithm to estimate the total number of tags in a system covered by multiple readers. On contrary to the A-MRCE algorithm, the S-MRCE algorithm is a distributed approach while readers perform interrogation and estimate the number of tags individually. Then, they transmit their estimation to the controller. The controller just sums up those numbers to obtain the estimation of the total number of tags. The computation load of estimation is at the reader's side. The S-MRCE algorithm implements (1) using the synchronous exclusive estimator. Algorithm 3 shows the S-MRCE algorithm. In this algorithm, all the readers are assumed to be synchronized with a central clock.

We divide the system time into several time periods and each reader is assigned a period number. Each reader performs an interrogation within the assigned period. To prevent collision between neighboring readers, we use a scheduling algorithm based on graph coloring [30]. We assign different colors to various readers such that two neighboring readers are not assigned the same color. The chromatic number of the system is the minimum number of colors needed to color all the readers. Let C denote the chromatic number of our system. The colors are interpreted as different time periods of the system. This provides a time division multiple access method for interrogation of all the readers. There are C interrogation periods assigned to the readers. Let s_r denote the period number which is assigned to the reader r. Each period is long enough for an interrogation process. At the beginning, the controller informs the persistence probability p, frame size f, the random seed vector $\mathbf{q} = (q_1, \ldots, q_M)$, where $q_i \neq q_j, \forall\ i, j$, and the slot number to all the readers (Step 1). We call the C slots together an interrogation round. Within an interrogation round, all the readers perform the interrogation once. To estimate the number of tags, all the readers perform interrogation twice in two consecutive rounds. In the first

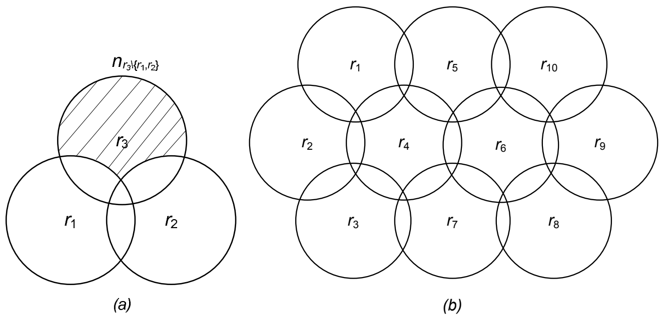


Fig. 2. Network topologies: (a) Three readers, and (b) Ten readers.

round, all the readers announce their seed value and then perform interrogation. In the second round, however, the readers just perform the interrogation and do not announce the seed value. For the reader r, the first interrogation results in measuring interrogation vector v_r while all the tags within range of reader r are interrogated with the same seed value. In the second interrogation, the reader r measures vector u_r while the tags in the range of reader r which are in the range of other readers have different seed value compared to those exclusively in the range of reader r. To make sure that neighboring readers are not using the same seed value, reader with slot number s_r used the s_r-th seed value. We assume that the chromatic number C is less that the number of samples M. To obtain M samples, the interrogation rounds are repeated M times with different seed values. We call this synchronous process performed by every reader the *periodic interrogation process* (Step 4).

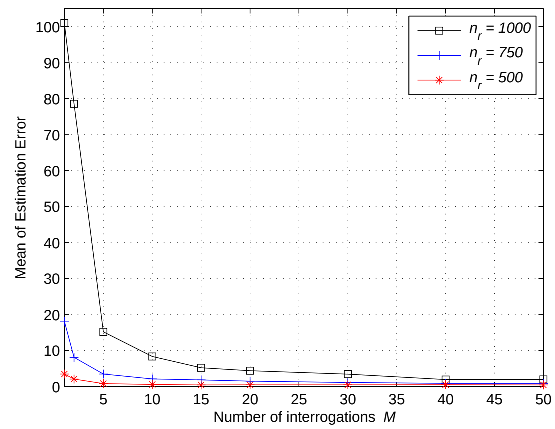
After performing the interrogations by all the readers, each reader r uses the vectors obtained by announcing the seed value to estimate n_r. Reader r calculates the estimated value of $n_{r\backslash\mathcal{W}}$ using the estimation of n_r and also two sets of interrogation vectors v_r and u_r (Step 8). Reader r sends the estimation of $n_{r\backslash\mathcal{W}}$ to the controller (Step 9). The controller can calculate the estimation of the total number of tags by adding up all values received from the readers (Step 11).

