# Full-duplex Relaying for D2D Communication in mmWave based 5G Networks

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***Abstract*—Device-to-device (D2D) communication, which can offload data from base stations by direct transmission between mobile devices, is a promising technology for the fifth generation (5G) wireless networks. However, the limited battery capacity of mobile devices is a barrier to fully exploit the benefits of D2D communication. Meanwhile, high data rate D2D communication is required to support the increasing traffic demand of emerging applications. In this paper, we study relay-assisted D2D communication in millimeter wave (mmWave) based 5G networks to address these issues. Multiple D2D user pairs are assisted by full-duplex relays that are equipped with directional antennas. To design an efficient relay selection and power allocation scheme, we formulate a multi-objective combinatorial optimization problem, which balances the trade-off between total transmit power and system throughput. The problem is transformed into a weighted bipartite matching problem. We then propose a centralized relay selection and power allocation algorithm and prove that it can achieve a Pareto optimal solution in polynomial time. We further propose a distributed algorithm based on stable matching. Simulation results show that our proposed algorithms substantially reduce the total transmit power and improve the system throughput compared to two existing algorithms in the literature.**

***Index Terms*—Full-duplex relaying, D2D communication, multi-objective optimization, matching theory.**

## I. Introduction

Device-to-device (D2D) communication, which is regarded as a promising technology for the fifth generation (5G) wireless networks, allows mobile devices to communicate with each other directly. D2D communication can provide high data rate transmission and offload data traffic from cellular base stations [1]. Relays in D2D networks can further reduce the energy consumption of mobile devices, enhance the quality of data transmission, assist connection establishment among devices, and increase the range of D2D communication [2]. The recently developed full-duplex techniques allow relays to simultaneously transmit and receive signals by enabling loop-interference cancellation. By using full-duplex relaying in D2D communication, where relays operate in full-duplex mode, the spectrum efficiency can be improved over traditional half-duplex relaying systems [3], [4].

Resource allocation in D2D networks has been widely studied in the literature [5]–[11]. Li *et al.* in [5] investigated uplink resource allocation for D2D networks. They proposed a scheme for transmission mode selection and resource sharing by formulating a coalition formation game. In [6], Wang *et al.* studied a joint channel and power allocation problem for D2D communication and proposed an iterative algorithm to optimize the energy efficiency. Qiao *et al.* in [7] proposed a resource sharing scheme to enable concurrent transmissions for D2D communication in millimeter wave (mmWave) wireless networks. In [8], Nguyen *et al.* proposed fair scheduling policies to exploit spatial and frequency reuse in D2D-enabled cellular systems. Xing *et al.* in [9] proposed resource management algorithms to improve the spectrum efficiency of D2D communication in cellular networks. In [10], Sheng *et al.* proposed an iterative algorithm based on fractional programming and Lyapunov optimization to balance the trade-off between energy efficiency and delay in D2D communication. Zhang *et al.* in [11] studied the channel allocation problem by using hypergraph theory, where D2D users are allowed to share the uplink channels with cellular users.

Several existing studies show that relaying can enhance the performance of D2D communication [12]–[16]. In [12], Shi *et al.* studied the energy-efficient spectrum sharing problem in D2D wireless networks. They proposed a mechanism for relay-assisted networks to optimize energy consumption and channel utilization. Hasan *et al.* studied a multi-user relay-assisted D2D network in [13] and formulated a robust optimization problem by considering the channel uncertainties. They showed that relay-assisted communication can improve the aggregate data rate. Wang *et al.* in [14] studied the feasibility of enabling full-duplex capability to D2D communication in heterogeneous networks. They provided solutions to address the interference mitigation issue in full-duplex D2D networks. In [15], Zhang *et al.* proposed a power allocation scheme which aims to maximize the data rate of D2D users in a relay-assisted D2D network underlaying the cellular system. Al-Hourani *et al.* in [16] derived a closed-form expression for energy saving geometrical zone where relaying is efficient. The aforementioned existing works reveal the benefits of using relays in D2D communication. Nevertheless, none of them jointly consider the importance of energy saving for battery-operated devices and system throughput for high data rate applications.

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Similar resource allocation problems are studied in cognitive radio networks (CRNs). Gharehshiran *et al*. in [17] considered subchannel allocation for orthogonal frequency-division multiplexing (OFDM) based CRNs. Xu *et al*. in [18] proposed flexible cooperation schemes to coordinate the primary and secondary users in orthogonal frequency-division multiple access (OFDMA) based CRNs. In [19], Dadallage *et al.* formulated a joint beamforming, power, and channel allocation problem in CRNs. However, the aforementioned resource allocation problems in CRNs do not consider the characteristics of full-duplex relay-assisted D2D communication with directional transmission [17]–[19]. Therefore, it is essential to utilize the distinct features of full-duplex relay-assisted communications to improve the system throughput and energy saving for D2D communication.

D2D communication, which can offload data from the base station and utilize possible direct inter-device connections, is promising for 5G systems. However, the limited battery capacity of mobile devices and occasional poor link quality may affect the aforementioned benefits of D2D communication. Full-duplex relays with mmWave technology and directional transmission can save the power of mobile devices and improve the link quality and data rate. Furthermore, the relay selection schemes proposed for half-duplex relays may not be efficient in full-duplex relay-assisted D2D communication, since loop-interference does not exist in half-duplex relaying systems. Utilizing an efficient relay selection scheme designed for full-duplex systems is essential to reduce the transmit power of mobile devices, extend their battery lifetime, and increase the data rate. Nonetheless, most of the existing works (e.g., [12]–[19]) do not consider the features of mmWave based full-duplex relaying in D2D communication.

In this paper, we study relay-assisted D2D communication for 5G wireless networks, where mobile devices transmit data using directional antennas in mmWave frequency band. We consider that the relays have full-duplex capability, while D2D users transmit in half-duplex mode. The relays can assist device discovery, connection establishment, and data transmission for the potential D2D communication. We also consider that loop-interference cannot be fully eliminated due to imperfect self channel estimation and hardware constraints. The contributions of this paper are as follows:

- *Full-duplex relaying in D2D communication*: We design a joint relay selection and power allocation scheme for full-duplex relay-assisted D2D communication in mmWave based wireless networks. We formulate a multi-objective combinatorial optimization problem, which considers the impact of loop-interference in full-duplex relaying systems. The formulated problem aims to reduce the total transmit power and improve the system throughput while satisfying certain quality of service (QoS) requirements and physical constraints.
- *Low complexity centralized and distributed algorithms based on matching theory*: We first transform the problem into a one-to-one weighted bipartite matching problem. We then propose a centralized algorithm that solves the matching problem optimally in polynomial time. We prove that the centralized algorithm achieves a Pareto optimal solution of the multi-objective optimization problem. Based on stable matching, we further propose a distributed algorithm, which has a lower information exchange overhead than the centralized approach.
- *Enhanced performance*: Through extensive simulations, we evaluate the performance of our proposed algorithms under different network settings and compare them with existing algorithms proposed in the literature. Simulation results illustrate the trade-off between total transmit power and system throughput. Results also show that our proposed algorithms improve the total transmit power and system throughput substantially compared to the algorithms proposed in [20] and [21].

We extend the work in [22] from several aspects. We improve the channel modeling by considering the possible non-line-of-sight (NLOS) communications. In [22], the objective is to minimize the power consumption. In this paper, we also consider the system throughput as an important design goal. Different from the single-objective problem formulation in [22], our multi-objective optimization problem balances the trade-off between total transmit power and system throughput. Our proposed centralized algorithm is proved to achieve a Pareto optimal solution in polynomial time. We further propose a distributed algorithm to reduce the information exchange overhead compared to the centralized algorithm.

This paper is organized as follows. In Section II, we present the system model. In Section III, we formulate the joint relay selection and power allocation problem and transform the problem into a one-to-one weighted matching problem. We also propose the centralized and distributed algorithms in Section III. Simulation results are presented in Section IV. Conclusion is given in Section V. The key notations and variables used in this paper are listed in Table I.

## II. System Model

We consider an mmWave cellular network with full-duplex relay-assisted D2D communication as shown in Fig. 1. Full-duplex relays are deployed to assist D2D users[1] who either suffer from poor direct link quality or require an extended communication range. The base station sends control messages through the control channel to D2D users and relays to coordinate the resource allocation process. We denote the set of relays as $\mathcal{R}$. The sets of source and destination devices which are assisted by relays are denoted as $\mathcal{S}$ and $\mathcal{D}$, respectively, where $\mathcal{S} \bigcap \mathcal{D} = \emptyset$. We further denote the set of source-destination D2D user pairs as $\mathcal{L}$. The $i^{\text{th}}$ source device $s_i \in \mathcal{S}$ and the $i^{\text{th}}$ destination device $d_i \in \mathcal{D}$ form a source-destination user pair $l_i = (s_i, d_i) \in \mathcal{L}$.

We assume that the base station, D2D users, and relays are with the same service provider. The relays which assist D2D user pairs share parts of the channel resources that the service provider owns, while those users that do not require assistance from relays use different spectral resources. A relay can assist multiple D2D communication pairs using different channels. Each relay is equipped with two sets of antennas that

[1] In the remaining parts of this paper, we use the terms "device" and "user" interchangeably.

TABLE I
LIST OF KEY NOTATIONS AND VARIABLES.

| Symbols | Meaning |
|---|---|
| $B$ | Bandwidth of each channel |
| $C_{l_i,r_j}$ | Throughput of D2D user pair $l_i$ assisted by relay $r_j$ |
| $C_{l_i}^{\text{min}}$ | Minimum throughput requirement for D2D user pair $l_i$ |
| $\mathcal{D}$ | Set of destination devices |
| $\mathcal{E}$ | Set of edges |
| $\mathcal{F}$ | A matching |
| $\mathcal{G}$ | Bipartite graph |
| $h_{i,j}$ | Channel gain between transmitter $i$ and receiver $j$ |
| $h_{LI}$ | Loop-interference channel gain |
| $\mathcal{L}$ | Set of source-destination user pairs |
| $L(z)$ | Path loss function |
| $N_0$ | Background noise power |
| $N_{r_j}$ | Number of channels that relay $r_j$ can use |
| $P_{s_i,r_j}$ | transmit power of source device $s_i$ to relay $r_j$ |
| $P_{r_j,d_i}$ | transmit power of relay $r_j$ to destination device $d_i$ |
| $P_r^{\text{max}}$ | Maximum transmit power of each relay |
| $P_s^{\text{max}}$ | Maximum transmit power of a source device |
| $\boldsymbol{P}_s$ | transmit power matrix of source devices |
| $\mathcal{R}$ | Set of relays |
| $\mathcal{R}^v$ | Set of virtual relays |
| $\mathcal{R}_{l_i}^v$ | Set of virtual relays that D2D user pair $l_i$ requests |
| $\mathcal{S}$ | Set of source devices |
| $w_{l_i,r_j}$ | Weight of edge $(l_i, r_j)$ |
| $\boldsymbol{W}$ | Weighting matrix of the edges |
| $\boldsymbol{W}_{l_i}$ | Weighting vector of the feasible edges of D2D user pair $l_i$ |
| $x_{l_i,r_j}$ | Relay selection indicator for user pair $l_i$ and relay $r_j$ |
| $\boldsymbol{X}$ | Relay selection matrix |
| $\lambda_1$ | Non-negative coefficient for total transmit power |
| $\lambda_2$ | Non-negative coefficient for system throughput |

enable full-duplex operation. Decode-and-forward protocol is employed by the relays. As discussed in 3GPP specification [23, pp. 10] and related studies [24]–[26], each D2D user pair is assigned a non-overlapping orthogonal channel in dedicated mode, where interference among users is avoided. In this paper, the D2D user pairs communicate in dedicated mode and are allocated non-overlapping orthogonal channels.

