Optimal Data Transmission and Channel Code Rate Allocation in Multipath Wireless Networks

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***Abstract*— Wireless links are often *unreliable* and prone to transmission error especially when network users are mobile. These can degrade the performance in wireless networks, particularly for applications with tight quality-of-service requirements. A common remedy to this problem is *channel coding*. However, this *per-link* solution can compromise the link data rate, leading to an undesired *end-to-end* performance. In this paper, we show that this shortcoming can be mitigated if the end-to-end transmission rates and channel code rates are selected properly over *multiple* routing paths. We formulate a joint channel coding and end-to-end data rate allocation problem in multipath wireless networks with *max-min fairness* as the objective function. Our goal is to maximize the minimum throughput available among the network users. To cope with the fast and frequent changes in dynamic environments typical for vehicular networks, we address both *adaptive* and *non-adaptive* channel coding scenarios. Unlike similar formulations in *single-path* routing networks, in the multipath routing case we face an optimization problem that is *non-convex* and is usually difficult to solve. We tackle the non-convexity by using *function approximation* and iterative techniques from *signomial programming*. Simulation results confirm that by using channel coding jointly with multipath routing, we can significantly improve end-to-end network performance, compared to the case when only one of them is used in the network. Non-adaptive channel coding is also shown to achieve high degree of optimality with much less complexity.**

***Keywords*: Link reliability, multipath routing, throughput maximization, max-min fairness, adaptive and non-adaptive channel coding, non-convex optimization, signomial programming.**

I. INTRODUCTION

Recent advances and technological developments in wireless communication, digital electronics, and radio frequency systems have placed wireless networks at the forefront of today's data transmission systems. However, unlike wired networks, links in wireless networks can be unreliable and prone to transmission error due to channel imperfections, background noise, environmental obstacles, weather conditions, and user mobility [1]. Unreliable links can degrade network performance particularly for applications with tight quality-of-service requirements such as voice-over-IP and video streaming [2]. Therefore, it is crucial to develop efficient strategies in order to improve the reliability of data transmission in wireless networks [3].

Different approaches are used to make wireless networks more reliable. They include rate allocation [4], [5], channel coding [4], network coding [6], [7], and multipath routing [8]–[10]. Many rate allocation approaches are based on variations of the network utility maximization (NUM) [11]–[15].

Channel coding is commonly used as a tool to leverage reliable transmissions over *lossy* wireless links. With channel coding, the transmitter node of each link encodes the transmitted packets by adding *auxiliary* or *redundant* bits, which can increase the distance among the codewords and decrease the *packet error probability*. If the number of extra bits is the same across all links, then channel coding is *non-adaptive*. On the other hand, if we change the amount of redundant bits for each link based on its current state, then channel coding is *adaptive*. Adaptive channel coding may result in better performance compared to non-adaptive channel coding; however, it entails a higher complexity. In general, channel coding usually introduces a *tradeoff* between reliability and data transmission rate. In fact, by changing the *code rate*, i.e., the ratio of data bits to data plus redundant bits, we can change the data rate at which the information is transmitted over each wireless link. In particular, the code rate can be decreased in order to improve (reduce) the probability of error at the cost of having lower data rates. Similarly, we can increase the code rate to increase the transmission data rate, but at the cost of increasing the probability of error. Adaptive channel coding has been used in [4] to enhance the network reliability, when single-path routing is being used. The rate-reliability tradeoff introduced through channel coding is studied in [4], [16]–[18].

Multipath routing can be used to compensate for the data rate reduction due to channel coding. This is done by distributing the load over multiple routing paths. Multipath routing can provide *fault tolerance* against link failures and also achieve *load balancing* in order to better utilize the available network capacity [19]–[21]. Multipath routing has been studied in both wired [22] and wireless networks [8], [9]. However, none of the above work address *jointly* the use of multipath routing and channel coding for reliability improvement.

In this paper, our focus is to *jointly* use channel coding and multipath routing in an optimization-based framework to further improve reliability compared to using *only* channel coding or *only* multipath routing. We are interested in answering the

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following question: *How shall we select the end-to-end data transmission rates over different paths and per-link channel code rates in order to achieve the optimal rate-reliability tradeoff in multipath wireless networks?*

Our main contribution is to use channel coding in multipath routing wireless multihop networks to provide fair resource allocation among the network users. In this regard, our work is closely related to [4]. However, here we introduce three key extensions. First, Lee *et al.* in [4] assume that the links in the network are either *wired* or *interference-free* wireless. On the contrary, here we have explicitly incorporated the impact of wireless interference. Second, unlike the system model in [4] which addresses only single-path routing, here we consider the case where there are multiple end-to-end routing paths available across the network. Clearly, this includes single-path routing as a special case. Third, we formulate the problem such that the minimum throughput among the individual users is maximized. This leads to fairness provisioning which is of great importance in certain applications such as vehicular networks where vehicles frequently switch among stationary mesh nodes to receive connectivity. In this case, different mesh nodes must be provided with fair and consistent data rates. The aforementioned three extensions introduce several challenges in solving the formulated optimization problem and have not been addressed before. Those are due to various *non-convexities* that cannot be directly transformed into a convex optimization problem using the well-known logarithmic change of variables as in [4]. Although our proposed method is centralized, it may be used in vehicular network applications such as those in which stationary access points provide connectivity for the vehicles in their coverage zone. Moreover, it can shed light on how per-link channel coding can improve end-to-end performance in a multipath routing wireless network. The centralized solution may also be used as a benchmark for evaluating distributed approaches which may be developed in the future. To the best of our knowledge, rate allocation with the goal of fairness and reliability enhancement using multipath routing and channel coding has not been addressed in any prior work.

In [23], we consider multipath routing and channel coding for reliability improvement but aiming at maximizing the aggregate throughput in the network. This paper is different because fairness is not considered in [23]. We also consider throughput maximization in [24] while minimizing the end-to-end delay in the network. However, in [23], [24] we do not consider mobility in vehicular networks nor the fast convergence in the presence of dynamic changes in the network. In our design, we also consider the case where there are frequent changes in the network (e.g., in the number of users and traffic patterns due to mobility in dynamic environments such as vehicular networks) and adjust our proposed algorithm such that it converges faster to the optimal solution. Moreover, packet re-transmission is not considered in the modeling in [23] and [24].

The contributions of this paper are as follows.

- We formulate max-min fair resource allocation in multipath wireless networks employing channel coding as an optimization problem. Our system model includes both *adaptive* and *non-adaptive* channel coding.
- We tackle the non-convexity of the formulated optimization problem in two steps. First, we use *function approximations* to reformulate the problem as a *signomial programming* problem (which is still non-convex). Next, we develop an iterative algorithm to solve the signomial programming problem by solving a chain of tractable *geometric programming* problems. We introduce a non-adaptive channel coding scheme with much lower degree of complexity, which can find a sub-optimal solution. We design our algorithm such that it can quickly find the new solution, whenever there is a change in the network topology and the number of users.
- To motivate the joint use of multipath routing and channel coding, we show through simulations that our proposed scheme significantly improves the network performance when compared to the case with multipath routing, but *without* channel coding. We also show that our joint scheme outperforms channel coding in single-path routing systems.
- We investigate the convergence properties of the proposed algorithm as well as its efficiency. The latter is studied particularly by evaluating the impact of the approximations made in the derivation of the algorithm.
- We compare the *adaptive* coding scheme with the *non-adaptive* coding scheme with less computational complexity. We evaluate the proposed algorithm in a dynamic vehicular environment where the data traffic pattern changes due to mobility. Finally, we study the effects of fading on the performance of the algorithm.

The rest of this paper is organized as follows. We present the system model and formulate the joint data rate and channel code rate allocation problem in Section II. In Section III, we reformulate the problem as a geometric programming problem and propose a reliability-based rate allocation algorithm to solve it. We also introduce a non-adaptive channel coding scheme as a sub-optimal solution with lower complexity. Simulation results are presented in Section IV. The paper is concluded in Section V.

