# An Incentive Framework for Mobile Data Offloading Market under Price Competition

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**Abstract**—Mobile data offloading can help the mobile network operator (MNO) cope with the explosive growth of cellular traffic, by delivering mobile traffic through third-party access points. However, the access point owners (APOs) would need proper incentives to participate in data offloading. In this paper, we consider a data offloading market that includes both price-taking and price-setting APOs. We formulate the interactions among the MNO and these two types of APOs as a three-stage Stackelberg game, and study the MNO's profit maximization problem. Due to a non-convex strategy space, it is in general a non-convex game. Nevertheless, we transform the strategy space into a convex set and prove that a unique subgame perfect equilibrium exists. We further propose iterative algorithms for the MNO and price-setting APOs to obtain the equilibrium. Employing the proposed algorithms, the APOs do not need to obtain full information about the MNO and other APOs. Through numerical studies, we show that the MNO's profit can increase up to three times comparing with the no-offloading case. Furthermore, our proposed incentive mechanism outperforms an existing algorithm by $18\%$ in terms of the MNO's profit. Results further show that price competition among price-setting APOs drives the equilibrium market prices down.

**Index Terms**—Mobile data offloading, network economics, non-convex game, Stackelberg game, subgame perfect equilibrium.

## 1 Introduction

### 1.1 Background

Cellular data traffic has been growing at an unprecedented rate over the past few years. The increasing data traffic forces the mobile network operators (MNOs) to employ different methods to fill the gap between the fast growing demands and the slow growing capacity of their deployed networks. Acquiring more spectrum licenses, installing new macro/micro/femto-cell base stations (BSs), and deploying new technologies can alleviate network congestion and increase the MNO's capacity. Nevertheless, these methods are both costly and time consuming to implement. Mobile data offloading, which refers to delivering data traffic of the MNO to third-party networks, is a promising alternative to address this gap. The MNO can deliver traffic of its own subscribers through WiFi, femtocell, or microcell networks to support the growing traffic demand. Mobile traffic offloaded onto WiFi and femtocell networks exceeded the cellular traffic for the first time in 2015 [1].

The performance benefit of mobile data offloading [2], [3], [4], [5], [6] has motivated the MNOs to deploy their own WiFi networks [7]. However, a ubiquitous deployment of WiFi access points (APs) with a good coverage can be very expensive due to the dynamic behavior of mobile traffic, the small coverage area of each AP as well as the cost of APs deployment and site acquisition. To fully exploit the benefit of data offloading, the MNO can take advantages of the third-party APs. In return, the MNO should provide proper economic incentives to the access point owners (APOs), in order to compensate their cost in terms of energy consumption, backhaul cost, and the potential impact on their own customers.

### 1.2 Motivations and Contributions

Several existing studies on mobile data offloading market (e.g., [8], [9], [10]) considered the leader-follower model assuming that the MNO always has a larger market power than the APOs. In these works, data offloading market is modeled as a two-stage Stackelberg game, where the MNO acts as the leader and sets the prices, while the APOs are *price-taking* followers. However, in practice, some APOs can have significant market power and hence are *price-setting* instead of pricing-taking players. The aforementioned existing models cannot be used to manage such markets. It is thus important to understand how the MNO should interact with both price-taking and price-setting APOs in a mobile data offloading market.

In this paper, we propose an incentive framework which allows the MNO to interact with both price-setting and price-taking APOs. Residential users or small companies which own WiFi/femtocell APs are considered as price-taking players. The MNO has the priority to determine market prices for this type of APOs. On the other hand, price-setting players are those APOs who have more market power than MNO. As examples, large network providers such as BT WiFi [11] with over 5 million hotspots in United Kingdom and chain stores such as Starbucks Corp. may have more market power than a single MNO. The following

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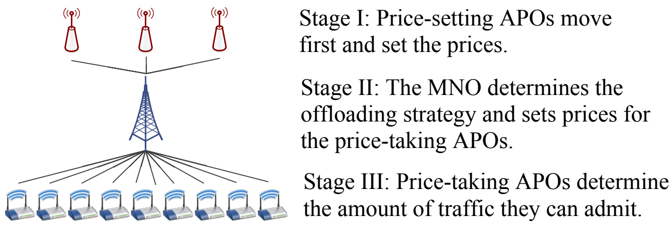


Fig. 1: Three-stage Stackelberg game model. Price-taking and price-setting APOs are represented by different icons for presentation clarity.

considerations sharply distinct our work from the existing literature:

(a) The MNO negotiates with both price-setting and price-taking APOs.
(b) The MNO aims to maximize its own profit, which is obtained by increasing the aggregate network capacity and simultaneously reducing the cost.

To facilitate the analysis, we formulate the interactions between the MNO and APOs as a three-stage Stackelberg game as shown in Fig. 1. Through this game, we study the price competition among price-setting APOs as well as the competition between the price-taking APOs for the amount of traffic they can deliver[1]. In the first stage, the price-setting APOs determine their prices with the goal of maximizing their own payoffs. In the second stage, the MNO decides whether to utilize the price-setting APOs by either accepting or declining their offered prices, and how much traffic to offload in case of accepting the prices. It then sets the prices for the price-taking APOs with the goal of maximizing its own profit. In the third stage, price-taking APOs follow the MNO's prices to determine how much traffic they can admit to maximize their own payoffs.

We consider two versions of this game formulation with different MNO objective functions, namely: i) *cost reduction*, and ii) *network expansion*. In the cost reduction problem, given a fixed admitted traffic demand, the MNO offloads mobile traffic to the APOs to reduce the cost of data delivery. The network expansion problem further allows the MNO to optimize the amount of traffic to be admitted into the system, which will be delivered through its own macrocell BSs and third-party APs. The network expansion problem can be viewed as a generalization of the cost reduction problem. For the sake of presentation clarity, we first study the cost reduction problem and then extend the analysis to study the network expansion problem.

In summary, the key contributions of this paper are as follows:

- *Unique incentive framework*: We model a data offloading market that incorporates both price-taking and price-setting APOs. We model the interactions among the two types of APOs and MNO as a three-stage Stackelberg game, and derive the corresponding subgame perfect equilibrium which determines the market prices.
- *Equilibrium Analysis*: We show that the game is in general a non-convex game due to a non-convex strategy space. Nevertheless, we transform the strategy space into a compact convex set via bijection, based on which we prove the existence of a subgame perfect equilibrium. We further prove that a unique equilibrium strategy can be obtained in the cost reduction problem.
- *Algorithm Design*: To obtain the subgame perfect equilibrium, we propose iterative algorithms, one for price-setting APOs to determine their best response strategies and one for the MNO to facilitate information exchange among the APOs. Utilizing the proposed algorithms, the price-setting APOs do not need to obtain full information about the MNO and other APOs, at the expense of a small communication overhead. We prove that the proposed algorithms converge to the unique subgame perfect equilibrium strategy.
- *Equilibrium Efficiency*: We evaluate the efficiency of the equilibrium through extensive numerical studies, by comparing the social welfare obtained by the equilibrium with the social welfare of a market without price competition. Results show that the equilibrium efficiency increases as the number of APOs increases due to a higher competition in the market. Moreover, our proposed incentive framework is able to achieve a close-to-optimal social welfare when a large number of APOs participate in the data offloading market.
- *Performance Evaluation*: Simulation results show that the proposed market model can significantly improve the MNO's profit. Our proposed framework increases the profit by up to three times comparing to the no-offloading case in the cost reduction problem. Moreover, our proposed framework outperforms the scheme proposed in [8] by 18% in terms of the MNO's profit. For the network expansion problem, our proposed framework can increase the MNO's profit by up to four times through delivering 30% more traffic. We also show that a higher data delivery cost of the MNO leads to higher payoffs for the APOs. Furthermore, we show that the proposed iterative algorithms converge quickly to the unique subgame perfect equilibrium strategy.

This paper is organized as follows. In Section 2, we review the related literature. In Section 3, we introduce the system model. We formulate the three-stage Stackelberg game in Section 4. In Section 5, we study the existence and uniqueness of equilibrium and develop the iterative algorithms. We evaluate the performance of our framework through extensive simulations in Section 6. Finally, we conclude in Section 7.

## 2 Related Literature

The economic aspects of mobile data offloading have recently been studied on two different approaches, namely *user-initiated* and *operator-initiated* offloading. The former approach considers the scenarios where mobile subscribers negotiate with the MNO and APOs to offload their traffic. The MNO has to lease the APs' bandwidth and provide incentives for mobile users to initialize the offloading [9], [10], [12], [13], [14], [15]. In particular, the mechanisms proposed in [9], [10] are based on leader-follower games. Lee *et al*. in [9] considered cellular traffic offloading via freely available WiFi networks. Zhang *et al*. in [10] modeled the data offloading market as a leader-follower game,

1. It should be noted that in the existing works (e.g., [8], [9], [10]), it was assumed that the APOs only compete for the amount of traffic they can offload. However, utilizing our proposed framework, we are able to model the price competition among the APOs as well.

where macrocell, small cell, and WiFi networks owners are leaders, and mobile subscribers are price-taking followers. In addition to the aforementioned studies in mobile data offloading, leader-follower games have been widely used to model pricing and economic aspects in different wireless networks [16], [17], [18], [19], [20]. These works modeled the markets as two-stage Stackelberg games.

The operator-initiated offloading approach focuses on the offloading decisions made by the MNO and APOs on behalf of the users. Such offloading decision is transparent to the mobile users. Although several incentive mechanisms have been proposed for operator-initiated offloading, none of them considered price-setting and price-taking players simultaneously. Gao *et al*. in [8] proposed a market-based data offloading solution considering only price-taking APOs. Gao *et al*. in [21] further considered a bargaining-based mobile data offloading approach, where the MNO is given the authority to initiate the market interaction. Wang *et al.* in [22] proposed a distributed incentive mechanism to model the interactions between offloading service providers (i.e., APOs) and offloading service consumers (i.e., data flows). Kang *et al.* in [23] proposed an incentive mechanism to motivate WiFi APOs to deliver the MNO's traffic. In this work, WiFi APOs are rewarded not only based on the amount of traffic they deliver but also based on the quality of their offloading service. The works in [24], [25], [26] proposed several auction mechanisms for data offloading, where the APOs are assumed to be price-taking. In these studies, the objective of the MNO is to minimize the cost of data delivery. In our work, however, we study a more general problem of improving the MNO's profit by admitting more traffic and reducing the cost simultaneously. Such a distinction makes our analysis of the network expansion problem much more challenging than those studied in the literature.

In addition to the economic aspects of data offloading, Chen *et al.* [27] studied energy-efficiency oriented traffic offloading mechanisms. They proposed an online reinforcement learning framework for the problem of traffic offloading in a stochastic heterogeneous cellular network. Their objective was to minimize the total energy consumption of the heterogeneous cellular network while maintaining the quality-of-service experienced by mobile users. However, they did not consider the strategic behavior of APOs.

## 3 System Model

We focus on the interactions between a macrocell BS of an MNO and a set of third-party APOs[2]. The APs have overlapping coverage area with the BS, hence the MNO may offload its traffic to them. There are $N^t$ price-taking and $N^s$ price-setting APOs, denoted by sets $\mathcal{N}^t = \{1, \ldots, N^t\}$ and $\mathcal{N}^s = \{N^t+1, \ldots, N^t+N^s\}$, respectively. Thus, we have the set $\mathcal{N} = \mathcal{N}^t \bigcup \mathcal{N}^s$ of third-party APOs, and $|\mathcal{N}| = N^t + N^s$. Since each APO's coverage area is relatively small, we assume the APOs are spatially non-overlapping and they do not interfere with each other[3], similar as the models in [8], [21], [28]. We further assume that APOs operate in spectrums different from the BS, and hence do not generate interference to the BS. In particular, WiFi APs operate in the unlicensed bands, and femtocell APs are allocated with different frequency bands from the BS's spectrum band [29], [30]. Fig. 2 illustrates an example of the network with one macrocell BS, two pricing-setting APOs, and seven price-taking APOs.

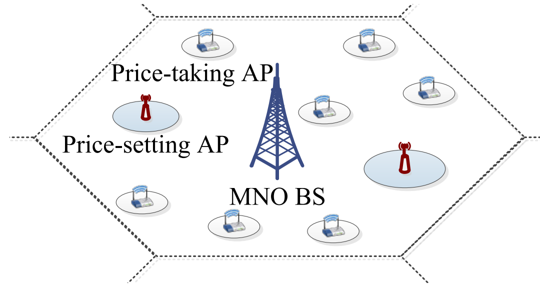


Fig. 2: A system with one macrocell BS, two price-setting APOs, and seven price-taking APOs.

We divide the coverage area of the BS into $|\mathcal{N}| + 1$ regions represented by the set $\mathcal{N} \bigcup \{0\}$. Among them, $|\mathcal{N}|$ regions are associated with APOs, where traffic of mobile subscribers can either be served by the BS or be offloaded to the corresponding APO. Set $\{0\}$ represents the region which is only covered by the BS but not by any APOs, where offloading is not possible.

The total downlink traffic demand in a region $m \in \mathcal{N} \bigcup \{0\}$ is denoted by $S_m$, which is initially steered towards the MNO[4]. Hence, the traffic demand vector of the MNO is

$$\boldsymbol{S} = \left(S_0, \ldots, S_{|\mathcal{N}|}\right).$$

The traffic demand $S_m$ varies over time due to the stochastic nature of mobile subscribers traffic. We consider a quasi-static network scenario, and analyze the market mechanism in a data offloading period (e.g., two seconds), during which $S_m$ remains unchanged for all $m$. For different approaches to measure and estimate the traffic demand, see [31], [32].

We define the amount of data traffic that can be delivered by one unit of spectrum resource (in Hz) per unit time as the *transmission efficiency*. The transmission efficiency of each link depends on various factors such as path loss, shadowing, and fading. However, we use the transmission efficiency to determine the aggregate bandwidth required to deliver the traffic demand to all users within each region. We do not specify how the transmission efficiency is computed, as our framework is applicable for general transmission models. The transmission efficiency can be obtained through measurements and various prediction methods [33], [34], [35]. Let $\theta_n$ denote the transmission efficiency (in bits/sec/Hz) of communications links between

2. Our analysis can be extended to the case of multiple macrocell BSs. To do so, the cost (to be introduced later) imposed to all BSs by delivering traffic to mobile users as well as the total traffic delivered through all BSs should be considered.

3. Extension of the current model to a general model with overlapping APOs requires considering the interference among the APOs. Moreover, the number of distinct overlapping regions partially covered by different APOs grows exponentially with $|\mathcal{N}|$, which further makes the analysis complicated. We leave this extension as future work.