Since the error of synchronous exclusive estimator has normal distribution with zero mean, the estimation of N obtained by using the S-MRCE algorithm has normal error distribution which is asymptotically unbiased. The upper bound for the variance of estimation error for the S-MRCE algorithm can be calculated using a similar approach employed to calculate the variance of the A-MRCE algorithm. The upper bound can be written as:

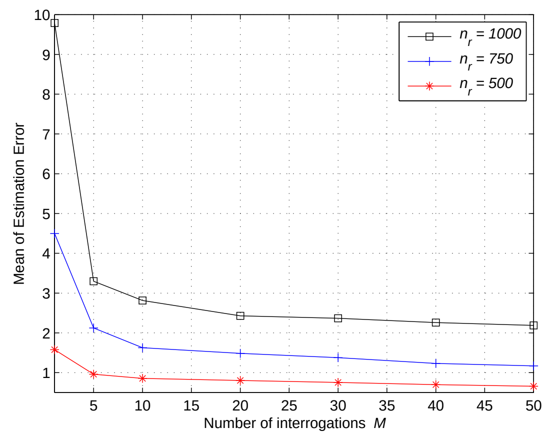
$$\sigma^2_{\text{syn}} \leq \sum_{r\in\mathcal{R}} \sum_{\mathcal{W}\in\mathcal{Q}_r} P_{n_{r\backslash\mathcal{W}}} \sigma^2_{n_{r\backslash\mathcal{W}}}, \tag{17}$$

where $\sigma^2_{n_{r\backslash\mathcal{W}}}$ is derived in (16). Compared to the A-MRCE algorithm, the S-MRCE algorithm provides better performance in terms of the estimation of error. The probability of error of asynchronous exclusive estimator increases exponentially with $n_{\mathcal{W}}$. When the number of neighboring readers increases, the probability of error for asynchronous exclusive estimator increases rapidly. However, the error of synchronous exclusive estimator depends on the number of tags within the zone of the reader, but not the neighboring readers. The cost which is paid to achieve this better performance is the need of the synchronous operation of readers.

The estimation error of A-MRCE and S-MRCE algorithms depends on the choice of design parameters f and p. To choose these parameters appropriately, the controller requires to know the number of tags in the system. In case that the controller does not have a priori information about the number of tags in the system, the controller can choose predetermined values for f and p and perform a round of interrogations. Then, based on the estimation of the number of tags, the controller chooses f and p for the next round of interrogation.



(a)



(b)

Fig. 3. Mean of estimation error for different number of interrogations M, (a) asynchronous exclusive estimator, and (b) synchronous exclusive estimator.

V. Performance Evaluation

We used MATLAB and developed a discrete-event RFID simulator to validate the analytical models and to evaluate the performance of the exclusive estimators and the MRCE algorithms.

A. Performance Evaluation for Exclusive Estimators

In this section, we investigate the performance of the exclusive estimators, validate the models, and compare them in terms of the mean and variance of the estimation error. We first consider the topology given in Fig. 2 (a). The interrogation zone of each reader is 15 m. The tags are randomly deployed within the zone of readers. The number of tags within the zone of all readers is equal. The average number of tags shared between any two of readers is 15% of n_r. Also, 5% of the tags are shared among three of them. We use asynchronous and synchronous exclusive estimators to estimate $n_{r_3\backslash\mathcal{W}}$ where $\mathcal{W} = \{r_1, r_2\}$. The estimation of $n_{\mathcal{W}}$, is obtained as $\tilde{n}_{\mathcal{W}} = \tilde{n}_{r_1} + \tilde{n}_{r_2\backslash r_1}$, where $\tilde{n}_{r_1}$ and $\tilde{n}_{r_2\backslash r_1}$ are obtained by using (2) and (5), respectively. We notice that $n_{r\backslash\mathcal{W}} = 0.75 n_r$.