II. System Model And Problem Formulation

A. System Model

Consider an ad-hoc wireless network. We can model the network topology as a directed graph $G(\mathcal{V},\mathcal{E})$, where $\mathcal{V} = \{1,2,\ldots,V\}$ is the set of nodes and $\mathcal{E}$ is the set of wireless links. Let $\mathcal{I} = \{1,2,\ldots,I\}$ denote the set of all unicast sessions in the network. For each session $i \in \mathcal{I}$, the source and destination nodes are denoted by s_i and t_i, respectively. Furthermore, we denote $\mathcal{K}_i = \{1,2,\ldots,K_i\}$ as the set of all available routing paths from source node s_i to destination node t_i. For each session $i \in \mathcal{I}$, each link $e \in \mathcal{E}$, and each $k \in \mathcal{K}_i$, we have

$$a_i^{ek} = \begin{cases} 1, & \text{if link } e \text{ belongs to } k^{\text{th}} \text{ routing path} \\ & \text{for session } i, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In this paper, we assume that static routing is used and the routing information is given *a priori*.

For each session $i \in \mathcal{I}$, let α_i^k denote the data rate of source s_i on its k^{th} routing path, $k \in \mathcal{K}_i$. The aggregate *transmission* rate for session i is obtained as

$$\sum_{k \in \mathcal{K}_i} \alpha_i^k. \tag{2}$$

Since the packets are retransmitted whenever they are lost in the network, the effective *receiving* rate at destination node t_i is the same as (2).

Channel coding can improve reliability on lossy channels by adding redundant bits to the data packets transmitted. In this regard, we define R_e as the *code rate* of link $e \in \mathcal{E}$, i.e., the ratio of the data bits to data plus redundant bits. Notice that if no channel coding is performed, then $R_e = 1$ as there will be no redundant bits in the packet.

Let $R_{0e} \leq 1$ denote the *cut-off rate* on wireless link $e \in \mathcal{E}$. We assume that the rate R_e of the adopted coding schemes (e.g., convolutional codes) is limited by the cutoff rate [25]. Given code rate $R_e \leq R_{0e}$ if random coding based on *M*-ary binary coded signals is used, we can bound the packet error probability on link e to be less than $2^{-T(R_{0e}-R_e)}$ as [4], [16], [17], [25]. Therefore, in the worst case, we have

$$P_e = 1 - 2^{-T(R_{0e}-R_e)}, \tag{3}$$

where P_e is the successful packet transmission probability on link e and T is the coding block length. In general, the cut-off rate R_{0e} depends on the signal-to-noise ratio (SNR) and the modulation scheme being used. For example, for a *binary phase shift keying* (BPSK) waveform [25], we have

$$R_{0e} = 1 - \log_2(1 + e^{-\gamma_e}), \tag{4}$$

where γ_e denotes the SNR at the receiver node of wireless link $e \in \mathcal{E}$. In particular, we have

$$\gamma_e = \Gamma_e \times d_e^{-\sigma} \times |f_e|^2, \quad \forall\ e \in \mathcal{E}, \tag{5}$$

where Γ_e depends only on the SNR at transmitter, d_e is the distance between the transmitter and receiver of link e, σ is the path loss exponent (e.g., between 2 and 5), and f_e is the small-scale fading gain. Assuming re-transmission after a packet is lost in the network until reaching a successful transmission, each packet must be sent $1/P_e$ times on average over each link e. Given the source transmission rates $\boldsymbol{\alpha} = (\alpha_i^k,\ \forall\ i \in \mathcal{I},\ k \in \mathcal{K}_i)$, successful transmission probabilities $\boldsymbol{P} = (P_e,\ \forall\ e \in \mathcal{E})$, and the link code rates $\boldsymbol{R} = (R_e,\ \forall\ e \in \mathcal{E})$, we can model the aggregate traffic load on link $e \in \mathcal{E}$ as

$$u_e = \frac{1}{R_e P_e} \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} a_i^{ek}\ \alpha_i^k. \tag{6}$$

From (6), the smaller the code rate R_e, the more redundant data is added to the transmitted packets on link $e \in \mathcal{E}$ leading to more reliable transmission (i.e., transmission with lower error probability). However, this will be at the cost of increasing the traffic load on the link.

We can model the mutual interference among the wireless links in a network by using a *contention graph* $G_C(\mathcal{V}_C, \mathcal{E}_C)$. In the contention graph G_C, the set of vertices $\mathcal{V}_C$ represents the set of all wireless links $\mathcal{E}$ in the network graph G. There exists an edge between any two vertices in set $\mathcal{V}_C$ if wireless links corresponding to two vertices mutually interfere with each other (i.e., the receiver node of one link is within the interference range of the sender node of the other link). Given the contention graph, each *complete* subgraph (i.e., a subgraph in which all vertices are connected to all other vertices) is called a *clique*. A *maximal clique* is then defined as a clique which is *not* a subgraph of any other clique [26]. Denote the set of all maximal cliques in contention graph G_C by $\mathcal{Q}_C$. Only one link among all the links corresponding to the vertices of a maximal clique $Q \in \mathcal{Q}_C$ can be active at a time.

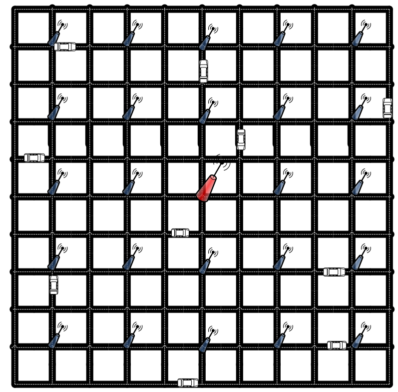

Fig. 1. An example downtown area with 25 access points (forming a wireless mesh infrastructure). The access point at the center serves as the gateway. There are 10 vehicles in the system, each one uses the nearest access point to connect to the Internet.

For the data link layer, we assume that time division multiple access (TDMA) is used. Let c_e denote the *nominal* data rate of link $e \in \mathcal{E}$. The ratio $\frac{u_e}{c_e}$ denotes the proportion of time at which link $e \in \mathcal{E}$ is active when it is used at data rate c_e. It is required that

$$\sum_{e \in Q} \frac{u_e}{c_e} \leq \nu, \qquad \forall\ Q \in \mathcal{Q}_C, \tag{7}$$

where $\nu \in (0, 1]$ is called the *clique capacity*. Note that if $\nu = 1$, then (7) is only a necessary constraint. It is shown that inequality (7) is a sufficient constraint when $\nu = 2/3$ [27].

Now we show how the provided modeling covers the vehicular environment. Consider Fig. 1 in which an example downtown area is shown. There is an access point in every other cross section and the one at the center of the area is denoted as the gateway. Access points correspond to nodes in set $\mathcal{V}$. There is a wireless link $e \in \mathcal{E}$ between two adjacent access points. Vehicles move in the streets continuously. Each vehicle at each instant of time finds the nearest access point and connects to it to transmit data to the gateway. The access point $i \in \mathcal{I}$ which is connected to a vehicle, represents source node s_i for flow i. The gateway corresponds to destination node t_i. During the time that vehicles move in the area the set of sources and thus the data traffic pattern changes.

B. Problem Formulation

Considering (2), (3), (6), and (7), the *rate-reliability tradeoff* can be explained as follows. For each link $e \in \mathcal{E}$, by *increasing* the code rate R_e we can reduce the traffic load per transmission on each link. Thus, *higher* transmission rates will be allowed with the *same* clique capacity. However, this is at the cost of less reliability and leads to more re-transmission attempts as in (6). On the other hand, by *decreasing* the code rate R_e, we can *reduce* the error probability in (3) which leads to *higher* probability of successful transmission along each routing path. Therefore, we may select either higher transmission rates, but with more packets being prone to error, or lower transmission rates, but with higher percentage of correctly received packets. In this regard, the key question to be answered is: *What transmission rates* $\boldsymbol{\alpha}$ *and code rates* $\boldsymbol{R}$ *should be selected to achieve optimal performance?*

To answer the above question, we formulate the following optimization problem.

