Advanced Demand Side Management for the Future Smart Grid Using Mechanism Design

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***Abstract*—In the future smart grid, both users and power companies can potentially benefit from the economical and environmental advantages of smart pricing methods to more effectively reflect the fluctuations of the wholesale price into the customer side. In addition, smart pricing can be used to seek social benefits and to implement *social objectives*. To achieve social objectives, the utility company may need to collect various information about users and their energy consumption behavior, which can be challenging. In this paper, we propose an efficient pricing method to tackle this problem. We assume that each user is equipped with an *energy consumption controller* (ECC) as part of its smart meter. All smart meters are connected to not only the power grid but also a communication infrastructure. This allows two-way communication among smart meters and the utility company. We analytically model each user's preferences and energy consumption patterns in form of a *utility function*. Based on this model, we propose a Vickrey-Clarke-Groves (VCG) mechanism which aims to maximize the *social welfare*, i.e., the aggregate utility functions of all users minus the total energy cost. Our design requires that each user provides some information about its energy demand. In return, the energy provider will determine each user's electricity bill payment. Finally, we verify some important properties of our proposed VCG mechanism for demand side management such as *efficiency*, *user truthfulness*, and *nonnegative transfer*. Simulation results confirm that the proposed pricing method can benefit both users and utility companies.**

***Keywords*: Demand side management, VCG mechanism design, energy consumption control, smart grid.**

I. Introduction

To achieve the high reliability required in power systems, utility companies need to design the grid for the *peak demand* rather than the average demand. This may result in an under-utilized system. With the increasing expectations of the customers both in quantity and quality [1], the limited energy resources, and the lengthy and expensive process of exploiting new resources, there is an essential need to improve utilization in power grids. In addition, the emergence of new types of loads such as plug-in hybrid electric vehicles (PHEVs), which can potentially double the average residential load, has further increased the need for development of new methods for *demand side management* (DSM).

There is a high concern regarding various environmental issues in the current power systems. The inefficient use of power in most buildings (e.g., due to poor thermal insulation) results in wasting a large amount of natural resources, since most of the electricity consumption occurs in buildings [2]. On the other hand, in some countries such as in the US, where oil and coal fired power plants are widely used to meet the peak demands, a large amount of SO_2, CO_2, and other greenhouse gases are emitted which could potentially be avoided with an efficient DSM program in place.

DSM has been practiced since the early 1980s [3]–[5]. It can be used as a tool for load shaping, where the electricity demand is re-distributed over a certain period of time (e.g., time-of-day, day-of-week). Among different techniques considered for DSM (e.g., voluntary load management programs [6]–[8] or direct load control [9]), *smart pricing* is one of the most effective tools that can encourage users to consume wisely and more efficiently. Given the recent increases in price of energy, users are now more willing to participate in DSM programs and try to shift the energy consumption schedule of their high-load household appliances to off-peak hours to reduce their energy expenses.

Wholesale prices (i.e., the prices set by the generators to regional electricity retailers through a bidding process) vary drastically between the low-demand times of a day and the high demand periods. In particular, the electricity prices are lower at low demand hours and higher at high demand hours. However, these changes in the wholesale prices are currently unnoticed by end users in most regions. That is, users are usually charged with some average prices, and thus, there is no incentive for them to change their power consumption to utilize the available generation capacity efficiently. Mapping the wholesale electricity prices to the retail users can mitigate this problem and encourage users to conserve energy at high demand hours or shift the operation of their high load appliances from peak hours to off-peak hours. Several pricing methods have already been proposed in the literature (e.g., flat pricing, peak load pricing, adaptive pricing, etc. [10]–[13]). Considering the enhancement of the current power transmission and distribution systems with communication facilities and information technologies, real-time and adaptive pricing attract more attention. Adaptive pricing and peak load pricing have been practiced for many years [10]–[13]. In peak load pricing, the operating cycle is divided into several periods and a distinct price is determined for each period. The prices

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are then announced ahead of time at the beginning of the operation cycle [12]. However, in adaptive pricing, the exact price for each period is selected in *real-time* and is announced only at the beginning of each time period, not at the beginning of the whole operation cycle.

In real-time pricing, random events and the reaction of the users to the previous prices will influence the price to be set in the upcoming operation periods [10]. However, given the *two-way* communication capabilities of smart grid, it has become possible to add new functionalities to the current power system, adopt more effective pricing methods, and provide users with improved customer services [14]–[17]. The level of success for different pricing methods depends on various factors such as the amount of information being provided to each user, the effectiveness of the mapping of the wholesale prices to the retail prices, and the knowledge and abilities of users to respond to price information. Another factor is the effectiveness of the home automation systems. For example, it is important whether the decisions about the schedule and the amount of power consumption are made automatically or manually. Some examples showing the limitations of manual control can be found in [14].

In this paper, we focus on developing a novel pricing method for DSM to encourage efficient energy consumption among users to achieve certain *social objectives*. However, it is difficult because of its computational complexity to achieve these social objectives, if all appliances of all users are to be jointly scheduled. To tackle this problem, a *top-down* control approach is devised. That is, first, the total power consumption of each user in each time slot is determined. This is referred to as user level control. Then, each user tries to schedule the operation of its own appliances to meet the desired power consumption level. This is referred to as appliance level control [18], [19]. In this study, we consider the problem of scheduling the total power consumption of each user at different time slots. We show that achieving social objectives is challenging even in user level control and requires collecting various information about the energy consumption behavior of the users, the price elasticity of the users, and the benefit that each user obtains by consuming a certain amount of energy. However, in general, users are not willing to reveal such information, unless there is an incentive for them to do so. Therefore, elaborate design rules (*mechanisms*) are needed such that it is in each user's self interest to reveal its local information. This problem has already been considered for smart grid [20] and in other contexts such as in telecommunication networks [21]–[23]. However, the prior works assumed that users are *price taker* who *accept* the prices as fixed parameters. That is, they do not consider the possibility that their actions may affect the price. For systems with built-in automated control units, this assumption may no longer be valid. Therefore, here, we consider the case where users can *anticipate* the impact of their actions on price values.