### A. Channel Model

Recently, over 10 GHz of spectrum above 24 GHz has been made available by the Federal Communications Commission (FCC) for 5G wireless communications [27]. One of the mmWave frequency bands is the 38 GHz band [27]–[29]. We consider a relay-assisted D2D communication system that operates on the frequency of 38 GHz. This band is selected based on its signal propagation feature and licensing issue. The channel model of mmWave communication is different from the current cellular channel model. One important difference is that mmWave communications require directional antennas. We adopt the channel model introduced in [30] and consider the impact of blockage and reflections for both line-of-sight (LOS) and NLOS cases. If the LOS is blocked, we consider potential NLOS communications due to reflections [31], [32]. Let $z$ denote the distance between the transmitter and receiver.

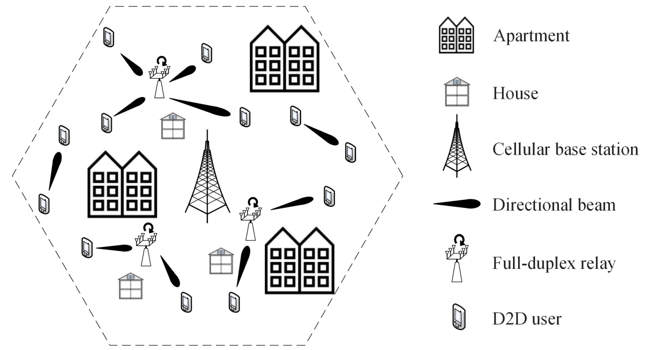



Fig. 1. An mmWave based D2D network with full-duplex relays and directional transmissions. D2D user pairs can choose whether or not to use full-duplex relays. The base station coordinates the resource allocation in the network.

The path loss function $L(z)$ in dB is

$$L(z) = \begin{cases} \overline{L_{\text{LOS}}}(z_0) + 10\alpha_{\text{LOS}} \log(z) + Z_{\sigma_{\text{LOS}}}, \\ \text{if LOS exists,} \\ \overline{L_{\text{NLOS}}}(z_0) + 10\alpha_{\text{NLOS}} \log(z) + Z_{\sigma_{\text{NLOS}}}, \\ \text{if LOS is blocked,} \end{cases} \tag{1}$$

where $\overline{L_{\text{LOS}}}(z_0)$ and $\overline{L_{\text{NLOS}}}(z_0)$ are the free-space path loss at reference distance $z_0$ for LOS and NLOS signals, respectively. Moreover, $\alpha_{\text{LOS}}$ and $\alpha_{\text{NLOS}}$ are the path loss exponents for LOS and NLOS cases, and $Z_{\sigma_{\text{LOS}}}$ and $Z_{\sigma_{\text{NLOS}}}$ are zero-mean Gaussian random variables with standard deviations $\sigma_{\text{LOS}}$ and $\sigma_{\text{NLOS}}$ that model the shadowing effects of LOS and NLOS environments, respectively. Channel estimation is used to determine the aforementioned parameters. The transmitter and receiver perform channel estimation periodically via transmitting and analyzing orthogonal pilot sequences [33].

In mmWave technology, directional antenna is used to improve the antenna gain. A sectored directional transmitting antenna model is proposed in [34]. We use this sectored antenna model, where the antennas achieve a constant high gain in the main lobe and a constant low gain in the side lobe. Let $\Theta^t$ represent the angle of departure of signals. The transmitting antenna gain is given as follows:

$$G^t(\Theta^t) = \begin{cases} M^t, & 0^\circ \le \Theta^t \le \Theta^t_{\text{HPBW}}, \\ m^t, & \Theta^t_{\text{HPBW}} < \Theta^t \le 180^\circ, \end{cases} \tag{2}$$

where $M^t, m^t$, and $\Theta^t_{\text{HPBW}}$ are the main lobe gain, side lobe gain, and half power beamwidth for the transmitting antenna, respectively. Similarly, let $\Theta^r$ represent the angle of arrival of signals. The receiving antenna gain is given as follows:

$$G^r(\Theta^r) = \begin{cases} M^r, & 0^\circ \le \Theta^r \le \Theta^r_{\text{HPBW}}, \\ m^r, & \Theta^r_{\text{HPBW}} < \Theta^r \le 180^\circ, \end{cases} \tag{3}$$

where $M^r, m^r$, and $\Theta^r_{\text{HPBW}}$ are the main lobe gain, side lobe gain, and half power beamwidth for the receiving antenna, respectively. The antenna gain between devices $i$ and $j$ is denoted as $G_{i,j} = G^t(\Theta^t_{i,j})G^r(\Theta^r_{j,i})$, where $\Theta^t_{i,j}$ is the angle of departure of signal from transmitter $i$ to receiver $j$, and $\Theta^r_{j,i}$ is the angle of arrival of signal in receiver $j$ sent from transmitter $i$. If $i$ and $j$ belong to a pair of communicating devices, $\Theta_{i,j}^t$ and $\Theta_{j,i}^r$ are both $0^o$, since we assume the transmitting and receiving antennas are accurately aligned. The total gain (including channel and antenna gains) between devices $i$ and $j$ can be represented as $h_{i,j} = G_{i,j}/L(z_{i,j})$, where $z_{i,j}$ is the distance between the corresponding devices.

### B. Throughput of a User Pair

To obtain the throughput of a user pair, we need to determine the signal-to-interference plus noise ratio (SINR) in each hop of the relayed communication. For user pair $l_i = (s_i, d_i)$ which is assisted by relay $r_j$, we denote $P_{s_i,r_j}$ as the transmit power of source device $s_i$ to relay $r_j$. We denote the transmit power of relay $r_j$ to destination device $d_i$ as $P_{r_j,d_i}$. The SINR from source $s_i$ to relay $r_j$ is

$$\mathrm{SINR}_{s_i,r_j} = \frac{h_{s_i,r_j} P_{s_i,r_j}}{h_{LI} P_{r_j,d_i} + N_0}, \tag{4}$$

where $h_{LI} P_{r_j,d_i}$ represents the loop-interference received by full-duplex relay $r_j$ and $N_0$ is the noise power. Similar to [15], [35], [36], we use the loop-interference channel gain $h_{LI}$ to determine the loop-interference power received by the full-duplex relay. The channel gain $h_{LI}$ is defined as the ratio between the received loop-interference power and transmit power of the full-duplex relay. It characterizes the result of power leakage from the transmitter of the full-duplex relay to its receiver due to imperfect loop-interference cancellation. The mutual interference among different user pairs is avoided since each user pair is allocated an orthogonal channel. The SINR from relay $r_j$ to destination device $d_i$ is

$$\mathrm{SINR}_{r_j,d_i} = \frac{h_{r_j,d_i} P_{r_j,d_i}}{h_{s_i,d_i} P_{s_i,r_j} + N_0}, \tag{5}$$

where $h_{s_i,d_i} P_{s_i,r_j}$ is the interference induced by source device $s_i$. Note that both the source-to-relay SINR and relay-to-destination SINR have included the effects of directional transmission and possible NLOS due to reflection in mmWave communications. In a full-duplex relaying system using decode-and-forward protocol, the throughput of user pair $l_i \in \mathcal{L}$ assisted by relay $r_j \in \mathcal{R}$ can be obtained from [35]

$$B \min\left(\log_2(1 + \mathrm{SINR}_{s_i,r_j}), \log_2(1 + \mathrm{SINR}_{r_j,d_i})\right), \tag{6}$$

where $B$ is the bandwidth of each channel.

## III. Problem Formulation

In this section, we formulate a joint relay selection and power allocation problem by taking both the total transmit power and the system throughput into consideration. This is motivated from the fact that the limited battery capacity of mobile devices requires a low transmit power, while applications such as video sharing require a high throughput data transmission. Our objective is to minimize the total transmit power and simultaneously maximize the system throughput of the D2D network. We formulate the problem as a multi-objective optimization problem to consider both of the aforementioned factors. We consider the impact of loop-interference in full-duplex relaying systems and derive a closed-form expression of the throughput of a user pair. Note that for full-duplex relays, a higher transmit power of a relay increases the received power in the destination device of D2D user pairs. However, it also induces a higher loop-interference at the same time. This means that transmitting with full power does not necessarily increase the throughput. According to (6), the throughput of a user pair assisted by a relay is the minimum throughput of the source-to-relay and relay-to-destination links. Thus, the transmit power of the source device is wasted if the source-to-relay throughput is greater than the relay-to-destination throughput. The same situation holds regarding the transmit power of the relays. In other words, the power of a full-duplex relay can be adjusted according to the allocated transmit power of the source devices. The source-to-relay throughput should be equal to the relay-to-destination throughput in order to save transmit power [36], [37]. This helps us to obtain a closed-form expression for the source-to-destination throughput. Consider D2D user pair $l_i = (s_i, d_i) \in \mathcal{L}$ and relay $r_j \in \mathcal{R}$, the above condition implies that $\mathrm{SINR}_{s_i,r_j} = \mathrm{SINR}_{r_j,d_i}$. In this case, from (4) and (5), $P_{r_j,d_i}$ can be expressed as a function of $P_{s_i,r_j}$ as follows:

$$P_{r_j,d_i}(P_{s_i,r_j}) = \frac{\sqrt{f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h_{r_j,d_i}^2} - N_0 h_{r_j,d_i}}{2 h_{LI} h_{r_j,d_i}}, \tag{7}$$

where

$$\begin{aligned} &f_{r_j,d_i}(P_{s_i,r_j}) \\ &= 4 h_{s_i,r_j} h_{LI} h_{r_j,d_i} h_{s_i,d_i} + 4 N_0 h_{s_i,r_j} h_{LI} h_{r_j,d_i} P_{s_i,r_j}. \end{aligned} \tag{8}$$

Therefore, when we substitute (5) and (7) into (6), the throughput of user pair $l_i \in \mathcal{L}$ assisted by relay $r_j \in \mathcal{R}$ can be expressed as follows:

$$C_{l_i,r_j}(P_{s_i,r_j}) = B \log_2\left(1 + \frac{h_{r_j,d_i} P_{r_j,d_i}(P_{s_i,r_j})}{h_{s_i,d_i} P_{s_i,r_j} + N_0}\right). \tag{9}$$

To introduce our objectives, we consider the transmit power of mobile devices. Since the relays are plugged into the power source and have sufficient power supply, the objective is to minimize the total transmit power of the transmitting D2D devices by selecting relays efficiently. We denote $\boldsymbol{P}_s = (P_{s_i,r_j})_{s_i \in \mathcal{S}, r_j \in \mathcal{R}}$ as the transmit power matrix. The total transmit power can be represented as:

$$f_1(\boldsymbol{P}_s) = \sum_{s_i \in \mathcal{S}} \sum_{r_j \in \mathcal{R}} P_{s_i,r_j}. \tag{10}$$

Another important design objective is to maximize the system throughput. Maximizing the system throughput is equivalent to minimizing the following function:

$$f_2(\boldsymbol{P}_s) = -\sum_{l_i \in \mathcal{L}} \sum_{r_j \in \mathcal{R}} C_{l_i,r_j}(P_{s_i,r_j}). \tag{11}$$

We should note that different power consumption models will result in different trade-off between transmit power and throughput. However, our proposed mechanism is applicable to general power consumption models. We will later address the trade-off between transmit power and throughput when formulating the multi-objective optimization problem.