Max-Min Fairness Problem:

$$\begin{aligned}
\underset{\boldsymbol{\alpha}\succeq\mathbf{0},\ \mathbf{0}\prec\boldsymbol{R}\preceq\boldsymbol{R_0}}{\text{maximize}} \quad & \underset{i\in\mathcal{I}}{\text{minimum}} \sum_{k\in\mathcal{K}_i} \alpha_i^k \\
\text{subject to} \quad & \sum_{e\in Q} \frac{1}{P_e R_e\ c_e} \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k \le \nu, \\
& \qquad\qquad\qquad\qquad \forall\ Q\in\mathcal{Q}, \\
& P_e = 1 - 2^{-T(R_{0e}-R_e)}, \qquad \forall\ e\in\mathcal{E},
\end{aligned} \tag{8}$$

where $\boldsymbol{R_0} = (R_{0e},\ \forall\ e\in\mathcal{E})$ denotes the vector of cut-off rates for all links in the network. The objective function in (8) is the *minimum* receiving rate among all sessions in the network, where for each session $i\in\mathcal{I}$, the receiving rate is as in (2). By solving (8), we can find $\boldsymbol{\alpha}$ and $\boldsymbol{R}$ such that the minimum throughput across all sessions is maximized. Notice that we could also maximize the aggregate network throughput. However, the aggregate network throughput maximization problem does *not* take into account any notion of fairness as the objective is to maximize the *total* network throughput. As a result, the optimal solution may lead to starvation in some sessions. Max-min fairness solution avoids starving any of the sessions and balances the performance in the network. We will discuss solving problem (8) in Section III.

III. Optimal Transmission Rate and Channel Code Rate Allocation

A. Max-Min Fairness

In this section, we propose an *iterative* algorithm to solve the max-min fairness optimization problem to achieve *optimal* allocation of source transmission rates $\boldsymbol{\alpha}$ as well as *optimal* channel code rates $\boldsymbol{R}$ in the network. In general, problem (8) is *non-convex* and difficult to solve. Note that the non-convexities in problem (8) come from the following three sources: (a) The *minimum* term in the objective function. (b) The *exponential* forms in the equality constraints with respect to error probabilities. (c) The *fractional* forms in the inequality constraints with respect to clique capacities.

Most of these challenges are caused by the fact that, unlike many of the existing related work in the literature on rate-reliability tradeoff (e.g., in [4]), we take into account multipath routing and wireless interference. For example, if the network is wired such that no interference occurs among transmissions, then the clique capacity constraints would reduce to several *linear* link capacity constraints such that for each link $e\in\mathcal{E}$ we have

$$\frac{1}{P_e R_e\ c_e} \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k \le 1 \Rightarrow \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k \le P_e R_e\ c_e. \tag{9}$$

However, these techniques are not applicable where multipath routing is used and wireless transmissions incur interference. Infact, we need to go through more elaborate steps in order to be able to solve problem (8) in the general case as will be explained in detail next.

Recall that problem (8) is a non-convex optimization problem due to the three reasons listed earlier, where one of them is the exponential forms in the equality constraints with respect to error probabilities. We start by tackling this source of non-convexity. First, we replace this equality with an inequality. This does not degrade the performance of the algorithm because it overestimates the unreliability in the network. For notational simplicity, we rewrite the error probability (3) as

$$P_e \le 1 - X_e\ \exp\left(L_e\ R_e\right), \qquad \forall\ e\in\mathcal{E}, \tag{10}$$

where $X_e = 2^{-TR_{0e}}$, and $L_e = T\ln 2$.

Recall that for each link $e\in\mathcal{E}$, we have $0 < R_e \le R_{0e}$. We use *Taylor series expansion* to write inequality (10) as

$$P_e \le 1 - X_e \sum_{n=0}^{\infty} \frac{(L_e\ R_e)^n}{n!}, \qquad \forall\ e\in\mathcal{E}. \tag{11}$$

Clearly, for some *bounded* integer $N_e \gg 1$, we have

$$P_e \le 1 - X_e \sum_{n=0}^{N_e} \frac{(L_e\ R_e)^n}{n!}, \qquad \forall\ e\in\mathcal{E}. \tag{12}$$

Unlike the *exponential* error probability model in (10), the model in (12) is in *polynomial* form. For (12) to approximate (11) accurately, we need N_e to be large enough such that $(L_e R_e)^{N_e} \ll N_e!$. We investigate the value of N_e necessary for obtaining a good approximation in Section IV-D.1.

By exploiting the *worst-case* packet error probability (12) in problem (8), we rewrite the max-min fairness problem as

$$\begin{aligned}
\underset{\boldsymbol{\alpha}\succ\mathbf{0},\ \mathbf{0}\prec\boldsymbol{R}\preceq\boldsymbol{R_0},\boldsymbol{P}\succ\mathbf{0}}{\text{maximize}} \quad & \underset{i\in\mathcal{I}}{\text{minimum}} \sum_{k\in\mathcal{K}_i} \alpha_i^k \\
\text{subject to} \quad & \sum_{e\in Q} \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k\ P_e^{-1}\ R_e^{-1}\ c_e^{-1} \le \nu, \\
& \qquad\qquad\qquad\qquad \forall\ Q\in\mathcal{Q}, \\
& \frac{P_e}{1-X_e} + \frac{X_e}{1-X_e} \sum_{n=1}^{N_e-1} \frac{(L_e R_e)^n}{n!} \le 1, \\
& \qquad\qquad\qquad\qquad \forall\ e\in\mathcal{E}.
\end{aligned} \tag{13}$$

The objective in (13) is to maximize the utility of the transmission session with the minimum value. We can replace the minimum function in the objective function by introducing a new *auxiliary* variable t and a set of new constraints as

$$
\begin{aligned}
\underset{t>0,\ \boldsymbol{\alpha}\succ\mathbf{0},\ \mathbf{0}\prec\boldsymbol{R}\preceq\boldsymbol{R_0},\boldsymbol{P}\succ\mathbf{0}}{\text{minimize}} \quad & t^{-1} \\
\text{subject to} \quad & t \leq \sum_{k\in\mathcal{K}_i} \alpha_i^k, \qquad \forall\ i\in\mathcal{I}, \\
& \sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k\ P_e^{-1}\ R_e^{-1}\ c_e^{-1} \leq \nu, \\
& \qquad \forall\ Q\in\mathcal{Q}, \\
& \frac{P_e}{1-X_e} + \frac{X_e}{1-X_e}\sum_{n=1}^{N_e-1}\frac{(L_eR_e)^n}{n!} \leq 1, \\
& \qquad \forall\ e\in\mathcal{E}.
\end{aligned}
\tag{14}
$$

The objective function and constraints in problem (14) are *signomials*, i.e., polynomials with *both* positive and negative terms. Therefore, we can apply *signomial programming* techniques [28] to solve problem (14).