Vickrey-Clarke-Groves (VCG) mechanism is a pricing method to elicit local information from rational users. For determining the price charged to each user, users are asked to declare their energy demand information. The payments of the users are then structured such that the users have incentive to declare their local information truthfully. We note that the VCG pricing mechanism has already been applied to resource management problems, e.g., in computer and communication networks [21]. However, since we consider a different problem formulation in the context of smart grid, many of the existing results, e.g., in [21] and [24], are not directly applicable and need to be revised as will be explained throughout this paper. The contributions of this paper are summarized as follows:

- We propose a VCG mechanism for DSM programs to encourage efficient energy consumption among users. In our system model, each user reveals its demand information to the energy provider. By running a centralized mechanism, the energy provider computes the optimal energy consumption level for each user, and advertises a specific electricity payment for each user.
- We formulate an optimization problem to maximize the aggregate utility of all users while minimizing the total cost imposed on the energy provider.
- We investigate some of the desired properties of our problem formulation. First, *truthfulness* and *efficiency* of the proposed mechanism are proved. Then, for our model, we show the property of *nonnegative transfer* which means that the users always make *nonnegative payments*.
- We compare our efficient VCG DSM method with the case where users are price taker. We study the differences of these two systems, especially from the user payment perspective, and show that for the VCG mechanism users have to pay less.
- Simulation results confirm that both the users and the energy provider will benefit from the proposed scheme.

The rest of this paper is organized as follows. The system model and problem formulation are presented in Section II. The VCG mechanism and its different properties are discussed in Section III. In Section IV, we provide a performance evaluation of the proposed pricing scheme, and conclusions are drawn in Section V.

II. System Model and Problem Formulation

A. Power System

As in [15]–[17], we consider a smart power system with a single energy provider and several load subscribers or users as part of the general wholesale electricity market as shown in Fig. 1. For each user, we assume that there is an *energy consumption controller* (ECC) unit which is embedded in the user's smart meter. The role of the ECC is to control the user's power consumption, and to coordinate each user with the energy provider. All ECC units are connected to the energy provider through a communication infrastructure such as a local area network.

The intended time cycle for the system's operation is divided into K time slots, where $K \triangleq |\mathcal{K}|$, and $\mathcal{K}$ is the set of all time slots. This division can be based on the behavior of the users and their power demand pattern: on-peak time slots, off-peak time slots, and mid-peak time slots. Let $\mathcal{N}$ denote the set of all users and $N \triangleq |\mathcal{N}|$. In each time slot, we classify the load demand into two types, *must-run*

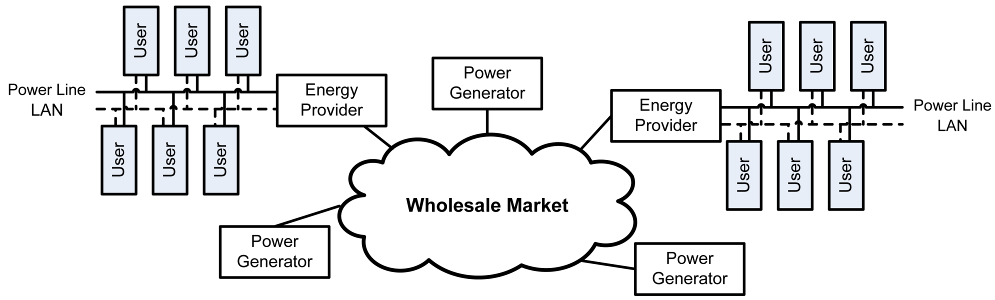

Fig. 1. An illustration of the regional energy providers, several users, and multiple power generators as parts of the general wholesale energy market.

loads and *controllable* loads [25]. Must-run loads are price-inelastic. For example, a refrigerator *always* needs to be *on* during the day. On the other hand, controllable loads can be stopped, adjusted, or shifted to other time slots and include the demand for services such as charging PHEVs. In each time slot k, we use M_n^k and m_n^k to denote the maximum and minimum power level for each user n, respectively. We use $\mathbf{M}_n \triangleq (M_n^1, \ldots, M_n^K)$ and $\mathbf{m}_n \triangleq (m_n^1, \ldots, m_n^K)$ to denote the vectors of the maximum and minimum power levels of user n in all time slots, respectively. We also define $\mathbf{M} \triangleq (\mathbf{M}_1, \ldots, \mathbf{M}_N)$ and $\mathbf{m} \triangleq (\mathbf{m}_1, \ldots, \mathbf{m}_N)$. We denote the minimum total energy requirements of user n as E_n and the vector of the minimum total energy requirements of all users as $\mathbf{E} \triangleq (E_1, \ldots, E_N)$, where for each user n, we have $E_n \geq \sum_{k\in\mathcal{K}} m_n^k$. For the users, it is difficult to determine their required demand information, i.e., the minimum and the maximum power requirement in each time slot, the minimum total energy requirement, and the benefit obtained by consuming a certain amount of energy. However, *machine learning* and *stochastic signal processing* techniques can be adopted in each user's ECC unit to help the user determine its required demand information. The normal pattern of the users' power consumption can be fed into appropriate machine learning algorithms to extract the demand information of the users. In order to provide the required energy for each user n within the operation cycle, it is required that

$$\sum_{k\in\mathcal{K}} x_n^k \geq E_n, \tag{1}$$

where x_n^k is the power consumption level of user n in time slot k. Furthermore, we define $\mathbf{x}_n \triangleq (x_n^1, \ldots, x_n^K)$ as the vector of energy consumption of user n. The *feasible* energy consumption controlling set of user n is defined as

$$\mathcal{X}_n \triangleq \left\{ \mathbf{x}_n \,\middle|\, \sum_{k\in\mathcal{K}} x_n^k \geq E_n, \quad m_n^k \leq x_n^k \leq M_n^k, \quad \forall\, k \in \mathcal{K} \right\}. \tag{2}$$

Our key assumption is that users have *price-elastic* load. That is, they may shift or change their energy consumption in response to price values [26]–[28].

B. User Preference and Utility Function

Each user is assumed to be an independent decision maker. The energy demand of each user may vary based on different parameters. For example, we can take into account the climate conditions and the price of electricity. The energy demand also depends on the type of the users. Residential users may have different *responses* to the same price than industrial users. Even users within the same category may not be identical. The different responses of different users to various price scenarios can be modeled by using *utility functions* from microeconomics [29]. In fact, we can model the behavior of different users through their different choices of utility functions [7]. For each user n, we represent the corresponding utility function as $U_n(\sum_{k\in\mathcal{K}} x_n^k) \triangleq U(\sum_{k\in\mathcal{K}} x_n^k, \omega_n)$, where x_n^k is the power consumption level of user n in time slot k and ω_n is a parameter, which may vary among users, representing the value of electricity for each user. For each user, the utility function represents the *level of satisfaction* obtained by the user as a function of its total power consumption throughout the operation period[1].