Both design objectives in (10) and (11) are important. A single-objective formulation that solely considers either (10) or (11) is insufficient to capture the complete design goals of the system. However, these objectives are conflicting in the sense that a higher throughput may result in a higher transmit power. This fact motivates us to formulate a multi-objective problem, which considers both objectives simultaneously.

To formulate the multi-objective optimization problem, we introduce the QoS requirements for different applications as well as the physical constraints of the devices and relays. We denote $C_{l_i}^{\min}$ as the minimum throughput requirement for D2D user pair $l_i$. To guarantee that the minimum data rate requirement is satisfied for each D2D user pair, we introduce the following constraint:

$$\sum_{r_j \in \mathcal{R}} C_{l_i,r_j}(P_{s_i,r_j}) \geq C_{l_i}^{\min}, \quad \forall\ l_i = (s_i, d_i) \in \mathcal{L}. \tag{12}$$

We define matrix $\boldsymbol{X} = (x_{l_i,r_j})_{l_i \in \mathcal{L}, r_j \in \mathcal{R}}$ to indicate the relay selection for the user pairs, where binary variable $x_{l_i,r_j} = 1$ if user pair $l_i$ selects relay $r_j$. Otherwise, $x_{l_i,r_j} = 0$. Then, the following constraint ensures that each user pair can be assisted by only one relay.

$$\sum_{r_j \in \mathcal{R}} x_{l_i,r_j} = 1, \quad \forall\ l_i \in \mathcal{L}. \tag{13}$$

Furthermore, the number of D2D user pairs assisted by relay $r_j$ should be less than or equal to the number of channels that $r_j$ can use. Let $N_{r_j}$ denote the number of channels in relay $r_j$. We have

$$\sum_{l_i \in \mathcal{L}} x_{l_i,r_j} \leq N_{r_j}, \quad \forall\ r_j \in \mathcal{R}. \tag{14}$$

We denote $P_s^{\max}$ as the maximum transmit power of each mobile device, and $P_r^{\max}$ as the maximum transmit power of each relay. To ensure the transmit powers of mobile devices and relays do not exceed the maximum transmit power allowed, we introduce the following constraints:

$$0 \leq P_{s_i,r_j} \leq x_{l_i,r_j} P_s^{\max}, \forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}, \tag{15}$$

$$0 \leq P_{r_j,d_i}(P_{s_i,r_j}) \leq x_{l_i,r_j} P_r^{\max}, \forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}. \tag{16}$$

Notice that since $P_{s_i,r_j} \geq 0$, $P_{r_j,d_i}(P_{s_i,r_j})$ is always non-negative. Let $P_{r_j,d_i}^{-1}(\cdot)$ denote the inverse of function $P_{r_j,d_i}(P_{s_i,r_j})$. Since $P_{r_j,d_i}(P_{s_i,r_j})$, given in (7), is strictly increasing, the inverse function always exists. Thus, from (16), we have

$$0 \leq P_{s_i,r_j} \leq P_{r_j,d_i}^{-1}(x_{l_i,r_j} P_r^{\max}), \forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}. \tag{17}$$

When $x_{l_i,r_j} = 1$, $P_{r_j,d_i}^{-1}(x_{l_i,r_j} P_r^{\max}) = P_{r_j,d_i}^{-1}(P_r^{\max})$, while $P_{s_i,r_j} = 0$ when $x_{l_i,r_j} = 0$. Therefore, inequality (17) can be rewritten as:

$$0 \leq P_{s_i,r_j} \leq x_{l_i,r_j} P_{r_j,d_i}^{-1}(P_r^{\max}), \forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}. \tag{18}$$

Consequently, constraint (16) is equivalent to the following constraint:

$$0 \leq P_{s_i,r_j} \leq x_{l_i,r_j} \widetilde{P}_s^{\max}, \forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}, \tag{19}$$

where constant

$$\widetilde{P}_s^{\max} = P_{r_j,d_i}^{-1}(P_r^{\max}). \tag{20}$$

By combining (15) and (19), we have

$$\begin{aligned} &0 \leq P_{s_i,r_j} \leq x_{l_i,r_j} \min\left(P_s^{\max}, \widetilde{P}_s^{\max}\right), \\ &\forall\ l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}. \end{aligned} \tag{21}$$

Then, the multi-objective relay selection and power allocation problem can be formulated as:

$$\underset{\boldsymbol{X}, \boldsymbol{P}_s}{\text{minimize}} \quad F(\boldsymbol{P}_s) = \left(f_1(\boldsymbol{P}_s), f_2(\boldsymbol{P}_s)\right)^T \tag{22a}$$

$$\text{subject to} \quad x_{l_i,r_j} \in \{0,1\}, \quad \forall\ l_i \in \mathcal{L}, r_j \in \mathcal{R}, \tag{22b}$$
$$\text{constraints} \ \ (12) - (14),\ \text{and}\ (21).$$

Problem (22) is a multi-objective combinatorial optimization problem. The weighted sum approach is commonly used to transform a multi-objective optimization problem into a scalar optimization problem [38]. Note that in problem (22), we take the impact of the loop-interference channel gain into consideration when introducing constraints (12) and (21). In addition, the directional antenna gain and channel gain for mmWave communications are incorporated in the data rate in constraint (12).

### A. Reformulation Using the Weighted Sum Method

In this subsection, we reformulate problem (22) using the weighted sum method. The weighted sum approach considers a linear combination of all design objectives and is commonly used in solving multi-objective optimization problems [39]. *Pareto optimality* is an important solution concept for the multi-objective optimization problems. An outcome is Pareto optimal when a single design objective (e.g, $f_1(\boldsymbol{P}_s)$) cannot be improved without degrading the other objective (e.g, $f_2(\boldsymbol{P}_s)$). Problem (22), which considers both total transmit power and system throughput, can be reformulated as a weighted sum problem. The Pareto optimal solution of problem (22) can be obtained by solving the following problem:

$$\underset{\boldsymbol{X}, \boldsymbol{P}_s}{\text{minimize}} \quad \lambda_1 f_1(\boldsymbol{P}_s) + \lambda_2 f_2(\boldsymbol{P}_s) \tag{23}$$
$$\text{subject to} \ \ \text{constraints} \ \ (12) - (14),\ (21),\ \text{and}\ (22b),$$

where $\lambda_1$ and $\lambda_2$ are non-negative coefficients to adjust the weights of objectives $f_1$ and $f_2$. For example, if $\lambda_1 = 1, \lambda_2 = 0$, problem (23) minimizes the total transmit power. By changing the value of $\lambda_1$ and $\lambda_2$, different Pareto optima can be obtained. Problem (23) can be solved using methods such as branch-and-bound, generalized Benders decomposition, or outer approximation. However, none of the aforementioned methods can guarantee to obtain the solution in polynomial time. In the next subsection, we transform the multi-objective combinatorial optimization problem into a matching problem [40] to obtain the Pareto optimal solution. We then propose a Pareto optimal relay selection and power allocation algorithm.

### B. Bipartite Graph Construction

Matching theory can provide tractable solutions for combinatorial problems. For resource allocation in wireless net-

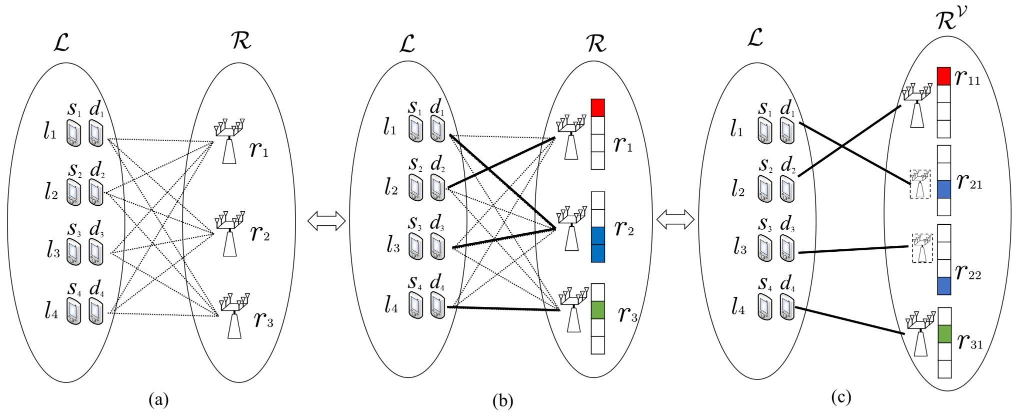




Fig. 2. (a) A bipartite graph with four D2D user pairs and three relays. (b) A matching example with four edges $(l_1, r_2), (l_2, r_1), (l_3, r_2)$, and $(l_4, r_3)$. (c) The one-to-one matching with virtual relays in dashed frames.

works, matching theory can address how resources can be allocated to users [40]. Users and resources are considered as vertices in disjoint sets that will be matched to each other. In this paper, we regard vertex sets as the set of D2D user pairs $\mathcal{L}$ and the set of relays $\mathcal{R}$. We consider all possible relay selections as different *matchings*. The goal is to find the best matching (i.e., relay selection) between D2D user pairs and relays, which results in the optimal solution of problem (23) and is also the Pareto optimal solution of problem (22). To achieve this goal, we construct a *bipartite graph*, which consists two disjoint vertex sets and edges, as shown in Fig. 2(a). A matching is represented by a set of distinct edges. We use tuple $(l_i, r_j)$ to denote the edge that connects D2D user pair $l_i$ with relay $r_j$. For instance, as shown in Fig. 2(b), the graph with four solid edges $(l_1, r_2), (l_2, r_1), (l_3, r_2)$, and $(l_4, r_3)$ corresponds to a matching example. A weight is allocated to each edge of the graph. The purpose of constructing the weighted graph is to transform problem (23) into an equivalent matching problem. Thus, we will determine the weight of each edge in order to achieve this goal. We also define the minimum weighted matching as a matching where the sum of the weights of those edges selected in the matching has the minimum value. We then obtain the minimum weighted matching, from which we can determine the optimal solution of problem (23).

To determine the weight of each edge in the graph, we first introduce the matching rules. These rules guarantee that the optimal matching is within the feasible region of problem (23). By considering constraint (13), only a single edge can be connected with a D2D user pair in the matching. Meanwhile, we allow at most $N_{r_j}$ edges to be connected with relay $r_j \in \mathcal{R}$ in order to satisfy constraint (14). Constraints (13) and (14) indicate that the equivalent matching problem is a many-to-one matching. We further consider (12) and (21) which are related to the transmit power variables. According to (13), each D2D user pair can only be assisted by one relay. We assume that D2D user pair $l_i$ is assisted by relay $r_j$ (i.e., $x_{l_i,r_j} = 1,\ x_{l_i,r_k} = 0,\ \forall r_k \in \mathcal{R} \setminus \{r_j\}$). If there exists a transmit power $P_{s_i,r_j}$ that satisfies both of the following inequalities:

$$C_{l_i,r_j}(P_{s_i,r_j}) \geq C_{l_i}^{\min}, \tag{24}$$

and

$$0 \leq P_{s_i,r_j} \leq \min\left(P_s^{\max}, \widetilde{P}_s^{\max}\right), \tag{25}$$

then $P_{s_i,r_j}$ is in the feasible region determined by constraints (12) and (21), and $r_j$ is a feasible relay for D2D user pair $l_i$. Otherwise, we regard relay $r_j$ as an infeasible relay. That is, D2D user pair $l_i$ will not use relay $r_j$. In this case, infeasible relay $r_j$ is excluded from the consideration of D2D user pair $l_i$, and edge $(l_i, r_j)$ will not be selected in the matching. By doing so, the optimal matching will be in the feasible region of problem (23) and satisfies all of its constraints.