4. Similar to other operator-initiated offloading mechanisms (e.g., [8], [15], [21], [22], [23], [24]), we consider the total traffic demand of each region, as the mobile subscribers are not directly involved in market negotiations.

the BS and mobile subscribers located in region $n \in \mathcal{N}$, which is covered by APO $n$. Since the coverage area of each APO is relatively small, we assume that different mobile subscribers in the same region $n$ have the same transmission efficiency. Furthermore, $\theta_0$ represents the *average* transmission efficiency between the BS and those mobile subscribers in region $\{0\}$ (hence, not covered by any APOs)[5]. The BS's bandwidth resource consumed for delivering one bit of data in region $m \in \mathcal{N} \bigcup \{0\}$ within one unit of time is $\frac{1}{\theta_m}$ (in Hz). The *transmission efficiency profile* of the BS in different regions is

$$\boldsymbol{\theta} \triangleq (\theta_0, \ldots, \theta_{|\mathcal{N}|}) .$$

We also denote the average transmission efficiency between APO $n \in \mathcal{N}$ and mobile subscribers located in the corresponding region by $\phi_n$. The *transmission efficiency profile* of APOs is

$$\boldsymbol{\phi} \triangleq (\phi_1, \ldots, \phi_{N^t}, \phi_{N^t+1}, \ldots, \phi_{N^t+N^s}) .$$

The transmission efficiency profiles may vary over time due to mobility of the subscribers. We consider a quasi-static network scenario, where the transmission efficiency profiles remain unchanged within a single time period and may change in different periods. We also assume that they can be measured by the BS and the corresponding APOs[6]. The above model and assumptions about transmission efficiency have been widely used in mobile data offloading studies for operator-initiated mechanisms (e.g., [8], [15], [21], [24]).

### 3.1 MNO Modeling

The MNO leases the bandwidth from third-party APOs to offload the traffic of mobile subscribers. The MNO chooses its offloading strategy to maximize its own profit, which is the total revenue (obtained from providing network bandwidth to the mobile subscribers) minus the cost (of delivering mobile traffic). Let $x_n^t$ denote the amount of traffic (in terms of total delivered bits in offloading period) offloaded to a price-taking APO in region $n \in \mathcal{N}^t$. Similarly, $x_n^s$ denotes the amount of traffic offloaded to a price-setting APO in region $n \in \mathcal{N}^s$. Then, we use $y_n$ to denote the traffic in region $n \in \mathcal{N}$, that is not offloaded to any APO but served by the MNO directly. Notice that $x_n^t + y_n$ ($x_n^s + y_n$, respectively), which represents the total amount of traffic in region $n \in \mathcal{N}^t$ ($n \in \mathcal{N}^s$, respectively), is equal to $S_n$ and is a constant in the later formulated cost reduction problem. However, in the network expansion problem, $x_n^t + y_n$ ($x_n^s + y_n$) is the total admitted traffic by the entire system in region $n$, and is a decision variable instead of a constant. Since each region $n \in \mathcal{N}$ is associated with an APO, we use the terms region and APO interchangeably for any $n = 1, \ldots, |\mathcal{N}|$. We denote the amount of traffic of subscribers located in region $\{0\}$ as $y_0$, which can only be served by the BS. The MNO's offloading strategy is captured by vectors $\boldsymbol{y} = (y_m)_{m \in \mathcal{N} \bigcup \{0\}}$ and $\boldsymbol{x} = [\boldsymbol{x}^t, \boldsymbol{x}^s]$, where $\boldsymbol{x}^t = (x_n)_{n \in \mathcal{N}^t}$ and $\boldsymbol{x}^s = (x_n)_{n \in \mathcal{N}^s}$.

The cost of MNO consists of the resource consumption cost and the payment provided to the APOs for offloading the MNO's traffic. The resource consumption cost is due to delivering un-offloaded traffic, which consumes the following amount of resources in the BS.

$$\sum_{m \in \mathcal{N} \bigcup \{0\}} \frac{y_m}{\theta_m}. \quad (1)$$

Notice that for each $m$, $y_m/\theta_m$ is the amount of bandwidth resources required for transmitting $y_m$ bits in one unit of time. The resource consumption cost of the MNO, denoted by $c(\boldsymbol{y})$, is

$$c(\boldsymbol{y}) = c_b \left( \sum_{m \in \mathcal{N} \bigcup \{0\}} \frac{y_m}{\theta_m} \right). \quad (2)$$

We assume that function $c_b(\cdot)$ is strictly increasing and convex (i.e., $c_b'(\cdot) > 0, c_b''(\cdot) > 0$) [36]. This convex cost function indicates that the marginal cost for delivering one more unit of data to the mobile subscribers is increasing. We further assume that the marginal cost function and it first derivative are weakly convex[7].

For the APOs, they need to receive proper payments from the MNO to be compensated for their costs incurred for offloading the traffic. A price-setting APO can determine such a payment, while a price-taking APO needs to decide whether to accept or reject the payment determined by the MNO. Although the payment functions are usually assumed to be linear in the demand, there exist several studies that consider nonlinear payment functions of the demand (e.g., [37], [38], [39], [40], [41], [42]). In this paper, we use the following different payment functions to reflect the market power of different players.

- *Price-setting APOs*: MNO's payment function for APO $n \in \mathcal{N}^s$ is $q_n^s(x_n^s) = p_n^s \left(x_n^s\right)^2$, where $p_n^s$ is the price set by the APO.
- *Price-taking APOs*: MNO's payment function for APO $n \in \mathcal{N}^t$ is $q_n^t(x_n^t) = p_n^t x_n^t$, where $p_n^t$ is the price set by the MNO.

The convex payment function related to price-setting APOs reflects their market power and the desire to get compensated more as they act as leaders for the MNO. However, for price-taking APOs, the linear payment function expresses the MNO's interest of paying less, since the MNO acts as the leader for these APOs and has more market power to set the prices.

We denote the price vector of price-taking and price-setting APOs as $\boldsymbol{p}^t = (p_n^t)_{n \in \mathcal{N}^t}$ and $\boldsymbol{p}^s = (p_n^s)_{n \in \mathcal{N}^s}$, respectively. We further denote the revenue of MNO obtained from delivering $z$ bits to the mobile subscribers within the offloading period as $r(z)$, which is an increasing weakly concave function. We define $z(\boldsymbol{x}, \boldsymbol{y}) \triangleq \sum_{m \in \mathcal{N} \cup \{0\}} y_m + \sum_{n \in \mathcal{N}^t} x_n^t + \sum_{n \in \mathcal{N}^s} x_n^s$, which represents the amount of

5. We assume region $\{0\}$ is relatively large and many mobile subscribers exist within this region. In this case, the average transmission efficiency $\theta_0$ accurately reflects the consumed resources.

6. An imperfect measurement of transmission efficiency profiles may affect the strategy of the players and degrade the performance of the framework. We may consider a robust optimization framework to address such imperfect information, and we will leave this extension as future work as the current model is already rich enough.

7. The cost functions used in the existing works [6], [8], [9], [10] also satisfy these conditions.

total mobile traffic delivered through the MNO and different APOs. The MNO's profit is

$$V(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p}) = r\left(z(\boldsymbol{x}, \boldsymbol{y})\right) - c(\boldsymbol{y}) - \sum_{n \in \mathcal{N}^s} q_n^s(x_n^s) - \sum_{n \in \mathcal{N}^t} q_n^t(x_n^t), \tag{3}$$

where vector $\boldsymbol{p} = (\boldsymbol{p}^t, \boldsymbol{p}^s)$.

### 3.2 APO Modeling

The APs are managed by selfish owners. They share their bandwidth with the MNO to maximize their own profits. We assume that each APO $n \in \mathcal{N}$ has a maximum capacity of $B_n$, which can be assigned to serve the MNO traffic as well as its own subscribers' traffic. We denote APO $n$'s profit from serving its own subscribers as $r_n(\cdot)$, which is a function of the APO's available capacity and may vary across different offloading periods. The APO determines the profit function $r_n(\cdot)$ considering its own subscribers' traffic. Similar to [43], we assume that $r_n(\cdot)$ is a non-decreasing and weakly concave function in the APO's available capacity.

APO $n \in \mathcal{N}$ will incur a profit loss from local subscribers when it admits $x$ bits of traffic from the MNO. We denote the APO's profit loss in this case by $J_n(x)$ as follows:

$$J_n(x) \triangleq r_n\left(B_n - \frac{x}{\phi_n}\right) - c_n\left(\frac{x}{\phi_n}\right), \tag{4}$$

where $c_n(\cdot)$ is the APO $n$'s cost of delivering the traffic of MNO. In each APO $n \in \mathcal{N}$, the amount of bandwidth resources required for transmitting $x$ bits in a unit of time is $x/\phi_n$, and $B_n - x/\phi_n$ represents the available capacity which can be allocated to the APO's subscribers. Similar to [36], we assume that $c_n(\cdot)$ is differentiable, increasing, and convex, which reflects the fact that the marginal cost for admitting one more unit of data is non-decreasing. Therefore, the function $J_n(\cdot)$ is a decreasing concave function, where $J_n(0) > 0$ and $J_n(\phi_n B_n) < 0$. We further assume that $J_n'(\cdot)$, which represents the marginal profit loss, and its first and second derivatives are weakly concave as well[8].

An APO's payoff obtained from both offloading MNO traffic and delivering its own subscribers' traffic is as follows:

$$V_n^t(x_n^t, p_n^t) = J_n(x_n^t) + p_n^t x_n^t, \qquad n \in \mathcal{N}^t \tag{5}$$

$$V_n^s(x_n^s, p_n^s) = J_n(x_n^s) + p_n^s \left(x_n^s\right)^2, \qquad n \in \mathcal{N}^s. \tag{6}$$

A feasible offloading strategy needs to satisfy the inequalities $x_n^t \leq \phi_n B_n$ and $x_n^s \leq \phi_n B_n$.

### 3.3 Data Offloading Game

We model the interactions between the MNO and APOs as a three-stage Stackelberg game as shown in Fig. 1. In each stage, each player determines its strategy with the goal of maximizing its payoff. We formally define the following game:

- Stage I: *Players*: price-setting APOs $n \in \mathcal{N}^s$; *Strategy*: price vector $\boldsymbol{p}^s$; *Payoff*: $V_n^s(x_n^s, p_n^s)$ given in (6).
- Stage II: *Player*: MNO; *Strategy*: offloading vectors $\boldsymbol{x}^s$, $\boldsymbol{y}$, and price vector $\boldsymbol{p}^t$; *Payoff*: $V(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p})$ given in (3).
- Stage III: *Players*: price-taking APOs $n \in \mathcal{N}^t$; *Strategy*: offloading vector $\boldsymbol{x}^t$; *Payoff*: $V_n^t(x_n^t, p_n^t)$ given in (5).

Our goal is to determine the *subgame perfect equilibrium* (SPE) of the multi-stage game, where neither the MNO nor the APOs have incentives to deviate unilaterally.

**Definition 1 (Subgame Perfect Equilibrium [44]).** *A strategy profile $\left(\boldsymbol{x}^{NE}, \boldsymbol{y}^{NE}, \boldsymbol{p}^{NE}\right)$ including the offloading strategies $\boldsymbol{x}^{NE}$ and $\boldsymbol{y}^{NE}$ and price vector $\boldsymbol{p}^{NE}$ is a subgame perfect equilibrium if it represents a Nash equilibrium (NE) in every subgame of the original game.*

In the next section, we will formulate the three-stage Stackelberg game for both cost reduction and network expansion problems. We will use backward induction [44] to obtain the corresponding SPE.

## 4 Three-stage Game Formulation

In this section, we start with Stage III and analyze the behavior of price-taking APOs. We then turn to Stage II to obtain the MNO's strategy. Finally, we study the price-setting APOs' strategies in Stage I.

### 4.1 Stage III (Price-taking APOs)

In Stage III, given the price-vector $\boldsymbol{p}^t$ set by the MNO, the price-taking APOs determine how much traffic they can admit in order to maximize their own payoffs. In particular, each APO $n \in \mathcal{N}^t$ selects its strategy $x_n^t$ within the strategy space $\mathcal{E}_{x_n^t} = [0, \phi_n B_n]$ to maximize its payoff $V_n^t\left(x_n^t, p_n^t\right)$. This leads to the following optimization problem.

$$\text{AP}^t\text{: } \underset{x_n^t \geq 0}{\text{maximize}} \; V_n^t\left(x_n^t, p_n^t\right) \tag{7a}$$

$$\text{subject to } x_n^t \leq \phi_n B_n. \tag{7b}$$

We can show that problem (7) is a concave maximization problem. Let $x_n^{t^*}\left(p_n^t\right)$ denote the unique optimal solution of problem (7), which is given in Theorem 1.

**Theorem 1.** *The optimal offloading traffic decision of price-taking APO $n$ is*

$$x_n^{t^*}\left(p_n^t\right) = \begin{cases} 0, & \text{if } 0 \leq p_n^t < p_n^{t,\min} \\ L_n(p_n^t), & \text{if } p_n^{t,\min} \leq p_n^t \leq p_n^{t,\max} \\ \phi_n B_n, & \text{if } p_n^t > p_n^{t,\max}, \end{cases} \tag{8}$$

*where $p_n^{t,\min} \triangleq -J_n'(0)$ and $p_n^{t,\max} \triangleq -J_n'(\phi_n B_n)$ with $J_n'(x) = \frac{dJ_n}{dx}$. Furthermore, $L_n(p_n^t)$ is an increasing function in $p_n^t$ and we know $L_n^{(-1)}(x) = -J_n'(x)$.*

The proof of Theorem 1 is given in Appendix A. We now characterize the properties of the optimal strategy $x_n^{t^*}\left(p_n^t\right)$ as a corollary of Theorem 1. These properties clarify the behavior of the price-taking APOs and will be used later in Stage II.

**Corollary 1.** *$x_n^{t^*}\left(p_n^t\right)$ is an increasing and concave function in $p_n^t$ over interval $[p_n^{t,\min}, p_n^{t,\max}]$.*

The proof of Corollary 1 can be found in Appendix B.

8. Weakly concave $r_n'(\cdot)$ and $r_n''(\cdot)$ and weakly convex $c_n'(\cdot)$ and $c_n''(\cdot)$ can satisfy these conditions although they are not necessarily required. The revenue and cost functions used in the existing works [6], [8], [9], [10] satisfy these conditions. However, we assume that the marginal profit loss and its first and second derivatives are weakly concave, which is less restrictive than the assumptions used in the aforementioned existing works.

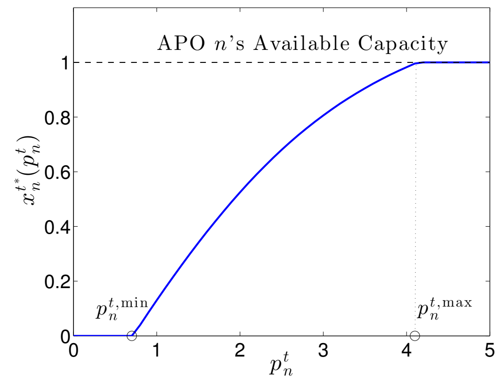


Fig. 3: The optimal strategy $x_n^{t^*}(p_n^t)$. APO $n$ will not admit traffic of the MNO for any price less than $p_n^{t,\min}$. Moreover, the MNO will not announce any price greater than $p_n^{t,\max}$, since APO $n$ is not able to deliver traffic more than its capacity.

*Observations*: Fig. 3 shows the optimal strategy of a price-taking APO. As proven in Corollary 1 and illustrated in Fig. 3, $x_n^{t^*}(p_n^t)$ is concave and increasing for any $p_n^t \in [p_n^{t,\min}, p_n^{t,\max}]$. Notice that the MNO will not set any price greater than $p_n^{t,\max}$, since the APO is not able to admit more traffic. Therefore, the strategy space of the MNO is limited to $p_n^t \in [p_n^{t,\min}, p_n^{t,\max}]$. Moreover, we will show that the analysis of Stage II becomes complicated due to the concavity of optimal strategy of price-taking APOs.

