Repeated Inter-Session Network Coding Games: Efficiency and Min-Max Bargaining Solution

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***Abstract*— Recent results have shown that selfish and strategic users do not have an incentive to participate in inter-session network coding in a *static* non-cooperative game setting. Because of this, the worst-case network efficiency (i.e., the *price-of-anarchy*) can be as low as 20%. In this paper, we show that if the same game is played *repeatedly*, then the price-of-anarchy can be significantly improved to 36%. In this regard, we design a grim-trigger strategy that encourages users to cooperate and participate in the inter-session network coding. A key challenge here is to determine a common cooperative coding rate that the users should mutually agree on. We resolve the conflict of interest among the users through a *bargaining* process, and obtain tight upper bounds for the price-of-anarchy which are valid for any possible bargaining scheme. Moreover, we propose a simple and efficient *min-max* bargaining solution that can achieve these upper bounds, as confirm through simulation studies. The co-existence of multiple selfish network coding sessions as well as the co-existence of selfish network coding and routing sessions are also investigated. Our results represent a first step towards designing practical inter-session network coding schemes which achieve reasonable performance for selfish users.**

I. Introduction

Since the seminal paper by Ahlswede *et al.* [1], a rich body of work has been reported on how network coding can improve performance in both wired and wireless networks [2]–[4]. In general, network coding is performed by jointly encoding multiple packets either from the *same* user (i.e., intra-session network coding, e.g., as in [1], [2]) or from *different* users (i.e., inter-session network coding, e.g., as in [3]–[5]). A common assumption in most existing network coding schemes is that the users are cooperative and do not pursue their own interests. However, this assumption can be violated in practice.

In *non-cooperative* network coding, each user individually decides on *whether* to use and *how* to use network coding to maximize its payoff. However, in inter-session network coding, users rely on each other as they need to receive some *remedy* packets to decode the coded data that they receive at their destinations. This leads to a *game* among users. Recent results in [6], [7] show that if the inter-session network coding game is played once (i.e., as a static game), then users do not have the incentive to provide each other with the remedy packets. Hence, no network coding is performed at Nash equilibrium. This significantly affects the network performance; the *price-of-anarchy* (PoA), i.e., the worst-case efficiency compared with the *optimal* network performance, can be only 20% [6].

In this paper, we study the more realistic scenario where the inter-session network coding game in [6] is likely to be played *repeatedly*. This reflects the case where users have many packets to transmit. As users continue sending more packets, they can take into account the history of the game (e.g., whether the other users have provided the needed remedy packets in the past) and plan their future actions accordingly.

It is well known that repeated interactions can encourage cooperation [8]–[11]. However, the key challenge in our model is that it is not immediately clear for the inter-session network coding users *how* they can cooperate. This introduces a *bargaining* problem among users to search for a mutually acceptable coding rate. We show that a "good" bargaining solution together with a *grim-trigger strategy* can encourage cooperation in inter-session network coding. We also analyze the general properties of *all* possible bargaining schemes, and provide universal upper bounds on the PoA for any bargaining method. Finally, we show that the PoA in the repeated game can be improved by using a min-max bargaining scheme. The contributions of this paper are as follows:

- *New Formulation*: To the best of our knowledge, we are the first to formulate non-cooperative inter-session network coding as a *repeated* resource allocation game.
- *Equilibrium Strategy Design*: We show that a *grim-trigger strategy* can form a subgame perfect equilibrium for the repeated inter-session network coding game, as long as the network coding users can agree on the inter-session network coding rate. Reaching such an agreement is non-trivial in general. It involves solving a bargaining problem that resolves the conflict of interest among users.
- *Performance Bounds for All Bargaining Schemes*: We show that, for *any* bargaining method, the PoA of the repeated inter-session network coding game in the studied network is upper-bounded by 36%, 44%, and 48% when the network has one network coding pair and several routing sessions, two network coding pairs, and one network coding pair, respectively. Our results show that the improvement in the PoA of repeated inter-session network coding games is not as drastic as most typical repeated games. We explain the reasons behind this observation.
- *Efficient Bargaining*: We propose a novel min-max bargaining method, which can reach the obtained performance upper bounds for the α-fair utility functions.

The results in this paper are different from the existing

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Manuscript received on January 17, 2011; revised on August 30, 2012 and April 27, 2013; and accepted on June 10, 2013. This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada. The review of this paper was coordinated by Prof. Suhas Diggavi.

This paper has been presented in part in the *IEEE International Conference on Computer Communications (INFOCOM'10)*, San Diego, CA, March 2010.

TABLE I

SUMMARY OF THE RESULTS ON THE ACHIEVABLE PoA FOR STATIC AND REPEATED INTER-SESSION NETWORK CODING GAMES.

Players	Static Game	Repeated Game
One Network Coding Pair	22%	48%
Two Network Coding Pairs	-	44%
Network Coding and Routing Sessions	20%	36%
Reference	[6]	This Paper

results on network coding games, e.g., [6], [7], [12]–[21]. The studies in [12]–[16], [20], [21] focused on intra-session network coding, while here we address inter-session network coding. Similar to [7], [17], we study inter-session network coding in a butterfly network topology. However, we further investigate the impacts of users' utility functions, link costs, and the PoA. Moreover, unlike the system models in [7], [17], [18], [22], we address the case where the network includes both network coding and pure routing users. Finally, we study repeated games, while the results in [6], [7], [12]–[19] are for static network coding games. A key motivation of this study is our prior work on static network coding games in [6]. A comparison of the main results in [6] and this paper is given in Table I. Compared to [6], the extension in this paper is non-trivial and reveals several interesting properties of repeated inter-session network coding games. For example, we show that even if a grim-trigger strategy and an arbitrarily large discount factor are used, "no coding" can still remain a dominant strategy under certain repeated game scenarios. Moreover, unlike the system model in [6] that includes only a single network coding pair, here we also consider the case with more than one network coding pair. The new model allows us to study the co-existence of multiple selfish inter-session network coding pairs, in addition to studying the co-existence of selfish network coding pairs and selfish routing sessions.