Consider the first constraint in (14). We follow the signomial programming techniques [28] to approximate the polynomial on the right-hand side of this inequality, which is a function of only $\boldsymbol{\alpha}$, as a *monomial*, i.e., a polynomial with only *one* term and *positive* multiplier. This approximation can be performed around some initial point $\hat{\boldsymbol{\alpha}}$. For a parameter $f_s > 1$, which is close to 1, we have

$$
\sum_{k\in\mathcal{K}_i}\alpha_i^k \approx \left(\sum_{k\in\mathcal{K}_i}\hat{\alpha}_i^k\right)\prod_{k\in\mathcal{K}_i}\left(\frac{\alpha_i^k}{\hat{\alpha}_i^k}\right)^{\hat{\alpha}_i^k/\left(\sum_{k'\in\mathcal{K}_i}\hat{\alpha}_{i'}^{k'}\right)}, \qquad \forall\ \boldsymbol{\alpha}\in[\hat{\boldsymbol{\alpha}}/f_s, f_s\hat{\boldsymbol{\alpha}}],
\tag{15}
$$

for any $i\in\mathcal{I}$, where $[\hat{\boldsymbol{\alpha}}/f_s, f_s\hat{\boldsymbol{\alpha}}]$ is a small neighborhood around initial point $\hat{\boldsymbol{\alpha}}$. The closer f_s is to 1, the more accurate the approximation of (15) will be at the cost of slower convergence of the algorithm. For simplicity of notation, for any $i\in\mathcal{I}$, we define $\hat{\Lambda}_i$ which only depends on the initial point $\hat{\boldsymbol{\alpha}}$, as

$$
\hat{\Lambda}_i^{-1} = \sum_{k\in\mathcal{K}_i}\hat{\alpha}_i^k.
\tag{16}
$$

From (15) and (16), the first constraint can be approximated around the initial point $\hat{\boldsymbol{\alpha}}$ as

$$
\hat{\Lambda}_i t \prod_{k\in\mathcal{K}_i}\left(\frac{\alpha_i^k}{\hat{\alpha}_i^k}\right)^{-\hat{\alpha}_i^k\hat{\Lambda}_i} \leq 1, \qquad \forall\ i\in\mathcal{I}.
\tag{17}
$$

The above constraint is a *posynomial*, i.e., a polynomial with only *positive* terms. Replacing the first constraint in (14) with (17), the max-min fairness problem becomes

$$
\begin{aligned}
\underset{t>0,\ \hat{\boldsymbol{\alpha}}/f_s\preceq\boldsymbol{\alpha}\preceq f_s\hat{\boldsymbol{\alpha}},\ \mathbf{0}\prec\boldsymbol{R}\preceq\boldsymbol{R_0},\ \boldsymbol{P}\succ\mathbf{0}}{\text{minimize}} \quad & t^{-1} \\
\text{subject to} \quad & \hat{\Lambda}_i t \prod_{k\in\mathcal{K}_i}\left(\frac{\alpha_i^k}{\hat{\alpha}_i^k}\right)^{-\hat{\alpha}_i^k\hat{\Lambda}_i} \leq 1, \\
& \qquad \forall\ i\in\mathcal{I}, \\
& \frac{1}{\nu}\sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} a_i^{ek}\ \alpha_i^k\ R_e^{-1}\ c_e^{-1} \leq 1, \\
& \qquad \forall\ Q\in\mathcal{Q}, \\
& \frac{P_e}{1-X_e} + \frac{X_e}{1-X_e}\sum_{n=1}^{N_e-1}\frac{(L_eR_e)^n}{n!} \leq 1, \\
& \qquad \forall\ e\in\mathcal{E}.
\end{aligned}
\tag{18}
$$

The above problem is a *geometric program*, which can be converted into a *convex* problem (cf. [28], [29]). Thus, problem (18) is a tractable optimization problem that can be solved efficiently using *convex programming* techniques such as the *interior point method* [30]. We can solve the signomial programming problem (14) by iteratively solving (18).

We now present Algorithm 1 to solve the max-min fairness problem in (8). Algorithm 1 starts by initializing various system parameters. The initial end-to-end transmission rates $\hat{\boldsymbol{\alpha}}$ are selected such that problem (18) is feasible. Several iterations are performed, where in each iteration, we solve the geometric programming problem (18) in Line 5 by using the interior point method [30]. Given the optimal transmission rates $\boldsymbol{\alpha}_{opt}$ in each iteration, we update parameters $\hat{\Lambda}_i$ for any $i\in\mathcal{I}$ according to (16) and correspondingly reformulate problem (18) to be solved again in the next iteration. The iterations continue until the optimal objective value t_{opt} which is obtained in the current iteration does *not* change compared to the optimal objective value t_{old} in the previous iteration. The convergence of the algorithm in each iteration is guaranteed since the interior point method is used [31]. The convergence of Algorithm 1 is also guaranteed [28, p. 115].

Algorithm 1 : Algorithm to solve max-min fair resource allocation problem (8).

1: Initialize f_s, T, N_e, R_{0e}, X_e, L_e, c_e, ν, and $\hat{\alpha}_i^k$ for each $e\in\mathcal{E}$, $i\in\mathcal{I}$, and $k\ \in\ \mathcal{K}_i$.
2: Set $t_{opt} := -\infty$; $\epsilon := 10^{-5}$.
3: **repeat**
4: $\quad t_{old} := t_{opt}$.
5: $\quad$ Solve problem (18) to obtain $\boldsymbol{\alpha_{opt}}$, $\boldsymbol{R_{opt}}$, and t_{opt}.
6: $\quad$ Update $\hat{\boldsymbol{\alpha}} := \boldsymbol{\alpha}_{opt}$ and update $\hat{\Lambda}_i$ as in (16) for each $i\in\mathcal{I}$.
7: **until** $|t_{opt}-t_{old}|\leq\epsilon$.
8: Optimal end-to-end data rates := $\boldsymbol{\alpha}_{opt}$; Optimal per-link code rates := $\boldsymbol{R}_{opt}$.

In case any change happens in the network (e.g., change in the network topology, the number of network users or the traffic pattern), the input parameters of the formulated problem are updated and the corresponding new solution is obtained. Clearly, this can be time consuming if the changes are very frequent. To cope with frequent changes in dynamic environments, we modify the proposed algorithm such that it updates the last end-to-end data rate vector $\boldsymbol{\alpha}_{opt}$ to obtain the new initial point $\hat{\boldsymbol{\alpha}}$ for the new problem. This improves the convergence speed of the algorithm compared to the case where the existing solution is ignored and the problem is solved from scratch. The update process is to remove the entries for the users who left the network and also to add new entries for the new users who have just joined the network. The new entries must be chosen such that the problem remains feasible (i.e., small values must be chosen). In case of topology changes, the algorithm finds new routing paths and updates $\hat{\boldsymbol{\alpha}}$ accordingly. As mentioned before, the algorithm is executed in a central node (e.g., the gateway) and the required information (e.g., channel state information, location of users) is transferred through control messages. In case of the example vehicular network in Fig. 1, the problem is reformulated and solved in specific time instants in the gateway and the solution is passed on to the access points through control messages.

We note that Algorithm 1 needs to be used to update the code rates as well as end-to-end data rates whenever new channel measurements are available, particularly in a fading or mobile environment. We will investigate the impact of our design in a fast fading environment in Section IV-H. Moreover, we will discuss non-adaptive channel coding in Section III-B for the case when parameters change faster than the time required for the algorithm to converge.