For all the users, we define $\boldsymbol{\omega} \triangleq (\omega_1, \ldots, \omega_N)$. We assume that the utility functions $U(x, \omega)$ satisfy the following properties:

Property 1: Utility functions are *non-decreasing*. This implies that the *marginal benefit* is nonnegative:

$$\frac{\partial U(x,\omega)}{\partial x} \geq 0. \tag{3}$$

Property 2: The marginal benefit of users is a non-increasing function. That is,

$$\frac{\partial^2 U(x,\omega)}{\partial x^2} \leq 0. \tag{4}$$

In other words, the utility functions are *concave*. While the class of utility functions that satisfy (3) and (4) is very large, it is convenient to have a linear marginal benefit [6], [7].

Property 3: For a fixed consumption level x, a larger ω gives a larger $U(x,\omega)$, which can be expressed as

$$\frac{\partial U(x,\omega)}{\partial \omega} > 0. \tag{5}$$

Property 4: When the consumption level is zero,

$$U(0,\omega) = 0, \quad \forall\, \omega > 0. \tag{6}$$

[1]The value of electricity for the users may also vary at different times of a day. Large loads that participate in wholesale electricity markets discriminate the value of electricity in different time slots through their different choices of utility functions at different times of a day [30], [31]. Considering the advances in home automation systems, it is conceivable that in the near future, instead of specifying only the value of their total power consumption, residential users will be able to determine the value of electricity for each time slot. In that case, the utility function of each user n can be replaced by $\sum_{k\in\mathcal{K}} \tilde{U}_n^k(x_n^k)$, where $\tilde{U}_n^k(\cdot)$ is the utility function of user n in time slot k.

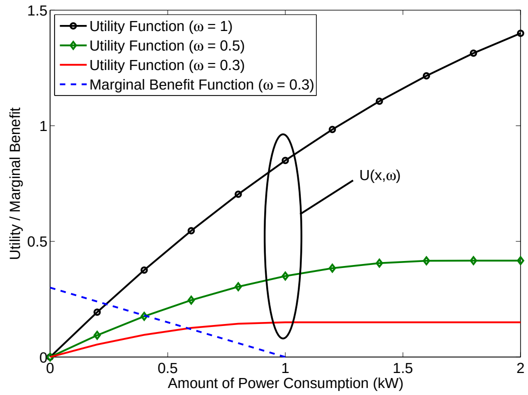

Fig. 2. Sample utility functions for power users ($\alpha = 0.3$).

We note that the operation of each individual appliance is meant to achieve a *goal* or to finish a *task*. For example, the air conditioning system is used to keep the temperature in a predetermined range. Thus, the total power consumption of each user can be considered as the aggregate power consumption required to complete different tasks. In this paper, since we define the utility functions for the *aggregate load* of different tasks, rather than for the power consumption of each individual appliance, the utility functions do not decrease. This is because users can complete more tasks if they consume more power. Furthermore, it is reasonable to assume that users prioritize their tasks. Therefore, as the prices increase, they tend to ignore some less important tasks or switch to a different mode of operation with lower power consumption. This implies a decreasing marginal benefit and a concave and increasing utility function for the total power consumption of different tasks. In addition, we assume that users are able to specify how much they value energy through the proper choice of parameter ω, i.e., a higher ω implies a higher utility value. Finally, as utility functions quantify the level of satisfaction of the users, intuitively zero power consumption should result in a zero utility value. Recent reports indicate that the behavior of power users can indeed be accurately modeled by certain utility functions [6]. In this paper, we consider *quadratic utility* functions corresponding to *linearly decreasing marginal benefit* [8]:

$$U(x,\omega) = \begin{cases} \omega x - \frac{\alpha}{2}x^2, & \text{if} \quad 0 \leq x < \frac{\omega}{\alpha}, \\ \frac{\omega^2}{2\alpha}, & \text{if} \quad x \geq \frac{\omega}{\alpha}, \end{cases} \tag{7}$$

where α is a pre-determined parameter. A few example utility functions from this class are shown in Fig. 2. The point where the utility function gets saturated and does not change corresponds to the maximum power requirement of the user.

C. Energy Cost Model

We consider a *cost function* $C_k(L_k)$ indicating the cost of providing L_k units of energy offered by the energy provider in each time slot k. We make the following assumptions:

Assumption 1: The cost functions are *increasing* with respect to the total offered energy capacity.

Assumption 2: The cost functions are *strictly convex*.

Assumption 3: There exists a differentiable, convex, non-decreasing function $p_k(q)$ over $q \geq 0$ for each $k \in \mathcal{K}$, with $p_k(0) \geq 0$ and $p_k(q) \to \infty$ as $q \to \infty$, such that for $q \geq 0$

$$C_k(q) = \int_0^q p_k(z)dz. \tag{8}$$

Note that *quadratic functions* are among several practical examples for cost functions that satisfy Assumptions 1-3, and are considered throughout this paper [15], [32]:

$$C_k(L_k) = a_k L_k^2 + b_k L_k + c_k, \tag{9}$$

where $a_k > 0$, $b_k \geq 0$, and $c_k \geq 0$ are fixed parameters.

D. Problem Formulation and Efficient Allocations

In this section, we consider the problem of power consumption level selection. From a social fairness point of view, it is desirable to utilize the available generated power provided by the energy provider in such a way that the sum of the utility functions of *all* users is maximized and the cost imposed on the energy provider is minimized. If centralized control is feasible and we can collect all information about the users' utility functions, an efficient energy consumption schedule can be characterized as the solution of the following problem:

$$\underset{\mathbf{x}_n \in \mathcal{X}_n,\, n \in \mathcal{N}}{\text{maximize}} \sum_{n \in \mathcal{N}} U_n\left(\sum_{k \in \mathcal{K}} x_n^k\right) - \sum_{k \in \mathcal{K}} C_k\left(\sum_{n \in \mathcal{N}} x_n^k\right), \tag{10}$$

where $\mathbf{x}_n$ is the vector of power consumptions of user n, $U_n(\cdot)$ is as in (7), and $C_k(\cdot)$ is defined in (9). The objective function in problem (10) is the sum of all utility functions *minus* the total energy cost in the system.