To find the feasible relays for a D2D user pair, we first consider (24) and (25). By substituting (9) into (24), we have

$$B\log_2\left(1 + \frac{h_{r_j,d_i}P_{r_j,d_i}(P_{s_i,r_j})}{h_{s_i,d_i}P_{s_i,r_j} + N_0}\right) \geq C_{l_i}^{\min}, \tag{26}$$

which can be rewritten as

$$\frac{h_{r_j,d_i}P_{r_j,d_i}(P_{s_i,r_j})}{h_{s_i,d_i}P_{s_i,r_j} + N_0} \geq 2^{(C_{l_i}^{\min}/B)} - 1. \tag{27}$$

Then, by substituting (7) into (27), we obtain

$$\frac{\sqrt{f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h_{r_j,d_i}^2} - N_0 h_{r_j,d_i}}{2h_{LI}(h_{s_i,d_i}P_{s_i,r_j} + N_0)} \geq 2^{(C_{l_i}^{\min}/B)} - 1. \tag{28}$$

The left-hand side of (28) is an increasing function of $P_{s_i,r_j}$. When relay $r_j$ is selected to assist user pair $l_i$, we can find the minimum transmit power of source device $s_i$. To achieve the minimum data rate requirement $C_{l_i}^{\min}$, the minimum transmit power of source device $s_i$ with assistance of relay $r_j$, denoted by $P_{s_i,r_j}^{\min}$, can be obtained from (28). We have

$$P_{s_i,r_j}^{\min} \triangleq \frac{h_{LI}N_0\left(2^{(C_{l_i}^{\min}/B)} - 1\right)^2 + N_0 h_{r_j,d_i}\left(2^{(C_{l_i}^{\min}/B)} - 1\right)}{h_{s_i,r_j}h_{r_j,d_i} - h_{LI}h_{s_i,d_i}\left(2^{(C_{l_i}^{\min}/B)} - 1\right)^2}. \tag{29}$$

From (25), we can determine the maximum transmit power of source $s_i$ to relay $r_j$ denoted by $P_{s_i,r_j}^{\max}$, where

$$P_{s_i,r_j}^{\max} \triangleq \min\left(P_s^{\max}, \widetilde{P}_s^{\max}\right). \tag{30}$$

If $P_{s_i,r_j}^{\min} > P_{s_i,r_j}^{\max}$, there is no feasible transmit power that satisfies both (24) and (25). In this case, relay $r_j$ is an infeasible relay for D2D user pair $l_i$ and edge $(l_i, r_j)$ is an infeasible edge in the graph. In order to exclude such infeasible edge, we set its weight to $+\infty$ so that the edge will not be considered when using the minimum weighted matching method. After excluding all infeasible edges from consideration, the remaining edges are the feasible relays for the corresponding D2D user pairs.

We now determine the weight of each feasible edge in the graph. We assume $r_j$ is a feasible relay for D2D user pair $l_i$ (i.e, $P_{s_i,r_j}^{\min} \le P_{s_i,r_j}^{\max}$). We denote the weight of edge $(l_i, r_j)$ by $w(l_i, r_j)$, where $l_i = (s_i, d_i) \in \mathcal{L},\ r_j \in \mathcal{R}$. To determine $w(l_i, r_j)$, we assume that only D2D user pair $l_i$ and its feasible relay $r_j$ exist in the network. In this case, the bipartite graph only has one edge (i.e., edge $(l_i, r_j)$). This assumption implies that $x_{l_i,r_j} = 1$, $x_{l_m,r_n} = 0$ and $P_{s_m,r_n} = 0$ for all $(l_m, r_n) \neq (l_i, r_j), l_m = (s_m, d_m) \in \mathcal{L}, r_n \in \mathcal{R}$. We can obtain the optimal transmit power of source device $s_i$ by solving the following problem:

$$\underset{P_{s_i,r_j}}{\text{minimize}} \quad \lambda_1 P_{s_i,r_j} - \lambda_2 C_{l_i,r_j}(P_{s_i,r_j}) \tag{31a}$$

$$\text{subject to} \quad P_{s_i,r_j}^{\min} \le P_{s_i,r_j} \le P_{s_i,r_j}^{\max}. \tag{31b}$$

For a particular edge $(l_i, r_j)$, the weight $w(l_i, r_j)$ is determined by the optimal value of problem (31). We will later show that solving problem (23) is equivalent to finding the minimum weighted matching, where the sum of the weights of the edges selected in the matching has the minimum value. Let $P_{s_i,r_j}^*$ denote the optimal solution of problem (31) that uniquely exists due to the following theorem.

**Theorem 1.** *Problem (31) is a strictly convex optimization problem, which has a unique optimal solution.*

The proof of Theorem 1 is given in the Appendix.

Thus, the optimal solution $P_{s_i,r_j}^*$, which represents the optimal power of D2D user pair $l_i = (s_i, d_i)$ assisted by relay $r_j$, can be obtained by using techniques such as interior-point method. Once $P_{s_i,r_j}^*$ has been obtained, the optimal throughput, denoted by $C_{l_i,r_j}^* = C_{l_i,r_j}(P_{s_i,r_j}^*)$, can be found as well. We can then compute the weight of edge $(l_i, r_j)$, when we assume that user pair $l_i$ is assisted by relay $r_j$. The weight of edge $(l_i, r_j)$ is set to $g_{l_i,r_j}(P_{s_i,r_j}^*) = \lambda_1 P_{s_i,r_j}^* - \lambda_2 C_{l_i,r_j}^*$.

Given a matching, the resource allocation matrix $\boldsymbol{X}$ is fixed and the constraints of problem (23) are all satisfied. For example, in the matching shown in Fig. 2(b) that includes edges $(l_1, r_2)$, $(l_2, r_1)$, $(l_3, r_2)$, and $(l_4, r_3)$, we have $x_{l_1,r_2} = x_{l_2,r_1} = x_{l_3,r_2} = x_{l_4,r_1} = 1$. Let $\mathcal{F}$ denote a matching and $\mathcal{E}$ denote the set of all edges of the bipartite graph. For matching $\mathcal{F} \subseteq \mathcal{E}$, we show that the minimum weighted sum of transmit power and throughput is equivalent to the summation of the weights of all edges in the matching. To do so, we formulate the following problem which minimizes the weighted sum of transmit power and throughput for matching $\mathcal{F}$:

$$\underset{\boldsymbol{P}_s}{\text{minimize}} \quad \lambda_1 \sum_{(l_i,r_j)\in\mathcal{F}} P_{s_i,r_j} - \lambda_2 \sum_{(l_i,r_j)\in\mathcal{F}} C_{l_i,r_j}(P_{s_i,r_j}) \tag{32a}$$

$$\text{subject to} \quad P_{s_i,r_j}^{\min} \le P_{s_i,r_j} \le P_{s_i,r_j}^{\max}, \quad \forall\ (l_i, r_j) \in \mathcal{F}, \tag{32b}$$

where $l_i = (s_i, d_i)$. Since $\lambda_1$ and $\lambda_2$ are constants, the objective function is equal to

$$\begin{aligned} &\sum_{(l_i,r_j)\in\mathcal{F}} \left(\lambda_1 P_{s_i,r_j} - \lambda_2 C_{l_i,r_j}(P_{s_i,r_j})\right) \\ &= \sum_{(l_i,r_j)\in\mathcal{F}} g_{l_i,r_j}(P_{s_i,r_j}). \end{aligned} \tag{33}$$

The objective of problem (32) is to minimize the summation of all user pairs' weighted sum for a given matching. According to (32) and (33), the optimal transmit power of each source device is independent from others when we know the matching. Therefore, given matching $\mathcal{F}$, for each edge $(l_i, r_j) \in \mathcal{F}$, we can obtain the optimal transmit power of source device $s_i$, denoted by $P_{s_i,r_j}^*$, by solving problem (31). The optimal value of problem (31) for edge $(l_i, r_j)$ is $g_{l_i,r_j}(P_{s_i,r_j}^*)$. Thus, given matching $\mathcal{F}$, the optimal value of problem (32) is

$$\sum_{(l_i,r_j)\in\mathcal{F}} g_{l_i,r_j}(P_{s_i,r_j}^*). \tag{34}$$

Notice that $w_{l_i,r_j} = g_{l_i,r_j}(P_{s_i,r_j}^*), \forall\ (l_i, r_j) \in \mathcal{F}$. Thus, the optimal value of problem (32) can also be represented as $\sum_{(l_i,r_j)\in\mathcal{F}} w_{l_i,r_j}$, which is the summation of the weights of all edges in matching $\mathcal{F}$. Up to now, given a matching, we have shown that the minimum weighted sum of transmit power and throughput equals to the summation of the weights of all edges in the matching.

We now focus on finding the optimal matching (i.e., minimum weighted matching). We construct bipartite graph $\mathcal{G} = (\mathcal{L}, \mathcal{R}, \mathcal{E}, \boldsymbol{W})$, where D2D user pair set $\mathcal{L}$ and relay set $\mathcal{R}$ are the sets of vertices, and $\boldsymbol{W} = (w_{l_i,r_j})_{l_i\in\mathcal{L}, r_j\in\mathcal{R}}$ is the weighting matrix of the edges. The minimum weighted matching can be obtained as follows:

$$\mathcal{F}^* = \arg \min_{\mathcal{F}\subseteq\mathcal{E}} \sum_{(l_i,r_j)\in\mathcal{F}} w_{l_i,r_j}. \tag{35}$$

By constructing the bipartite graph, we have transformed the combinatorial relay selection and power allocation problem into a many-to-one matching problem. However, it is still difficult to obtain the optimal solution for this many-to-one matching problem in an efficient manner. Although each relay can assist multiple D2D user pairs, each user pair is allocated a non-overlapping channel. By utilizing this feature, we can further transform the many-to-one matching problem into a one-to-one matching problem which can be solved optimally in polynomial time. Since each relay $r_j$ has $N_{r_j}$ channels, we replace $r_j$ with $N_{r_j}$ virtual relays, which are located at the same location. Each virtual relay is assigned a non-overlapping channel, as shown in Fig. 2(c). We use $r_{jk}$ to represent the virtual relay that operates on the $k^{\text{th}}$ channel of relay $r_j$. We denote the set of virtual relays as $\mathcal{R}^v = \{r_{11}, \cdots, r_{1N_{r_1}}, \cdots r_{|\mathcal{R}|1}, \cdots, r_{|\mathcal{R}|N_{r_{|\mathcal{R}|}}}\}$, which represents a new vertex set. For ease of exposition, we denote $r_j^v$ as the $j^{\text{th}}$ virtual relay in set $\mathcal{R}^v$. We denote the new set of edges as $\mathcal{E}^v$ and the new weighting matrix as $\boldsymbol{W}^v$. By doing so, we transform the many-to-one matching problem into a one-to-one matching problem denoted by a new bipartite graph $\mathcal{G}^v = (\mathcal{L}, \mathcal{R}^v, \mathcal{E}^v, \boldsymbol{W}^v)$. Note that one-to-one matching problems can be solved in polynomial time [41, Ch. 3].