The rest of this paper is organized as follows. In Section II, we introduce the system model and review the static game results in [6]. The repeated game is formulated in Section III. Our results on subgame perfect equilibrium, bargaining, and the PoA bounds are given in Section IV. The min-max bargaining solution and its PoA are discussed in Section V. Numerical results are presented in Section VI. Conclusions and directions for future work are provided in Section VII.

II. SYSTEM MODEL AND BACKGROUND

Consider the network topology in Fig. 1, which is usually referred to as a butterfly network in the network coding literature[1]. It consists of $N \geq 2$ end-to-end users and $2M+1$ wired links, where $M \geq 1$. The bottleneck link (i,j) is shared by all users $\mathcal{N} = \{1, \ldots, N\}$. For each user $n \in \mathcal{N}$, the source and the destination nodes are denoted by s_n and t_n, respectively. We first distinguish two different types of users:

- *Network Coding Users* in set $\mathcal{M} = \{1, \ldots, M, N-M+1, \ldots, N\}$, who can perform inter-session network coding.

[1]Although the network coding scenario in Fig. 1 is simple, it can be used as a building block for more general scenarios. For example, [2], [3] showed that a network can be modeled as a *superposition* of several butterfly networks. Thus, understanding Fig. 1 is a key to study more general networks.

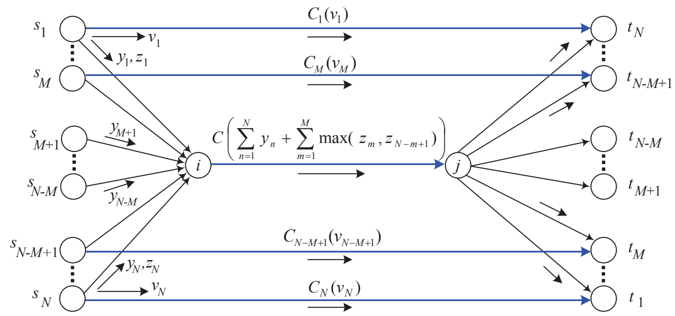

Fig. 1. A butterfly network with N unicast sessions, called users. Users $1, \ldots, M$ and $N-M+1, \ldots, N$ are network coding users. Users $M+1, \ldots, N-M$ are routing users. As an example, users 1 and N can perform inter-session network coding over links (i,j), (s_1, t_N), and (s_N, t_1). Packet $X_1 \oplus X_N$ is obtained by joint encoding of packets X_1 and X_N.

- *Routing Users* in set $\mathcal{N} \backslash \mathcal{M} = \{M+1, \ldots, N-M\}$, who cannot perform inter-session network coding.

Network coding users can mark their packets (e.g., by setting a single-bit flag in the packet header) for either routing or network coding. However, routing users can setup all their packets only for routing. At the intermediate node i, all packets that are marked for routing are simply forwarded to node j through link (i,j). However, the packets that are marked for network coding are treated differently. Consider users 1 and N that form a network coding pair. Let X_1 and X_N denote two packets which are marked for network coding and are sent from nodes s_1 and s_N to node i, respectively. Node i can *encode* packets X_1 and X_N (e.g., using XOR encoding [23]), and send the resulting packet, denoted by $X_1 \oplus X_N$, to node j (and from there to t_1 and t_N). Given the *remedy* data X_1 from side link (s_1, t_N) and the remedy data X_N from side link (s_N, t_1), nodes t_N and t_1 can decode their own packets X_N and X_1 based on the encoded data they receive from the intermediate node j. Clearly, the benefit of network coding is to reduce the traffic on bottleneck link (i,j), by sending only one packet $X_1 \oplus X_N$ rather than two packets X_1 and X_N.

We define the following notations for data rates in Fig. 1:

- y_n: Transmission rate of *routing* packets sent from node s_n to node i, for each user $n \in \mathcal{N}$.
- z_n: Transmission rate of *network coding* packets sent from node s_n to node i, for each user $n \in \mathcal{M}$.
- v_n: Transmission rate of *remedy* packets sent from node s_n over its side link, for each user $n \in \mathcal{M}$.

In this paper, we make the following key assumption:

Assumption 1: Users are *autonomous* and have full control over their transmission rates. The network coding users individually indicate, via marking, whether their packets should be encoded or simply be forwarded over link (i,j).

Node i encodes packets at rate $\min\{z_m, z_{N-m+1}\}$, for each $m \in \mathcal{M}$, and forwards the rest of the packets, without encoding their contents at rate $\sum_{n=1}^{N} y_n + \sum_{m=1}^{M} |z_m - z_{N-m+1}|$. Thus, the *total* rate on bottleneck link (i,j) becomes

$$\textstyle\sum_{n=1}^{N} y_n + \sum_{m=1}^{M} \max\{z_m, z_{N-m+1}\}. \qquad (1)$$

Destination nodes t_n, for $n = M+1, \ldots, N-M$, receive information at rate y_n while nodes t_m and t_{N-m+1}, for $m =$

$1,\ldots,M$, receive information at rates $y_m+\min\{z_m, v_{N-m+1}\}$ and $y_{N-m+1}+\min\{z_{N-m+1}, v_m\}$, respectively.