First, we investigate the performance of the models by varying the number of interrogations M from 1 to 50. We set the frame size f to 500 and the persistence probability p to 1. We measure the mean and variance of the estimation error for both asynchronous and synchronous exclusive estimators. These errors contain the errors in estimation of $\tilde{n}_{\mathcal{W}}$ and $\tilde{n}_3$ as well. Figs. 3 (a) and (b) show that both estimators are asymptotically unbiased and the mean of the error approaches zero rapidly when M increases in the system. Figs. 4

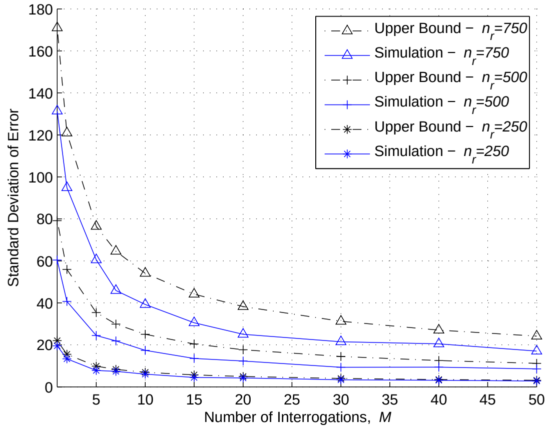


Fig. 4. Standard deviation of estimation error for different number of interrogations M when using an *asynchronous* exclusive estimator.

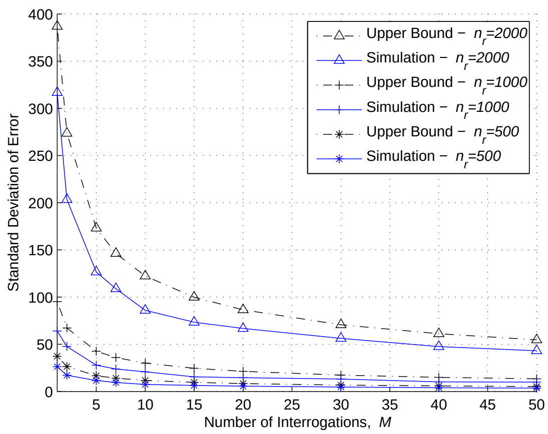


Fig. 5. Standard deviation of estimation error for different number of interrogations M when using the *synchronous* exclusive estimator.

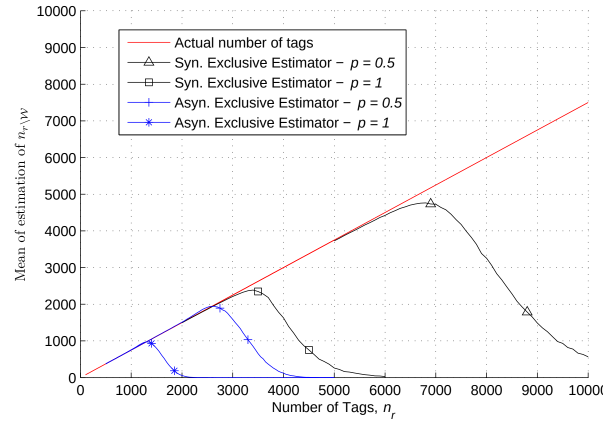


Fig. 6. The mean of estimation error for varying number of tags, asynchronous and synchronous exclusive estimators.

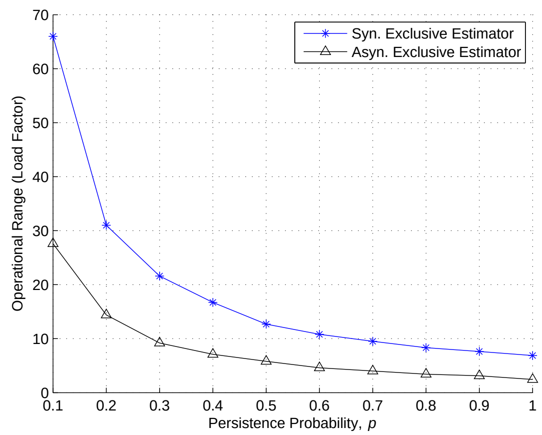
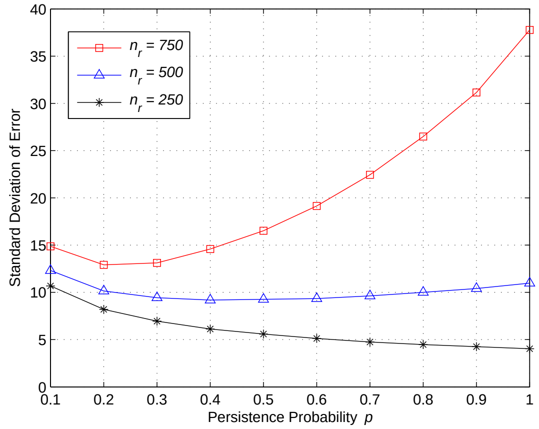


Fig. 7. Operational range of asynchronous and synchronous exclusive estimators for varying persistence probabilities.