### 4.2 Stage II (MNO Problem)

We now analyze the MNO's behavior in Stage II. We first formulate the cost reduction problem. We then extend the results and formulate the network expansion problem.

#### 4.2.1 Cost Reduction Problem

In the cost reduction problem, we have $x_n^t + y_n = S_n$ for all $n \in \mathcal{N}^t$ and $x_n^s + y_n = S_n$ for all $n \in \mathcal{N}^s$. Since $S_n$ is a constant for each APO $n$, the total delivered traffic $S = \sum_{m\in\mathcal{N}\bigcup\{0\}} S_m$ is also a constant. Thus, the MNO's profit is

$$V^{\text{crp}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{p}) = r(S) - c(\boldsymbol{y}) - \sum_{n\in\mathcal{N}^s} p_n^s (x_n^s)^2 - \sum_{n\in\mathcal{N}^t} p_n^t x_n^t, \tag{9}$$

where $r(S)$ is the constant revenue obtained in this case.

The MNO's strategy space is $\mathcal{E} = \mathcal{E}_{\boldsymbol{x}^s} \times \mathcal{E}_{\boldsymbol{y}} \times \mathcal{E}_{\boldsymbol{p}^t}$, where $\mathcal{E}_{\boldsymbol{x}^s} = \{[0, S_n]\}_{n\in\mathcal{N}^s}$, $\mathcal{E}_{\boldsymbol{y}} = \{[0, S_m]\}_{m\in\mathcal{N}\bigcup\{0\}}$, while according to Corollary 1, we have $\mathcal{E}_{\boldsymbol{p}^t} = \{[p_n^{t,\min}, p_n^{t,\max}]\}_{n\in\mathcal{N}^t}$. Given the strategies of the price-setting APOs (i.e., $\boldsymbol{p}^s$) and with the knowledge of how the MNO strategy would affect the price-taking APOs' strategies (i.e., $x_n^{t^*}(p_n^t), n \in \mathcal{N}^t$), the MNO maximizes its own profit in Stage II. This leads to the following optimization problem.

$$\text{MNO: } \underset{(\boldsymbol{x}^s,\boldsymbol{y},\boldsymbol{p}^t)\in\mathcal{E}}{\text{maximize}}\; V^{\text{crp}}\left(\left(\boldsymbol{x}^{t^*}(\boldsymbol{p}^t), \boldsymbol{x}^s\right), \boldsymbol{y}, (\boldsymbol{p}^t, \boldsymbol{p}^s)\right) \tag{10a}$$

$$\text{subject to } x_n^{t^*}(p_n^t) + y_n = S_n, \quad \forall\, n \in \mathcal{N}^t \tag{10b}$$

$$x_n^s + y_n = S_n, \quad \forall\, n \in \mathcal{N}^s. \tag{10c}$$

The optimal solution of problem (10) exists and is unique if the problem is a strictly concave maximization problem. Concavity of this problem depends on the concavity of the objective function $V^{\text{crp}}$ given by (10a) and convexity of its constraints. According to Corollary 1, we know that constraint (10b) is not an affine equality (i.e., convex) constraint, as $x_n^{t^*}(p_n^t)$ is concave in $p_n^t$. Thus, problem (10) is not a concave maximization problem.

To transform problem (10) into a concave maximization problem, we replace constraint (10b) by an affine equality constraint. To do so, we introduce new variables $\tilde{\boldsymbol{x}}^t = (\tilde{x}_n^t)_{n\in\mathcal{N}^t}$, which belong to the space set $\mathcal{E}_{\tilde{\boldsymbol{x}}^t} \triangleq \{[0, \phi_n B_n]\}_{n\in\mathcal{N}^t}$. We then replace $x_n^{t^*}(p_n^t)$ by $\tilde{x}_n^t$ for any $n \in \mathcal{N}^t$ in constraint (10b), and obtain the following equality which is an affine constraint.

$$\tilde{x}_n^t + y_n = S_n, \quad \forall\, n \in \mathcal{N}^t. \tag{11}$$

We further replace $p_n^t, n \in \mathcal{N}^t$ by $x_n^{t^*(-1)}(\tilde{x}_n^t)$ in the objective function of problem (10), where $x_n^{t^*(-1)}$ is the inverse of function $x_n^{t^*}(p_n^t)$. Inverse of $x_n^{t^*} : [p_n^{t,\min}, p_n^{t,\max}] \mapsto [0, \phi_n B_n]$ exists due to Corollary 1. We now transform problem (10) into the following equivalent problem.

$$\underset{(\boldsymbol{x}^s,\boldsymbol{y},\tilde{\boldsymbol{x}}^t)\in\tilde{\mathcal{E}}}{\text{maximize}}\; r(S) - c(\boldsymbol{y}) - \sum_{n\in\mathcal{N}^s} p_n^s (x_n^s)^2 - \sum_{n\in\mathcal{N}^t} \tilde{x}_n^t x_n^{t^*(-1)}(\tilde{x}_n^t) \tag{12a}$$

$$\text{subject to (10c), (11),} \tag{12b}$$

where $\tilde{\mathcal{E}} = \mathcal{E}_{\boldsymbol{x}^s} \times \mathcal{E}_{\boldsymbol{y}} \times \mathcal{E}_{\tilde{\boldsymbol{x}}^t}$. Since problems (10) and (12) are equivalent, we can obtain the MNO's optimal strategy by solving problem (12). Through the following theorem, we prove that problem (12) is a strictly concave maximization problem.

**Theorem 2.** *Problem (12) is a strictly concave maximization problem under any fixed $\boldsymbol{p}^s$ and has a unique optimal solution.*

The proof of Theorem 2 is given in Appendix C. We denote the optimal solution of problem (12) as $\left(\boldsymbol{x}^{s^*}(\boldsymbol{p}^s), \boldsymbol{y}^*(\boldsymbol{p}^s), \tilde{\boldsymbol{x}}^{t^*}(\boldsymbol{p}^s)\right)$. This solution depends on the strategy of price-setting APOs, i.e., $\boldsymbol{p}^s$, which will be obtained in Stage I.

#### 4.2.2 Network Expansion Problem

The network expansion problem considers a more general scenario than the cost reduction problem. Through network expansion, the MNO maximizes its own profit by reducing the cost and expanding the network, where the total delivered mobile traffic can be higher than the MNO's own network capacity.

To obtain the strategy of the MNO, we first determine its profit. Note that the total delivered traffic is no longer a constant in this case. Similar to the cost reduction problem, we introduce new variables $\tilde{\boldsymbol{x}}^t = (\tilde{x}_n^t)_{n\in\mathcal{N}^t}$ and replace the optimal strategy of price-taking APO $n$ (i.e., $x_n^{t^*}(p_n^t)$) by $\tilde{x}_n^t$. The MNO's profit is

$$V^{\text{nep}}((\tilde{\boldsymbol{x}}^t, \boldsymbol{x}^s), \boldsymbol{y}, \boldsymbol{p}^s) = r\left(z\left(\tilde{\boldsymbol{x}}^t, \boldsymbol{x}^s\right)\right) - c(\boldsymbol{y}) - \sum_{n\in\mathcal{N}^s} p_n^s (x_n^s)^2 - \sum_{n\in\mathcal{N}^t} \tilde{x}_n^t x_n^{t^*(-1)}(\tilde{x}_n^t), \tag{13}$$

where the first term in (13) is the revenue obtained from delivering data to mobile subscribers. The MNO's optimal strategy can be determined from the following problem, where we take the limited capacity of the BS into account.

$$\text{MNO: } \underset{(\boldsymbol{x}^s,\boldsymbol{y},\tilde{\boldsymbol{x}}^t)\in\tilde{\mathcal{E}}}{\text{maximize}}\ V^{\text{nep}}\left((\tilde{\boldsymbol{x}}^t,\boldsymbol{x}^s),\boldsymbol{y},\boldsymbol{p}^s\right) \tag{14a}$$

$$\text{subject to } \tilde{x}_n^t + y_n \geq S_n, \qquad \forall\, n\in\mathcal{N}^t \tag{14b}$$

$$x_n^s + y_n \geq S_n, \qquad \forall\, n\in\mathcal{N}^s \tag{14c}$$

$$y_0 \geq S_0, \tag{14d}$$

$$\sum_{m\in\mathcal{N}\bigcup\{0\}} \frac{y_m}{\theta_m} \leq B, \tag{14e}$$

where $B$ denotes the capacity of the BS, and constraint (14e) ensures that the total resources consumed in the BS is less than its available capacity. Since the network expansion problem predicts how much traffic the MNO can ultimately deliver, constraints (14b)–(14d) ensure that the total traffic is greater than or equal to the admitted traffic. The network capacity obtained through network expansion can either be allocated to the best effort traffic or be used to admit more mobile subscribers.

The feasible region of problem (14) is different from (12). However, we can show that it is still compact and convex. Notice that all constraints are affine. Therefore, similar to Theorem 2, problem (14) is a strictly concave maximization problem and has a unique optimal solution. Similarly, we denote the optimal solution of problem (14) as $\left(\boldsymbol{x}^{s^*}(\boldsymbol{p}^s),\boldsymbol{y}^*(\boldsymbol{p}^s),\tilde{\boldsymbol{x}}^{t^*}(\boldsymbol{p}^s)\right)$. Notice that if the inequality constraints of problem (14) are replaced by equality constraints and constraint (14e) is removed, then the problem will be transformed into the cost reduction problem.

The MNO's profit maximization problems in both cost reduction and network expansion problems are shown to be concave maximization problems. Therefore, their optimal solutions can be obtained using standard optimization techniques such as the interior-point method [45]. Moreover, the payoff maximization problems of price-taking APOs formulated in Section 4.1 and the one of price-setting APOs which will be introduced in Section 4.3 have only one variable (i.e., the APO's strategy). These together imply the scalability of the proposed incentive framework.

### 4.3 Stage I (Price-setting APOs)

We now analyze the price competition among price-setting APOs, which determine their strategies with the knowledge of how their strategies would affect the MNO strategy. For price-setting APO $n\in\mathcal{N}^s$, the optimal response of the MNO obtained in Stage II (i.e., $x_n^{s^*}(\boldsymbol{p}^s)$) depends not only on price $p_n^s$, but also on the prices submitted by other players. This reflects the interdependence of APO's pricing decisions. Let $\boldsymbol{p}_{-n}^s$ denote the strategy of the price-setting APOs excluding $n$. Thus, we have $\boldsymbol{p}^s = \left(p_n^s,\boldsymbol{p}_{-n}^s\right)$. Since the strategies of price-setting players are coupled, we form the price-setting non-cooperative game (PS-NCG) $\mathcal{G}^s(\mathcal{N}^s,\mathcal{E}^s,V_n^s)$, in which $\mathcal{E}^s$ represents the strategy space. Moreover, the APO $n$'s payoff function is

$$V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right) = J_n\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right) + p_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right)^2, \tag{15}$$

where the optimal strategy of the MNO (i.e., $x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$) obtained in Stage II is substituted into the payoff function. To obtain the optimal strategies of price-setting APOs, we first introduce the concept of best response strategy and NE.

**Definition 2** (**Best Response Strategy** [46]). *Given $\boldsymbol{p}_{-n}^s$, player $n$'s best response strategy is:*

$$p_n^{s^*} = \underset{p_n^s\geq 0}{\operatorname{argmax}}\ V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right) \tag{16a}$$

$$\text{subject to } x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\leq \phi_n B_n, \tag{16b}$$

*which is the choice of $p_n^s$ that maximizes $V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right)$.*

**Definition 3** (**Nash Equilibrium** [46]). *A strategy profile $\boldsymbol{p}^{s^{\text{NE}}} = (p_1^{s^{\text{NE}}},\ldots,p_{N^s}^{s^{\text{NE}}})$ is an NE if it is a fixed point of best responses, i.e., for all $p_n^{s'}\geq 0, n\in\mathcal{N}^s$*

$$V_n^s\left(x_n^{s^*}\left(p_n^{s^{\text{NE}}},\boldsymbol{p}_{-n}^{s^{\text{NE}}}\right),p_n^{s^{\text{NE}}}\right)\geq V_n^s\left(x_n^{s^*}\left(p_n^{s'},\boldsymbol{p}_{-n}^{s^{\text{NE}}}\right),p_n^{s'}\right).$$

To obtain the best response strategies of price-setting APOs, we first need to know $x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$, which is the MNO's optimal strategy in response to the price vector $\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$. We determine the MNO's optimal strategy and its properties through Lemmas 1 to 3 to analyze the existence and uniqueness of the NE in Section 5.

**Lemma 1.** *In the cost reduction problem, given $\boldsymbol{p}_{-n}^s$, the MNO's optimal strategy is*

$$x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right) = \begin{cases} S_n, & \text{if } p_n^s < p_n^{s,\min},\\ \frac{L_n(p_n^s,\boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s+1}, & \text{otherwise,}\end{cases} \tag{17}$$

*where $L_n(p_n^s,\boldsymbol{p}_{-n}^s)$ is a function of $p_n^s$ and $\boldsymbol{p}_{-n}^s$, and $p_n^{s,\min}\geq 0$ and $\kappa_n>0$ are user dependent constants.*

The proof of Lemma 1 is given in Appendix D, where we also obtain $L_n(p_n^s,\boldsymbol{p}_{-n}^s)$. We now derive the following properties of MNO's optimal strategy given in (17).

**Lemma 2.** *Given $\boldsymbol{p}_{-n}^s$, the MNO's optimal strategy $x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ obtained in (17) satisfies the following properties.*

1) *$x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ is decreasing in $p_n^s$ for $p_n^s\geq p_n^{s,\min}$;*
2) *$x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ is convex in $p_n^s$ for $p_n^s\geq p_n^{s,\min}$;*
3) *$L_n(p_n^s,\boldsymbol{p}_{-n}^s)$ is decreasing and convex in $p_n^s$ for $p_n^s\geq p_n^{s,\min}$.*

The proof of Lemma 2 is given in Appendix E.

**Lemma 3.** *In the network expansion problem, the MNO's optimal strategy $x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ satisfies the following properties.*

1) *$\lim_{p_n^s\to 0} x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)=\infty$ for any $\boldsymbol{p}_{-n}^s$.*
2) *$x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ is strictly decreasing in $p_n^s$ for any $\boldsymbol{p}_{-n}^s$.*

The proof of Lemma 3 can easily be obtained similar to the proof of Lemma 2 given in Appendix E, hence is omitted due to page limit.

*Observations*: Fig. 4(a) illustrates the optimal solution $x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s)$ in the cost reduction problem, which confirms the properties in Lemma 2. According to Fig. 4(a), price-setting APO $n$ will never set its price $p_n^s < p_n^{s,\min}$, since the MNO offloads the same amount of traffic even when setting $p_n^s = p_n^{s,\min}$. In other words, the payoff function $V_n^s\left(x_n^{s^*}(p_n^s,\boldsymbol{p}_{-n}^s),p_n^s\right)$ is less than

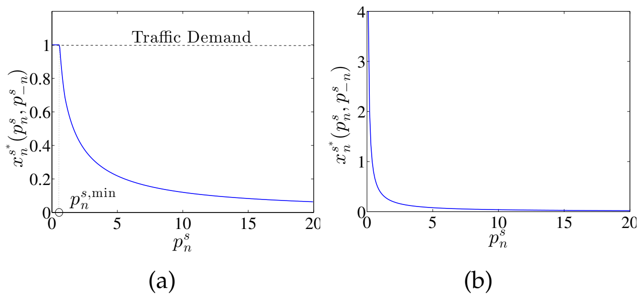


Fig. 4: MNO's optimal strategy, which determines the amount of data offloaded to APO $n$ (a) $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ in the cost reduction problem. APO $n$ does not submit any price less than $p_n^{s,\min}$, since the MNO does not offload more traffic than its demand. (b) $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ in the network expansion problem. The amount of offloaded traffic approaches infinity when $p_n^s \to \infty$.