A. Utility, Cost, and Price Functions

Each user $n\in\mathcal{N}$ has a utility function U_n, representing its evaluation of its achieved data rate. Link (i,j) has a cost function C, which depends on its total traffic load $\sum_{n=1}^{N} y_n + \sum_{m=1}^{M}\max\{z_m, z_{N-m+1}\}$. Similarly, each set of side links (s_m, t_{N-m+1}) and (s_{N-m+1}, t_m) have cost functions C_m and C_{N-m+1}, which depend on their loads v_m and v_{N-m+1}.

Assumption 2: The utility functions $U_1,\ldots,U_N$ are *concave*, *non-negative*, *increasing*, and *differentiable* [24].

Assumption 3: The link cost functions are given as $C(q)=\frac{a}{2}q^2$ and $C_m(q)=\frac{b_m}{2}q^2$ for all $m\in\mathcal{M}$, where $a>0$, $b_m>0$, and $q\geq 0$. These convex quadratic cost functions are related to linear price functions $p(q)=aq$ and $p_m(q)=b_m q$. In fact, $C(q)=\int_0^q p(\theta)d\theta$ and $C_m(q)=\int_0^q p_m(\theta)d\theta$ for all $m\in\mathcal{M}$.

Quadratic cost and linear price functions are the only cost and price functions that satisfy the four axioms of *rescaling*, *consistency*, *positivity*, and *additivity* in cost-sharing systems [25]. They are often used in network resource management (cf. [26]–[32]) to model either actual transmission cost (e.g., in dollars) or simply the queueing delay on each link.

B. Optimization-based Resource Allocation

Let $\boldsymbol{y}=(y_n,\ \forall n\in\mathcal{N})$, $\boldsymbol{v}=(v_m,\forall m\in\mathcal{M})$, and $\boldsymbol{z}=(z_m,\forall m\in\mathcal{M})$. The *network aggregate surplus* is defined as the total utility of the users minus the total cost of the links:

$$\begin{aligned}\mathbb{S}(\boldsymbol{y},\boldsymbol{z},\boldsymbol{v}) = &\textstyle\sum_{m=1}^{M}\left[U_m\left(y_m+\min\{z_m, v_{N-m+1}\}\right)\right.\\ &\left.+\,U_{N-m+1}\left(y_{N-m+1}+\min\{z_{N-m+1}, v_m\}\right)\right]\\ &+\textstyle\sum_{n=M+1}^{N-M} U_n\left(y_n\right)\\ &-\textstyle\sum_{m=1}^{M}\left[C_m(v_m)+C_{N-m+1}(v_{N-m+1})\right]\\ &-C(\textstyle\sum_{n=1}^{N} y_n+\sum_{m=1}^{M}\max\{z_m, z_{N-m+1}\}).\end{aligned}$$

Given complete knowledge and centralized control of the network in Fig. 1, we can compute the efficient rate allocation by solving the following optimization problem:

Problem 1 (Network Surplus Maximization Problem):

$$\begin{aligned}&\underset{\boldsymbol{y},\boldsymbol{z},\boldsymbol{v}}{\textbf{maximize}} && \mathbb{S}(\boldsymbol{y},\boldsymbol{z},\boldsymbol{v})\\ &\textbf{subject to} && y_n\geq 0,\quad n\in\mathcal{N},\quad z_m, v_m\geq 0,\quad m\in\mathcal{M}.\end{aligned}$$

Let $\boldsymbol{y}^S=(y_1^S,\ \forall n\in\mathcal{N})$, $\boldsymbol{v}^S=(v_m^S,\ \forall m\in\mathcal{M})$, and $\boldsymbol{z}^S=(z_m^S,\ \forall m\in\mathcal{M})$ be an optimal solution for Problem 1. We can verify that $v_m^S=z_m^S$ for all $m\in\mathcal{M}$, i.e., the network coding users send the coded and remedy packets at the same rate for the optimal rate allocation.

C. Pricing and Resource Allocation Game

If the network has no centralized controller and Assumption 1 holds, *pricing* can be used to encourage efficient resource allocation in a distributed fashion [24]. Given the rate vectors $\boldsymbol{y}$ and $\boldsymbol{z}$ from the users, the shared link (i,j) can set a price

$$\mu(\boldsymbol{y},\boldsymbol{z})=\left(\textstyle\sum_{n=1}^{N} y_n+\sum_{m=1}^{M}\max\{z_m, z_{N-m+1}\}\right)\qquad(2)$$

for any *uncoded* data rate it carries, where price function $p(q)$ is described in Assumption 3. For coded packets, however, the bottleneck link can set a separate reduced price

$$\sigma(\boldsymbol{y},\boldsymbol{z})=\beta\, p\left(\textstyle\sum_{n=1}^{N} y_n+\sum_{m=1}^{M}\max\{z_m, z_{N-m+1}\}\right).\qquad(3)$$

Here, $\beta\in(0,1]$ is the *price discrimination* parameter, and the intuition is to charge less for coded packets to encourage network coding. Note that only the choice of $\beta=\frac{1}{2}$ can avoid over- or under-charging of network coding users [6].

Assumption 4: Throughout this paper, we set $\beta=\frac{1}{2}$.