### *C. Pareto Optimal Relay Selection*

Up to now, we have constructed the bipartite graph to obtain a solution of problem (23) using a one-to-one matching. In the following, we prove that the obtained solution is a Pareto optimal resource allocation in terms of total transmit power and system throughput. We first formally define the Pareto optimal resource allocation in Definition 2 with the help of Definition 1 [42, Ch. 1]:

**Definition 1.** *In a minimization problem, given resource allocation decision matrices* $\mathbf{A}$ *and* $\mathbf{A}'$*, the allocation outcome* $\big(f_1(\mathbf{A}), f_2(\mathbf{A})\big)$ *is dominated by* $\big(f_1(\mathbf{A}'), f_2(\mathbf{A}')\big)$ *if* $f_1(\mathbf{A}') \leq f_1(\mathbf{A})$*,* $f_2(\mathbf{A}') \leq f_2(\mathbf{A})$ *and* $\big(f_1(\mathbf{A}), f_2(\mathbf{A})\big) \neq \big(f_1(\mathbf{A}'), f_2(\mathbf{A}')\big)$.

**Definition 2.** *A resource allocation decision* $\mathbf{A}$ *is Pareto optimal if and only if there does not exist another resource allocation* $\mathbf{A}'$ *dominating* $\mathbf{A}$.

Based on the definition of Pareto optimal resource allocation, we have the following theorem.

**Theorem 2.** *The solution of problem (23), denoted by* $(\boldsymbol{X}^*, \boldsymbol{P}_s^*)$*, is a Pareto optimal resource allocation.*

*Proof.* Assume $(\boldsymbol{X}^*, \boldsymbol{P}_s^*)$ is the solution of problem (23) and is not Pareto optimal. Then, there exists an allocation $(\widetilde{\boldsymbol{X}}, \widetilde{\boldsymbol{P}_s})$ such that $f_1(\widetilde{\boldsymbol{P}_s}) \leq f_1(\boldsymbol{P}_s^*)$ and $f_2(\widetilde{\boldsymbol{P}_s}) \leq f_2(\boldsymbol{P}_s^*)$. Given $\lambda_1$ and $\lambda_2$, we have $\lambda_1 f_1(\widetilde{\boldsymbol{P}_s}) + \lambda_2 f_2(\widetilde{\boldsymbol{P}_s}) \leq \lambda_1 f_1(\boldsymbol{P}_s^*) + \lambda_2 f_2(\boldsymbol{P}_s^*)$. This means that $(\boldsymbol{X}^*, \boldsymbol{P}_s^*)$ is not the optimal solution of problem (23). Here, we have the contradiction which completes the proof. □

### *D. Algorithm Design*

In this subsection, we develop the centralized and distributed algorithms.

*1) Centralized Relay Selection and Power Allocation Algorithm:* We first propose a centralized relay selection and power allocation algorithm as shown in Algorithm 1 based on the Hungarian method [43] to solve the one-to-one matching problem. As proven in Theorem 2, our proposed algorithm achieves a Pareto optimal solution of problem (23). We define $\boldsymbol{X}^{v*}$ as the optimal virtual relay selection matrix. The algorithm contains two main steps. As shown in Algorithm 1, we first create the weighted bipartite graph by computing the weights of all edges in the graph (Lines 2 to 16). By computing the minimum value of the weighted sum of transmit power and minus throughput, we obtain the weight of each edge (Line 7). If the minimum data rate requirement cannot be satisfied, we set the corresponding weight to $+\infty$ (Line 9). Since the Hungarian algorithm used in this algorithm is to find the minimum sum of all weights, those edges with weight $+\infty$ will not be considered in the Hungarian algorithm (i.e., Algorithm 2). Thus, we exclude the virtual relays which would result in an infeasible solution for the corresponding D2D user

**Algorithm 1:** Centralized relay selection and power allocation algorithm

1 **input** $\mathcal{L}$, $\mathcal{S}$, $\mathcal{D}$, $\mathcal{R}^v$, $C_{l_i}^{\min}, \forall l_i \in \mathcal{L}$, $P_s^{\max}$, $P_r^{\max}$, $\lambda_1$, $\lambda_2$, $h_{LI}$, $h_{s_i,r_j}$, $h_{r_j,d_i}$, $h_{s_i,d_i}$ $\forall\ l_i = (s_i, d_i) \in \mathcal{L}$, $r_j \in \mathcal{R}$
2 **for** $l_i \in \mathcal{L}$ **do**
3     **for** $r_j^v \in \mathcal{R}^v$ **do**
4         Calculate $P_{s_i,r_j^v}^{\min}$ and $P_{s_i,r_j^v}^{\max}$ using (29) and (30)
5         **if** $P_{s_i,r_j^v}^{\min} \leq P_{s_i,r_j^v}^{\max}$ **then**
6             Solve problem (31) to obtain $P_{s_i,r_j^v}^*$
7             $w_{l_i,r_j^v}^v := \lambda_1 P_{s_i,r_j^v}^* - \lambda_2 C_{l_i,r_j^v}^*(P_{s_i,r_j^v}^*)$
8         **else**
9             $w_{l_i,r_j^v}^v := +\infty$
10         **end**
11     **end**
12     **if** $P_{s_i,r_j^v}^{\min} > P_{s_i,r_j^v}^{\max}\ \forall\ r_j^v \in \mathcal{R}^v$ **then**
13         Return infeasible
14     **end**
15 **end**
16 $\boldsymbol{W}^v := (w_{l_i,r_j^v})_{l_i \in \mathcal{L}, r_j^v \in \mathcal{R}^v}$
17 $\boldsymbol{X}^{v*} :=$ Hungarian($\boldsymbol{W}^v$)
18 **output**: Virtual relay selection decision $\boldsymbol{X}^{v*}$, and transmit power matrix $\boldsymbol{P}_s^*$.

pairs. If the minimum data rate requirement for a user pair cannot be satisfied regardless of which virtual relay the user pair uses (Line 12), this user pair will have to communicate using the cellular base station (instead of D2D mode) and operate under the resource allocation rules for regular cellular users. If the minimum data rate requirement can be satisfied, then the Hungarian method is used to obtain the optimal virtual relay selection matrix $\boldsymbol{X}^{v*}$ (Line 17).

The optimal matching that minimizes the summation of the weights can be obtained by the Hungarian method [43]. For the sake of completeness, we present the Hungarian method in Algorithm 2. We use $\boldsymbol{z}^* = (z_{l_1}^*, \cdots, z_{l_i}^*, \cdots, z_{l_{|\mathcal{L}|}}^*)$ to indicate the current matching result. For example, $z_{l_3}^* = r_5^v$ means that user pair $l_3$ is matched with virtual relay $r_5^v$. We define $\boldsymbol{m} = (m_{l_i,r_j^v})_{l_i \in \mathcal{L}, r_j^v \in \mathcal{R}^v}$ as a marking indicator of the edges of the bipartite graph (i.e., the elements of $\boldsymbol{W}^v$). This indicator is used to mark the element that has the minimum weight. The edge corresponding to the marked element is regarded as a potential edge in the optimal matching. Element $w_{l_i,r_j^v}$ is marked when $m_{l_i,r_j^v}$ is set to 1. If $m_{l_i,r_j^v} = 0$, element $w_{l_i,r_j^v}$ is not marked. We define $\boldsymbol{cr} = (cr_{l_i})_{l_i \in \mathcal{L}}$ and $\boldsymbol{cc} = (cc_{r_j^v})_{r_j^v \in \mathcal{R}^v}$ as the row cover indicator and column cover indicator, respectively. The purpose of covering a row (or a column) is to exclude the elements of that row (or column) from further operations. For example, row $i$ of the weighting matrix $\boldsymbol{W}^v$ is covered and excluded from operations if $cr_{l_i} = 1$. The main steps of the algorithm are as follows. First, the minimum value of each row of $\boldsymbol{W}^v$ is subtracted from all elements in its row (Lines 3 to 5). Then, zeros in the modified matrix $\boldsymbol{W}^v$ are found and their indices are recorded in $\boldsymbol{z}^*$ (Lines 6 to 10). Next, the algorithm verifies whether all user pairs and virtual relays are matched by checking the number of zeros in $\boldsymbol{z}^*$ (Lines 15 to 17). If there is no zero in $\boldsymbol{z}^*$, the matching is obtained. Otherwise, the weighting

**Algorithm 2:** Hungarian method

1 **input** Weighting matrix $\boldsymbol{W}^v$
2 **initialize** $\boldsymbol{z}^* = (z^*_{l_i})_{l_i \in \mathcal{L}} := \{\mathbf{0}\}_{1\times|\mathcal{L}|}$, $\boldsymbol{X}^{v*} = (x^*_{l_i,r^v_j})_{l_i\in\mathcal{L},r^v_j\in\mathcal{R}^v} := \{\mathbf{0}\}_{|\mathcal{L}|\times|\mathcal{R}^v|}$, $\boldsymbol{m} = (m_{l_i,r^v_j})_{l_i\in\mathcal{L},r^v_j\in\mathcal{R}^v} := \{\mathbf{0}\}_{|\mathcal{L}|\times|\mathcal{R}^v|}$, $\mathcal{A} := \mathcal{L}$, and $\mathcal{B} := \mathcal{R}^v$,
3 **for** $l_i \in \mathcal{L}$ **do**
4     Set $w_{l_i,r^v_j} := w_{l_i,r^v_j} - \min_{r^v_k\in\mathcal{R}^v}\{w_{l_i,r^v_k}\}$, $\forall\ r^v_k \in \mathcal{R}^v$
5 **end**
6 **while** $\mathcal{A} \neq \emptyset$ **do**
7     Set $z^*_{l_i} := r^v_j,\ \forall\ l_i \in \mathcal{A},\ r^v_j \in \mathcal{B}$, such that $w_{l_i,r^v_j} = 0$
8     Set $\mathcal{A} := \mathcal{A} \setminus \{l_i\}$
9     Set $\mathcal{B} := \mathcal{B} \setminus \{r^v_j\}$
10 **end**
11 **while** true **do**
12     Set $\boldsymbol{cr} = (cr_{l_i})_{l_i\in\mathcal{L}} := \{\mathbf{0}\}_{1\times|\mathcal{L}|}$
13     Set $\boldsymbol{cc} = (cc_{r^v_j})_{r^v_j\in\mathcal{R}^v} := \{\mathbf{0}\}_{1\times|\mathcal{R}^v|}$
14     Set $cr_{l_i} := 1, \forall\ l_i \in \mathcal{L}$, such that $z^*_{l_i} > 0$
15     **if** $z^*_{l_i} \neq 0\ \forall\ l_i \in \mathcal{L}$ **then**
16         Break
17     **end**
18     **while** true **do**
19         Set $m_{l_i,r^v_j} := 1\ \ \forall\ l_i \in \mathcal{L},\ r^v_j \in \mathcal{R}^v$, such that $cc_{r^v_j} = 0$ and $w_{l_i,r^v_j} = 0$
20         **if** $\nexists\ (l_i, r^v_j)\ \forall\ l_i \in \mathcal{L},\ r^v_j \in \mathcal{R}^v$, such that $m_{l_i,r^v_j} = 1,\ cr_{l_i} = 0,$ and $w_{l_i,r^v_j} = 0$ **then**
21             Break
22         **else**
23             Set $cr_{l_i} := 1,\ cc_{r^v_j} := 0\ \ \forall\ l_i \in \mathcal{L},\ r^v_j \in \mathcal{R}^v$, such that $m_{l_i,r^v_j} = 1,\ cr_{l_i} = 0$, and $w_{l_i,r^v_j} = 0$
24         **end**
25         Set $\mathcal{J} := \{(l_i, r^v_j) \in \mathcal{L}\times\mathcal{R}^v \mid cr_{l_i} = cc_{r^v_j} = 0\}$
26         Set $a := \min_{(l_i,r^v_j)\in\mathcal{J}}\{w_{l_i,r^v_j}\}$
27         Update $w_{l_i,r^v_j} := w_{l_i,r^v_j} + a\ \ \forall\ l_i \in \mathcal{L}$, such that $cr_{l_i} = 0$
28         Update $w_{l_i,r^v_j} := w_{l_i,r^v_j} - a\ \ \forall\ r^v_j \in \mathcal{R}^v$, such that $cc_{r^v_j} = 0$
29     **end**
30     Set $z^*_{l_i} := r^v_j\ \ \forall\ l_i \in \mathcal{L},\ r^v_j \in \mathcal{R}^v$, such that $w_{l_i,r^v_j} = 0$, $m_{l_i,r^v_j} = 1$, and $cr_{l_i} = cc_{r^v_j} = 0$
31 **end**
32 **for** $l_i \in \mathcal{L}$ **do**
33     Set $x^*_{l_i,r^v_j} := 1\ \ \forall\ r^v_j \in \mathcal{R}^v$, such that $z^*_{l_i} = r^v_j$
34 **end**
35 **output**: Virtual relay selection decision $\boldsymbol{X}^{v*}$