$V_n^s\left(x_n^{s^*}(p_n^{s,\min}, \boldsymbol{p}_{-n}^s), p_n^{s,\min}\right)$ for any $p_n^s < p_n^{s,\min}$, since $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = x_n^{s^*}(p_n^{s,\min}, \boldsymbol{p}_{-n}^s)$. Therefore, we can limit the strategy space of APO $n$ to $p_n^s \in [p_n^{s,\min}, \infty)$, where the MNO's optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is decreasing and convex. Fig. 4(b) shows the optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ in the network expansion problem. According to Fig. 4, the MNO's optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ in Stage II can be greater than the capacity of the APO (i.e., $\phi_n B_n$) for small values of the price $p_n^s$. This would not be a problem however, as price-setting APO $n$ will choose its price $p_n^s$ in Stage I such that the amount of traffic that MNO chooses to offload in Stage II will be no larger than the APO's capacity.

## 5 Equilibrium Analysis

In this section, we study the existence and uniqueness of the equilibrium in PS-NCG. We then develop iterative algorithms for the price-setting APOs and MNO in the cost reduction problem and prove that they converge to the unique NE.

### 5.1 Existence of Nash Equilibria

We study PS-NCG formulated in Stage I for price-setting APOs, to show whether there exists an NE strategy. To derive the NE of the game, we can compute the APO's best response strategies in this game by solving problem (16). However, PS-NCG is generally a non-convex game due to the non-convexity of the payoff function. Furthermore, constraint (16b) may also be non-convex in the network expansion problem. To tackle this issue, we use the strategy space mapping and transform the non-convex strategy space into a compact convex one via bijection.

To transform constraint (16b) into a convex constraint, we introduce a new variable $\tilde{x}_n^s$ for each price-setting APO $n \in \mathcal{N}^s$, and replace $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ by $\tilde{x}_n^s$. We further replace $p_n^s$ by $x_n^{s^*(-1)}(\tilde{\boldsymbol{x}}^s)$ in problem (16), where $\tilde{\boldsymbol{x}}^s = (\tilde{x}^s)_{n\in\mathcal{N}^s}$. The inverse function $x_n^{s^*(-1)}(\tilde{\boldsymbol{x}}^s)$ always exists due to Lemmas 2 and 3. Therefore, the APO $n$'s strategy space is mapped one-to-one from $p_n^s \in [0, +\infty)$ into $\tilde{x}_n^s \in [0, +\infty)$. The strategy space is further limited to $\tilde{x}_n^s \in [0, \phi_n B_n]$, to take the limited capacity of APO into account. Thus, problem (16) can be converted into the following equivalent problem, from which the best response strategy of APO $n$ will be obtained.

$$\text{AP}^s\text{: } \tilde{x}_n^{s^*} = \underset{\tilde{x}_n^s \geq 0}{\text{argmax}}\ V_n^s(\tilde{x}_n^s, x_n^{s^*(-1)}(\tilde{\boldsymbol{x}}_n^s)) \tag{18a}$$

$$\text{subject to } \tilde{x}_n^s \leq \phi_n B_n. \tag{18b}$$

Since there is a one-to-one mapping between $\tilde{x}_n^s$ and price $p_n^s$, the best pricing strategy of player $n$ can be obtained from $p_n^{s^*} = x_n^{s^*(-1)}(\tilde{\boldsymbol{x}}^{s^*})$. Although the constraint of problem (18) is affine, the objective function may not be concave. We prove the existence of an NE using the Brouwer's fixed point theorem [46, Ch. 3].

**Theorem 3.** *There exists an NE in Game PS-NCG.*

*Proof.* To prove this theorem, we first show that the fixed point solution of the best response strategies for all price-setting APOs exists. To determine the fixed point solution, we first represent the APO $n$'s best response strategy obtained from problem (18) as $\tilde{x}_n^s = f_n(\tilde{\boldsymbol{x}}_{-n}^s)$, where $\tilde{\boldsymbol{x}}_{-n}^s$ is the amount of traffic offloaded to the price-setting APOs excluding $n$. We further define the function $\tilde{\boldsymbol{x}}^s = \mathbf{F}(\tilde{\boldsymbol{x}}^s)$, where $\mathbf{F} = (f_n)_{n\in\mathcal{N}^s} : \{[0, \phi_n B_n]\}_{n\in\mathcal{N}^s} \to \{0, \phi_n B_n]\}_{n\in\mathcal{N}^s}$.

We now apply the Brouwer's fixed point theorem [46, Ch. 3], which states that for any continuous function $\mathbf{F}$ that maps a closed convex set into itself, there is a point $\boldsymbol{x}_0$ such that $\mathbf{F}(\boldsymbol{x}_0) = \boldsymbol{x}_0$. It is clear that the set $\{[0, \phi_n B_n]\}_{n\in\mathcal{N}^s}$ is closed and convex and $\mathbf{F}$ is continuous. Therefore, there exists a fixed point solution for function $\mathbf{F} = (f_n)_{n\in\mathcal{N}^s} : \{[0, \phi_n B_n]\}_{n\in\mathcal{N}^s} \to \{0, \phi_n B_n]\}_{n\in\mathcal{N}^s}$, which corresponds to the best response strategies of price-setting APOs as it is obtained from problem (18). According to Definition 3, we prove the existence of an NE. ■

### 5.2 Uniqueness of the Nash Equilibrium

In this subsection, we focus on the cost reduction problem and show that a unique NE exists for this problem. Notice that in practice, the MNO determines the offloading strategy based on the cost reduction problem, while the network expansion problem is only formulated to predict how much traffic the MNO can ultimately deliver. To obtain the NE, we determine the best response strategies of price-setting APOs through the following problem, when we substitute $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ given by (17) into problem (16).

$$p_n^{s^*} = \underset{p_n^s \geq 0}{\text{argmax}}\ J_n\left(\frac{L_n(p_n^s, \boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s + 1}\right) + p_n^s\left(\frac{L_n(p_n^s, \boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s + 1}\right)^2 \tag{19a}$$

$$\text{subject to } \frac{L_n(p_n^s, \boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s + 1} \leq \phi_n B_n. \tag{19b}$$

Due to Lemma 2, constraint (19b) is a convex constraint. However, the objective function may not be concave as $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = \frac{L_n(p_n^s, \boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s + 1}$ is convex in $p_n^s$. Nevertheless, through the following lemma, we prove that problem (19) is a strictly concave maximization problem and admits a unique optimal solution.

**Lemma 4.** *There exists $p_n^{s,\max} \geq p_n^{s,\min}$, such that problem (19) is a strictly concave maximization problem over the closed interval $[p_n^{s,\min}, p_n^{s,\max}]$, and the objective function is decreasing for $p_n^s >$*

$p_n^{s,\max}$. *Thus, problem (19) has a unique optimal solution over the same interval.*

The proof of Lemma 4 is given in Appendix F. Utilizing Lemma 4, we can replace the feasible region of problem (19) by $[p_n^{s,\min}, p_n^{s,\max}]$, which is introduced in Appendix F. Through the following theorem, we now prove the uniqueness of the NE in PS-NCG.

**Theorem 4.** *There exists a unique NE in Game PS-NCG.*

*Proof.* The proof is based on the following lemma [47].

**Lemma 5** ([47])**.** *A unique NE exists in Game PS-NCG if for all* $n \in \mathcal{N}^s$

- *The strategy space is a nonempty, convex, and compact subset of some Euclidean space.*
- *Player $n$'s payoff $V_n^s$ is continuous and strictly concave in its own strategy $p_n^s$.*

According to Lemma 4, PS-NCG satisfies the above properties, and we can conclude the uniqueness of the NE. ■

### 5.3 Algorithm Design

In this subsection, we develop iterative algorithms for price-setting APOs and the MNO to obtain the NE of PS-NCG under the cost reduction problem. Utilizing the proposed algorithms, price-setting APOs do not need to obtain full information about the MNO (e.g., inner operation of the MNO, traffic demand, and cost). The proposed algorithm for the MNO also facilitates information exchange among the price-setting APOs in a distributed manner. We prove that the proposed algorithms converge to the unique NE determined in Section 4.

We first develop an iterative algorithm for the price-setting APOs to update their best response strategies. We consider a price-setting APO $n \in \mathcal{N}$, whose best response strategy can be obtained by solving problem (19). Let $p_n^{s^{(t)}}$ and $x_n^{s^{*(t)}}$ denote the APO $n$'s and MNO's strategies at the $t$-th iteration. We update the APO $n$'s strategy using the following rule, which is obtained based on the gradient method:

$$p_n^{s^{(t+1)}} = \mathcal{P}_n\Bigg(p_n^{s^{(t)}} + \varphi^{(t)} \frac{\partial x_n^{s^*}}{\partial p_n^s}\Big|_{p_n^{s^{(t)}}} \times \left(\frac{\mathrm{d}J_n}{\mathrm{d}x}\Big|_{x_n^{s^{*(t)}}} + 2p_n^{s^{(t)}} x_n^{s^{*(t)}}\right) + \left(x_n^{s^{*(t)}}\right)^2\Bigg),$$

where $\varphi^{(t)}$ is the step size at iteration $t$ and $\mathcal{P}_n$ denotes the projection onto the feasible region of problem (19). We denote this update rule as

$$p_n^{s^{(t+1)}} = \mathcal{F}_n\left(p_n^{s^{(t)}}\right). \tag{20}$$

The proposed gradient-based algorithm is illustrated in Algorithm 1. Price-setting APO $n$ first randomly initializes $p_n^{s^{(0)}}$ (Step 1). In each iteration, the APO submits its price to the MNO via the communication link established between the APO and MNO (Step 3). Meanwhile, the MNO computes $x_n^{s^{*(t)}}$ and $\frac{\partial x_n^{s^*}}{\partial p_n^s}\Big|_{p_n^{s^{(t)}}}$ and announces them to the APO. Upon reception of the MNO's responses (Step 4), the APO updates the price based on rule (20) (Step 5). This procedure continues until convergence. The APO checks the termination criterion (Step 7) and stops the algorithm when the relative changes of the prices during consecutive iterations are sufficiently small (as determined by the positive constant $\epsilon$). Utilizing this iterative algorithm, price-setting APOs only need to know the MNO's responses to their strategies. The proposed iterative algorithm has a relatively small communication overhead, since a few messages need to be exchanged in the market.

**Algorithm 1:** $BR_n(\boldsymbol{p}_{-n}^s)$: Iterative Best Response Adaptation for a Price-setting APO $n$.

1 **initialization**: $t \leftarrow 0$, $p_n^{s^{(0)}}$, and $\epsilon$
2 **do**
3 The APO submits the price strategy $p_n^{s^{(t)}}$ to the MNO.
4 The APO collects the MNO's responses $x_n^{s^{*(t)}}(p_n^{s^{(t)}}, \boldsymbol{p}_{-n}^s)$ and $\frac{\partial x_n^{s^*}}{\partial p_n^s}$.
5 The APO updates its price strategy. $p_n^{s^{(t+1)}} \leftarrow \mathcal{F}_n\left(p_n^{s^{(t)}}\right)$ as in (20)
6 $t \leftarrow t+1$
7 **while** $|p_n^{s^{(t)}} - p_n^{s^{(t-1)}}| > \epsilon$
8 **output**: $p_n^{s^{(t)}}$

**Algorithm 2:** Distributed Iterative Algorithm for Information Exchange among the MNO and APOs.

1 **initialization**: $k \leftarrow 0$, conv_flag $\leftarrow 0$, and $\epsilon'$
2 **while** conv_flag $= 0$
3 The MNO collects $p_n^{s^{(k)}}$ from all APOs $n \in \mathcal{N}^s$.
4 Each APO $n \in \mathcal{N}^s$ updates its best response strategy via Algorithm 1. $p_n^{s^{(k+1)}} \leftarrow BR_n\left(\boldsymbol{p}_{-n}^{s^{(k)}}\right), \forall\, n \in \mathcal{N}^s$
5 The MNO checks the termination criterion.
 **if** $|p_n^{s^{(k+1)}} - p_n^{s^{(k)}}| < \epsilon', \forall\, n \in \mathcal{N}^s$ **then**
  conv_flag $\leftarrow 1$
 **end**
6 $k \leftarrow k+1$
7 **end**
8 **output**: $\boldsymbol{p}^{s^{(k)}}$

Utilizing Algorithm 2, the MNO facilitates information exchange among the price-setting APOs. The MNO response to each price-setting APO reflects the strategies of other APOs. As a result, the price-setting APOs do not need to know (or measure) the strategies of other APOs. In each iteration, the MNO shares the information needed by the price-setting APOs and receives their updated best response strategies. This procedure continues until convergence.

We now prove the convergence of our algorithms. As stated in Lemma 4, problem (19) is a striclty concave maximization problem. Therefore, the gradient-based algorithm shown in Algorithm 1 converges to the optimal solution of this problem [48]. To study the convergence of Algorithm 2, we follow the rationale of the proof in [49], which has been widely used in the literature (e.g., [50]).

**Theorem 5.** *The proposed iterative algorithm shown in Algorithm 2 converges to the unique NE of PS-NCG.*

*Proof.* Algorithm 2 converges when the following conditions are satisfied [49]: First, a fixed point solution (that is NE) must exist. Second, the payoff function $V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right)$ is concave in $\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$. Notice that according to [49, Def. 1], concavity is sufficient for the convergence, while it is not a necessary condition. Similar to Lemma 4, we now show that $V_n^s$ is concave in the feasible region of the payoff maximization problem around the NE.

**Lemma 6.** *For each price-setting player $n \in \mathcal{N}^s$, the payoff function $V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right)$ is concave in $\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$.*

The proof of Lemma 6 is given in Appendix G. As stated in Theorem 4, a unique NE exists for PS-NCG. Moreover, Lemma 6 states that $V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right)$ is concave. These together complete the proof of Theorem 5. ∎

## 6 Performance Evaluation

In this section, we evaluate the MNO's profit when offloading its subscribers' traffic to the third-party APOs, and APOs' payoffs by admitting the MNO traffic. Through extensive numerical studies, we compare the SPE with the outcome of data offloading game (DOFF) proposed in [8], which only considers motivating price-taking APOs to admit the MNO's traffic.

To validate the SPE's efficiency, we compare our proposed framework with the socially optimal solution. The efficiency of SPE is defined as the ratio between the social welfare obtained by the SPE and the maximum social welfare. This value is always less than or equal to 1, and illustrates how the market outcome degrades due to players' selfish behaviors in problems considered in this paper.