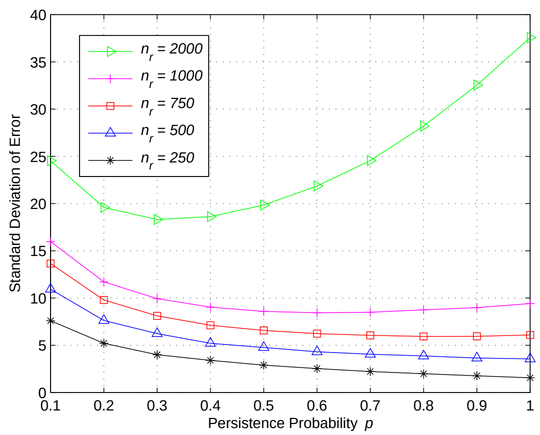
and 5 show the standard deviation of error obtained from simulations and compare these values with analytical upper bounds obtained in equations (8) and (16). For both estimators, the variance of error approaches zero for large values of M. As shown in Figs. 4 and 5, the upper bounds are tight when the number of tags within the zone of readers is small.

Next, we investigate the behavior of these estimators for different number of tags within the range of readers. We use the notion of operational range for comparison. The operational range of the estimators is defined as the the range of n_r such that the mean of estimation error is within a certain threshold of the actual value. We use $\pm 1\%$ as the threshold. The lower bound of the range is always zero and the upper bound depends on frame size f and persistence probability p. Lowering the persistence probability p can decrease the effective number of tags transmitting in an interrogation process and it can increase the operational range. The analytical models for the estimators are valid as long as n_r is within the operational range. We vary the number of tags within the zone of each reader from 100 to 10,000 by steps of 50. Fig. 6 shows the mean of estimated value for asynchronous and synchronous exclusive estimators for two values of persistence probability, $p = 1$ and $p = 0.5$. The mean of estimation for the asynchronous exclusive estimator is within the $\pm 1\%$ of the actual value of $n_{3\backslash\mathcal{W}}$ for values of n_3 less than 1,100 and 2,200 for $p = 1$ and $p = 0.5$, respectively. These indicated the operational range of the estimators. The values beyond these thresholds are out of the operational range of the estimator. These values for synchronous exclusive estimator are 2,600 and 5,500 for $p = 1$ and $p = 0.5$, respectively.

The operational range of the exclusive estimators is a function of the persistence probability. Fig. 7 shows the operational range of the asynchronous and synchronous exclusive estimators for various values of p. The operational range is extended when the probability is decreased. Although decreasing p can increase the operational range, it may increase the estimation error under some circumstances, especially for the systems with a low number of tags. Figs. 8 (a) and (b) show the behavior of the standard deviation of the error versus the persistence probability for the asynchronous and synchronous exclusive estimators, respectively. We notice that n_r equals to 1000 and 2000 are not in the operational range of the asynchronous exclusive estimator. Therefore, Fig. 8 (a) does not include the curves for 1000 and 2000 nodes. Both estimators have similar behavior in terms of variance of error. For the systems with a small number of tags compared to the frame size, the variance of the estimation error increases when p decreases. It means, $p = 1$ is a suitable choice. On the other hand, for large values of n_r compared to the frame size, decreasing the persistence probability can improve the error to some extent. However, for very small persistence probabilities, the error start increasing again. There is a trade off between the operational range and the variance of error in choosing the design parameter p.



(a)



(b)

Fig. 8. Standard deviation of error for varying persistence probabilities: (a) asynchronous exclusive estimator, (b) synchronous exclusive estimator.

B. Performance Evaluation for A-MRCE and S-MRCE Algorithms

In this section, we first investigate the performance of the A-MRCE and S-MRCE algorithms for a system with ten readers shown in Fig. 2 (b). The interrogation zone of each reader has a range equal to 15 m. The tags are randomly deployed within the interrogation zone of the readers. We set the frame size f to be 500 time slots, the persistence probability p to 1, and M to 50. Then, we increase the total number of tags in the system from 500 to 5000 with steps of 500. These numbers are chosen such that the single reader and exclusive estimators work in their operational range (i.e., the mean of estimation error is zero). Fig. 9 shows the standard deviation of error for the A-MRCE and S-MRCE algorithms and compares the analytical results obtained in (10) and (17) with the simulation results. The simulation results are averaged over 1000 iterations. As we expect, the analytical values provide upper bounds on the simulation results for both algorithms. In the worst case, the analytical results are 40% and 30% higher than the simulation results. As Fig. 9 shows, the standard deviation of error is negligible compared to the actual number of tags in the system, which proves the accuracy of the algorithms. Moreover, it can be seen that the S-MRCE algorithm outperforms the A-MRCE algorithm substantially in terms of estimation error. However, we notice that S-MRCE is not suitable for cases when readers cannot perform synchronously in the system.