matrix is updated according to the mark indicator $\boldsymbol{m}$ and cover indicators $\boldsymbol{cr}$ and $\boldsymbol{cc}$ (Lines 18 to 29). Specifically, the minimum uncovered element is subtracted from each covered row, and then added to each uncovered column. By doing so, new zeros will be resulted. The algorithm then finds and records the indices of the new zeros (Line 30) and runs the while loop (Lines 11 and 31) again. If $\boldsymbol{z}^*$ has no zeros, the recorded indices will be returned as the matching results of the algorithm (Lines 32 to 34). The convergence of the Hungarian method is proven in [43].

We now discuss the computational complexity of Algorithm 1. The computational complexity of the Hungarian method is $O(|\mathcal{L}|^3)$ [41]. The complexity of computing the weights of all edges (Lines 2 to 16) is $O(|\mathcal{L}||\mathcal{R}^v|)$, which is polynomial. Thus, Algorithm 1 can obtain a Pareto optimal solution in polynomial time. However, the centralized algorithm needs to exchange the control messages between the relays and base stations. Each relay needs to send the channel state information (CSI) of its links to the base station. In return, the base station sends the resource allocation decision to all relays.

**Algorithm 3:** Distributed relay selection and power allocation algorithm

1 **input** $\mathcal{L}$, $\mathcal{S}$, $\mathcal{D}$, $\mathcal{R}^v$, $C^{\min}_{l_i}, \mathcal{R}^v_{l_i}, \forall l_i \in \mathcal{L}$, $P^{\max}_s$, $P^{\max}_r$, $\lambda_1$, $\lambda_2$, $h_{LI}$, $h_{s_i,d_i}\ \forall\ l_i = (s_i, d_i) \in \mathcal{L}$
2 **for** $l_i \in \mathcal{L}$ **do**
3     D2D user pair $l_i$ sends relay request. Relays that receive the request join set $\mathcal{R}^v_{l_i}$.
4     **for** $r^v_j \in \mathcal{R}^v_{l_i}$ **do**
5         Relay $r^v_j$ estimates $h_{s_i,r^v_j}$, $h_{r^v_j,d_i}$, $\forall\ l_i = (s_i, d_i) \in \mathcal{L}$.
6     **end**
7 **end**
8 **for** $l_i \in \mathcal{L}$ **do**
9     **for** $r^v_j \in \mathcal{R}^v_{l_i}$ **do**
10         Relay $r^v_j$ calculates $P^{\min}_{s_i,r^v_j}$ and $P^{\max}_{s_i,r^v_j}$ using (29) and (30)
11         **if** $P^{\min}_{s_i,r^v_j} \leq P^{\max}_{s_i,r^v_j}$ **then**
12             Solve problem (31) to obtain $P^*_{s_i,r^v_j}$
13             $w^v_{l_i,r^v_j} := \lambda_1 P^*_{s_i,r^v_j} - \lambda_2 C^*_{l_i,r^v_j}(P^*_{s_i,r^v_j})$
14         **else**
15             $\mathcal{R}^v_{l_i} := \mathcal{R}^v_{l_i} \setminus \{r^v_j\}$
16         **end**
17     **end**
18     **if** $\mathcal{R}^v_{l_i} = \emptyset$ **then**
19         Return infeasible
20     **end**
21     $\boldsymbol{W}^v_{l_i} := (w_{l_i,r^v_j})_{r^v_j\in\mathcal{R}^v_{l_i}}$
22 **end**
23 $\boldsymbol{X}^{v*} = (x^*_{l_i,r^v_j})_{l_i\in\mathcal{L},r^v_j\in\mathcal{R}^v} := \{\mathbf{0}\}_{|\mathcal{L}|\times|\mathcal{R}^v|}$,
24 **while** *rank*$(\boldsymbol{X}^{v*}) \neq |\mathcal{L}|$ **do**
25     Find the smallest $b$ such that $w_{b,r^v_k} = 0,\ \forall\ r^v_k \in \mathcal{R}^v$, and set $l_i := b$
26     $a := r^v_j$ such that $w_{l_i,r^v_k} = \min_{r^v_k\in\mathcal{R}^v}\{w_{l_i,r^v_k}\},\ \forall\ r^v_k \in \mathcal{R}^v_{l_i}$
27     **for** $l_k \in \mathcal{L} \setminus \{l_i\}$ **do**
28         **if** There exists $(x^*_{l_k,r^v_j})_{l_k\in\mathcal{L}\setminus\{l_i\},r^v_j\in\mathcal{R}^v} = a$ **then**
29             **if** $w^v_{l_i,r^v_j} < w^v_{l_k,r^v_j}$ **then**
30                 $x^*_{l_i,r^v_j} := r^v_j$
31                 $x^*_{l_k,r^v_j} := 0$
32             **else**
33                 $x^*_{l_i,r^v_j} := 0$
34                 $\mathcal{R}^v_{l_i} := \mathcal{R}^v_{l_i} \setminus \{r^v_j\}$
35             **end**
36         **else**
37             $x^*_{l_i,r^v_j} := r^v_j$
38         **end**
39     **end**
40 **end**
41 **output**: Virtual relay selection decision $\boldsymbol{X}^{v*}$ and transmit power matrix $\boldsymbol{P}^*_s$.

*2) Distributed Relay Selection and Power Allocation Algorithm:* We now propose a suboptimal distributed relay selection and power allocation algorithm to reduce the communication overhead imposed by exchanging control messages. In this algorithm, D2D user pairs and relays act in a distributed manner. As shown in Algorithm 3, D2D user pairs first broadcast their requests for potential relays locally. We denote the set of relays which receive the request of D2D user pair $l_i$ as $\mathcal{R}^v_{l_i}$ (Line 3). Each relay in set $\mathcal{R}^v_{l_i}$ estimates the CSI of the corresponding source-relay and relay-destination links (Lines 4 to 6). Note that the CSI of all links are not shared globally. The next step is to determine the feasible relays for each D2D user pair (Lines 8 to 22). We exclude the virtual relays which would result in an infeasible solution for the corresponding D2D user pair (Line 15). After excluding the infeasible relays, each D2D user pair can determine the set of all feasible relays. The weight of the feasible edges for D2D user pair $l_i$ is stored in a vector denoted by $\boldsymbol{W}^v_{l_i}$ (Line 21). Then, the distributed relay selection is executed based on the idea of stable matching (Lines 24 to 40) [44]. The relay allocation is completed when all D2D user pairs are allocated one virtual relay (Line 24). Any user pair that has not been allocated a virtual relay will send a request to its preferred virtual relay (Line 26). That virtual relay will accept the request if no other user pair is allocated to the relay. If the virtual relay (e.g., $r^v_j$) has been allocated to another D2D user pair (Line 28), the virtual relay will autonomously compare the weights of two edges. These weights are $w^v_{l_k,r^v_j}$ (i.e., the weight of the edge from $r^v_j$ to its currently allocated D2D user pair $l_k$), and $w^v_{l_i,r^v_j}$ (i.e., the weight of the edge from $r^v_j$ to D2D user pair $l_i$ which is requesting to be assisted by $r^v_j$) (Line 29). Virtual relay $r^v_j$ will choose to assist the D2D user pair with a smaller weight (Lines 29 to 35). By repeating Lines 25 to 39 until convergence, each user pair will be allocated a virtual relay and a stable relay selection will be achieved [44].

We now discuss the computational complexity of Algorithm 3. The complexity of computing the weights of all edges (Lines 2 to 22) is $O(|\mathcal{L}||\mathcal{R}^v|)$. The complexity of stable matching is $O(|\mathcal{L}|^2)$ [44]. The distributed algorithm does not require each relay to send (or receive) any CSI to (or from) the coordinator. The centralized algorithm requires information exchange to allocate the resources and incurs at least $2|\mathcal{L}|$ more message exchange compared to the distributed algorithm.

## IV. Performance Evaluation

In this section, we evaluate the performance of our proposed algorithms and compare them with two recently proposed relay selection algorithms, namely all relays selection (ARS) algorithm [20] and distributed relay selection (DRS) algorithm [21]. Each relay can use four channels and communicate with up to four D2D user pairs simultaneously (i.e., $N_{r_j} = 4, \forall r_j \in \mathcal{R}$) [45]. The bandwidth of each channel $B = 100$ MHz [1], and the noise power spectral density is $-174$ dBm/Hz. We also consider a cell radius of 500 m [34]. Other simulation parameters are as follows [30], [35]: $P^{\max}_s = 2$ Watt, $P^{\max}_r = 10$ Watt, $C^{\min}_{l_i} = 400$ Mbps, $\forall\ l_i \in \mathcal{L}$, $M^t = M^r = 10$ dB, $m^t = m^r = -5$ dB, $\Theta^t_{\text{HPBW}} = \Theta^r_{\text{HPBW}} = 15^o$, $\alpha = 2$, and $\sigma = 1.5$. We use Monte Carlo simulations and calculate the average value of the total transmit power and the system throughput over different network settings.