The social welfare is defined as the summation of all players' payoffs as follows.

$$\Psi(\boldsymbol{x},\boldsymbol{y}) = r\left(z(\boldsymbol{x},\boldsymbol{y})\right) - c(\boldsymbol{y}) + \sum_{n\in\mathcal{N}^t} J_n(x_n^t) + \sum_{n\in\mathcal{N}^s} J_n(x_n^s).$$

The maximum social welfare, denoted by $\Psi^*$, can be obtained from the following problem.

$$\Psi^* = \underset{(\boldsymbol{x},\boldsymbol{y})\in\mathcal{R}}{\text{maximize}}\ \Psi(\boldsymbol{x},\boldsymbol{y}),$$

where $\mathcal{R}$ is the feasible region of $(\boldsymbol{x},\boldsymbol{y})$. We denote the equilibrium strategies derived in our model as $\boldsymbol{x}^{\text{NE}}$ and $\boldsymbol{y}^{\text{NE}}$.

**Definition 4 (SPE's Efficiency).** *The SPE's efficiency is the ratio between the social welfare obtained by the equilibrium and the maximum social welfare, and equals*

$$\frac{\Psi(\boldsymbol{x}^{NE},\boldsymbol{y}^{NE})}{\Psi^*}. \tag{21}$$

The price-of-anarchy (PoA) is another concept in game theory that measures how the efficiency of a system degrades due to selfish behavior of its agents. For the cost reduction problem that admits a unique SPE, the PoA is the inverse of SPE's efficiency.

### 6.1 Simulation Setup

We consider an MNO, represented by a macrocell BS. Unless stated otherwise, there are $N^t = 10$ price-taking WiFi APOs and $N^s = 2$ price-setting WiFi APOs. We now describe the measurement-based models (similar as in [9], [24]) used to evaluate the performance of our proposed framework. We consider a model of WiFi connection probability based on the measurements in [2], and a model of traffic demand of mobile subscribers based on the information from [1], [51], [52]. There are a total of $400$ mobile subscribers. The APOs are located in those areas with high mobile subscribers density, and hence high traffic demand. According to [2], the average WiFi contact probability is $0.7$. Similar to [8], [21], we consider the normalized transmission efficiency profiles of MNO and APOs, which follow a uniform distribution in the intervals $[0.3, 1]$ and $[0.4, 1]$, respectively. We further assume that the bandwidth of macrocell BS is 20 MHz, while the normalized bandwidth of each WiFi AP is randomly chosen from $\{1, 2, 5.5, 11\}$ MHz similar to [21]. The achievable data rate is the product of bandwidth and transmission efficiency.

To generate realistic traffic demand, we refer to the measurements of mobile data reported in [51], [52] and the projection of future mobile data predicted by Cisco [1]. The average smartphone will generate $4.4$ GB of traffic per month by 2020 [1]. According to [51], [52], the user traffic volume follows an upper-truncated power-law distribution. We choose the parameters of such a distribution to match the per-month average data traffic of $4.4$ GB[9].

We assume that the revenue and cost functions for the MNO are $r(z) = \bar{r}z$ and $c_b(y) = \bar{c}y^2$, respectively, where $\bar{r} = 1$ \$/Mb and the constant $\bar{c}$ is the MNO cost factor. For each APO $n \in \mathcal{N}$, we assume $r_n(x) = a_n \log(1+x)$ and $c_n(x) = \bar{c}_n x^2$, where $a_n$ and $\bar{c}_n$ are chosen randomly and uniformly from the intervals $[1, 1.5]$ and $[0.2, 0.5]$, respectively. We study the market for an offloading period that corresponds to two seconds.

### 6.2 Cost Reduction Problem

We first evaluate the MNO's profit and the total traffic delivered to mobile subscribers. Fig. 5 shows the profit for different values of the MNO cost factor (i.e., $\bar{c}$), while Fig. 6 illustrates the corresponding amount of traffic delivered by the macrocell BS and offloaded to the APOs. From these figures, we can observe that data offloading significantly increases the MNO's profit. The profit is increased up to three times when $\bar{c} = 0.04$, whereas $25\%$ of traffic is offloaded to the APOs. When the cost of data delivery in the MNO increases, the MNO leases more resources from APOs to offload more traffic. Moreover, the proposed three-stage game outperforms the DOFF game by up to $18\%$ when $\bar{c} = 0.04$. Such improvement is achieved by the participation of price-setting APOs, which provide more resources and drive the prices for price-taking APOs down.

We then evaluate the efficiency of SPE. Fig. 7 shows the ratio between the social welfare obtained at an SPE and the maximum social welfare. As the number of APOs increases, the efficiency ratio increases due to a higher competition in the market. Furthermore, the SPE's efficiency degrades when the cost of data delivery becomes higher. This is because more traffic will be delivered to the APOs, which

9. The daily traffic of each mobile subscriber follows the power-law distribution $f_S(s) = (1-\sigma)s^{-\sigma}/S^{\max^{1-\sigma}}$ for $0 \le s \le S^{\max}$, where $\sigma = 0.57$ as observed in [52] and $S^{\max}$ is determined such that the per-month average data traffic is 4.4 GB assuming mobile users are active at daytime (i.e., 8:00 a.m. – 8:00 p.m.)

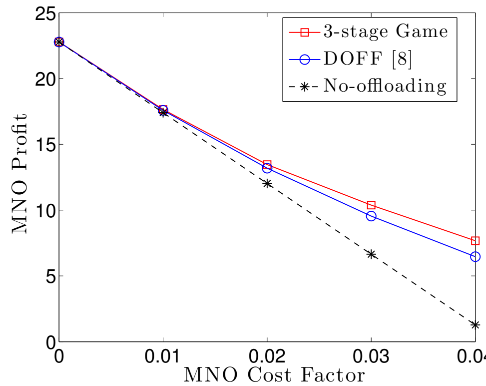


Fig. 5: The MNO's profit for different values of MNO cost factor. The proposed framework significantly improves the MNO's profit in comparison with DOFF [8] and no-offloading case.

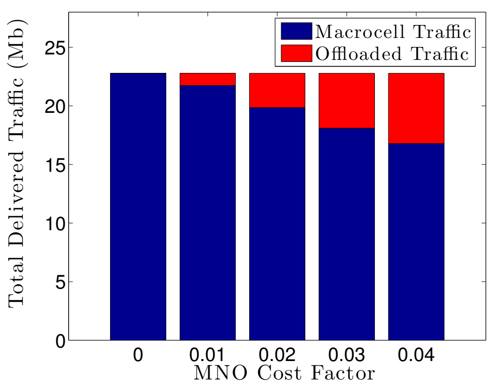


Fig. 6: The corresponding total delivered traffic for different values of MNO cost factor. More traffic will be offloaded to the APOs as the cost of data delivery in the MNO increases.

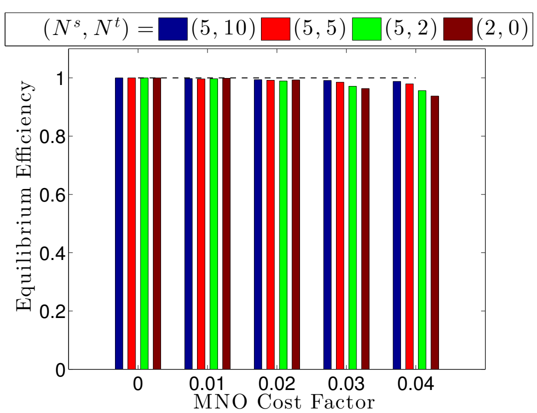


Fig. 7: Efficiency of equilibrium. Three-stage game efficiently generates a close-to-optimal social welfare.

can manipulate the market and make the SPE less efficient. When the cost of data delivery is zero, the social welfare at the SPE is the same as the maximum social welfare. In this case, we have $(\boldsymbol{x}^{\rm NE}, \boldsymbol{y}^{\rm NE}) = (\boldsymbol{0}, \boldsymbol{S})$, which results in the efficiency of 1.

We now investigate the payoffs achieved by the APOs when delivering the traffic. To evaluate the payoffs of different APOs in a fair manner, we choose the same set of parameters for all APOs. Fig. 8 illustrates the average

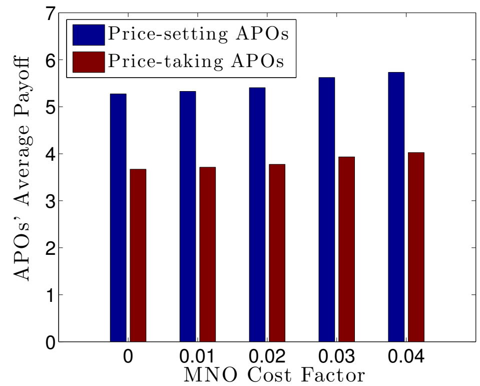


Fig. 8: Average payoffs per APO for price-setting and price-taking APOs versus the MNO cost factor.

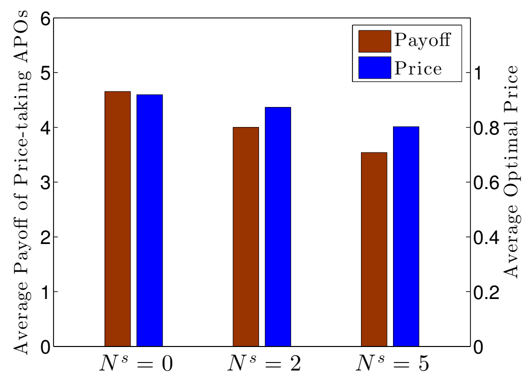


Fig. 9: Average payoff that a price-taking APO can obtain for different values of $N^s$ and the average of optimal price $\boldsymbol{p}^t$ set by the MNO for price-taking APOs. ($N^t = 10, \bar{c} = 0.04$)

payoffs per APO obtained by price-setting and price-taking APOs. The larger cost of data delivery of the MNO, the higher payoff that the APOs can obtain. Furthermore, the price-setting APOs can achieve a higher average payoff than the price-taking APOs due to more market power.

We further evaluate how the price competition among price-setting APOs affects the market prices set by the MNO for price-taking APOs (i.e., $\boldsymbol{p}^t$). Fig. 9 shows the average payoffs of price-taking APOs and the corresponding prices set by the MNO, when different number of price-setting APOs participate in the market. From this figure, we can observe that the participation of price-setting APOs will drive the market prices down as they have more market power. Consequently, the price-taking APOs obtain less payoffs.

We finally investigate the convergence rate of Algorithm 2. In each iteration, the price-setting APOs update and submit their best response strategies to the MNO. Fig. 10 illustrates the price dynamics chosen by the price-setting APOs, and shows that Algorithm 2 converges fairly quickly. As Theorem 5 states that this algorithm converges to the unique NE of Game PS-NCG, we conclude that Algorithm 2 converges quickly to the SPE.

### 6.3 Network Expansion Problem

The network expansion problem studies how much traffic the system can deliver, when the MNO is able to expand the

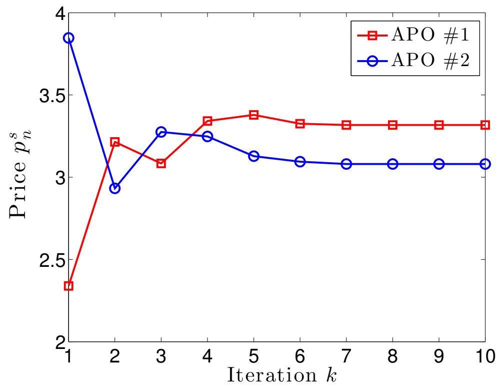


Fig. 10: The prices chosen by price-setting APOs in different iterations as obtained by Algorithm 2. ($\epsilon = \epsilon' = 0.01, \varphi^{(t)} = 1/t^2$)

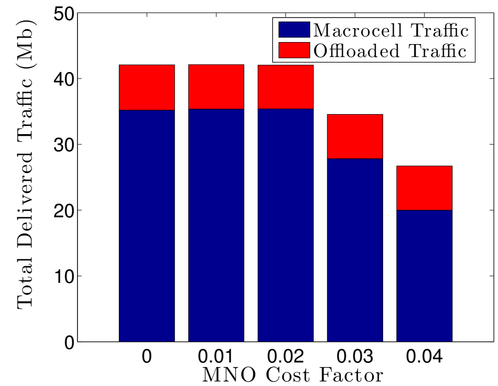


Fig. 12: The amount of total traffic that can be ultimately delivered to the mobile subscribers.

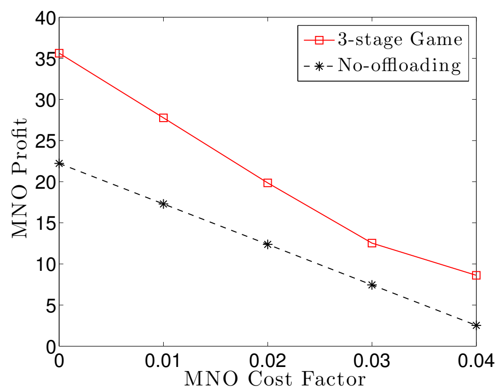


Fig. 11: The MNO's profit for different values of $\bar{c}$. The proposed framework significantly improves the profit in comparison with no-offloading case.

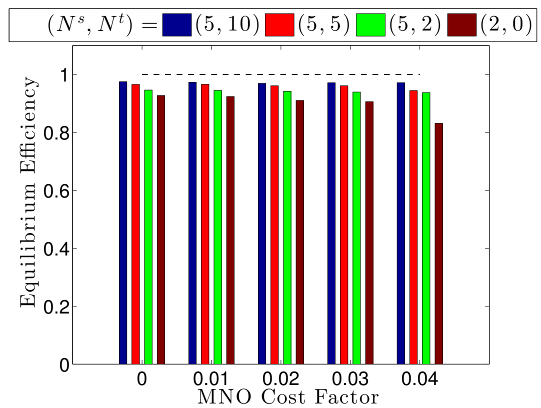


Fig. 13: Efficiency of equilibrium versus $\bar{c}$. The equilibrium efficiency degrades when the cost of data delivery becomes higher.

network by offloading more mobile traffic to the APOs. Fig. 11 shows the MNO's profit obtained by data offloading, and Fig. 12 shows the total traffic delivered by the macrocell BS and offloaded to the APOs for different values of $\bar{c}$. Fig. 12 shows that $30\%$ more traffic can be delivered to the mobile subscribers by leasing the APOs' resources when $\bar{c} = 0.04$, under the network expansion equilibrium result. Fig. 11 shows the MNO can obtain up to $4$ times more profit in this case. By comparing Fig. 12 and Fig. 6, we see that the consideration of network expansion allows us to deliver $20\%$ more traffic when $\bar{c} = 0.04$, comparing with the case of not considering network expansion.

Fig. 13 shows the efficiency of the equilibrium of the network expansion problem. Since the amount of traffic offloaded to the APOs in the network expansion problem is higher than in the cost reduction problem, the equilibrium is less efficient in the network expansion problem. Unlike Fig. 7, the efficiency ratio is less than 1 even when $\bar{c} = 0$. This is because the MNO is still interested in leasing the APO's resources to expand its network, even if its cost of data delivery is zero.