Given data rates $\boldsymbol{v}$ for the remedy packets, for each $m=1,\ldots,M$, side links (s_m, t_{N-m+1}) and (s_{N-m+1}, t_m) set their prices as $p_m\left(v_m\right)$ and $p_{N-m+1}\left(v_{N-m+1}\right)$ for the data they carry. Network coding users are charged as follows:

- User m pays the following payment to link (i,j):

$$\begin{aligned}&\sigma(\boldsymbol{y},\boldsymbol{z})\ \min(z_m, z_{N-m+1})+\mu(\boldsymbol{y},\boldsymbol{z})\, y_m\\ &+\ \mu(\boldsymbol{y},\boldsymbol{z})\ \left(z_m-\min(z_m, z_{N-m+1})\right)\\ &=\ \mu(\boldsymbol{y},\boldsymbol{z})\left(y_m+z_m-(1-\beta)\min\{z_m, z_{N-m+1}\}\right),\end{aligned}$$

  and pays $v_m\, p_m(v_m)$ to link (s_m, t_{N-m+1}).
- User $N-m+1$ makes similar payments to links (i,j) and (s_{N-m+1}, t_m).

Each routing user $n\in\mathcal{N}\backslash\mathcal{M}$ pays $\mu(\boldsymbol{y},\boldsymbol{z})\, y_n$ to link (i,j). Users set their rates to maximize their surplus, i.e., utility minus charges [26], [29]. Each user's surplus also depends on the rates set by other users, leading to a game among users:

Game 1 (Non-cooperative Resource Allocation Game):

- *Players*: Users in set $\mathcal{N}$.
- *Strategies*: Transmission rates $\boldsymbol{y}$, $\boldsymbol{z}$, and $\boldsymbol{v}$.
- *Payoffs*: $P_1(\cdot),\ldots,P_N(\cdot)$, where for any $m=1,\ldots,M$:

$$\begin{aligned}&P_m(y_m, z_m, v_m;\boldsymbol{y}_{-1},\boldsymbol{z}_{-1},\boldsymbol{v}_{-1})=\\ &\quad U_m\left(y_m+\min\{z_m, v_{N-m+1}\}\right)-v_m p_m(v_m)\\ &\quad -\left(y_m+z_m-(1-\beta)\min\{z_m, z_{N-m+1}\}\right)\\ &\quad \times p\left(\textstyle\sum_{r=1}^{N} y_r+\sum_{r=1}^{M}\max\{z_r, z_{N-r+1}\}\right),\end{aligned}$$

$$\begin{aligned}&P_{N-m+1}(y_{N-m+1}, z_{N-m+1}, v_{N-m+1};\\ &\qquad\boldsymbol{y}_{-(N-m+1)},\boldsymbol{z}_{-(N-m+1)},\boldsymbol{v}_{-(N-m+1)})=\\ &\quad U_{N-m+1}\left(y_{N-m+1}+\min\{z_{N-m+1}, v_m\}\right)\\ &\quad -v_{N-m+1}p_{N-m+1}(v_{N-m+1})\\ &\quad -\left(y_{N-m+1}+z_{N-m+1}-(1-\beta)\min\{z_m, z_{N-m+1}\}\right)\\ &\quad \times p\left(\textstyle\sum_{r=1}^{N} y_r+\sum_{r=1}^{M}\max\{z_r, z_{N-r+1}\}\right),\end{aligned}$$

  and for the routing user $n\in\mathcal{N}\backslash\mathcal{M}$:

$$\begin{aligned}&P_n(y_n;\boldsymbol{y}_{-n},\boldsymbol{z},\boldsymbol{v})=U_n(y_n)\\ &\quad -y_n p\left(\textstyle\sum_{r=1}^{N} y_r+\sum_{m=1}^{M}\max\{z_m, z_{N-m+1}\}\right).\end{aligned}$$

Here, $\boldsymbol{y}_{-n}=(y_1,\ldots,y_{n-1},y_{n+1},\ldots,y_N)$ for any $n=1,\ldots,N$; $\boldsymbol{z}_{-m}=(z_1,\ldots,z_{m-1},z_{m+1},\ldots,z_M,z_{N-M+1},\ldots,z_N)$ and $\boldsymbol{v}_{-m}=(v_1,\ldots,v_{m-1},v_{m+1},\ldots,v_M,v_{N-M+1},\ldots,v_N)$ for any $m=1,\ldots,M$; and $\boldsymbol{z}_{-m}=(z_1,\ldots,z_M,z_{N-M+1},\ldots,z_{N-m},z_{N-m+2},\ldots,z_N)$ and $\boldsymbol{v}_{-m}=(v_1,\ldots,v_M,v_{N-M+1},\ldots,v_{N-m},v_{N-m+2},\ldots,v_N)$ for any $m=N-M+1,\ldots,N$. Game 1 is a static game and is played once. The repeated version of Game 1 will be formulated in Section III.

D. Efficiency and Price-of-Anarchy of Game 1

The selfish nature of the players in Game 1 leads to undesirable and inefficient network performance. To see this, we first introduce the following definitions for future reference.

Definition 1 (Nash equilibrium): The non-negative rates $\boldsymbol{y}^* = (y_n^*,\ \forall n \in \mathcal{N})$, $\boldsymbol{v}^* = (v_m^*,\ \forall m \in \mathcal{M})$, and $\boldsymbol{z}^* = (z_m^*,\ \forall m \in \mathcal{M})$ form a Nash equilibrium of Game 1 if *no* user $n \in \mathcal{N}$ can increase its payoff by *unilaterally* changing its data rates. The Nash equilibrium predicts how Game 1 will be played.