Next, we compare the A-MRCE and S-MRCE algorithms with two other algorithms: the lottery frame (LoF) [22] and EZB [25] algorithms. Both EZB and LoF algorithms can also be extended

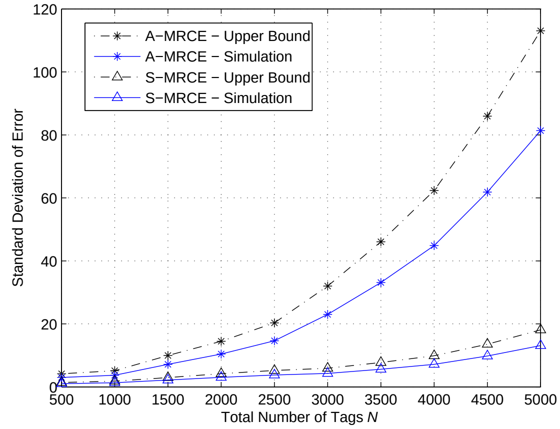
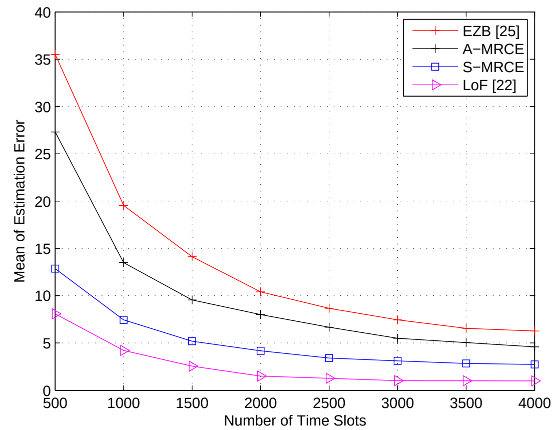


Fig. 9. Comparison of A-MRCE and S-MRCE in terms of standard deviation of error.

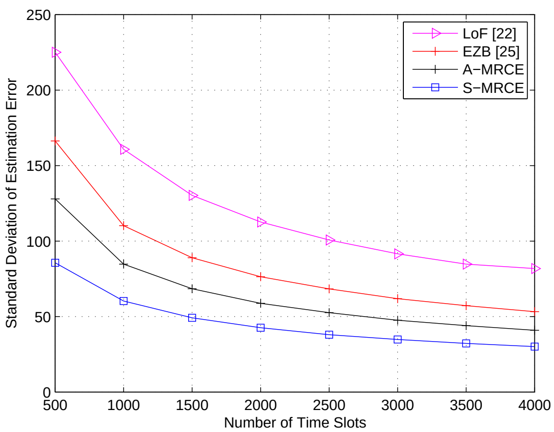
to estimate the number of tags for systems with multiple readers. To achieve that, the interrogation vectors of all the readers in the system are merged using slot-wise and operator. The resulting vector is similar to a vector obtained by a single reader interrogating all the tags in the system. Therefore, this vector can be used as an input for the EZB or LoF algorithm to estimate the total number of tags in the system. We compare these algorithms for a system with three readers and ten readers separately.

First, we compare the mean and variance of estimation error of A-MRCE, S-MRCE, LoF, and EZB algorithms for various interrogation times for an RFID system with three readers shown in Fig 2 (a). The number of tags within the range of each reader is 750. Figs. 10 (a) and (b) show the mean and standard deviation of error in estimating the total number of tags versus the interrogation time, respectively. The interrogation time is defined as the number of time slots that each reader requires to perform interrogation. The frame size for A-MRCE, S-MRCE, and EZB algorithms is set to 500 while it is set to 16 for LoF. We vary the number of interrogations for various algorithms (change M and the number of hash functions) and determine the mean and variance of estimation error. Fig. 10 (a) shows that LoF algorithm has a lower mean of estimation error compared to other algorithms for the same interrogation times. Then, LoF has a wider operational range compared to other schemes. However, as Fig. 10 (b) shows, the variance of estimation error is higher in LoF compared to other algorithms.