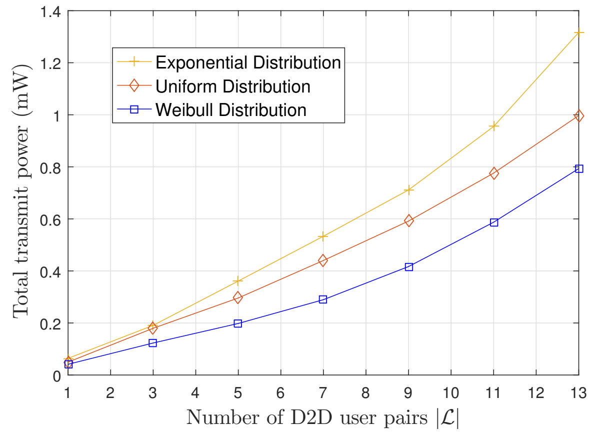



Fig. 3. Total transmit power versus number of D2D user pairs $|\mathcal{L}|$ with $|\mathcal{R}| = 4$ ($\lambda_1 = 1$ and $\lambda_2 = 0$).

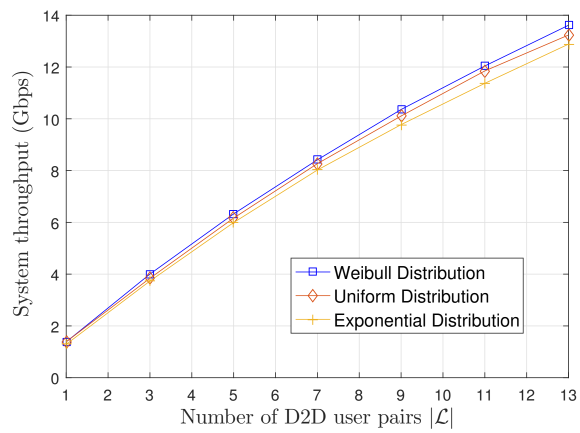



Fig. 4. System throughput versus number of D2D user pairs $|\mathcal{L}|$ with $|\mathcal{R}| = 4$ ($\lambda_1 = 0$ and $\lambda_2 = 1$).

We first investigate the performance of our proposed centralized algorithm for different distributions of the distance between the relays and base station. We consider Weibull, exponential, and uniform distributions. For the Weibull distribution, the cumulative distribution function (cdf) is $F(d_{rb}; k, \mu) = 1 - \mathrm{e}^{-(\frac{d_{rb}}{\mu})^k}$, where $d_{rb}$ is the distance between a relay and the base station, and $k$ and $\mu$ are constants. We set $k = 7$ and $\mu = 400$. For the exponential distribution, the cdf is $F(d_{rb}; \gamma) = 1 - \mathrm{e}^{-\frac{d_{rb}}{\gamma}}$. For fair comparison, we set the mean parameter $\gamma = 374$, which results in the same average distance as Weibull distribution. For the uniform distribution, relays are uniformly distributed in the cell area.

The total transmit power and the system throughput are shown in Figs. 3 and 4, respectively. As can be observed, different distributions result in different total transmit power and system throughput. However, our proposed algorithm is in general applicable for any distributions.

We now choose the Weibull distribution as an example to evaluate the performance of our proposed algorithms. We assume that the distance between relays and the base station, $d_{rb}$, follows a Weibull distribution with the aforementioned parameters. We first set coefficients $\lambda_1 = 1$ and $\lambda_2 = 0$ to study the single-objective optimization problem for minimiz-

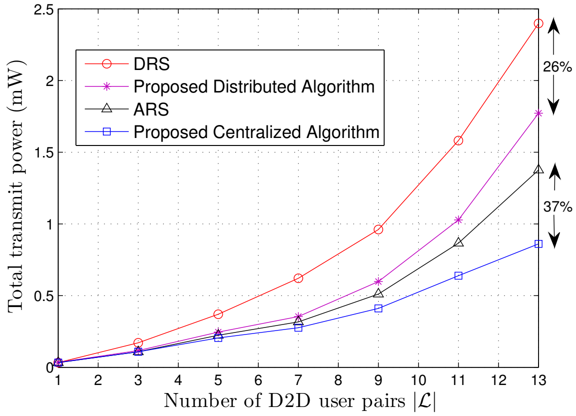



Fig. 5. Total transmit power versus number of D2D user pairs $|\mathcal{L}|$ with $|\mathcal{R}| = 4$.

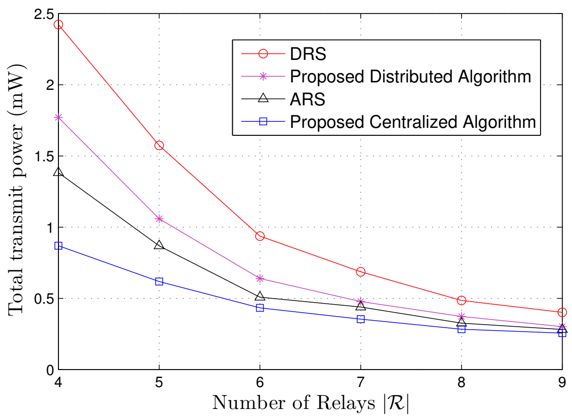



Fig. 6. Total transmit power versus number of relays $|\mathcal{R}|$ with $|\mathcal{L}| = 13$.

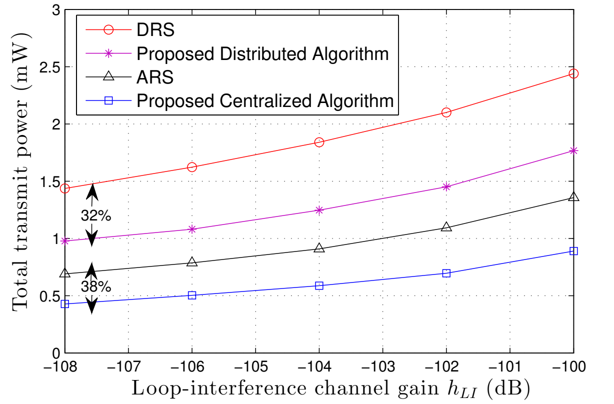



Fig. 7. Total transmit power versus loop-interference channel gain $h_{LI}$.

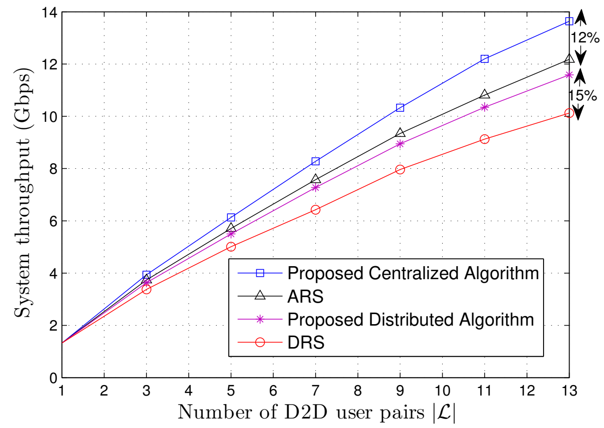



Fig. 8. System throughput versus number of D2D user pairs $|\mathcal{L}|$ with $|\mathcal{R}| = 4$.

ing the transmit power. In Fig. 5, we compare the total transmit power of devices in our proposed algorithms with that obtained by ARS and DRS algorithms for different number of D2D user pairs. As shown in this figure, our proposed centralized algorithm substantially outperforms ARS. The proposed distributed algorithm also outperforms DRS. When the number of D2D user pairs is 13, the centralized algorithm results in 37% less transmit power than ARS, and our distributed algorithm achieves 26% less transmit power than DRS. This is because the centralized algorithm achieves the optimal solution with minimum weighted matching and our proposed distributed algorithm based on stable matching can obtain a better solution than heuristic DRS.

In Fig. 6, we compare the total transmit power versus different number of relays when the number of D2D user pairs is 13. Fig. 6 shows that the total transmit power is decreasing in all algorithms as the number of relays increases. This is because each D2D user pair has more opportunity to select a nearby relay. Results also show that our proposed centralized algorithm significantly outperforms ARS, while our proposed distributed algorithm achieves a lower transmit power than DRS. The results indicates that our algorithms are more efficient, especially in the networks with few relays.

In Fig. 7, we compare the total transmit power versus different loop-interference channel gain. We consider the gain range from $-108$ dB to $-100$ dB [46]. As shown in this figure, the total transmit power increases as the loop-interference channel gain increases. This is because a stronger loop-interference will result in a higher transmit power. When the loop-interference channel gain $h_{LI} = -108$ dB, our proposed centralized algorithm achieves 38% lower transmit power compared to ARS. Furthermore, our distributed algorithm outperforms DRS by 32%.

We now set $\lambda_1 = 0$ and $\lambda_2 = 1$ to obtain the relay selection and power allocation for maximizing the system throughput. In Fig. 8, we compare the system throughput versus different number of D2D user pairs when there are four relays in the network. Results show that our proposed centralized algorithm achieves a higher throughput compared to ARS and our proposed distributed algorithm obtains a higher throughput compared to DRS under different number of D2D user pairs. When the number of D2D user pairs is 13, the centralized algorithm achieves 12% higher throughput than ARS and our distributed algorithm outperforms DRS by 15%. Results also show that the throughput improvement of our proposed algorithms over ARS and DRS increases as the number of D2D user pairs increases. This is because when the the number of D2D user pairs increases, it is more difficult for ARS and DRS to guarantee that each D2D user pair can use its preferred relay. This substantially degrades the performance of the system.

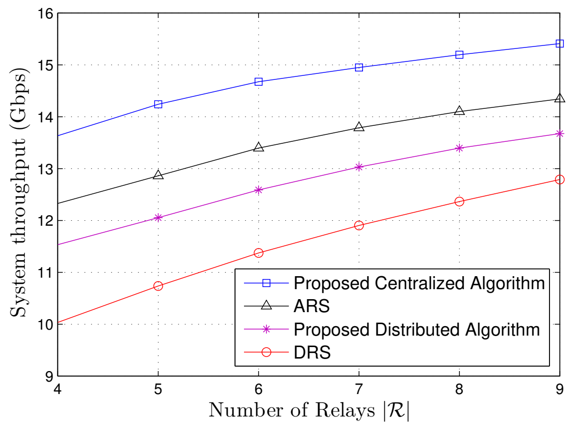



Fig. 9. System throughput versus number of relays $|\mathcal{R}|$ with $|\mathcal{L}| = 13$.