B. Non-adaptive Channel Coding

In this section, we simplify the system model in Section III-A and assume that the channel code rate is *fixed* and is no longer an optimization variable in our design. That is,

$$R_e = R, \quad \forall e \in \mathcal{E}. \tag{19}$$

The impact of such an assumption is two-fold. First, it can simplify the clique capacity constraints in problem (8) as for each maximal clique $Q \in \mathcal{Q}$, we have

$$\begin{aligned}\sum_{e\in Q}\frac{1}{R_e P_e\, c_e}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} a_i^{ek}\alpha_i^k &= \frac{1}{RP}\sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i}\frac{a_i^{ek}}{c_e}\,\alpha_i^k \le \nu \\ \Rightarrow \quad \sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i}\frac{a_i^{ek}}{c_e}\,\alpha_i^k &\le R\,P\,\nu,\end{aligned} \tag{20}$$

which is simply a *linear* inequality constraint. Second, since we are adding the extra equality constraints into problem (8), any solution we achieve would be *sub-optimal*. In the non-adaptive channel coding case, the max-min fair resource allocation problem (8) is reformulated as

$$\begin{aligned}\underset{\boldsymbol{\alpha}\succ\mathbf{0}}{\text{maximize}} \quad & \underset{i\in\mathcal{I}}{\text{minimum}} \sum_{k\in\mathcal{K}_i}\alpha_i^k \\ \text{subject to} \quad & \sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i}\frac{a_i^{ek}}{c_e}\,\alpha_i^k \le RP\nu, \qquad \forall\, Q\in\mathcal{Q},\end{aligned} \tag{21}$$

where $P = 2^{T(R-R_0)}$. By introducing an auxiliary variable t and considering the worst case for error probabilities, problem (21) becomes

$$\begin{aligned}\underset{\boldsymbol{\alpha}\succ\mathbf{0},\, t}{\text{maximize}} \quad & t \\ \text{subject to} \quad & t \le \sum_{k\in\mathcal{K}_i}\alpha_i^k, \qquad\qquad \forall\, i\in\mathcal{I}, \\ & \sum_{e\in Q}\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i}\frac{a_i^{ek}}{c_e}\,\alpha_i^k \le RP\nu, \qquad \forall\, Q\in\mathcal{Q},\end{aligned} \tag{22}$$

which is a *linear programming* problem. To find the best fixed code rate, we can solve problem (22) for different values of $R \in [0, R_0]$ and choose the solution with the highest objective value. With non-adaptive channel coding, we significantly decrease the computational complexity of solving the problem at some cost in performance. This can particularly help in dynamic environments where there are frequent changes in the system parameters.

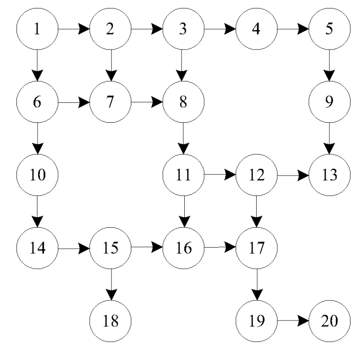

Fig. 2. A sample network topology with 20 nodes randomly located in a 5×5 grid. The network includes five sessions: $1 \to 16$, $3 \to 13$, $2 \to 8$, $14 \to 17$, and $6 \to 20$. There are 4, 2, 2, 1, and 3 routing paths available for these sessions, respectively.

IV. Performance Evaluation

In this section, we assess the performance of our proposed joint channel coding and transmission data rate allocation algorithm (Algorithm 1). In our simulation model, we consider network topologies where $V = |\mathcal{V}| = m(m-1)$ wireless nodes are positioned on an $m \times m$ square grid with *randomly* selected grid locations. As an example, for the network in Fig. 2, we have $m = 5$ and $V = 20$. The network includes m source and destination pairs, with potentially many available routing paths from the source node to the destination node. In Fig. 2, there are *four* available routing paths from source node 1 to destination node 16. They include: $\{(1,2),\ (2,3),\ (3,8),\ (8,11),\ (11,16)\}$, $\{(1,2),\ (2,7),\ (7,8),\ (8,11),\ (11,16)\}$, $\{(1,6),\ (6,7),\ (7,8),\ (8,11),\ (11,16)\}$, and $\{(1,6),\ (6,10),\ (10,14),\ (14,15),\ (15,16)\}$. Unless stated otherwise, the rest of the system parameters are selected as follows: $T = 10$, $N_e = 15$, $f_s = 1.1$, $R_{0e} = 1$, $\nu = \frac{2}{3}$ [27].

Without loss of generality, we choose the link capacity, c_e for each link $e \in \mathcal{E}$, to be equal to 1. Therefore, the transmission data rates, $\boldsymbol{\alpha}$, obtained in the optimal point can be interpreted as the vector of *normalized* transmission rates. If the algorithm is being executed for the first time, we set the initial data rates to be *small*, i.e., $\hat{\alpha}_i^k = 0.01$ for all $i \in \mathcal{I}$ and any $k = 1, \ldots, K_i$, in order to guarantee a feasible starting point for Algorithm 1, as we already discussed in Section III-A. Otherwise, in case of updating the current rate vector, we

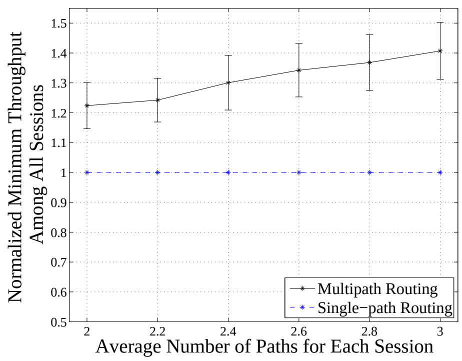

Fig. 3. Comparison between the performance of adaptive channel coding in single-path and multipath routing networks in terms of the achieved normalized minimum throughout.

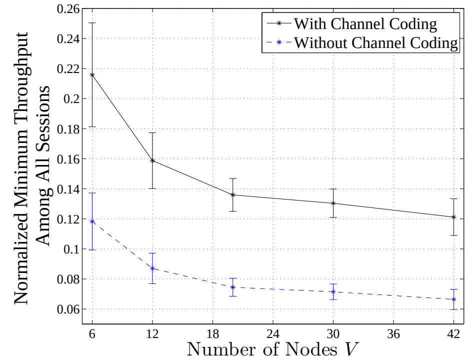

Fig. 4. Comparison between the performance of multipath routing *with* and *without* per-link channel coding in terms of the achieved normalized minimum throughput among the end-to-end sessions, when the scale of the network increases and the number of nodes varies from 6 to 42.

set the new entries for the new routing paths equal to 0.01. To solve the geometric programming problems, we use the MOSEK software [32].

A. Multipath vs. Single-path Routing

We first study the performance enhancement achieved by using multipath routing compared to single-path routing. In the latter case, each source only uses one (out of possibly several) of the available shortest paths to its corresponding destination. We compare our proposed algorithm with the one in [4], where both channel coding and transmission rate allocation is performed in a single-path routing system.

By solving the max-min fair resource allocation problem (8) for the single-path routing (as in [4]) and also for the multipath routing cases (as in our proposed design), the optimal end-to-end data rates are obtained. Recall that the objective value in problem (8) is the *minimum* throughput among all five sessions. In Fig. 3, each point represents the averaged performance gain over 50 random topologies. We can see that the performance gain (i.e., the ratio of the averaged performance under multipath routing to the averaged performance under single-path routing) directly depends on the number of available (and used) routing paths. It monotonically increases as the number of available routing paths increases. This increase is due to the availability of additional paths, the algorithm can distribute the load to the paths which experience less interference. Therefore, the sending rates are increased. In this case, the minimum network throughput can be enhanced by 22% on average when the average number of paths for each session is *only two*. This enhancement increases to 40% when the average number of routing paths increases to three. This is because the algorithm can inject the packets into the paths experiencing less interference.

B. Channel Coding vs. No Channel Coding

Next, we study how channel coding can improve the achieved network throughput in a *multipath* routing system. Since equality (3) models the worst case condition (i.e., provides upper bound on the error probability) and the error probability is equal to 1 in the absence of channel coding, we use the following exact successful packet transmission probability model for BPSK modulation for the case without channel coding:

$$P_e = \left(1 - Q(\sqrt{2\gamma_e})\right)^T, \tag{23}$$

where $Q(.)$ denotes the Gaussian Q-function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-\frac{u^2}{2}) du, \tag{24}$$

and γ_e denotes the SNR at the receiver node of wireless link $e \in \mathcal{E}$. For a received SNR equal to 3 dB, we have $P_e = 0.4$ for $T = 40$. Our comparison reveals the *minimum* achievable performance gain by the use of channel coding. This is because we use the exact P_e for the case without channel coding but a lower bound (worst case) for the case with channel coding. As shown in Fig. 4, a major performance gain can be achieved with channel coding. The achieved performance degrades in both cases when the size of the network increases. This is because as the number of users increases, the interference in the network increases. For the results in Fig. 4, each point represents the normalized throughput averaged over 50 randomly generated network topologies.