Definition 2 (Efficiency): For a certain choice of system parameters, the *efficiency* at Nash equilibrium $(\boldsymbol{y}^*, \boldsymbol{z}^*, \boldsymbol{v}^*)$ is the ratio of the achieved network aggregate surplus $\mathbb{S}(\boldsymbol{y}^*, \boldsymbol{z}^*, \boldsymbol{v}^*)$ to the optimal network aggregate surplus $\mathbb{S}(\boldsymbol{y}^S, \boldsymbol{z}^S, \boldsymbol{v}^S)$.

Definition 3 (Price-of-anarchy): The price-of-anarchy, denoted by $\text{PoA}(\text{Game 1}, \text{Problem 1})$, is the *worst-case* efficiency of a Nash equilibrium among *all* possible choices of *parameters* (number of users and the utility, cost, and price functions).

Next, we notice that payoff $P_m(\cdot)$ for any $m \in \mathcal{M}$ is *decreasing* in v_m. Thus, a *selfish* network coding user m will always choose to send *no* remedy packets to avoid payments over its side link. Being aware of this issue, all network coding users will *not* participate in network coding, as they cannot decode any encoded packets without the remedy packets. The following results are from [6, Theorem 11].

Theorem 1: (a) Game 1 has a unique Nash equilibrium.
(b) At Nash equilibrium of Game 1, we have

$$v_m^* = z_m^* = 0 \quad \forall\, m \in \mathcal{M}. \tag{4}$$

(c) If $N = 2$ and $M = 1$ then

$$\text{PoA}\,(\text{Game 1}, \text{Problem 1}) = \frac{2}{9} \approx 22\%. \tag{5}$$

(d) If $N > 2$ and $M = 1$ then

$$\text{PoA}\,(\text{Game 1}, \text{Problem 1}) = \frac{1}{5} = 20\%. \tag{6}$$

The PoA results in Theorem 1 are significantly less than the 67% PoA for a similar game with routing users only that is showed in [26]. The results in Theorem 1 imply that although inter-session network coding can potentially improve network performance, it is more *sensitive* to selfish behavior than routing. Next, we show that we can design better strategies with better PoAs when Game 1 is played repeatedly.

III. Repeated Inter-session Network Coding Game

Consider the case where Game 1 is played *repeatedly*. That is, *every* time users play Game 1 (called one stage), they will play Game 1 again with a probability δ. Parameter δ is the *discount factor* [8]. A repeated game formulation is natural if users have many packets to transmit. If Game 1 is played multiple times, then the strategy space of the users will expand to include their data rates at each stage of the game. Let $\boldsymbol{y}^k = (y_n^k,\ \forall n \in \mathcal{N})$, $\boldsymbol{z}^k = (z_m^k,\ \forall m \in \mathcal{M})$, and $\boldsymbol{v}^k = (v_m^k,\ \forall m \in \mathcal{M})$ denote the actions chosen by users at stage $k \geq 1$. At the beginning of stage k, the rates that have been already played in stages $1, \ldots, k-1$ form the *history* of the game, while the rates to be played in stages $k, k+1, \ldots$ are strategies of users. For notational simplicity, for each $1 \leq l \leq k$, we define

$$\mathcal{R}_{t=l}^{k} = \left\{\boldsymbol{y}^t, \boldsymbol{z}^t, \boldsymbol{v}^t\right\}_{t=l}^{k}. \tag{7}$$

In this regard, $\mathcal{R}_{t=1}^{k-1}$ denotes the history and $\mathcal{R}_{t=k}^{\infty}$ denotes the strategies of the users, at each stage $k \geq 1$.

***Game 2** (**Repeated Game 1**):*

- *Players*: Users in set $\mathcal{N}$.
- *Histories*: Data rates $\mathcal{R}_{t=1}^{k-1}$, at each stage $k \geq 1$.
- *Strategies*: Contingency plans for selection of rates $\mathcal{R}_{t=k}^{\infty}$ at each stage $k \geq 1$ for any given history profile $\mathcal{R}_{t=1}^{k-1}$.
- *Payoffs*: $Q_n(\cdot)$ for each user $n \in \mathcal{N}$, where at each $k \geq 1$,

$$Q_n(\mathcal{R}_{t=k}^{\infty} \mid \mathcal{R}_{t=1}^{k-1}) = \textstyle\sum_{t=k}^{\infty} (\delta)^{t-k}\, P_n(\boldsymbol{y}^t, \boldsymbol{z}^t, \boldsymbol{v}^t).$$

In Game 2, the *single-stage* payoffs $P_1(\cdot), \ldots, P_N(\cdot)$ are the same as in Game 1. Payoffs $Q_1(\cdot), \ldots, Q_N(\cdot)$ are the *discounted* summations of the users' payoffs in the future. The term $(\delta)^{t-k}$ denotes the probability that Game 2 is played at stage $t > k$, given that it is currently played at stage $k \geq 1$.

Definition 4 (Subgame): Given a history profile $\mathcal{R}_{t=1}^{k-1}$ at stage $k \geq 1$ of Game 2, the rest of the repeated game at stages $k, k+1, \ldots$ is defined as a *subgame* at stage k.

The solution concept for a repeated game is the *subgame perfect equilibrium* which is defined as follows [8]:

Definition 5 (Subgame Perfect Equilibrium): A strategy profile $\mathcal{R}_{k=1}^{\infty}$ is a subgame perfect equilibrium of Game 2, if at any stage k, the restricted strategy profile $\mathcal{R}_{t=k}^{\infty}$ is a Nash equilibrium for any subgame at stage k formed by every given history $\mathcal{R}_{t=1}^{k-1}$. That is, at any stage and for any history profile, no user $n \in \mathcal{N}$ can increase its payoff $Q_n(\cdot)$ by unilaterally changing its own data rates in future stages.