Problem (10) is a concave maximization problem and can be solved in a centralized fashion using *convex programming* techniques such as the interior point method (IPM) [33]. Since it is assumed that parameters ω_n, $\mathbf{m}_n$, $\mathbf{M}_n$, and E_n for each user n are local information, the energy provider may not have sufficient information to solve problem (10). Each user aims to optimize its local objective. To align these individual objectives with the social objective, some elaborately designed pricing scheme is needed. In general, users may have different approaches in responding to the price values set by the energy provider. This can lead to different equilibriums among users. We are interested in analyzing *competitive equilibrium* and *Nash equilibrium*. In competitive equilibrium, each user acts as a *price taker*. That is, it does not consider the effect of its actions on the price. However, in Nash equilibrium, we assume that users are price anticipator, i.e., they consider the effect of their actions on the price set by the energy provider.

1) Price Taking Users: If users are price taker, i.e., they do not consider the effect of their actions on the price, then we need to analyze the *competitive equilibrium* among the users and the energy provider. Given a price vector $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_K)$, where λ_k is the price in time slot k, a user who consumes x_n^k kW electricity in time slot k is charged

$\lambda_k x_n^k$ dollars for that time slot. Since users treat the prices as fixed values, the *payoff* function for each user n becomes

$$P_n(\mathbf{x}_n) = U_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) - \sum_{k\in\mathcal{K}} \lambda_k x_n^k, \tag{11}$$

where the first term represents the utility of user n as a function of its power consumption, and the second term represents its payment to the energy provider.

We call a pair $(\mathbf{x}, \boldsymbol{\lambda})$, where $\mathbf{x} \triangleq (\mathbf{x}_1, \ldots, \mathbf{x}_N)$, a *competitive equilibrium* if each user n maximizes its own payoff function defined in (11) for a given price vector $\boldsymbol{\lambda}$, i.e.,

$$P_n(\mathbf{x}_n) \geq P_n(\bar{\mathbf{x}}_n), \quad \bar{\mathbf{x}}_n \in \mathcal{X}_n,\ n \in \mathcal{N}, \tag{12}$$

where vector $\mathbf{x}$ is the solution to the problem defined in (10). It has been shown that under Properties 1-4 and Assumptions 1-3, a competitive equilibrium always exists [20], [34], and the results are summarized in the following proposition.

Proposition 1: There exists a competitive equilibrium $(\mathbf{x}, \boldsymbol{\lambda})$, where $\mathbf{x}$ is an optimal solution to problem (10).

The assumption that users are price taker is usually considered when the number of users is large, the amount of information provided to each user is limited, and the computer programs running the decentralized algorithm are embedded in the computer operating system and are not tampered with by the vast majority of the users. However, as some individual users such as large industrial users may have a significant impact on the power system, or some parts of the power system may act autonomously as in microgrids for household users in which the number of users is much lower than the number of users in the whole grid, the price taking assumption may not always be valid. When the price taking assumption is violated, the model changes into a game, and the assumptions required for the validity of Proposition 1 do not hold. We investigate this scenario in the next sub-section.

2) Price Anticipating Users: If users are price anticipator, i.e., they do consider the effect of their actions on the price, then we need to analyze the Nash equilibrium of the *game* which is played among multiple users who compete for the available power provided by the energy provider. In this game theoretic model [35], the strategies of the users represent their power consumption level. We consider the following pricing scheme for resource allocation. Given $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$, the energy provider sets a single price $\mu_k(\mathbf{x}) = p_k(\sum_{n\in\mathcal{N}} x_n^k)$ for time slot k. User n then pays $x_n^k p_k(\sum_{n\in\mathcal{N}} x_n^k)$ for that time slot. We use the notation $\mathbf{x}_{-n}$ to denote the vector of all consumption powers chosen by users other than user n, i.e., $\mathbf{x}_{-n} = (\mathbf{x}_1, \ldots, \mathbf{x}_{n-1}, \mathbf{x}_{n+1}, \ldots, \mathbf{x}_N)$. Then, given $\mathbf{x}_{-n}$, the payoff of each user n is obtained as

$$Q_n(\mathbf{x}_n; \mathbf{x}_{-n}) = U_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) - \sum_{k\in\mathcal{K}} x_n^k p_k \left(\sum_{m\in\mathcal{N}} x_m^k\right). \tag{13}$$

The payoff function Q_n is similar to P_n, defined for price-taking users in (11). The only difference is that while the payoff function P_n takes the price λ_k as a fixed parameter, price anticipating users realize that the price is set according to $p_k(\sum_{m\in\mathcal{N}} x_m^k)$, and adjust their payoffs accordingly.

From (13), the payoff of each user depends on its power consumption as well as the power consumptions of other users. Hence, we have the following game among the users:

- Players: Registered users in set $\mathcal{N}$.
- Strategies: Each user $n \in \mathcal{N}$ selects its energy consumption level $\mathbf{x}_n \in \mathcal{X}_n$ to maximize its payoff.
- Payoffs: $Q_n(\mathbf{x}_n; \mathbf{x}_{-n})$ for each user $n \in \mathcal{N}$ as in (13).

A *Nash equilibrium* of the game defined by $(Q_1, \ldots, Q_N)$ is a vector $\mathbf{x}$ such that for all $n \in \mathcal{N}$,

$$Q_n(\mathbf{x}_n; \mathbf{x}_{-n}) \geq Q_n(\bar{\mathbf{x}}_n; \mathbf{x}_{-n}), \quad \bar{\mathbf{x}}_n \in \mathcal{X}_n. \tag{14}$$

It can be shown that a Nash equilibrium exists for this game, and the results are summarized in the following proposition. However, the details of the proof can be found in [35].

Proposition 2: Suppose that Properties 1-4 and Assumptions 1-3 hold. There exists a Nash equilibrium $\mathbf{x}$ for the game defined by $(Q_1, \ldots, Q_N)$.

In general, the Nash equilibrium of a resource allocation game may not be optimal [35], [36]. That is, the energy consumption profile obtained at the Nash equilibrium in a distributed pricing scenario may not necessarily be the same as the optimal solution of the optimization problem in (10). Next, we investigate how the price values can be set carefully by the utility company such that the system performance becomes optimal at the aforementioned Nash equilibrium.

III. Applying the Vickrey-Clarke-Groves Mechanism

In the previous section, we considered a method (mechanism) which uses only a single price in each time slot for all users to allocate the provided power. Despite its simplicity, the introduced mechanism suffers from a loss in efficiency if users are indeed price anticipator, and evaluate the effect of their actions on the price function. As mentioned before, the main obstacle in solving problem (10) is the lack of information about the utility functions of the users and their feasible set of power consumptions. However, if we remove the restriction that the mechanism only chooses a single price, we can elicit the local information of the users. One possible approach to convince users to declare their utility functions and constraint parameters truthfully is the VCG mechanism [37].