C. Convergence Properties of Algorithm 1

Recall that each iteration of Algorithm 1 includes a function approximation step and a geometric programming step. Considering the network topology in Fig. 2, the convergence of the objective value for problem (8), when Algorithm 1 is used, is shown in Fig. 5. The objective value for problem (8) is the minimum throughput among all sessions. From the results in Fig. 5, Algorithm 1 converges after around 50 iterations. Similar results can be obtained for other network topologies.

D. Impact of Various Design and System Parameters

1) Parameter N_e: In Section III, we use the approximation in (12) to convert problem (8) into a tractable geometric programming problem as in (13). We can improve the accuracy of the approximation in (12) by increasing the value of N_e. However, this would be at the cost of making problem (13) more complicated to solve. In this section we are interested in choosing N_e to obtain a reasonable accuracy with low computational complexity. Considering 50 random topologies,

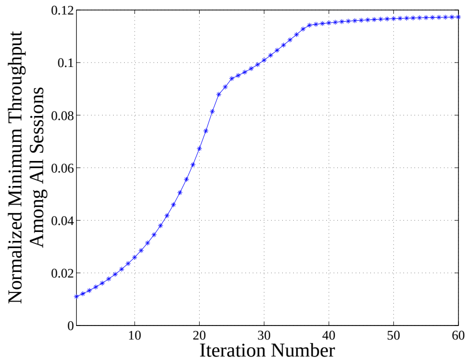

Fig. 5. Convergence of Algorithm 1 with respect to solving problem (8). We can see that the algorithm converges after 50 iterations.

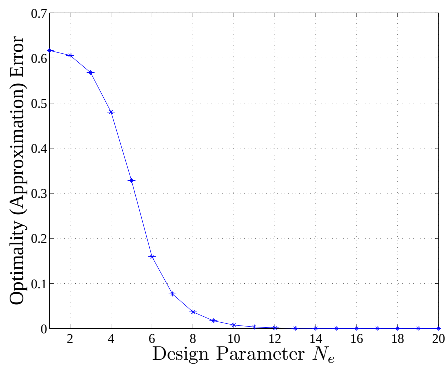

Fig. 6. The impact of the choice of design parameter N_e in approximation (12). The average optimality error decreases as N_e increases. It becomes almost zero for $N_e > 12$.

the simulation results, when N_e varies from 1 to 20, are shown in Fig. 6, where each point indicates the average *optimality error* observed for all 50 topologies. By obtaining the difference between the achieved network throughput at a particular choice of N_e and that at $N_e = 20$ (as the optimal throughput) and computing the ratio of this difference to the optimal throughput, we can define a measure for assessing the *optimality error*. Fig. 6 shows that the optimality error approaches *zero* when N_e is around 12 or higher.

2) Parameter f_s: Another approximation in Section III is the monomial approximation in (15). The approximation is made at each iteration within a close neighborhood of initial point $\hat{\alpha}$. The size of the neighborhood is denoted by design parameter f_s. In general, although we can increase the speed of convergence by increasing the value of f_s, it would be at the cost of a lower accuracy in the approximation. Considering such a tradeoff and based on our simulation results, we select $f_s = 1.1$, for a relatively good performance in terms of approximation accuracy, with a fast convergence speed.

3) Parameter T: In general, when we increase the coding block length T for a given code rate, the probability of error decreases. This can be seen in (3). By increasing T, one can allocate a higher code rate to a wireless link, while achieving the same probability of error, i.e., the same reliability measure. On the other hand, the more reliable links let the algorithm allocate higher end-to-end data rates, leading to improved optimal objective values in problem (8). This is shown for three

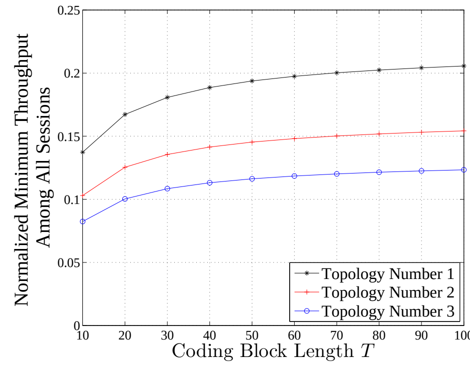

Fig. 7. The impact of choosing different coding block lengths T on the network performance for three different random topologies. We observe that the performance improves when the coding block length increases.

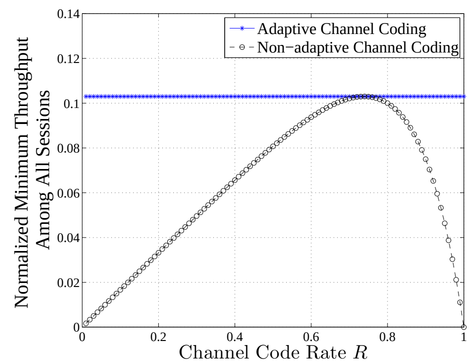

Fig. 8. Comparison between adaptive and non-adaptive channel coding.

random network topologies in Fig. 7, where the coding block length T varies from 10 to 100. The minimum throughput in the network increases in all three topologies when the coding block length (and thus the reliability) increases.

E. Adaptive vs. Non-adaptive Channel Coding

In this section, we show how choosing the code rate for each link individually (i.e., adaptive channel coding) can lead to different optimality and computational complexity results, compared to the case when channel coding is non-adaptive. Recall from Section III-B that in a non-adaptive channel coding scenario, we assume that all wireless links use the *same* code rate R as expressed in (19). In this case, for each fixed R, problem (8) becomes a *linear* programming problem. This can significantly reduce the computational complexity, but it may result in a loss in performance.

Consider the network topology in Fig. 2. Here, we examine various choices of non-adaptive code rate R within the feasible range $[0, R_0]$. We can see in Fig. 8 that by using non-adaptive channel coding, the highest throughput is achieved when the code rate on all links is equal to 0.74. At this point, we reach almost the optimal value that is achievable by using adaptive channel coding. It is also interesting to investigate the distribution of the optimal adaptive code rates of all wireless links, compared to the optimal non-adaptive code rate. We can see in Fig. 9 that in the adaptive channel coding case, the optimal code rates for various links can be significantly different. It is interesting to note that the code

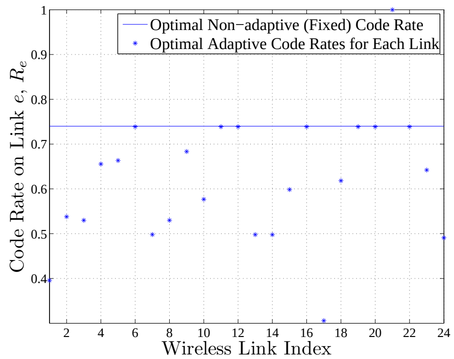

Fig. 9. Optimal non-adaptive code rate versus adaptive code rate distribution among all wireless links of the network topology in Fig. 2.

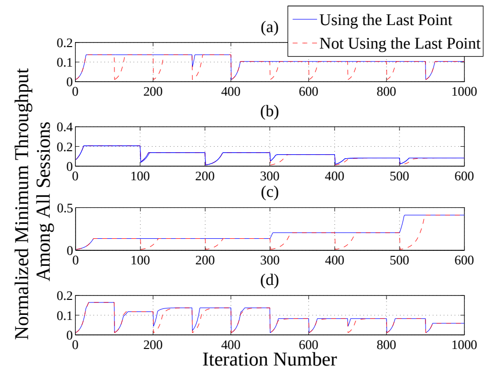

Fig. 10. Convergence speed for Algorithm 1 in presence of dynamic changes in the network. We compare two cases where the previous solution is exploited and the case where the previous solution is ignored. Every 100 time slots: (a) five links are added and five links are removed, (b) a new pair of source-destination nodes is added, (c) a pair of source-destination node is removed, (d) a random pair is added and another random pair is removed.