## 7 Conclusion

In this paper, we studied the economics of mobile data offloading in cellular networks, where an MNO interacts with both price-setting and price-taking APOs to offload the mobile traffic. We proposed an incentive framework for the data offloading market, and modeled the interactions in the market as a three-stage Stackelberg game. We analyzed two games with different MNO objective functions via the cost reduction and network expansion problems. In the cost reduction problem, given fixed total admitted traffic demand, the MNO aims to offload its traffic to reduce the cost of data delivery. Through the network expansion problem, the MNO is able to increase the aggregate network capacity and simultaneously reduce the cost of data delivery. Numerical results showed our proposed incentive framework outperforms an existing market model by $18\%$ in terms of the MNO's profit in the cost reduction problem. Moreover, the MNO can obtain $4$ times more profit through network expansion. Results further showed that price competition among price-setting APOs will drive the market prices down, comparing to the case where all APOs are price-taking.

Although we only considered a single representative BS in this model, the results can be directly extended to the case where an MNO has multiple BSs. For future work, we will consider a market when multiple MNOs compete to offload their traffic to different types of APOs. Moreover, we will consider interference induced by overlapping APs, when they are densely deployed. We will further take the latency and reliability into account to enable data offloading in future fifth generation (5G) wireless systems.

## ACKNOWLEDGMENTS

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## APPENDIX A
## PROOF OF THEOREM 1

The optimal solution of problem (7) can be obtained by Karush-Kuhn-Tucker (KKT) analysis, which satisfies the following conditions.

$$J_n'(x_n^{t^*}) + p_n^t - \lambda_n^* \frac{1}{\phi_n} = 0, \tag{22a}$$

$$\lambda_n^* \left(x_n^{t^*} - \phi_n B_n\right) = 0, \tag{22b}$$

$$x_n^{t^*} \leq \phi_n B_n, \tag{22c}$$

$$\lambda_n^* \geq 0, \tag{22d}$$

where $J_n'(x_n^{t^*}) \triangleq \frac{dJ_n}{dx}|_{x_n^{t^*}}$ and $\lambda_n^* \geq 0$ is the optimal Lagrange multiplier. According to the KKT conditions, we consider two different cases:

*Case 1.* $\lambda_n^* = 0$: In this case, the optimal solution satisfies

$$J_n'(x_n^{t^*}) + p_n^t = 0. \tag{23}$$

We denote the value of $x_n^{t^*}(p_n^t)$ obtained from (23) as $L_n(p_n^t)$, where we know $L_n^{(-1)}(x) = -J_n'(x)$. Although it is not in closed form, we still can extract the required properties. If $p_n^t < p_n^{t,\min} \triangleq -J_n'(0)$, (23) does not have any solution and we obtain $x_n^{t^*} = 0$.

*Case 2.* $\lambda_n^* > 0$: In this case, according to the complementary slackness condition given by (22b), the optimal solution is $x_n^{t^*} = \phi_n B_n$. It means that the total capacity of APO $n$ is allocated to the MNO's traffic. This condition is satisfied when $p_n^t > p_n^{t,\max} \triangleq -J_n'(\phi_n B_n)$.

This completes the proof of Theorem 1. ∎

## APPENDIX B
## PROOF OF COROLLARY 1

The optimal strategy $x_n^{t^*}$ is increasing for $p_n^t \in [p_n^{t,\min}, p_n^{t,\max}]$ since $-J_n'$, and hence $L_n(p_n^t)$, are increasing. To show that $x_n^{t^*}$ is concave, we take the derivative twice from both sides of (23). We have

$$\frac{\mathrm{d}^2 x_n^{t^*}}{\mathrm{d}p_n^{t2}} \frac{\mathrm{d}J_n'}{\mathrm{d}x}(x_n^{t^*}) + \left(\frac{\mathrm{d}x_n^{t^*}}{\mathrm{d}p_n^t}\right)^2 \frac{\mathrm{d}^2 J_n'}{\mathrm{d}x^2}(x_n^{t^*}) = 0. \tag{24}$$

According to the assumptions mentioned in Section 3, we know that $J_n'(\cdot) < 0$, $\frac{\mathrm{d}J_n'}{\mathrm{d}x} < 0$, and $\frac{\mathrm{d}^2 J_n'}{\mathrm{d}x^2} \leq 0$. Therefore, we can conclude that $\frac{\mathrm{d}^2 x_n^{t^*}}{\mathrm{d}p_n^{t2}} \leq 0$, which completes the proof. ∎

## APPENDIX C
## PROOF OF THEOREM 2

The strategy space of the MNO is a compact and convex set. In addition, all constraints of problem (12) including (11) are affine constraints. Therefore, to prove that problem (12) is a strictly concave maximization problem, we need to show that its objective function, given as follows, is concave.

$$V^{\mathrm{crp}}\left((\tilde{\boldsymbol{x}}^t, \boldsymbol{x}^s), \boldsymbol{y}, \boldsymbol{p}^s\right) = r(S) - c(\boldsymbol{y}) - \sum_{n\in\mathcal{N}^s} p_n^s \left(x_n^s\right)^2 - \sum_{n\in\mathcal{N}^t} \tilde{x}_n^t x_n^{t^*(-1)}(\tilde{x}_n^t). \tag{25}$$

Let $h(\tilde{\boldsymbol{x}}^t) \triangleq -\sum_{n\in\mathcal{N}^t} \tilde{x}_n^t x_n^{t^*(-1)}(\tilde{x}_n^t)$. To investigate the concavity of (25), we first represent the variable set of problem (12) as $\boldsymbol{\eta} = (\boldsymbol{y}, \boldsymbol{x}_n^s, \tilde{\boldsymbol{x}}^t)$ and form the Hessian as follows.

$$\mathbf{H} = \left[\frac{\partial^2 V^{\mathrm{crp}}}{\partial \eta_i \partial \eta_j}\right]_{\eta_i,\eta_j\in\boldsymbol{\eta}} = \begin{pmatrix} \mathbf{H}^{\boldsymbol{y}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{\boldsymbol{x}^s} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}^{\tilde{\boldsymbol{x}}^t} \end{pmatrix}, \tag{26}$$

where

$$\begin{aligned} \mathbf{H}^{\boldsymbol{y}} &= -\left[\frac{\partial^2 c(\boldsymbol{y})}{\partial y_m \partial y_k}\right]_{m,k\in\mathcal{N}\bigcup\{0\}} \\ &= -c''(\boldsymbol{y}) \begin{pmatrix} \frac{1}{\theta_0} \\ \vdots \\ \frac{1}{\theta_{|\mathcal{N}|}} \end{pmatrix} \left(\frac{1}{\theta_0}, \dots, \frac{1}{\theta_{b,|\mathcal{N}|}}\right) = -c''(\boldsymbol{y})\boldsymbol{\theta}^T\boldsymbol{\theta}, \\ \mathbf{H}^{\boldsymbol{x}^s} &= -2\,\mathbf{diag}\left(p_1^s, \dots, p_{N^s}^s\right), \\ \mathbf{H}^{\tilde{\boldsymbol{x}}^t} &= \mathbf{diag}\left(\frac{\partial^2 h}{\partial \tilde{x}_1^{t2}}, \dots, \frac{\partial^2 h}{\partial \tilde{x}_{N^t}^{t2}}\right). \end{aligned}$$

It is clear that $\mathbf{H}$ is negative semidefinite, if $\mathbf{H}^{\boldsymbol{y}}$, $\mathbf{H}^{\boldsymbol{x}^s}$, and $\mathbf{H}^{\tilde{\boldsymbol{x}}^t}$ are all negative semidefinite. We first focus on $\mathbf{H}^{\boldsymbol{y}}$. Given any non-zero vector $\boldsymbol{w} = (w_0, \dots, w_{|\mathcal{N}|})$, we calculate

$$\boldsymbol{w}\mathbf{H}^{\boldsymbol{y}}\boldsymbol{w}^T = -c''(\boldsymbol{y}) \left(\boldsymbol{\theta}_b \boldsymbol{w}^T\right)^T \left(\boldsymbol{\theta}_b \boldsymbol{w}^T\right). \tag{27}$$

Since $c''(\boldsymbol{y}) = c_b''\left(\sum_{n=0}^{|\mathcal{N}|} \frac{y_n}{\theta_n}\right) > 0$ and $\left(\boldsymbol{\theta}_b \boldsymbol{w}^T\right)^T \left(\boldsymbol{\theta}_b \boldsymbol{w}^T\right) = \left(\sum_{n=0}^{|\mathcal{N}|} \theta_n w_n\right)^2 > 0$, we have $\boldsymbol{w}\mathbf{H}^{\boldsymbol{y}}\boldsymbol{w}^T < 0$. Thus, matrix $\mathbf{H}^{\boldsymbol{y}}$ is negative semidefinite. Furthermore, we can easily show that $\mathbf{H}^{\boldsymbol{x}^s}$ is negative semidefinite. As mentioned in Section 4 and illustrated in Fig. 4, we always know $p_n^s > 0$ for all $n \in \mathcal{N}^s$. To prove that diagonal matrix $\mathbf{H}^{\tilde{\boldsymbol{x}}^t}$ is negative semidefinite, it is sufficient to show that for any $n$, $\frac{\partial^2 h(\tilde{\boldsymbol{x}})}{\partial \tilde{x}_n^{t2}} < 0$. We first prove the following lemma.

**Lemma 7.** *For any invertible concave function $f$, $f^{(-1)}(x)$ is convex if $f$ is an increasing function, where $f^{(-1)}$ is the inverse of function $f$.*

*Proof.* To prove Lemma 7, we derive the second derivative of $f^{(-1)}(x)$. The first derivative of $f^{(-1)}(x)$ is $\frac{1}{f'\left(f^{(-1)}(x)\right)}$, hence the second derivative is $-\frac{f''\left(f^{(-1)}(x)\right)}{\left(f'\left(f^{(-1)}(x)\right)\right)^3}$, which is positive when $f$ is an increasing and concave function. This completes the proof of Lemma 7. ∎

We now proceed the proof of Theorem 2. Each diagonal element of $\mathbf{H}^{\tilde{\boldsymbol{x}}^t}$ is

$$\frac{\partial^2 h(\tilde{\boldsymbol{x}}^t)}{\partial \tilde{x}_n^{t2}} = -2\frac{\mathrm{d}x_n^{t^*(-1)}}{\mathrm{d}\tilde{x}_n^t} - \tilde{x}_n^t\frac{\mathrm{d}^2 x_n^{t^*(-1)}}{\mathrm{d}\tilde{x}_n^{t2}}. \quad (28)$$

As stated in Corollary 1, $x_n^{t^*}(p_n^t)$ is an increasing concave function. According to Lemma 7, $x_n^{t^*(-1)}(\tilde{x}_n^t)$ is an increasing convex function. Thus, we can conclude that $\frac{\partial^2 h(\tilde{\boldsymbol{x}}^t)}{\partial \tilde{x}_n^{t2}} < 0$, which results in the negative semidefiniteness of $\mathbf{H}^{\tilde{\boldsymbol{x}}^t}$. Therefore, the objective function (25) is concave, which completes the proof. ■

## Appendix D
## Proof of Lemma 1

We first prove that there exists a $p_n^{s,\min} > 0$ such that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = S_n$ when $p_n^s \in [0, p_n^{s,\min})$. We then obtain $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ for $p_n^s \geq p_n^{s,\min}$. According to constraint (10c), we know that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) \leq S_n$. To prove that $p_n^{s,\min}$ exists, by contradiction, we assume that $x_n^{s^*} < S_n$ for $p_n^s = 0$. Therefore, we have $y_n^* > 0$ to satisfy $x_n^{s^*} + y_n^* = S_n$. We know that $(x_n^s, y_n) = (S_n, 0)$ is also a feasible solution of problem (12). By substituting $(x_n^s, y_n) = (S_n, 0)$ into the objective function (i.e., MNO's profit) of problem (12), we obtain a higher MNO's profit than $(x_n^{s^*}, y_n^*)$. Note that the payment $p_n^s x_n^{s^2}$ is zero when $p_n^s = 0$. Therefore, $x_n^{s^*} < S_n$ cannot be the optimal solution, and we have $x_n^{s^*} = S_n$ when $p_n^s = 0$.

To obtain the MNO's optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$, we form the Lagrangian of problem (12).

$$\begin{aligned}
&\mathcal{L}^{\mathrm{crp}}\left((\tilde{\boldsymbol{x}}^t, \boldsymbol{x}^s), \boldsymbol{y}, \boldsymbol{p}^s, \boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \\
&= r(S) - c(\boldsymbol{y}) - \sum_{n\in\mathcal{N}^s} p_n^s (x_n^s)^2 - \sum_{n\in\mathcal{N}^t} \tilde{x}_n^t x_n^{t^*(-1)}(\tilde{x}_n^t) \\
&\quad - \sum_{n\in\mathcal{N}^s} \lambda_n (x_n^s + y_n - S_n) - \sum_{n\in\mathcal{N}^t} \lambda_n (\tilde{x}_n^t + y_n - S_n) \\
&\quad + \sum_{n\in\mathcal{N}} \alpha_n y_n + \sum_{n\in\mathcal{N}^s} \beta_n x_n^s + \sum_{n\in\mathcal{N}^t} \beta_n \tilde{x}_n^t,
\end{aligned}$$

where $\boldsymbol{\lambda} = (\lambda_n)_{n\in\mathcal{N}}$, $\boldsymbol{\alpha} = (\alpha_n)_{n\in\mathcal{N}} \succeq \mathbf{0}$, and $\boldsymbol{\beta} = (\beta_n)_{n\in\mathcal{N}} \succeq \mathbf{0}$ are the Lagrange multipliers. Let $h_n(\tilde{x}_n^t) \triangleq x_n^{t^*(-1)}(\tilde{x}_n^t)$. The optimal solution satisfies the following KKT conditions.

$$\begin{aligned}
&(\mathcal{A}.1)\ \frac{\partial \mathcal{L}^{\mathrm{crp}}}{\partial y_n} = -\frac{1}{\theta_n} c'(\boldsymbol{y}^*) - \lambda_n^* + \alpha_n^* = 0, && \forall\, n \in \mathcal{N}, \\
&(\mathcal{A}.2)\ \frac{\partial \mathcal{L}^{\mathrm{crp}}}{\partial x_n^s} = -2p_n^s x_n^{s^*} - \lambda_n^* + \beta_n^* = 0, && \forall\, n \in \mathcal{N}^s, \\
&(\mathcal{A}.3)\ \frac{\partial \mathcal{L}^{\mathrm{crp}}}{\partial \tilde{x}_n^t} = -h_n'(\tilde{x}_n^{t^*}) - \lambda_n^* + \beta_n^* = 0, && \forall\, n \in \mathcal{N}^t, \\
&(\mathcal{A}.4)\ \alpha_n^* y_n^* = 0,\ \alpha_n^* \geq 0, && \forall\, n \in \mathcal{N}, \\
&(\mathcal{A}.5)\ \beta_n^* x_n^{s^*} = 0,\ \beta_n^* \geq 0, && \forall\, n \in \mathcal{N}^s, \\
&(\mathcal{A}.6)\ \beta_n^* \tilde{x}_n^{t^*} = 0,\ \beta_n^* \geq 0, && \forall\, n \in \mathcal{N}^t, \\
&(\mathcal{A}.7)\ \tilde{x}_n^{t^*} + y_n^* = S_n, && \forall\, n \in \mathcal{N}^t, \\
&(\mathcal{A}.8)\ x_n^{s^*} + y_n^* = S_n, && \forall\, n \in \mathcal{N}^s,
\end{aligned}$$

where $h_n' \triangleq \frac{dh_n}{d\tilde{x}_n^t}$. We focus on the MNO's optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ for $n \in \mathcal{N}^s$. We know $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = S_n$ when $p_n^s \in [0, p_n^{s,\min})$. In this case, we have $\beta_n^* = 0$ according to $(\mathcal{A}.5)$. When we further increase $p_n^s$, $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ decreases, whereas according to $(\mathcal{A}.2)$, $\lambda_n^* \neq 0$. Since $x_n^{s^*} + y_n^* = S_n$, we have $y_n^* > 0$, which forces $\alpha_n^*$ to be zero. In this case, by combining $(\mathcal{A}.1)$ and $(\mathcal{A}.2)$, we have

$$x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = \frac{c'(\boldsymbol{y}^*)}{2\theta_n p_n^s} = \frac{c_b'\left(\sum_{m\in\mathcal{N}\bigcup\{0\}} \frac{y_m^*}{\theta_m}\right)}{2\theta_n p_n^s}. \quad (29)$$