Next, we consider the system in Fig. 2 (b). The number of tags within the zone of different readers is $n_r = 500$. The frame size f is set to 500 time slots for A-MRCE, S-MRCE, and EZB algorithms while the frame size of the LoF is set to 16. The number of interrogations M is set to 20 for A-MRCE and EZB algorithms and it is set to 10 for S-MRCE algorithm. We use 600 various hash functions for the LoF algorithm. We notice that under this setting, the readers in all the algorithms have the same interrogation time. We also mention that the mean of estimation error is close to zero for all the estimators under this setting. At the beginning, we only consider reader 1 for the simulations. Then, we add the reader one by one to the system and investigate the performance of the algorithms in the presence of different number of readers. Fig. 11 shows the standard deviation of the estimation error for the total number of tags for various number of readers. It can be seen that the standard deviation of error in the A-MRCE and S-MRCE algorithms grows *linearly* with the number of readers while it increases *exponentially* in LoF and EZB algorithms. In the presence of multiple readers, EZB and LoF algorithms merge the interrogation vectors of several readers to obtain one interrogation vector. For a fixed number of tags within the range of the readers, increasing the number of readers would linearly increase the total number of tags in the system. Since both



(a)



(b)

Fig. 10. Comparing different algorithms in terms of interrogation time (number of time slots), (a) mean of estimation error, and (b) standard deviation of estimation error.

algorithms use the merged interrogation vector for estimation and the variance of the estimation error for either EZB and LoF algorithms increases exponentially with the number of tags, the variance of EZB and LoF algorithms increases exponentially as the number of readers linearly increases. However, for the A-MRCE and S-MRCE algorithms, whenever a reader is added to the system, only the error of the exclusive estimator which is used to estimate the tags within range of that reader is added to the total error. Therefore, the increase in the error is linear as the number of readers increases linearly.

VI. Conclusions

In this paper, we studied the problem of anonymous cardinality estimation in RFID systems with multiple readers. We proposed two ML estimators, namely an asynchronous exclusive estimator and a synchronous exclusive estimator, to estimate the number of tags which are exclusively located within the zone of a reader. We showed that these estimators are asymptotically unbiased and we derived upper bounds on the variance of estimation error. We proposed the A-MRCE and S-MRCE algorithms which can accurately estimate the tag population anonymously using the query replies of different readers and exclusive estimators. We derived the probability density function of the estimation error and showed that it can be approximated as a normal distribution with zero mean. The accuracy of the model and the approximations are validated via simulations. For performance comparisons, results showed that the variance of estimation error for A-MRCE and S-MRCE algorithms increase

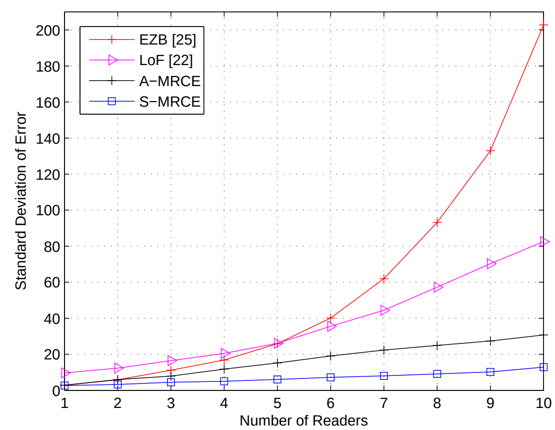


Fig. 11. Comparing the standard deviation of estimation error for A-MRCE, S-MRCE, EZB [25], and LoF [22] algorithms.

linearly with the number of readers while it increases exponentially for EZB [25] and LoF [22] algorithms. For future work, one can study the problem of choosing the design parameters f and p based on the initial estimation of the number of tags and study the trade off between the variance of estimation error and interrogation time.