Definition 6 (Efficiency): The *efficiency* at subgame perfect equilibrium $\mathcal{R}_{k=1}^{\infty}$ is defined as the average efficiency among all stages of Game 2, where the efficiency for rates $(\boldsymbol{y}^k, \boldsymbol{z}^k, \boldsymbol{v}^k)$ at stage k is defined according to Definition 2.

Definition 7 (Price-of-anarchy): The *price-of-anarchy*, denoted by $\text{PoA}(\text{Game 2}, \text{Problem 1})$, is the *worst-case* (i.e., the smallest) efficiency at a subgame perfect equilibrium of Game 2 among all possible choices of system parameters.

Before we end this section, we note that the parameters of Game 2 are assumed to be fixed at all stages of the game.

IV. Punishment and Bargaining in Inter-Session Network Coding

In this section, we analyze repeated Game 2 and show the following. First, a grim-trigger strategy encourages users to cooperate. Second, if the network coding users cooperate, they will select the same network coding rates. Third, the common network coding rate can be determined via bargaining. Finally, the PoA of Game 2 is better than that of Game 1.

A. Punishment and Grim-trigger Strategy

At the end of each stage of Game 2, user m, for each $m = 1, \ldots, M$, knows whether user $N-M+1$ has cooperated (i.e., sent enough remedy packets such that user m can decode all received encoded packets) during the current stage. Thus, user m can *punish* user $N-m+1$ in the next stage, if user $N-m+1$ has *cheated*. This is also true for user $N-m+1$.

Network coding users m and $N-m+1$ may consider various *punishment* strategies against a cheating user. For example, if user $N-m+1$ cheats at stage $k-1$ of Game 2, then user m may select its data rates (y_m^k, z_m^k, v_m^k) to minimize user $N-m+1$'s payoff in the next stage. Another option for user m is not to participate in network coding by setting $v_m^k = z_m^k = 0$. Punishment strategies can be either *limited scope*, lasting for only a few stages, or *unlimited scope*, lasting until the game ends. In this paper, we assume that the punishment is not to participate in network coding for the rest of the game. We will show that this punishment strategy can prevent cheating. To start with, we show that if users decide to cooperate, they must choose the same network coding rates.

Theorem 2: Assume that users select data rates $\boldsymbol{y}^k$, $\boldsymbol{z}^k$, and $\boldsymbol{v}^k$ at a stage k of repeated Game 2 with

$$v_m^k = z_m^k > v_{N-m+1}^k = z_{N-m+1}^k, \tag{8}$$

for any $m = 1, \ldots, M$. In this case, neither user m nor user $N-m+1$ cheat, but user m wants to participate in network coding with a higher rate than user $N-m+1$. Then, user m can switch to new rates $(\bar{y}_m^k, \bar{v}_m^k, \bar{z}_m^k)$ such that

$$\bar{y}_m^k = y_m^k + (z_m^k - z_{N-m+1}^k), \; \bar{v}_m^k = \bar{z}_1^k = v_{N-m+1}^k = z_{N-m+1}^k, \tag{9}$$

to strictly increase its own payoff at stage k, while keeping the payoff of all the other users unchanged at stage k.

The proof of Theorem 2 is given in Appendix A. From Theorem 2, if users m and $N-m+1$ do not plan to cheat and want to cooperate, they must choose the *same* coding rates:

$$z_m^k = v_m^k = z_{N-m+1}^k = v_{N-m+1}^k, \quad \forall k \geq 1. \tag{10}$$

These results can help us predict how network coding users behave if they choose to cooperate. However, we still need to answer two questions:

1) Which *common* network coding rate
$$z_m^k = v_m^k = z_{N-m+1}^k = v_{N-m+1}^k = z_{m,N-m+1} \geq 0 \tag{11}$$
should users m and $N-m+1$ choose in stage k?
2) How do network coding users m and $N-m+1$ enforce cooperation such that they both have the incentive to send remedy packets at the desired rate $z_{m,N-m+1} \geq 0$?

We will answer the second question first. The first question will be answered in Section IV-C when we discuss bargaining.

Next, we explain how users behave at each stage $k \geq 1$ of repeated Game 2 if (11) holds for a given $z_{m,N-m+1} \geq 0$. For the ease of exposition, we define a new *static* game which is derived from Game 1 and is parameterized with $z_{m,N-m+1}$.

Game 3 (Reduced Game 1 for a Given $z_{m,N-m+1} \geq 0$):

- *Players*: Users in set $\mathcal{N}$.
- *Strategies*: Transmission rates $\boldsymbol{y}$, when for each $m = 1, \ldots, M$, the network coding rates $\boldsymbol{v}$ and $\boldsymbol{z}$ are fixed at
$$z_m = v_m = z_{N-m+1} = v_{N-m+1} = z_{m,N-m+1}. \tag{12}$$
- *Payoffs*: $P_n(\cdot)$ for each user $n \in \mathcal{N}$ as in Game 1.

Games 1 and 3 differ only due to (12). Since the network coding rates are pre-determined, the strategy of users in Game 3 is *reduced* to routing rates $\boldsymbol{y}$ only. From Theorem 1(a), Game 1 has a *unique* Nash equilibrium. Clearly, the Nash equilibrium of Game 3 depends on the choice of parameter $z_{m,N-m+1}$.