A. VCG Mechanism

In the VCG class of mechanisms, each user is asked to specify its feasible set of power consumption and its utility function, which in case of the utility functions in (7) reduces to revealing a utility parameter ω_n. For each user n, we use $U_n(\sum_{k\in\mathcal{K}} x_n^k) \triangleq U(\sum_{k\in\mathcal{K}} x_n^k, \omega_n)$ and $\hat{U}_n(\sum_{k\in\mathcal{K}} x_n^k) \triangleq U(\sum_{k\in\mathcal{K}} x_n^k, \hat{\omega}_n)$ to denote the true and declared utility function and $\mathcal{X}_n$ and $\hat{\mathcal{X}}_n$ to denote the true and declared feasible set of power consumptions, respectively. We define

$$\mathbf{I}_n \triangleq \{\omega_n, \mathbf{M}_n, \mathbf{m}_n, E_n\} \tag{15}$$

and

$$\hat{\mathbf{I}}_n \triangleq \{\hat{\omega}_n, \hat{\mathbf{M}}_n, \hat{\mathbf{m}}_n, \hat{E}_n\} \tag{16}$$

to denote the true and declared demand parameters, respectively, where $\hat{\omega}_n$, $\hat{\mathbf{M}}_n$, $\hat{\mathbf{m}}_n$, and $\hat{E}_n$ are the declared values for ω_n, $\mathbf{M}_n$, $\mathbf{m}_n$, and E_n, respectively. For notational simplicity, we also define

$$\mathbf{I} \triangleq \{\boldsymbol{\omega}, \mathbf{M}, \mathbf{m}, \mathbf{E}\} \tag{17}$$

and

$$\hat{\mathbf{I}} \triangleq \{\hat{\boldsymbol{\omega}}, \hat{\mathbf{M}}, \hat{\mathbf{m}}, \hat{\mathbf{E}}\}, \tag{18}$$

where $\hat{\boldsymbol{\omega}}$, $\hat{\mathbf{M}}$, $\hat{\mathbf{m}}$, and $\hat{\mathbf{E}}$ are the declared values for vectors $\boldsymbol{\omega}$, $\mathbf{M}$, $\mathbf{m}$, and $\mathbf{E}$, respectively. If user n has a consumption vector $\mathbf{x}_n$, but has to pay t_n, then the payoff function of user n is

$$U_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) - t_n. \tag{19}$$

On the other hand, the social objective is in the form of

$$U_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) + \sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k\right), \tag{20}$$

where $\mathcal{N}_{-n}$ is the set of all users except user n. For a given vector of declared demand information $\hat{\mathbf{I}}$, the VCG mechanism chooses the energy consumption allocation $\mathbf{x}(\hat{\mathbf{I}})$ as an optimal solution to problem (10) and calculates optimal energy consumption vectors as

$$\mathbf{x}(\hat{\mathbf{I}}) = \underset{\mathbf{x}_n\in\hat{\mathcal{X}}_n,\, n\in\mathcal{N}}{\arg\max}\left\{\sum_{n\in\mathcal{N}} \hat{U}_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{n\in\mathcal{N}} x_n^k\right)\right\}, \tag{21}$$

and the payments are structured such that

$$t_n(\hat{\mathbf{I}}) = -\left(\sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k\right)\right) + h_n(\hat{\mathbf{I}}_{-n}), \tag{22}$$

where h_n is an arbitrary function of $\hat{\mathbf{I}}_{-n}$, i.e., the declared demand information of the users with user n excluded from the system. The true demand information of the users other than n is denoted by $\mathbf{I}_{-n}$. The definition of the payments in (22) aligns user objectives with the social planner's objective.

Remark 1: We note that the information in (15) is similar to the type of information submitted by large purchasers of electricity in a wholesale electricity market. Each purchaser in a wholesale electricity market makes a day-ahead bid based on its demand curve. However, in contrast to (21) and (22), the power share of each purchaser and the price of electricity in day-ahead markets are determined by clearing the demand against the supply offers. The dispatch of the power is then balanced in real-time on the day of dispatch [30], [31]. As a result, the proposed schemes in this paper can find interesting applications also in the wholesale electricity market.

The cost term $C_k(\cdot)$ in (20) couples the consumption power variables of all users $\mathbf{x}$. This term makes the whole problem not only a utility maximization but also a cost minimization problem, and thus, the *system objective* is different from the normal objective of VCG mechanisms studied in other contexts [21], [38], [39]. These changes in our problem formulation require the verification of some desired properties of the proposed VCG mechanism for the new scenario. To this end, we make the following proposition.

Proposition 3: If the VCG mechanism defined in (21) and (22) is used to select electricity payment values, then declaring $\hat{\mathbf{I}}_n = \mathbf{I}_n$ is a dominant strategy for each user n, and following this strategy results in an efficient allocation.

The proof of Proposition 3 is given in Appendix A. Proposition 3 highlights two main features of the proposed VCG mechanism. First, the payment of each user is structured such that regardless of other users' strategies, the intended user cannot do better than truthfully declaring its demand information. This feature significantly reduces the communication requirements of the method and eliminates the need for interaction among users. Second, if all users declare their demand truthfully, the proposed VCG system results in an efficient system, i.e., the utilities of all users are maximized and the cost imposed on the energy provider is minimized. For the following, we need to determine function h_n introduced in (22). Here, we will use a popular choice for this function which is referred to as Clarke tax [37],

$$h_n(\hat{\mathbf{I}}_{-n}) = \sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k(\hat{\mathbf{I}}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\hat{\mathbf{I}}_{-n})\right), \tag{23}$$

where $\mathbf{x}(\hat{\mathbf{I}}_{-n})$ is the VCG allocation choice in (21), but when user n is excluded from the system. Thus, the payment of user n is

$$\begin{aligned} t_n(\hat{\mathbf{I}}) = &-\left(\sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k(\hat{\mathbf{I}})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k(\hat{\mathbf{I}})\right)\right) \\ &+\left(\sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k(\hat{\mathbf{I}}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\hat{\mathbf{I}}_{-n})\right)\right). \end{aligned} \tag{24}$$

The payment of user n is the difference in the social welfare of the other users with and without the presence of user n.