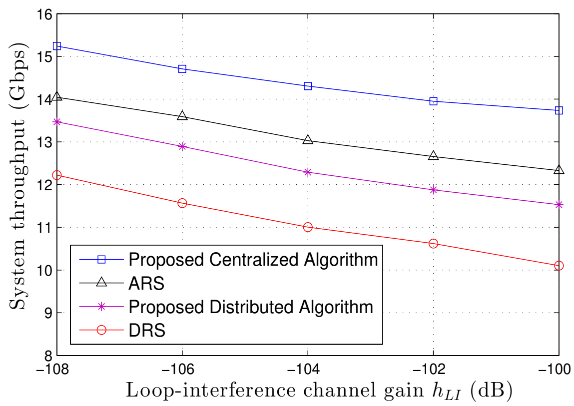
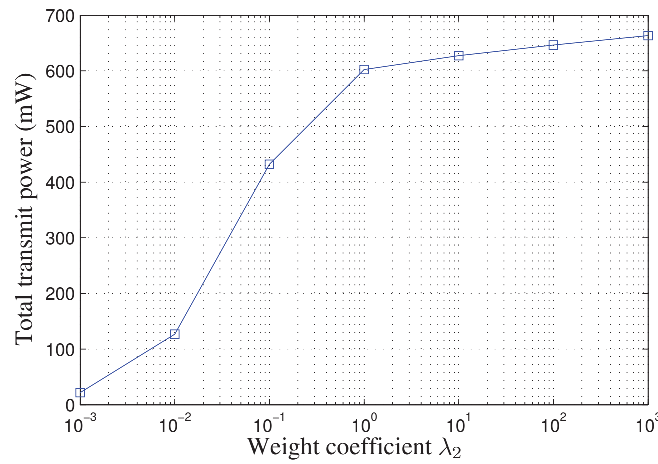
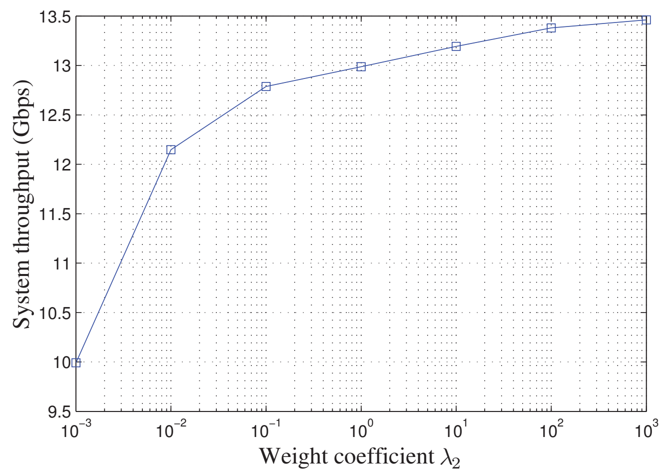



Fig. 10. System throughput versus loop-interference channel gain $h_{LI}$.




(a) Optimal power consumption




(b) Optimal system throughput

Fig. 11. Optimal total transmit power and system throughput versus weight coefficient $\lambda_2$ when $\lambda_1 = 1$ with $|\mathcal{R}| = 4$ and $|\mathcal{L}| = 13$.

In Fig. 9, we show the system throughput versus different number of relays when the number of D2D user pairs is 13. Our proposed centralized algorithm substantially improves the system throughput compared to ARS and our distributed algorithm outperforms DRS under different number of relays. Note that the improvement slightly decreases as the number of relays increases. This is because the ARS, DRS, and our distributed algorithm are more likely to allocate the optimal relay for each D2D user pair when more relays are deployed.

In Fig. 10, we compare the system throughput versus different loop-interference channel gain, which varies from $-108$ dB to $-100$ dB. As shown in this figure, the system throughput slightly decreases as the loop-interference channel gain increases. This shows that a stronger loop-interference will result in a lower throughput. However, our proposed centralized algorithm always achieves a higher throughput than ARS and our distributed algorithm always outperforms DRS.

We now consider the multi-objective optimization problem to study the trade-off between the total transmit power and the system throughput. In Fig. 11, we plot the optimal total transmit power and system throughput obtained by the centralized algorithm when $\lambda_1 = 1$ and we vary $\lambda_2$. From Fig. 11, we can observe that the optimal solution is sensitive to the weight coefficients. When $\lambda_2$ increases, improving the throughput becomes more important than reducing the power. In this case, the optimal solution tends to consume more power to achieve a higher throughput. Results also show that the increment of the throughput is not as fast as the increment of the power. In other words, the marginal throughput increment becomes smaller when the transmit power increases. This provides useful insights of the system design to balance the total transmit power and system throughput.

## V. Conclusion

In this paper, we studied the joint relay selection and power allocation problem for full-duplex relay-assisted D2D communication in mmWave based 5G networks. We first formulated a multi-objective optimization problem to balance the trade-off between total transmit power and system throughput. The formulated problem characterizes loop-interference cancellation in full-duplex relaying systems. It also considers the QoS requirements for different applications as well as the physical constraints of the devices and relays. The problem is a combinatorial optimization problem which is complex to solve using standard optimization techniques. To mitigate the complexity of the combinatorial problem, we transformed the problem into a one-to-one matching problem by constructing a weighted bipartite graph. We then proposed a centralized algorithm to find the solution in polynomial time. We proved that the solution obtained by the proposed centralized algorithm is Pareto optimal. We further proposed a distributed algorithm to reduce the overhead imposed by exchanging control messages. We evaluated the performance of our proposed algorithms through simulations. Results showed that our proposed algorithms substantially reduce the total transmit power and improve the system throughput compared to recently proposed algorithms in the literature. For future work, we will study the resource allocation problem when the mobile devices can also communicate in full-duplex mode.

## APPENDIX: PROOF OF THEOREM 1

Since $P_{s_i,r_j}$ is within a closed interval, constraint (31b) is a convex constraint. To prove that problem (31) is strictly convex, we need to show its objective function is strictly convex. We determine the second-order derivative of the objective function. Since the first term of the objective function is linear in $P_{s_i,r_j}$, the second-order derivative of the objective function is $-\lambda_2 C''_{l_i,r_j}(P_{s_i,r_j})$. We rewrite $C_{l_i,r_j}(P_{s_i,r_j})$ as follows:

$$C_{l_i,r_j}(P_{s_i,r_j}) = B\log_2\left(\Psi_{l_i,r_j}(P_{s_i,r_j})\right), \tag{36}$$

where

$$\Psi_{l_i,r_j}(P_{s_i,r_j}) = 1 + \frac{h_{r_j,d_i}P_{r_j,d_i}(P_{s_i,r_j})}{h_{s_i,d_i}P_{s_i,r_j}+N_0}. \tag{37}$$

Then, according to (36), $-C''_{l_i,r_j}(P_{s_i,r_j})$ can be written as:

$$B\frac{\Psi'^2_{l_i,r_j}(P_{s_i,r_j})}{\Psi^2_{l_i,r_j}(P_{s_i,r_j})} - B\frac{\Psi''_{l_i,r_j}(P_{s_i,r_j})}{\Psi_{l_i,r_j}(P_{s_i,r_j})}, \tag{38}$$

where

$$\begin{aligned}
&\Psi'_{l_i,r_j}(P_{s_i,r_j})\\
&= \frac{N_0}{\sqrt{f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h^2_{r_j,d_i}}}\\
&\quad\Bigg(h_{s_i,d_i}\left(\sqrt{f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h^2_{r_j,d_i}} - N_0 h_{r_j,d_i}\right)\\
&\quad + 2h_{s_i,r_j}h_{LI}h_{s_i,d_i}P_{s_i,r_j} + 2h_{s_i,r_j}h_{LI}N_0\Bigg)\\
&\quad \frac{h_{r_j,d_i}}{2h_{LI}\left(h_{s_i,d_i}P_{s_i,r_j}+N_0\right)^2\ln 2},
\end{aligned} \tag{39}$$

and

$$\begin{aligned}
&\Psi''_{l_i,r_j}(P_{s_i,r_j}) =\\
&\frac{-\Omega_{l_i,r_j}(P_{s_i,r_j})}{h_{LI}\left(h_{s_i,d_i}P_{s_i,r_j}+N_0\right)^3\left(f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h^2_{r_j,d_i}\right)^{\frac{3}{2}}\ln 2},
\end{aligned} \tag{40}$$

in which

$$\begin{aligned}
&\Omega_{l_i,r_j}(P_{s_i,r_j})\\
&\quad = N_0 h_{r_j,d_i}h^2_{s_i,d_i}\left(f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h^2_{r_j,d_i}\right)^{\frac{3}{2}}\\
&\quad\quad + 8N_0 h^2_{s_i,r_j}h^2_{LI}h^2_{r_j,d_i}h^3_{s_i,d_i}P^3_{s_i,r_j}\\
&\quad\quad + N_0 h_{r_j,d_i}\Bigg(18h^2_{s_i,r_j}h^2_{LI}N_0 h_{r_j,d_i}h^2_{s_i,d_i}\\
&\quad\quad - 6h_{s_i,r_j}h_{LI}N_0 h^2_{r_j,d_i}h^3_{s_i,d_i}\Bigg)P^2_{s_i,r_j}\\
&\quad\quad + N_0 h_{r_j,d_i}\Bigg(12h^2_{s_i,r_j}h^2_{LI}N_0^2 h_{r_j,d_i}h_{s_i,d_i}\\
&\quad\quad - 6h_{s_i,r_j}h_{LI}N_0^2 h^2_{r_j,d_i}h^2_{s_i,d_i}\Bigg)P_{s_i,r_j}\\
&\quad\quad - N_0^4 h^4_{r_j,d_i}h^2_{s_i,d_i} + 2h^2_{s_i,r_j}h^2_{LI}N_0^4 h^2_{r_j,d_i}.
\end{aligned} \tag{41}$$

Note that $\Psi_{l_i,r_j}(P_{s_i,r_j}) > 0$ and $\Psi'_{l_i,r_j}(P_{s_i,r_j}) > 0$ since $P_{s_i,r_j} \geq 0$. If $\Psi''_{l_i,r_j}(P_{s_i,r_j})$ is negative, then (38) is always positive. Note that $\Psi''_{l_i,r_j}(P_{s_i,r_j})$ is negative if $\Omega_{l_i,r_j}(P_{s_i,r_j}) > 0$. We now show that $\Omega_{l_i,r_j}(P_{s_i,r_j}) > 0$. We first calculate the first-order derivative of $\Omega_{l_i,r_j}(P_{s_i,r_j})$ as follows:

$$\begin{aligned}
&\Omega'_{l_i,r_j}(P_{s_i,r_j})\\
&= 6h_{s_i,r_j}h_{LI}N_0 h^2_{r_j,d_i}h_{s_i,d_i}\left(2h_{s_i,d_i}P_{s_i,r_j}+N_0\right)\\
&\quad\Bigg(h_{s_i,d_i}\sqrt{(f_{r_j,d_i}(P_{s_i,r_j}) + N_0^2 h^2_{r_j,d_i}} - N_0 h_{r_j,d_i}h_{s_i,d_i}\\
&\quad + 2h_{s_i,r_j}h_{LI}h_{s_i,d_i}P_{s_i,r_j} + 2h_{s_i,r_j}h_{LI}N_0\Bigg).
\end{aligned} \tag{42}$$

Since $\Omega_{l_i,r_j}(0) = 2h^2_{s_i,r_j}h^2_{LI}N_0^3 h_{r_j,d_i} > 0$ and $\Omega'_{l_i,r_j}(P_{s_i,r_j}) > 0$, we have $\Omega_{l_i,r_j}(P_{s_i,r_j}) > 0$. Thus, $\Psi''_{l_i,r_j}(P_{s_i,r_j}) < 0$. Given $B > 0$, $\Psi'^2_{l_i,r_j}(P_{s_i,r_j}) \geq 0$, $\Psi^2_{l_i,r_j}(P_{s_i,r_j}) > 0$ $\Psi''_{l_i,r_j}(P_{s_i,r_j}) < 0$, and $\Psi_{l_i,r_j}(P_{s_i,r_j}) > 0$, the second-order derivative of $-C_{l_i,r_j}(P_{s_i,r_j})$, shown in (38), is always positive, which proves the convexity of $-C_{l_i,r_j}(P_{s_i,r_j})$ and the objective function of problem (31). Considering the constraint is also convex, problem (31) is proved to be a convex problem.

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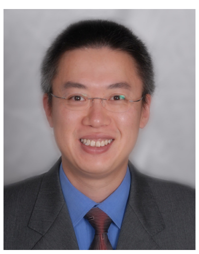

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