Given $z_{m,N-m+1} \geq 0$, we denote the Nash equilibrium of Game 3 by $\boldsymbol{y}^*(\boldsymbol{z})$. Therefore, the payoff for each user $n \in \mathcal{N}$ at Nash equilibrium of Game 3 is denoted by

$$P_n(\boldsymbol{y}^*(\boldsymbol{z}), z_{1,N}, \ldots, z_{M,N-M+1}). \tag{13}$$

For example, for network coding user 1, we have

$$\begin{aligned} P_1(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z}) = {} & U_1\left(y_1^*(\boldsymbol{z}) + z_{1,N}\right) - z_{1,N}\, p_1(z_{1,N}) \\ & - \left(y_1^*(\boldsymbol{z}) + \beta z_{1,N}\right) \\ & \times p\left(\textstyle\sum_{r=1}^N y_r^*(\boldsymbol{z}) + \sum_{m=1}^M z_{m,N-m+1}\right). \end{aligned}$$

We now return to repeated Game 2. Clearly, if the network coding users agree on selecting their network coding rates according to (11), then at each stage $k \geq 1$, the users simply select their routing data rates to be $\boldsymbol{y}^k = \boldsymbol{y}^*(\boldsymbol{z})$. This helps us to introduce a strategy profile that can enforce cooperation, answering our second question posed earlier in this section.

Definition 8 (Grim-trigger Strategy): Given a set of pre-determined coding rates $z_{m,N-m+1} \geq 0$, for all $m = 1, \ldots, M$, a *grim-trigger strategy* for Game 2 is defined as

Step 1: For network coding pairs m and $N-m+1$, where $m = 1, \ldots, M$, always *participate* in network coding adopting the coding rate $z_{m,N-m+1}$. All users choose their routing rates according to the Nash equilibrium of Game 3 for given $z_{1,N}, \ldots, z_{M,N-M+1}$. Go to Step 2 if users m or $N-m+1$, for any $m = 1, \ldots, M$, deviate from coding rate $z_{m,N-m+1}$.

Step 2: *Refuse* network coding *forever*. That is, at any future stage k, set network coding rates $v_m^k = z_m^k = 0$ for all $m \in \mathcal{M}$ and routing rates $y_n^k = y_n^*(\boldsymbol{0})$ for all $n \in \mathcal{N}$.

The above is an unlimited scope punishment. Both network coding and routing users participate in the punishment as they all set their rates according to the new Nash equilibrium of Game 3 with no network coding. We can show the following:

Theorem 3: Given fixed common coding rates $\boldsymbol{z} = (z_{m,N-m+1} \geq 0,\ m = 1, \ldots, M)$, there exists a $\delta_{\min} \in (0, 1]$ such that the grim-trigger strategy in Definition 8 forms a *subgame perfect equilibrium* for Game 2 if and only if the discount factor is $\delta_{\min} \leq \delta \leq 1$ and we have

$$P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z}) \geq P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0}), \quad \forall m \in \mathcal{M}. \tag{14}$$

The proof is given in Appendix B. If (14) holds, then all network coding users are better off to perform network coding at rate $\boldsymbol{z}$ as in Step 1 instead of no coding as in Step 2.

B. Examples and Comparison with Prisoner's Dilemma

As an example, consider Fig. 1, with one network coding pair ($N = 2$ and $M = 1$). The system parameters are set as

$$U_1(x) = \log(1+x), \qquad U_2(x) = 0.75 \log(1+x), \tag{15}$$

$$a = 1, \qquad b_1 = 0.5, \qquad b_2 = 0.25, \qquad \beta = 0.5. \tag{16}$$

TABLE II

PAYOFFS AT EACH STAGE OF GAME 2 WHEN (15) AND (16) HOLD.

		User 2	
	Strategy	Cooperate	Cheat
User 1	Cooperate	(0.19, 0.12)	(-0.08, 0.14)
	Cheat	(0.24, -0.10)	(0.12, 0.08)

TABLE III

PAYOFFS AT EACH STAGE OF GAME 2 WHEN (17) AND (18) HOLD.

		User 2	
	Strategy	Cooperate	Cheat
User 1	Cooperate	(0.27, -0.008)	(0.17, -0.006)
	Cheat	(0.28, -0.03)	(0.25, 0)

We can verify that if we select $z_{1,2} = 0.3$, then $y_1^*(z_{1,2}) = 0.128$, $y_2^*(z_{1,2}) = 0$, and at each stage of repeated Game 2, the users play a game according to Table II, where the numbers in each box indicate the payoffs for user 1 and user 2, respectively. In this example, the grim-trigger strategy is a subgame perfect equilibrium and the users always pay (Cooperate, Cooperate) if discount factor $\delta \geq \max\{\frac{0.24-0.19}{0.24-0.12}, \frac{0.14-0.12}{0.14-0.08}\} \approx 0.38$. We note that the payoffs in Table II resemble the payoffs in the *prisoner's dilemma* game [8, p. 110]. However, in general, Game 2 has two key differences with the prisoner's dilemma game. First, "cooperation" is not well-defined in Game 2, as it is not immediately clear for the network coding users which common (and cooperative) coding rate $z_{1,2}$ they should choose. We will address this issue when we discuss bargaining in Section IV-C. Second, even if no user cheats and both users transmit the remedy packets at the same rate as the coding packets, cooperation may still remain non-beneficial for one or both users. This second property can be seen in our next example where the system parameters are set as

$$U_1(x) = x, \qquad U_2(x) = 0.2x, \tag{17}$$

$$a = 1, \qquad b_1 = 0.1, \qquad b_2 = 0.2, \qquad \beta = 0.5. \tag{18}$$

If we select $z_{1,2} = 0.1$, then $y_1^*(z_{1,2}) = 0.425$, $y_2^*(z_{1,2}) = 0$, and at each stage of repeated Game 2, the users play a game according to Table III. In this case, while cooperation is beneficial for user 1, it is damaging for user 2, something that does not occur in a prisoner's dilemma game. Therefore, regardless of the value of the discount factor, users always pay (Cheat, Cheat) as the sub-game perfect equilibrium.