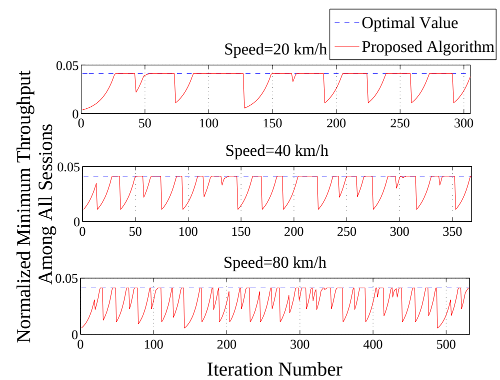

Fig. 11. The convergence of the algorithm is shown for different speeds. Recalculations occur every 5 seconds if needed.

rates corresponding to the links which are not in any routing path (i.e., link 21) are chosen to be 1. Moreover, links which are used in many routing paths have code rates close to the corresponding non-adaptive channel code rate (0.74).

F. The Effect of Dynamic Changes on the System Performance

In this section, we study the effect of dynamic topology changes as well as changes in the number of network users on the network performance. As mentioned in Section III-A, whenever the setting of the network changes, the algorithm solves the new problem by updating the last obtained end-to-end data rate vector, which is used as the new initial point for faster convergence. This may be beneficial especially in dynamic environments such as vehicular networks where the vehicles move constantly. Fig. 10 shows the convergence of the algorithm when changes happen in the network and compares it with the case when the algorithm does not use the available information related to the previous state of the network. In Fig. 10 (a), five randomly chosen links are added to and five random links are removed from the current topology every 100 time slots. In Fig. 10 (b), a new pair of source-destination nodes is added every 100 time slots while in Fig. 10 (c) a pair of source-destination node is removed from the network. Finally, in Fig. 10 (d), a randomly chosen pair is added and a randomly chosen pair is removed every 100 time slots. Fig. 10 shows that using the available information from the previous state of the network significantly increases the convergence speed of the algorithm.

G. The Effect of Mobility on the System Performance

In this section, we study the effect of mobility in a vehicular network on the performance of our proposed design. In a vehicular network, users (i.e., vehicles) are always moving and in each instant, they connect to the nearest access point (a mesh node) for network provisioning. This results in dynamic changes in the traffic pattern which in turn leads to performance degradation in the system. The degree of performance reduction depends to the coverage area of the access points as well as the speed at which the vehicles move. Consider the example downtown area shown in Fig. 1. There are ten cars which move in random directions. In each instant, they connect to the nearest access point to communicate with the gateway. The network recalculates the optimal data rates and channel code rates every 5 seconds based on the most recent topology characteristics of the network. Clearly, the higher the speed of the vehicles, the larger will be the changes in the network. This can lead to a performance degradation. Fig. 11 shows the convergence of the adaptive scheme compared to the optimal value when the vehicles move with velocities of 20, 40, and 80 km/h. The rates are updated 100 times in a 500 second period. It is shown in Fig. 11 that the number of instants where the performance of the network deviates from the optimal solution increases when vehicles speed up. It is interesting that while the vehicles move in the area, the optimal solution does not change. This is because the destination for all data flows is the gateway and therefore there is a bottleneck around that node. Thus, although the source nodes change, the bottleneck remains and the achieved aggregate throughput remains unchanged. However, the allocated rates corresponding to different access points change such that the minimum throughput also remains optimal.

The average minimum throughput of the network over 20 random scenarios is shown in Fig. 12 when the speed of the vehicles changes from 10 to 100 km/h. We observe that the average performance degrades when the speed increases under adaptive channel coding because more changes occur between two successive problem reformulation. However, the perfor-

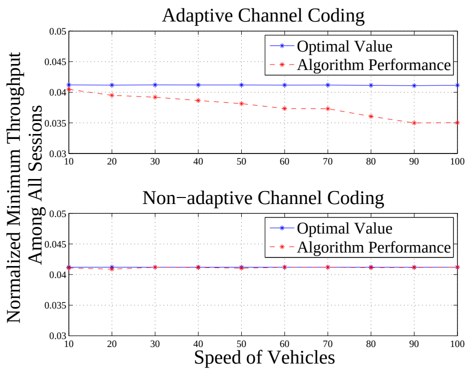

Fig. 12. Performance of the algorithm is studied in the downtown area of Fig. 1 when vehicles move with different speeds.

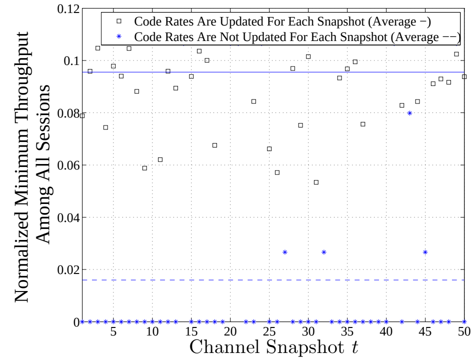

Fig. 13. Performance trend in a fading channel for 50 channel snapshots.

mance *remains optimal* under non-adaptive channel coding. This is because non-adaptive channel coding is less complex and converges faster to the final solution.

H. Impact of Fading

Finally, we study the impact of *fading* on the system performance when Algorithm 1 is used. Recall from Section III-A that we can incorporate the impact of fading by separately solving problem (8) for each wireless channel realization with fading gains f_e and corresponding cut-off rates as in (4) and (5). In this case, Algorithm 1 is invoked every time new channel measurement data becomes available. We refer to each channel measurement data as one *channel snapshot*.

Simulation results for the network topology in Fig. 2 for 50 different channel snapshots are shown in Fig. 13. In our simulation model, we generate the fading gains for each channel snapshot based on a random realization of the Rayleigh fading distribution. For the results in Fig. 13, we compare the performance of two design scenarios. The first design is an adaptive channel coding scheme based on the *average* fading information. That is, solving problem (8) only *once* by assuming that the fading gains take their average values within the Rayleigh fading distribution. On the other hand, in our second design, we solve problem (8) once for *each* channel snapshot. We can see that on average, the latter case (solid line) can improve the minimum throughput among all end-to-end sessions by a factor of 6 compared to the former one (dash line). The achieved performance improvement is at the cost of a significantly higher computational complexity due to the requirement of solving problem (8) for each snapshot, which may not always be desired in practice. The snapshots in which the minimum throughput among the sessions is zero denote the scenarios where there is at least one link in all paths of one session that has an instantaneous cutoff rate which is less than its assigned code rate. This does not happen if the code rates are updated according to the channel information in each snapshot.

In summary, we showed that the adaptive channel coding approach converges to the optimal solution in the presence of dynamic changes in the network due to channel variations and mobility. However, if the changes occur too frequently, the algorithm may fail to follow the changes fast enough and the performance degrades. On the other hand, we showed that non-adaptive channel coding is able to follow the dynamic changes and provides a high performance for the network without substantial sub-optimality.