To determine (29), we categorize the APOs excluding $n$ into the following sets: $\mathcal{Q}_n^1 = \{m \in \mathcal{N} \mid y_m^* = 0\}$, $\mathcal{Q}_n^2 = \{m \in \mathcal{N}\bigcup\{0\} \setminus \{n\} \mid y_m^* = S_m\}$, $\mathcal{Q}_n^3 = \{m \in \mathcal{N} \setminus \{n\} \mid 0 < y_m^* < S_m\}$. We now rewrite $\sum_{m\in\mathcal{N}\bigcup\{0\}} \frac{y_m^*}{\theta_m}$ as follows:

$$\begin{aligned}
\sum_{m\in\mathcal{Q}_n^2} \frac{S_m}{\theta_m} + \frac{S_n - x_n^{s^*}}{\theta_n} + \sum_{m\in\mathcal{N}^s\bigcap\mathcal{Q}_n^3} \frac{S_m - x_m^{s^*}}{\theta_m} \\
+ \sum_{m\in\mathcal{N}^t\bigcap\mathcal{Q}_n^3} \frac{S_m - \tilde{x}_m^{t^*}}{\theta_m}. \quad (30)
\end{aligned}$$

Notice that $x_m^{s^*} + y_m^* = S_m$ for all $m \in \mathcal{N}^s$, $\tilde{x}_m^{t^*} + y_m^* = S_m$ for all $m \in \mathcal{N}^t$, and $y_0^* = S_0$. Moreover, we know $x_m^{s^*} > 0$ for all $m \in \mathcal{N}^s\bigcap\mathcal{Q}_n^3$ and $\tilde{x}_m^{t^*} > 0$ for all $m \in \mathcal{N}^t\bigcap\mathcal{Q}_n^3$. According to $(\mathcal{A}.4)$–$(\mathcal{A}.6)$, we have $\alpha_m^* = 0$ and $\beta_m^* = 0$ for all $m \in \mathcal{Q}_n^3$. By combining $(\mathcal{A}.1)$–$(\mathcal{A}.3)$, we have

$$\theta_n p_n^s x_n^{s^*} = \theta_m p_m^s x_m^{s^*}, \quad \forall\, m \in \mathcal{N}^s\bigcap\mathcal{Q}_n^3, m \neq n, \quad (31)$$

$$\theta_n p_n^s x_n^{s^*} = h_m'(\tilde{x}_m^{t^*}), \quad \forall\, m \in \mathcal{N}^t\bigcap\mathcal{Q}_n^3. \quad (32)$$

We denote the inverse of function $h_m'(\cdot)$, which exists due to Corollary 1, as $h_m'^{(-1)}(\cdot)$. By substituting the above equations into (30), we obtain

$$\begin{aligned}
\sum_{m\in\mathcal{N}\bigcup\{0\}} \frac{y_m^*}{\theta_m} = A_n - \frac{x_n^{s^*}}{\theta_n} - \theta_n p_n^s x_n^{s^*} \sum_{m\in\mathcal{N}^s\bigcap\mathcal{Q}_n^3} \frac{1}{p_m^s \theta_m^2} \\
- \sum_{m\in\mathcal{N}^t\bigcap\mathcal{Q}_n^3} \frac{h_m'^{(-1)}(\theta_n p_n^s x_n^{s^*})}{\theta_m}, \quad (33)
\end{aligned}$$

where $A_n \triangleq \sum_{m\in\mathcal{Q}_n^2} S_m/\theta_m + S_n/\theta_n + \sum_{m\in\mathcal{N}\bigcap\mathcal{Q}_n^3} S_m/\theta_m$ is constant. By substituting (33) into (29) and some mathematical manipulations, we have

$$\begin{aligned}
&\frac{x_n^{s^*}}{\theta_n} + \theta_n p_n^s x_n^{s^*} \sum_{m\in\mathcal{N}^s\bigcap\mathcal{Q}_n^3} \frac{1}{p_m^s \theta_m^2} \\
&= A_n - c_b'^{(-1)}(2\theta_n p_n^s x_n^{s^*}) - \sum_{m\in\mathcal{N}^t\bigcap\mathcal{Q}_n^3} \frac{h_m'^{(-1)}(\theta_n p_n^s x_n^{s^*})}{\theta_m},
\end{aligned}$$

which can be rewritten as:

$$(\kappa_n p_n^s + 1) x_n^{s^*} = g_n(\theta_n p_n^s x_n^{s^*}), \quad (34)$$

where $\kappa_n \triangleq \sum_{m\in\mathcal{N}^s\bigcap\mathcal{Q}_n^3} \frac{\theta_n^2}{p_m^s \theta_m^2}$ and

$$\begin{aligned}
g_n(\theta_n p_n^s x_n^{s^*}) \triangleq \theta_n A_n - \theta_n c_b'^{(-1)}(2\theta_n p_n^s x_n^{s^*}) \\
- \theta_n \sum_{m\in\mathcal{N}^t\bigcap\mathcal{Q}_n^3} \frac{h_m'^{(-1)}(\theta_n p_n^s x_n^{s^*})}{\theta_m}. \quad (35)
\end{aligned}$$

Thus, by solving equation (34), the MNO's optimal strategy $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ can be obtained as follows:

$$x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = \begin{cases} S_n, & \text{if } p_n^s < p_n^{s,\min}, \\ \frac{L_n(p_n^s, \boldsymbol{p}_{-n}^s)}{\kappa_n p_n^s + 1}, & \text{otherwise,} \end{cases}$$

which completes the proof. ■

## APPENDIX E
## PROOF OF LEMMA 2

To prove that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is decreasing in $p_n^s$, by contradiction, we assume that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is non-decreasing. Thus, $p_n^s x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is increasing in $p_n^s$, while $\sum_{m\in\mathcal{N}\bigcup\{0\}} \frac{y_m^*}{\theta_m}$ given in (30) is decreasing in $p_n^s$. Notice that according to Corollary 1, $h_m'^{(-1)}$ is an increasing function. Therefore, the right hand side of (29) is decreasing in $p_n^s$, which contradicts the assumption that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is non-decreasing.

To show that the second and third properties stated in Lemma 2 also hold, we take the derivative twice from both sides of (34).

$$\begin{aligned}2\kappa_n \frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s} &+ (\kappa_n p_n^s + 1)\frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2} \\ &= \theta_n^2 \left(x_n^{s^*} + p_n^s \frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\right)^2 \frac{\mathrm{d}^2 g_n}{\mathrm{d}{p_n^s}^2} \\ &\quad + \theta_n \left(2\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s} + p_n^s \frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2}\right)\frac{\mathrm{d}g_n}{\mathrm{d}p_n^s}. \qquad (36)\end{aligned}$$

Thus,

$$\begin{aligned}\left(2\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s} + p_n^s \frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2}\right)\left(\kappa_n - \theta_n \frac{\mathrm{d}g_n}{\mathrm{d}p_n^s}\right) + \frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2} \\ = \theta_n^2 \left(x_n^{s^*} + p_n^s \frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\right)^2 \frac{\mathrm{d}^2 g_n}{\mathrm{d}{p_n^s}^2}. \qquad (37)\end{aligned}$$

To proceed, we need the following lemma.

**Lemma 8.** *The function* $g_n(x) = \theta_n A_n - \theta_n c_b'^{(-1)}(2x) - \theta_n \sum_{m\in\mathcal{N}^t \bigcap \mathcal{Q}_n^3} \frac{h_m'^{(-1)}(x)}{\theta_m}$ *is decreasing and convex.*

*Proof.* Since $c_b'(\cdot)$ is an increasing convex function, similar to Lemma 7, we can conclude that the inverse function $c_b'^{(-1)}(\cdot)$ is increasing but concave. We now show that $h_m'^{(-1)}$ is also an increasing concave function. Recall that $h_m(x) = x_m^{t^*(-1)}(x) = -J_m'(x)$. According to the assumptions mentioned in Section 3, $h_m'(\cdot)$ is an increasing convex function, while $h_m'^{(-1)}$ is an increasing concave function due to Lemma 7. Thus, $g_n(\cdot)$ is decreasing and convex. ■

According to Lemma 8, $-\frac{\mathrm{d}g_n}{\mathrm{d}p_n^s}$ and $\frac{\mathrm{d}^2 g_n}{\mathrm{d}{p_n^s}^2}$ are both positive. To show that $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is convex in $p_n^s$, by contradiction, we assume that there is at least one point, where $\frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2} < 0$. In this case, the left hand side of (37) becomes negative since $x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is non-increasing, while we know $\frac{\mathrm{d}^2 g_n}{\mathrm{d}{p_n^s}^2} \geq 0$. Therefore, we can conclude that $x_n^{s^*}$ is convex in $p_n^s$. Similarly, if there exists a point at which the second derivative of $L_n(p_n^s, \boldsymbol{p}_{-n}^s) = (\kappa_n p_n^s + 1)x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$, as shown in (36), is negative, the second derivative of $p_n^s x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ will be negative as well, since $x_n^{s^*}$ is convex. However, this contradicts with (36). Thus, $L_n(p_n^s, \boldsymbol{p}_{-n}^s)$ is convex in $p_n^s$.

We finally show that $L_n(p_n^s, \boldsymbol{p}_{-n}^s) = (\kappa_n p_n^s + 1)x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ is decreasing. When $p_n^s$ increases, $p_n^s x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s)$ increases as well, while $g_n(\theta_n p_n^s x_n^{s^*})$ decreases according to Lemma 8. Thus, $(\kappa_n p_n^s + 1)x_n^{s^*}(p_n^s, \boldsymbol{p}_{-n}^s) = g_n(\theta_n p_n^s x_n^{s^*})$, as given by (34), is decreasing in $p_n^s$. ■

## APPENDIX F
## PROOF OF LEMMA 4

According to Lemma 2, the feasible region of problem (19) is compact and convex. With the help of properties stated in Lemma 2, we show that there exists $p_n^{s,\max} \geq p_n^{s,\min}$ such that for any $p_n^s \in [p_n^{s,\min}, p_n^{s,\max}]$, the objective function is concave and the optimal price $p_n^{s^*}$ is located within this region. For the sake of presentation clarity, we abuse the notations and use $L_n(p_n^s)$ since $\boldsymbol{p}_{-n}^s$ is constant in problem (19). To obtain the optimal solution of problem (19), we set the first derivative of the objective function (19a) to zero.

$$\frac{\mathrm{d}V_n^s}{\mathrm{d}p_n^s} = \frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\left(J_n'(x_n^{s^*}) + 2p_n^s x_n^{s^*}\right) + \left(x_n^{s^*}\right)^2 = 0. \qquad (38)$$

We now show that the objective function $V_n^s$ is concave at any stationary points satisfying the above equation. The second derivative of $V_n^s$ is

$$\begin{aligned}\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2} &= \frac{\mathrm{d}^2 x_n^{s^*}}{\mathrm{d}{p_n^s}^2}\left(J_n'(x_n^{s^*}) + 2p_n^s x_n^{s^*}\right) \\ &\quad + \frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\left(\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s} J_n''(x_n^{s^*}) + 2\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\right) + 2x_n^{s^*}\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}. \qquad (39)\end{aligned}$$

By substituting $x_n^{s^*} = \frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}$ into (39) and some mathematical manipulations, $\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2}$ can be rewritten as follows:

$$\begin{aligned}\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2} &= \frac{L_n''(p_n^s)}{\kappa_n p_n^s + 1}\left(J_n'\left(\frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}\right) + 2p_n^s \frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}\right) \\ &\quad - \frac{2}{\kappa_n p_n^s + 1}\left(\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\left(J_n'(x_n^{s^*}) + 2x_n^{s^*}\right) + \left(x_n^{s^*}\right)^2\right) \\ &\quad + \left(\frac{L_n'(p_n^s)}{\kappa_n p_n^s + 1} - \frac{L_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\right) \\ &\quad \times \left(J_n''\left(\frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}\right)\left(\frac{L_n'(p_n^s)}{\kappa_n p_n^s + 1} - \frac{L_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\right)\right. \\ &\qquad \left. + \frac{2L_n(p_n^s)}{\kappa_n p_n^s + 1} + \frac{2p_n^s L_n'(p_n^s)}{\kappa_n p_n^s + 1} - \frac{2pL_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\right) \\ &\quad + \frac{2L_n(p_n^s)L_n'(p_n^s)}{(\kappa_n p_n^s + 1)^2}. \qquad (40)\end{aligned}$$

Due to (38), the second term of $\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2}$ is zero. We further rewrite $\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2}$ as follows:

$$\begin{aligned}\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}{p_n^s}^2} &= \frac{L_n''(p_n^s)}{\kappa_n p_n^s + 1} J_n'\left(\frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}\right) \\ &\quad + \left(\frac{L_n'(p_n^s)}{\kappa_n p_n^s + 1} - \frac{L_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\right)^2 J_n''\left(\frac{L_n(p_n^s)}{\kappa_n p_n^s + 1}\right) \\ &\quad + \frac{2L_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\left(p_n^s L_n''(p_n^s) + L_n'(p_n^s)\right) \\ &\quad + \left(\frac{L_n'(p_n^s)}{\kappa_n p_n^s + 1} - \frac{L_n(p_n^s)}{(\kappa_n p_n^s + 1)^2}\right)\end{aligned}$$

$$\times\left(\frac{2L_n(p_n^s)}{\kappa_n p_n^s+1}+\frac{2p_n^s L_n'(p_n^s)}{\kappa_n p_n^s+1}-\frac{2pL_n(p_n^s)}{(\kappa_n p_n^s+1)^2}\right). \quad (41)$$

The first and second terms of $\frac{\mathrm{d}^2 V_n^s}{\mathrm{d}p_n^{s\,2}}$ are both negative since $J_n(\cdot)$ is a decreasing concave function while $L_n(\cdot)$ is convex. Similar to Lemma 2, we can prove that $L_n'$ is also convex in $p_n^s$, while the proof is omitted due to page limit. Therefore, the third term of (41) is also negative since $p_n^s L_n''(p_n^s)+L_n'(p_n^s)\leq L_n'(2p_n^s)$ due to convexity of $L_n'$. Finally, the fourth term of (41) is negative as well, since it can be rewritten as $\frac{\mathrm{d}x_n^{s^*}}{\mathrm{d}p_n^s}\frac{\mathrm{d}}{\mathrm{d}p_n^s}(p_n^s x_n^{s^*})<0$. Therefore, $V_n^s$ is concave at any stationary points implying that there exists at most one such point. We now consider two possible cases:

*Case 1.* There is one stationary point as denoted by $p_n^{s^*}$. Thus, we can conclude that there exists an interval where the objective function $V_n^s$ is locally concave and $p_n^{s^*}$ belongs to this interval. Furthermore, $V_n^s$ is decreasing for $p_n^s > p_n^{s,\max}$ due to the uniqueness of the stationary point.