B. VCG Mechanism and Nonnegative Transfer

In general, if users can serve as a source of electricity at some time instances during the day, e.g., because they have local generation capability or they can transfer the power stored in their local batteries back to the grid, then such users may receive payments from the grid. Such payments can also be interpreted as negative payments made by the users. However, in the problem formulation considered in this paper, since users are only electricity consumers, this case does *not* arise, and the users' payments to the grid are always *nonnegative*. We will refer to this property as *nonnegative transfer*. In the following theorem, we show that for our problem formulation the nonnegative transfer property holds.

Theorem 2: Suppose Properties 1-4 and Assumptions 1-3 hold. Then, the VCG mechanism in (21) and (24) has the property of nonnegative transfer.

The proof of Theorem 2 is given in Appendix B.

C. VCG Mechanism and Market Clearing Price

The following theorem shows that the electricity payment of each user in the proposed VCG mechanism is less than its payment in a system which has price taking users and uses marginal cost pricing, i.e., the $\boldsymbol{\lambda}$ term in Proposition 1.

Theorem 3: Suppose Properties 1-4 and Assumptions 1-3 hold. For the VCG mechanism in (21) and (24), the payment of each user is $t_n \le \sum_{k \in \mathcal{K}} \lambda_k^* x_n^k(\mathbf{I})$, where $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_K^*)$ is the vector of market clearing prices for problem (10).

The proof is based on the assumptions that the utility functions are concave and the cost function is convex. Optimality conditions of the VCG allocation (21) are adopted to relate the VCG payment of each user to the market clearing price. The proof of Theorem 3 can be found in Appendix C.

IV. Performance Evaluation

In this section, we present simulation results and assess the performance of our proposed mechanism and the impact of different system parameters. In our simulations, we assume that all users have concave quadratic utility functions as described in (7), where parameter α is chosen as 0.5. We set the parameters of the cost function in (9) for each time slot to $a > 0$, $b = 0$, and $c = 0$.

A. Performance Gains from Real-time Interaction with Users

To have a baseline scheme to compare with, we consider a *peak load pricing* (PLP) method in which the price value for each time slot is calculated based on the average power consumption of the users in each time slot to maximize the payoff of the energy provider which is its revenue minus total energy cost. For the PLP method, we assume that the energy provider has some prior information about the distribution of parameter ω of the users. Here, we assume a uniform distribution. We assume there are $N = 50$ users. We consider $K = 24$ representing a 24-hour period. Parameter ω of each user is selected from the set $\{5, 6, \ldots, 15\}$. However, random events are modeled via a small perturbation in the ω value of each user. We set the parameter a of the cost function equal to 0.02, 0.3, and 0.5 for off-peak, mid-peak, and on-peak hours, respectively. We assume that each user has a minimum required energy in each operation period, E_n, which varies from 9 kWh to 21 kWh. The minimum power requirements of each user in each time slot, m_n^k, are set on average to 0.1 kW, 0.5 kW, and 1 kW for off-peak, mid-peak, and on-peak hours, respectively.

As illustrated in Fig. 3, the proposed VCG mechanism improves the performance of the system not only by reducing the power consumption of users but also by reducing the *peak-to-average ratio* from 1.51 to 1.21.

B. The Impact of Reflecting the Generating Cost

The proposed VCG mechanism is used to maximize the social welfare. Maximizing the aggregate utility of all users while minimizing the cost imposed on the energy provider is beneficial for both users and energy provider. The opportunity

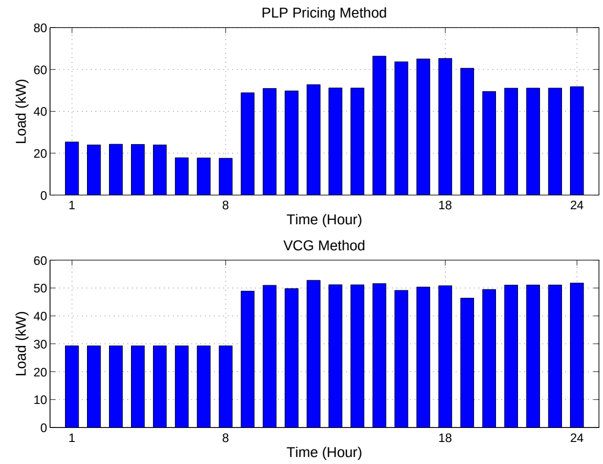

Fig. 3. Power consumption for the proposed VCG method and a peak load pricing (PLP) method.

of reflecting the fluctuations of the wholesale price into the customer side is one of the main advantages of the proposed VCG mechanism. This aspect becomes more important in situations where the cost imposed on the energy provider is high. To have a baseline scheme to compare with, we consider a system which has price anticipating users and employs marginal cost pricing. It has been shown that in a system with price taking users, marginal cost pricing not only maximizes the social welfare, but also maximizes the payoff of the energy provider [20]. As an upper bound on the payoff of the energy provider, we consider a system which has price taking users and employs marginal cost pricing. We assume there are 50 users, and parameter ω of each user is selected from the set $\{15, 25, 30, 40\}$. We assume that for each user n, parameter E_n varies from 10 kWh to 15 kWh and for different time slots, parameter m_n^k is set on average to 0.1 kW, 0.5 kW, and 1 kW for off-peak, mid-peak, and on-peak hours, respectively.

Furthermore, we assume that parameter a of the cost function is constant in all three time slots. The payoffs of the energy provider for the proposed VCG system, the system with price anticipating users, and the system with price taking users for different values of parameter a of the cost function are presented in Fig. 4. We can see that, since the VCG payment (24) is structured to consider the cost imposed on the energy provider, the payoff of the energy provider is higher compared to the system with price anticipating users. Note that the proposed VCG system and the price taking system are both efficient systems with the same power allocation. Hence, they have the same total power consumption.