V. Conclusion

We considered the problem of jointly using per-link channel coding in wireless networks and multipath routing. In this regard, we focused on per-link channel code rate selection and end-to-end transmission data rate allocation and formulated a max-min fairness optimization problem, which is of interest in vehicular network applications to offer fair and consistent data rates. Unlike the case of single-path routing, solving this problem in a multipath routing network is hard and involves nonconvex programming. We tackled the non-convexity by using appropriate function approximations and iterative techniques from signomial programming. We proposed a novel code and data rate selection algorithm which uses the available information related to the latest optimal solution in order to converge faster in highly changing conditions. Moreover, we studied different variations of our proposed per-link channel code rate selection and end-to-end data rate allocation algorithm in order to address both adaptive and non-adaptive channel coding and also the impact of fading. Simulation results confirm that by using channel coding jointly with multipath routing, we can significantly improve the end-to-end network performance compared to the case when only channel coding or only multipath routing is used. We also showed through simulations that as a sub-optimal approach with less complexity, non-adaptive channel coding achieves a high degree of optimality. Although our algorithm needs to be executed in a centralized manner, it can be applied in certain applications such as vehicular networks where stationary mesh nodes provide connectivity for moving vehicles. The centralized solution can also be used as a benchmark for distributed algorithms to be developed in the future. The investigation of distributed end-to-end data and channel code rate allocation approaches using Lyapunov stability theory is an interesting topic for future work. Another interesting extension of our work would be to include network coding across different end-to-end paths in our joint design which may introduce new challenges in terms of solving the formulated optimization problem.

REFERENCES

[1] C. B. R. Estrello, G. H. Valdez, and F. A. C. Perez, “System-level analysis of mobile cellular networks considering link unreliability,” *IEEE Trans. on Vehicular Technology*, vol. 58, no. 2, pp. 926–940, Feb. 2009.

[2] B. Wang, Z. He, and Y. Sun, “Distributed rate allocation and performance optimization for video communication over mesh networks,” in *Proc. of ICIP*, San Antonio, TX, Sept. 2007.

[3] A. Beljadid, A. S. Hafid, and A. Gendreau, “Design of wireless mesh networks: Expansion and reliability studies,” in *Proc. of IEEE Globecom*, New Orleans, LA, Dec. 2008.

[4] J. W. Lee, M. Chiang, and A. R. Calderbank, “Price-based distributed algorithms for rate-reliability based tradeoff in network utility maximization,” *IEEE J. Select. Areas Commun.*, vol. 24, no. 5, pp. 962–976, May 2006.

[5] Q. Gao, J. Zhang, and S. V. Hanly, “Cross-layer rate control in wireless networks with lossy links: Leaky-pipe flow, effective network utility maximization and hop-by-hop algorithms,” *IEEE Trans. on Wireless Communications*, vol. 8, no. 6, pp. 3068–3076, June 2009.

[6] M. Ghaderi, D. Towsley, and J. Kurose, “Reliability gain of network coding in lossy wireless networks,” in *Proc. of IEEE Infocom*, Phoenix, AZ, Apr. 2008.

[7] D. S. Lun, M. Médard, R. Koetter, and M. Effros, “On coding for reliable communication over packet networks,” *Physical Communication*, vol. 1, no. 1, pp. 3–20, Mar. 2008.

[8] Z. Ye, S. V. Krishnamurthy, and S. K. Tripathi, “A framework for reliable routing in mobile ad hoc networks,” in *Proc. of IEEE Infocom*, San Francisco, CA, Apr. 2003.

[9] Y. Fan, J. Zhang, and X. Shen, “Mobility-aware multi-path forwarding scheme for wireless mesh networks,” in *Proc. of IEEE WCNC*, Las Vegas, NV, Apr. 2008.

[10] S. Dulman, T. Nieberg, J. Wu, and P. Havinga, “Trade-off between traffic overhead and reliability in multipath routing for wireless sensor networks,” in *Proc. of IEEE WCNC*, New Orleans, LA, Mar. 2003.

[11] J. Huang, Z. Li, M. Chiang, and A. K. Katsaggelos, “Joint source adaptation and resource allocation for multi-user wireless video streaming,” *IEEE Trans. on Circuits and Systems for Video Technology,*, vol. 18, no. 5, pp. 582–595, May 2008.

[12] D. O’Neill, A. Goldsmith, and S. Boyd, “Wireless network utility maximization,” in *Proc. of MILCOM*, San Diego, CA, Nov. 2008.

[13] Y. Lin and V. W. S. Wong, “Adaptive tuning of MIMO-enabled 802.11e WLANs with network utility maximization,” in *Proc. of IEEE WCNC*, Las Vegas, NV, Apr. 2008.

[14] D. O’Neill, A. Goldsmith, and S. Boyd, “Optimizing adaptive modulation in wireless networks via utility maximization,” in *Proc. of IEEE ICC*, Beijing, China, May 2008.

[15] D. O’Neill, E. Akuiyibo, S. Boyd, and A. Goldsmith, “Optimizing adaptive modulation in wireless networks via multi-period network utility maximization,” in *Proc. of IEEE ICC*, Cape Town, South Africa, May 2010.

[16] J. W. Lee, M. Chiang, and A. R. Calderbank, “Distributed algorithms for optimal rate-reliability tradeoff in networks,” in *Proc. of IEEE ISIT*, Adelaide, Australia, Sept. 2005.

[17] D. O’Neill, B. S. Thian, A. Goldsmith, and S. Boyd, “Wireless NUM: Rate and reliability tradeoffs in random environments,” in *Proc. of IEEE WCNC*, Budapest, Hungary, Apr. 2009.

[18] Y. Li, M. Chiang, R. Calderbank, and S. N. Diggavi, “Optimal rate-reliability-delay tradeoff in networks with composite links,” *IEEE Trans. on Communications*, vol. 57, no. 5, pp. 1390–1401, May 2009.

[19] S. Huang and B. Mukherjee, “Adaptive reliable multi-path provisioning in WDM mesh networks,” in *Proc. of IEEE ICC*, Beijing, China, May 2008.

[20] H. Ma, D. Fayek, and P. H. Ho, “Availability-constrained multipath protection in backbone networks with double-link failure,” in *Proc. of IEEE ICC*, Beijing, China, May 2008.

[21] V. Bui, W. Zhu, A. Botta, and A. Pescape, “An MDP-based approach for multipath data transmission over wireless networks,” in *Proc. of IEEE ICC*, Beijing, China, May 2008.

[22] I. Cidon, R. Rom, and Y. Shavitt, “Analysis of multi-path routing,” *IEEE/ACM Trans. on Networking*, vol. 7, no. 6, pp. 885–896, Dec. 1999.

[23] K. Ronasi, A. H. Mohsenian-Rad, V. W. S. Wong, S. Gopalakrishnan, and R. Schober, “Optimal data and channel code rate allocation in wireless networks with multi-path routing,” in *Proc. of IEEE ICC*, Kyoto, Japan, June 2011.

[24] ——, “Delay-throughput enhancement in wireless networks with multi-path routing and channel coding,” *IEEE Trans. on Vehicular Technology*, vol. 60, no. 3, pp. 1116–1123, Mar. 2011.

[25] J. G. Proakis, *Digital Communications*. 4th edition, New York: McGraw-Hill, 2001.

[26] R. Gupta and J. Walrand, “Approximating maximal cliques in ad-hoc networks,” in *Proc. of IEEE PIMRC*, Barcelona, Spain, Sept. 2004.

[27] C. Shannon, “A theorem on coloring the lines in the network,” *J. Math. Phys.*, vol. 28, pp. 148–151, Sept. 1949.

[28] S. Boyd, S. J. Kim, L. Vandenberghe, and A. Hassibi, *A Tutorial on Geometric Programming*. Springer Science and Business Media, 2007.

[29] M. Chiang, *Geometric Programming for Communication Systems*. NOW, 2005.

[30] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[31] Y. Nesterov and A. Nemirovsky, *Interior Point Polynomial Algorithms in Convex Programming*. SIAM, 1994.

[32] “MOSEK software.” [Online]. Available: http://www.mosek.com

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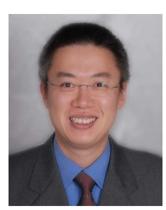

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