*Case 2.* There is no stationary point, which implies that the objective function is either increasing or decreasing. If the objective function is strictly decreasing, the optimal solution of problem (19) is $p_n^{s,\min}$ and we have $p_n^{s,\max}=p_n^{s,\min}$. If the objective function is increasing, the second term of (40) is always negative. Thus, $V_n^s$ is concave in $[p_n^{s,\min},+\infty)$ and $p_n^{s,\max}=+\infty$.

In conclusion, we can limit the feasible region of problem (19) to the interval $[p_n^{s,\min},p_n^{s,\max}]$, which completes the proof. ■

## Appendix G Proof of Lemma 6

To show that the payoff function $V_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right),p_n^s\right)$ is concave in $\boldsymbol{p}^s=\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$ over the feasible region of APO $n$'s payoff maximization problem, we use the first order condition for the concavity. The function $f(\boldsymbol{p}^s)$ is concave if for any $\boldsymbol{p}^s$ and $\hat{\boldsymbol{p}}^s$, we have

$$f\left(\hat{\boldsymbol{p}}^s\right)<f\left(\boldsymbol{p}^s\right)+\left(\hat{\boldsymbol{p}}^s-\boldsymbol{p}^s\right)^T\nabla f\left(\boldsymbol{p}^s\right). \quad (42)$$

Therefore, for each price-setting APO $n\in\mathcal{N}^s$, we need to show that

$$\begin{aligned} &J_n\left(x_n^{s^*}\left(\hat{p}_n^s,\hat{\boldsymbol{p}}_{-n}^s\right)\right)+\hat{p}_n^s\left(x_n^{s^*}\left(\hat{p}_n^s,\hat{p}_{-n}^s\right)\right)^2\\ &<\ J_n\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right)+p_n^s\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right)^2\\ &\quad+\sum_{m\in\mathcal{N}^s}(\hat{p}_m^s-p_m^s)\frac{\partial V_n^s}{\partial p_m^s}. \end{aligned}$$

Due to Lemma 4, $V_n^s$ is concave in $p_n^s$. Therefore, to prove that $V_n^s$ is concave in $\boldsymbol{p}_n^s$, it is sufficient to show that $\sum_{m\in\mathcal{N}^s\setminus\{n\}}(\hat{p}_m^s-p_m^s)\frac{\partial V_n^s}{\partial p_m^s}\geq 0$. Without loss of generality, we assume $\hat{\boldsymbol{p}}^s\succeq\boldsymbol{p}^s$. If $\frac{\partial V_n^s}{\partial p_m^s}\geq 0$ for all $m\in\mathcal{N}^s\setminus\{n\}$, the above condition holds. We now derive $\frac{\partial V_n^s}{\partial p_m^s}$ as follow:

$$\frac{\partial V_n^s}{\partial p_m^s}=\frac{\partial x_n^{s^*}}{\partial p_m^s}\left(J_n'\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right)+2p_n^s x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right).$$

As stated in Lemma 2, $x_m^{s^*}$ is non-increasing in $p_m^s$, while $p_m^s x_m^{s^*}$ is increasing. Thus, based on (31) given in Appendix D, $x_n^{s^*}$ is non-decreasing in $p_m^s$ and we have $\frac{\partial x_n^{s^*}}{\partial p_m^s}\geq 0$. Furthermore, according to (38), $J_n'\left(x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)\right)+2p_n^s x_n^{s^*}\left(p_n^s,\boldsymbol{p}_{-n}^s\right)$ is positive around the NE, which completes the proof. ■

## References


[1] Cisco, "Cisco visual networking index: Global mobile data traffic forecast update 2015-2020," Feb. 2016.

[2] K. Lee, J. Lee, Y. Yi, I. Rhee, and S. Chong, "Mobile data offloading: How much can WiFi deliver?" *IEEE/ACM Trans. on Networking*, vol. 21, no. 2, pp. 536–550, Aug. 2013.

[3] M. Bennis, M. Simsek, A. Czylwik, W. Saad, S. Valentin, and M. Debbah, "When cellular meets WiFi in wireless small cell networks," *IEEE Commun. Magazine*, vol. 51, no. 6, pp. 44–50, Jun. 2013.

[4] E. Bulut and B. K. Szymanski, "WiFi access point deployment for efficient mobile data offloading," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 17, no. 1, pp. 71–78, Jul. 2013.

[5] J. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. Reed, "Femtocells: Past, present, and future," *IEEE J. on Selected Areas in Commun.*, vol. 30, no. 3, pp. 497–508, Apr. 2012.

[6] L. Duan, J. Huang, and B. Shou, "Economics of femtocell service provision," *IEEE Trans. on Mobile Computing*, vol. 12, no. 11, pp. 2261–2273, Nov. 2013.

[7] AT&T Press Release, "New AT&T Wi-Fi solution connects businesses and their customers," Mar. 2015.

[8] L. Gao, G. Iosifidis, J. Huang, and L. Tassiulas, "Economics of mobile data offloading," in *Proc. of IEEE INFOCOM*, Turin, Italy, Apr. 2013.

[9] J. Lee, Y. Yi, S. Chong, and Y. Jin, "Economics of WiFi offloading: Trading delay for cellular capacity," *IEEE Trans. on Wireless Commun.*, vol. 13, no. 3, pp. 1540–1554, Mar. 2014.

[10] H. Zhang, M. Bennis, L. A. DaSilva, and Z. Han, "Multi-leader multi-follower Stackelberg game among Wi-Fi, small cell and macrocell networks," in *Proc. of IEEE GLOBECOM*, Austin, TX, Dec. 2014.

[11] BT WiFi, http://www.btwifi.com/.

[12] M. H. Cheung and J. Huang, "DAWN: Delay-aware Wi-Fi offloading and network selection," *IEEE J. on Selected Areas in Commun.*, vol. 33, no. 6, pp. 1214–1223, Jun. 2015.

[13] X. Zhuo, W. Gao, G. Cao, and S. Hua, "An incentive framework for cellular traffic offloading," *IEEE Trans. on Mobile Computing*, vol. 13, no. 3, pp. 541–555, Mar. 2014.

[14] C. Joe-Wong, S. Sen, and S. Ha, "Offering supplementary network technologies: Adoption behavior and offloading benefits," *IEEE/ACM Trans. on Networking*, vol. 23, no. 2, pp. 355–368, Apr. 2015.

[15] W. Wang, X. Wu, L. Xie, and S. Lu, "Femto-Matching: Efficient traffic offloading in heterogeneous cellular networks," in *Proc. of IEEE INFOCOM*, Hong Kong, China, Apr. 2015.

[16] L. Rose, E. V. Belmega, W. Saad, and M. Debbah, "Pricing in heterogeneous wireless networks: Hierarchical games and dynamics," *IEEE Trans. on Wireless Commun.*, vol. 13, no. 9, pp. 4985–5001, Sep. 2014.

[17] X. Kang and Y. Wu, "Incentive mechanism design for heterogeneous peer-to-peer networks: A Stackelberg game approach," *IEEE Trans. on Mobile Computing*, vol. 14, no. 5, pp. 1018–1030, May 2015.

[18] M. Haddad, P. Wicek, H. B. A. Sidi, and E. Altman, "An automated dynamic offset for network selection in heterogeneous networks," *IEEE Trans. on Mobile Computing*, vol. 15, no. 9, pp. 2151–2164, Sep. 2016.

[19] Z. Chen, Y. Liu, B. Zhou, and M. Tao, "Caching incentive design in wireless D2D networks: A Stackelberg game approach," in *Proc. of IEEE ICC*, Kuala Lumpur, Malaysia, May 2016.

[20] J. Li, H. Chen, Y. Chen, Z. Lin, B. Vucetic, and L. Hanzo, "Pricing and resource allocation via game theory for a small-cell video caching system," *IEEE J. on Selected Areas in Commun.*, vol. 34, no. 8, pp. 2115–2129, Aug. 2016.

[21] L. Gao, G. Iosifidis, J. Huang, L. Tassiulas, and D. Li, "Bargaining-based mobile data offloading," *IEEE J. on Selected Areas in Commun.*, vol. 32, no. 6, pp. 1114–1125, Jun. 2014.

[22] K. Wang, F. C. M. Lau, L. Chen, and R. Schober, "Pricing mobile data offloading: A distributed market framework," *IEEE Trans. on Wireless Commun.*, vol. 15, no. 2, pp. 913–927, Feb. 2016.

[23] X. Kang and S. Sun, "Incentive mechanism design for mobile data offloading in heterogeneous networks," in *Proc. of IEEE ICC*, London, UK, Jun. 2015.

[24] W. Dong, S. Rallapalli, R. Jana, L. Qiu, K. Ramakrishnan, L. Razoumov, Y. Zhang, and T. W. Cho, "iDEAL: Incentivized dynamic cellular offloading via auctions," *IEEE/ACM Trans. on Networking*, vol. 22, no. 4, pp. 1271–1284, Aug. 2014.

[25] S. Paris, F. Martignon, I. Filippini, and L. Chen, "An efficient auction-based mechanism for mobile data offloading," *IEEE Trans. on Mobile Computing*, vol. 14, no. 8, pp. 1573–1586, Aug. 2015.

[26] G. Iosifidis, L. Gao, J. Huang, and L. Tassiulas, "A double-auction mechanism for mobile data-offloading markets," *IEEE/ACM Trans. on Networking*, vol. 23, no. 5, pp. 1634–1647, Oct. 2015.

[27] X. Chen, J. Wu, Y. Cai, H. Zhang, and T. Chen, "Energy-efficiency oriented traffic offloading in wireless networks: A brief survey and a learning approach for heterogeneous cellular networks," *IEEE J. on Selected Areas in Commun.*, vol. 33, no. 4, pp. 627–640, Apr. 2015.

[28] L. Duan, J. Huang, and B. Shou, "Economics of femtocell service provision," *IEEE Trans. on Mobile Computing*, vol. 12, no. 11, pp. 2261–2273, Nov. 2013.

[29] J. D. Hobby and H. Claussen, "Deployment options for femtocells and their impact on existing macrocellular networks," *Bell Labs Technical Journal*, vol. 13, no. 4, pp. 145–160, Feb. 2009.

[30] N. Shetty, S. Parekh, and J. Walrand, "Economics of femtocells," in *Proc. of IEEE GLOBECOM*, Honolulu, HI, Nov. 2009.

[31] U. Paul, A. Subramanian, M. Buddhikot, and S. Das, "Understanding traffic dynamics in cellular data networks," in *Proc. of IEEE INFOCOM*, Shanghai, China, Apr. 2011.

[32] U. Paul, L. Ortiz, S. Das, G. Fusco, and M. Buddhikot, "Learning probabilistic models of cellular network traffic with applications to resource management," in *Proc. of IEEE DYSPAN*, McLean, VA, Apr. 2014.

[33] Y. J. Hong, J. Kim, and D. K. Sung, "Two-dimensional channel estimation and prediction for scheduling in cellular networks," *IEEE Trans. on Vehicular Technology*, vol. 58, no. 6, pp. 2727–2740, Jul. 2009.

[34] W. Xi, X. Yun, S. Nagata, and L. Chen, "Novel CQI update methods in CoMP transmission," in *Proc. of IEEE PIMRC*, London, UK, Sep. 2013.

[35] R. Margolies, A. Sridharan, V. Aggarwal, R. Jana, N. K. Shankaranarayanan, V. A. Vaishampayan, and G. Zussman, "Exploiting mobility in proportional fair cellular scheduling: Measurements and algorithms," *IEEE/ACM Trans. on Networking*, vol. 24, no. 1, pp. 355–367, Feb. 2016.

[36] K. Son, H. Kim, Y. Yi, and B. Krishnamachari, "Toward energy-efficient operation of base stations in cellular wireless networks," in *Green Communications: Theoretical Fundamentals, Algorithms and Applications*, J. Wu, S. Rangan, and H. Zhang, Eds. CRC Press, Taylor - Francis, pp. 435–474, Sep. 2012.

[37] H. Shen and T. Basar, "Optimal nonlinear pricing for a monopolistic network service provider with complete and incomplete information," *IEEE J. on Selected Areas in Commun.*, vol. 25, no. 6, pp. 1216–1223, Aug. 2007.

[38] S. Shakkottai, R. Srikant, A. Ozdaglar, and D. Acemoglu, "The price of simplicity," *IEEE J. on Selected Areas in Commun.*, vol. 26, no. 7, pp. 1269–1276, Sep. 2008.

[39] P. Hande, M. Chiang, R. Calderbank, and J. Zhang, "Pricing under constraints in access networks: Revenue maximization and congestion management," in *Porc. of IEEE INFOCOM*, San Diego, CA, Mar. 2010.

[40] R. B. Wilson, *Nonlinear Pricing*. Oxford University Press, 1993.

[41] N. El Karoui and M. Quenez, "Non-linear pricing theory and backward stochastic differential equations," *Lecture Notes in Mathematics, Financial Mathematics*, pp. 191–246, Oct. 2006.

[42] C. Bonnet and P. Dubois, "Inference on vertical contracts between manufacturers and retailers allowing for nonlinear pricing and resale price maintenance," *The RAND Journal of Economics*, vol. 41, no. 1, pp. 139–164, Jan. 2010.

[43] C. Courcoubetis and R. Weber, *Pricing Communication Networks: Economics, Technology and Modelling*. Wiley Press, 2003.

[44] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory*. Oxford University Press, New York, NY, 1995.

[45] A. Ben-Tal and A. S. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. Society for Industrial and Applied Mathematics (SIAM), 2001.

[46] Y. Shoham and K. Leyton-Brown, *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2008.

[47] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave $N$-person games," *Econometrica*, vol. 33, no. 3, pp. 347–351, Jun. 1965.

[48] S. Boyd, "Convex Optimization I," *Lecture Notes*. [Online]. Available: http://web.stanford.edu/class/ee364a/index.html

[49] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. on Selected Areas in Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[50] D. Niyato and E. Hossain, "A noncooperative game-theoretic framework for radio resource management in 4G heterogeneous wireless access networks," *IEEE Trans. on Mobile Computing*, vol. 7, no. 3, pp. 332–345, Mar. 2008.

[51] U. Paul, A. P. Subramanian, M. M. Buddhikot, and S. R. Das, "Understanding traffic dynamics in cellular data networks," in *Proc. of IEEE INFOCOM*, Shanghai, China, Apr. 2011.

[52] M. Z. Shafiq, L. Ji, A. X. Liu, and J. Wang, "Characterizing and modeling Internet traffic dynamics of cellular devices," in *Proc. of ACM SIGMETRICS*, San Jose, CA, Jun. 2011.

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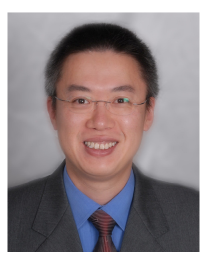

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