C. Communication Requirements of the VCG System

The communication requirements are among the main aspects considered for any pricing method. In this section, the number of messages exchanged between users and also the energy provider is considered as a measure to compare the proposed VCG system with a system which has price anticipating users. In the VCG system, each user is asked to declare its parameter ω and its feasible set of power consumption to the energy provider, and in return, the energy provider determines the payment and the allocated power of each user. In practice,

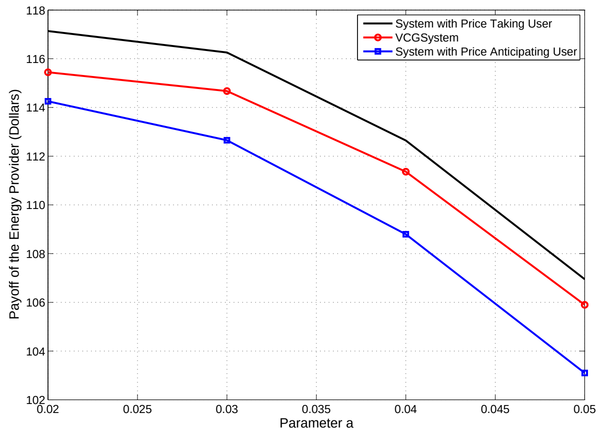

Fig. 4. Payoff of the energy provider for the proposed VCG system, the system with price anticipating users, and the system with price taking users.

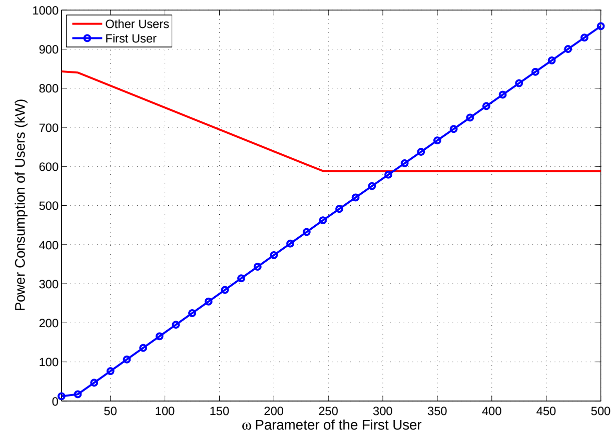

Fig. 5. Power consumption of the first user and aggregate power consumption of other users when ω of the first user is increased.

TABLE I
AVERAGE NUMBER OF EXCHANGED MESSAGES IN VCG SYSTEM AND SYSTEM WITH PRICE ANTICIPATING USERS.

Number of Users N	Price Anticipating System	VCG System
10	42767	20
20	79624	40
30	87808	60
40	135290	80
50	145718	100

it may be preferable for the users to communicate only with a trusted entity such as the energy provider. However, when users are price anticipator, they form a game and have to exchange messages with each other. Communication requirements become an important feature specially in situations where the cost imposed on the energy provider is low, and most of the users can compete in the power consumption game. In a system where users are price anticipator, we use the myopic best-response algorithm [37, Ch. 6] to compute the Nash equilibrium. In this system, each user informs other users whenever it changes its power consumption. Each time one of the users updates its power consumption information, a *message* is sent. We set the parameter a of the cost function equal to 0.02, 0.3, and 0.5 for off-peak, mid-peak, and on-peak hours, respectively. We assume that for each user n, parameter E_n varies from 10 kWh to 20 kWh and for different time slots, parameter m_n^k is set on average to 0.1 kW, 0.5 kW, and 1 kW for off-peak, mid-peak, and on-peak hours, respectively.

The average number of messages exchanged between the various entities in the VCG system and the system with price anticipating users for $K = 24$ is presented in Table I. As illustrated in Table I, the method used in the system with price anticipating users requires much more message exchanges to converge than the VCG mechanism.

D. Effect of Parameter ω

In this section, we explore the effect of parameter ω on different aspects of the power system for $N = 50$ users and $K = 3$ time slots. In this regard, we mainly focus on the power consumption of the system and the payments of the users. To understand how changes in the parameter ω of a single user can affect others, we consider the same ω for all the users and change it for the first user starting from 5 while keeping this parameter fixed equal to 20 for the other users. We set parameter a of the cost function equal to 0.02 for all time slot. For each user n, parameter E_n selected to be 12, and parameter m_n^k is set to zero for all time slots.

The simulation results for the power consumption of the users are presented in Fig. 5. We notice that as we increase the ω of the first user, the power consumption of the other users decreases until they reach their minimum power requirements. Intuitively, as the first user values energy more by increasing its ω value, it has a negative impact on the power consumption of its rivals in the system. For the VCG mechanism, the payments of the users are closely related to their power consumption, i.e., as the first user increases its ω, the power consumption of the other users reduces as well as their aggregate payment until they reach the point where they consume their minimum power requirements. After reaching this point, as they have a guaranteed amount of power consumption, their aggregate payment increases.

E. Exploring the Truthfulness Property

Truthfulness in dominant strategy for the proposed VCG mechanism means that regardless of other users' strategy, the intended user cannot do better than truthfully declare its demand information. In this section, we consider a system where there are $N = 10$ users and $K = 3$ time slots. We set parameter a of the cost function equal to 0.02 for all time slot. For each user n, parameter E_n is equal to 15 kWh and for different time slots, parameter m_n^k is set to zero for all time slots. We assume the true ω parameter of the users is $\boldsymbol{\omega} = [12, 6, 8, 8, 10, 10, 12, 12, 16, 20]$ and E_1=15. We explore the best response of the first user while other users declare their demand information truthfully. As illustrated in Fig. 6, the considered user (first user) with ω_1=12 and E_1=15 cannot do better than truthfully declare $\hat{\omega}_1$=12 and $\hat{E}_1$=15.

V. CONCLUSIONS

In this paper, we proposed a VCG mechanism for DSM in the future smart grid. The proposed mechanism aims to maximize the aggregate utility of all users while minimizing

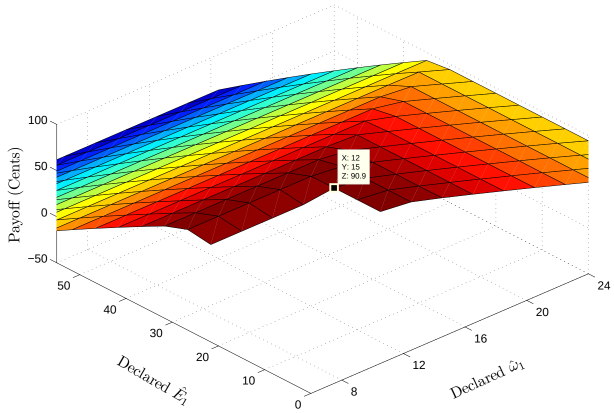

Fig. 6. The payoff of the first user for different values of *declared* $\hat{\omega}_1$ and $\hat{E}_1$ (the *true* ω_1 is equal 12 and the *true* E_1 is equal 15).

the total cost of power generation. We investigated some of the main properties of the proposed mechanism such as truthfulness, efficiency, and nonnegative transfer. Through simulation, we showed that the proposed VCG mechanism improves the performance of the system by encouraging users to reduce their power consumption and shift their loads to off-peak hours. The proposed VCG mechanism significantly reduces the communication overhead. We also analyzed the impact of some key parameters on our model through simulations. The simulations confirmed that by using our proposed VCG mechanism, in addition to maximizing the social welfare, the energy provider will benefit as well. The ideas developed in this paper can be extended in several directions. For example, a system with multiple energy providers can be considered. The effect of malicious users can be explored as well.