Appendix

A. Proof of Theorem 1

Let ϕ_z and ψ_z denote the event that z predetermined slots are nonempty in $\mathbf{v}_r$ and empty in $\mathbf{v}_w, \forall\ w \in \mathcal{W}$, respectively. Let θ_z denote the event that both events ϕ_z and ψ_z occur. We have $\mathrm{P}(\theta_z) = \mathrm{P}\left(\phi_z \cap \psi_z\right) = \mathrm{P}(\phi_z \mid \psi_z)\mathrm{P}(\psi_z)$, where

$$\begin{aligned}\mathrm{P}(\phi_z \mid \psi_z) \quad &= \quad \sum_{t=0}^{n_{r\setminus\mathcal{W}}} \left(\mathrm{P}(\phi_z \mid t \text{ tags pick these } z \text{ slots})\right. \\ &\qquad \left.\times \mathrm{P}\left(\ t \text{ tags pick these } z \text{ slots} \mid \psi_z\right)\right).\end{aligned}$$

The condition on observed ψ_z affects the upper bound of the summation. The probability that z time slots are non-empty if t tags choose them is as follows [31, p. 92]:

$$\mathrm{P}(\phi_z \mid t \text{ tags pick these } z \text{ slots}) = \sum_{k=0}^{z}(-1)^k \binom{z}{k}\left(1-\frac{z}{f}\right)^t.$$

We also have

$$\begin{aligned}&\mathrm{P}\left(\ t \text{ tags pick these } z \text{ slots} \mid \psi_z\right) \\ &\qquad = \binom{n_{r\setminus\mathcal{W}}}{t}\left(\frac{pz}{f}\right)^t\left(1-\frac{pz}{f}\right)^{(n_{r\setminus\mathcal{W}}-t)}.\end{aligned}$$

Hence, we have

$$\begin{aligned}\mathrm{P}(\phi_z) \quad &= \quad \sum_{t=0}^{n_{r\setminus\mathcal{W}}}\sum_{k=0}^{z}(-1)^k\binom{z}{k}\left(1-\frac{k}{f}\right)^t\binom{n_{r\setminus\mathcal{W}}}{t} \\ &\qquad \left(\frac{pz}{f}\right)^t\left(1-\frac{pz}{f}\right)^{(n_{r\setminus\mathcal{W}}-t)} \\ &= \quad \sum_{k=0}^{z}(-1)^k\binom{z}{k}\left(1-\frac{pk}{f}\right)^{n_{r\setminus\mathcal{W}}}.\end{aligned}$$

Therefore, we can write $\mathrm{P}(\theta_z)$ as

$$\mathrm{P}(\theta_z) = \left(1-\frac{pz}{f}\right)^{n_w}\sum_{k=0}^{z}(-1)^k\binom{z}{k}\left(1-\frac{pk}{f}\right)^{n_{r\setminus\mathcal{W}}}.$$

Let S_z denote the summation of probability of all possible occurrences of z events in a frame. This is equal to $S_z = \binom{f}{z} P(\theta_z)$. Let P_z denote the probability of having exactly z slots nonempty in $\mathbf{v}_r$ and empty in $\mathbf{v}_w, \forall\, w \in \mathcal{W}$. This probability can be calculated using [31, p. 96] as follows:

$$\begin{aligned} P_z &= \sum_{m=z}^{f} (-1)^{m-z} \binom{m}{z} S_m \\ &= \sum_{m=z}^{f} (-1)^{m-z} \binom{m}{z} \binom{f}{m} P(\theta_m). \end{aligned} \tag{18}$$

Equation (18) shows that this problem is similar to the occupancy problem investigated in [27]. It is shown in [27], [28] that the distribution for the occupancy problem is asymptotically normal. Using a similar approach, we can show that Z has a normal distribution for large values of f and n_r. To calculate the mean and variance of Z, we use the approach presented in [28]. We define an auxiliary random variable X_i which takes values from $\{0, 1\}$. X_i is equal to one if the i^{th} time slot is non-empty in $\mathbf{v}_r$ and empty in $\mathbf{v}_w,\ \forall w \in \mathcal{W}$. Therefore, we have $Z = \sum_{i=1}^{f} X_i$. The mean and variance of variable Z can be obtained using this auxiliary variable as follows:

$$\begin{aligned} \mu_z &= E[Z] = E\left[\textstyle\sum_{i=1}^{f} X_i\right] = fP_1 \\ &\approx f\left(1 - \exp\left(-\frac{pn_{r\setminus\mathcal{W}}}{f}\right)\right) \exp\left(-\frac{pn_{\mathcal{W}}}{f}\right). \\ \sigma_z^2 &= E\left[\left(\textstyle\sum_{i=1}^{f} X_i\right)^2\right] - \mu_z^2 \\ &= \textstyle\sum_{i=1}^{f} E\left[X_i^2\right] + 2E\left[\sum_{i=1}^{f}\sum_{j=i+1}^{f} X_i X_j\right] \\ &= fP_1 + f(f-1)P_2 - \mu_z^2 \\ &\approx \mu_z\left(1 - \tfrac{\mu_z}{f}\left(1 + \tfrac{p^2 n_{\mathcal{W}}}{f}\right)\right) \\ &\quad -p^2 n_{r\setminus\mathcal{W}} \exp\left(-\frac{2p}{f}(n_{r\setminus\mathcal{W}} + n_{\mathcal{W}})\right). \qquad \blacksquare \end{aligned}$$

B. Proof of Theorem 2

We replace $\frac{\sum_m z^m}{M}$ and $\tilde{n}_{\mathcal{W}}$ by $\mu_z + e_z^M$ and $n_{\mathcal{W}} + e_n^M$, respectively while e_z^M and e_n^M are random variables. Since variable Z has a normal distribution, variable e_z^M is a normal random variable with zero mean and variance σ_z^2/M. Moreover, it is assumed that $\tilde{n}_{\mathcal{W}}$ is a normal random variable and the estimation is asymptotically unbiased. Therefore, e_n^M is a normal random variable with zero mean. For large values of f and M, the variance of e_z^M and e_n^M approaches zero and we have

$$\begin{aligned} &\tilde{n}_{r\setminus\mathcal{W}} \\ &= -\frac{f}{p}\ln\left(1 - \frac{\sum_m z^m}{Mf}\exp\left(\frac{p\tilde{n}_{\mathcal{W}}}{f}\right)\right) \\ &= -\tfrac{f}{p}\ln\left(1 - \tfrac{\left(\mu_z + e_z^M\right)}{f}\exp\left(\tfrac{p\left(n_{\mathcal{W}} + e_n^M\right)}{f}\right)\right) \\ &\approx -\tfrac{f}{p}\ln\left(1 - \tfrac{\left(\mu_z + e_z^M\right)}{f}\left(1 + p/fe_n^M\right)\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right)\right) \\ &\approx -\tfrac{f}{p}\ln\left(1 - \tfrac{\mu_z}{f}\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right) - \tfrac{e_z^M + \mu_z p/fe_n^M}{f}\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right)\right) \\ &\approx -\tfrac{f}{p}\ln\left(1 - \tfrac{\mu_z}{f}\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right)\right) - \tfrac{e_z^M/p + \mu_z/fe_n^M}{1 - \mu_z/f\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right)}\exp\left(\tfrac{pn_{\mathcal{W}}}{f}\right) \\ &\approx n_{r\setminus\mathcal{W}} - \tfrac{e_z^M}{p}\exp\left(\tfrac{p\left(n_{\mathcal{W}} + n_{r\setminus\mathcal{W}}\right)}{f}\right) - e_n^M\left(\exp\left(\tfrac{pn_{r\setminus\mathcal{W}}}{f}\right) - 1\right). \end{aligned}$$

Since both variables e_z^M and e_n^M are normal random variables, variable $\tilde{n}_{\mathcal{W}}$ is a normal random variable for large values of M. Moreover, the mean of error approaches zero for large values of M. Variables e_z^M and e_n^M are dependent in general since they are both derived from the same vectors $\mathbf{v}_r$ and $\mathbf{v}_w$. However, the error of these two variables are not in the same direction. If the error of e_z^M is positive, it means that in these vectors, the tags are spread such that the number of empty slots in vectors $\mathbf{v}_w$, which overlap with non-empty slots, is higher than the expected value. In this case, the estimate of $n_{\mathcal{W}}$, which is obtained from those vectors, should be less than its expected value. Therefore, the error of e_n^M is positive. In general, the summation of variances for these two terms gives an upper bound for the variance of error as follows:

$$\begin{aligned} \sigma_{n_{r\setminus\mathcal{W}}}^2 &= \frac{1}{p^2}\exp\left(\frac{2p\left(n_{r\setminus\mathcal{W}} + n_{\mathcal{W}}\right)}{f}\right)\frac{\sigma_z^2}{M} \\ &\quad + \left(\exp\left(\frac{pn_{r\setminus\mathcal{W}}}{f}\right) - 1\right)^2 \sigma_{n_{\mathcal{W}}}^2. \qquad \blacksquare \end{aligned}$$

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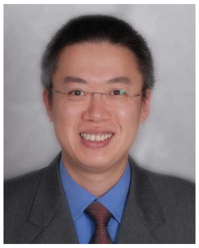

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