Appendix

A. Proof of Proposition 3

Given the payment in (22), since user n cannot affect the term h_n by changing $\hat{\mathbf{I}}_n$, it declares $\hat{\mathbf{I}}_n$ only to maximize

$$W_n(\mathbf{x}_n(\hat{\mathbf{I}}), t_n(\hat{\mathbf{I}})) = U_n\left(\sum_{k\in\mathcal{K}} x_n^k(\hat{\mathbf{I}})\right) + \sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k(\hat{\mathbf{I}})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k(\hat{\mathbf{I}})\right).$$

However, the above expression is bounded above by

$$\underset{\substack{\mathbf{x}_n\in\mathcal{X}_n,\, \mathbf{x}_m\in\hat{\mathcal{X}}_m,\\ m\in\mathcal{N}_{-n}}}{\text{maximize}} \quad U_n\left(\sum_{k\in\mathcal{K}} x_n^k\right) + \sum_{m\in\mathcal{N}_{-n}} \hat{U}_m\left(\sum_{k\in\mathcal{K}} x_m^k\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k\right).$$

Note that $\mathbf{x}(\hat{\mathbf{I}})$ satisfies (21), and user n can achieve the maximum payoff by truthfully declaring $\hat{\mathbf{I}}_n = \mathbf{I}_n$ for solving (21). Since this optimal strategy does not depend on the demand information declared by other users, it confirms the result that for VCG mechanisms, truthful declaration is a dominant strategy. ∎

B. Proof of Theorem 2

In the equilibrium, all users declare their demand information truthfully. Then, we can write the payment of user n as

$$t_n(\mathbf{I}) = -\left(\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k(\mathbf{I})\right)\right) + \left(\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\mathbf{I}_{-n})\right)\right),$$

where $\mathbf{x}(\mathbf{I}_{-n})$ is the optimal solution for the social objective when user n is excluded from the system. So, we have

$$\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\mathbf{I}_{-n})\right) \geq \sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\mathbf{I})\right). \tag{25}$$

Furthermore, from Assumption 1, $C_k(\cdot)$ is an increasing function. Therefore, we have

$$\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\mathbf{I}_{-n})\right) \geq \sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k(\mathbf{I})\right), \tag{26}$$

and thus (24) is nonnegative. ∎

C. Proof of Theorem 3

In the equilibrium, all users declare their demand information truthfully. So, the payment of user n is

$$t_n(\mathbf{I}) = -\left(\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}} x_m^k(\mathbf{I})\right)\right) + \left(\sum_{m\in\mathcal{N}_{-n}} U_m\left(\sum_{k\in\mathcal{K}} x_m^k(\mathbf{I}_{-n})\right) - \sum_{k\in\mathcal{K}} C_k\left(\sum_{m\in\mathcal{N}_{-n}} x_m^k(\mathbf{I}_{-n})\right)\right).$$

Since $\mathbf{x}(\mathbf{I})$ is the optimal solution for the social objective problem, the optimality conditions of (21) imply that

$$\begin{aligned} &\lambda_k^* = p_k\left(\textstyle\sum_{n\in\mathcal{N}} x_n^k(\mathbf{I})\right), \\ &U_n'\left(\sum_{k\in\mathcal{K}} x_n^k(\mathbf{I})\right) = \lambda_k^*, \quad \text{if } x_n^k(\mathbf{I}) > m_n^k \text{ and} \sum_{k\in\mathcal{K}} x_n^k(\mathbf{I}) > E_n, \\ &U_n'\left(\sum_{k\in\mathcal{K}} x_n^k(\mathbf{I})\right) \leq \lambda_k^*, \quad \text{if } x_n^k(\mathbf{I}) = m_n^k \text{ or} \sum_{k\in\mathcal{K}} x_n^k(\mathbf{I}) = E_n, \end{aligned} \tag{27}$$

where $p_k(\cdot)$ has been introduced in (8), and λ_k^* is the market clearing price for the problem (10).

By concavity of U_n we have

$$\begin{aligned} &U_n(x) \geq U_n'(x)x, \\ &\tfrac{U_n(x)-U_n(y)}{U_n'(x)} \leq x - y, \end{aligned} \tag{28}$$

and by convexity of C_k, we have

$$\begin{aligned} &C_k(q_1) \leq C_k^{'}(q_1) q_1, \\ &\tfrac{C_k(q_1) - C_k(q_2)}{C_k^{'}(q_1)} \geq q_1 - q_2. \end{aligned} \tag{29}$$

Then, from (27)-(29), we have

$$\begin{aligned} t_n \leq &\sum_{m \in \mathcal{N}_{-n}} \frac{\lambda_k^* \Big[U_m\big(\sum\limits_{k \in \mathcal{K}} x_m^k(\mathbf{I}) \big) - U_m\big(\sum\limits_{k \in \mathcal{K}} x_m^k(\mathbf{I}_{-n}) \big) \Big]}{U_m^{'}\big(\sum\limits_{k \in \mathcal{K}} x_m^k(\mathbf{I}) \big)} \\ &- \sum_{k \in \mathcal{K}} \frac{\lambda_k^* \Big[C_k\Big(\sum\limits_{m \in \mathcal{N}_{-n}} x_m^k(\mathbf{I}_{-n}) \Big) - C_k\Big(\sum\limits_{m \in \mathcal{N}} x_m^k(\mathbf{I}) \Big) \Big]}{p_k\Big(\sum\limits_{m \in \mathcal{N}} x_m^k(\mathbf{I}) \Big)} \\ \leq &\sum_{k \in \mathcal{K}} \lambda_k^* x_n^k(\mathbf{I}), \end{aligned}$$

which completes the proof. ∎

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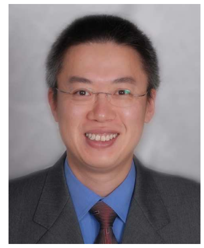

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