C. Bargaining

So far, we have assumed that the common network coding rate $z_{m,N-m+1} \geq 0$, for any $m = 1, \ldots, M$, is given. In this section, we will discuss how the network coding users m and $N-m+1$ can agree on the choice of $z_{m,N-m+1}$. Clearly, network coding user m prefers to choose $z_{m,N-m+1}$ to

$$\underset{z_{m,N-m+1}\geq 0}{\textbf{maximize}} \; P_m(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1}), \tag{19}$$

where $\boldsymbol{z}_{-(m,N-m+1)} = (z_{r,N-r+1}, r = 1, \ldots, m{-}1, m{+}1, \ldots, M)$. Similarly, user $N-m+1$ would prefer the solution of

$$\underset{z_{m,N-m+1}\geq 0}{\textbf{maximize}} \; P_{N-m+1}(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1}). \tag{20}$$

However, in either case, the solution may not be fair and mutually acceptable to *both* users m and $N-m+1$. A natural way to resolve this is *bargaining* in cooperative game theory [33], where two players negotiate on the details of cooperation.

One option is the well-known *Nash bargaining solution* [34]. However, this solution is usually computationally complex. Thus, we seek to find an alternative bargaining approach that is simple yet efficient (as we will see in Section V) and based on modifying problems (19) and (20). But first, we analyze some of the general properties that hold in a repeated inter-session network coding game for *any* bargaining solution.

D. Upper Bounds on Price-of-Anarchy of Game 2

For a pair of network coding users m and $N-m+1$, where $m = 1, \ldots, M$, assume that given $\boldsymbol{z}_{-(m,N-m+1)}$, either

$$P_m(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1}) < P_m(\boldsymbol{y}^*(0, \boldsymbol{z}_{-(m,N-m+1)}), 0), \quad \forall z_{m,N-m+1} > 0, \tag{21}$$

or

$$P_{N-m+1}(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1}) < P_{N-m+1}(\boldsymbol{y}^*(0, \boldsymbol{z}_{-(m,N-m+1)}), 0), \quad \forall z_{m,N-m+1} > 0. \tag{22}$$

Then the grim-trigger strategy in Definition 8 is a subgame perfect equilibrium if and only if the common coding rate is

$$z_{m,N-m+1} = 0, \tag{23}$$

for *any* value of discount factor $\delta \in (0, 1]$. That is, no network coding is performed at the subgame perfect equilibrium between users m and $N-m+1$ because at least one of them is worse off when it participates in network coding. In an extreme case, if (21) or (22) holds for *any* $m = 1, \ldots, M$ given $\boldsymbol{z}_{-(m,N-m+1)} = \boldsymbol{0}$, then regardless of the bargaining approach being used, at subgame perfect equilibrium we have:

$$\boldsymbol{z} = \boldsymbol{0}, \qquad \boldsymbol{y}^k = \boldsymbol{y}^*(\boldsymbol{0}), \qquad \forall k \geq 1. \tag{24}$$

That is, users simply play the Nash equilibrium of Game 1 at every stage of Game 2. Thus, *in such special cases*, efficiency at subgame perfect equilibrium of the repeated game is equal to efficiency at the Nash equilibrium of the static game.

It is shown in [6, Theorem 11] that the *worst-case* efficiency of static Game 1 occurs under the following conditions:

- The utility functions of the users are linear. That is,
$$U_n(x) = \gamma_n\, x, \quad \forall n \in \mathcal{N}. \tag{25}$$
- The cost parameters for *side* links are negligible. That is,
$$b_m \to 0, \quad \forall m \in \mathcal{M}. \tag{26}$$

The intuition behind (26) is clear: if the side links have low cost, then performing inter-session network coding can bring significant throughput gains to the users without significantly increasing the cost. Thus, not performing inter-session network coding in this case will hurt the system performance the most.

The above discussions imply that we expect to find tight upper bounds for PoA (Game 2, Problem 1) if we can obtain the worst-case efficiency among all choices of system parameters which satisfy (25), (26), and either (21) or (22). In this regard, we study three cases separately as we see next.

1) One Network Coding Pair ($N=2$ and $M=1$): Assume that the network in Fig. 1 has only two network coding users.

Proposition 1: Given $z_{1,2} \geq 0$, if conditions (25) and (26) hold, then a at Nash equilibrium of the reduced Game 3:
(a) If $0 \leq z_{1,2} < (2\gamma_2 - \gamma_1)/(a(1+\beta))$ then

$$\begin{bmatrix} y_1^*(z_{1,2}) \\ y_2^*(z_{1,2}) \end{bmatrix} = \begin{bmatrix} \frac{2\gamma_1-\gamma_2-a(1+\beta)z_{1,2}}{3a} \\ \frac{2\gamma_2-\gamma_1-a(1+\beta)z_{1,2}}{3a} \end{bmatrix}, \tag{27}$$

(b) If $(2\gamma_2 - \gamma_1)/(a(1+\beta)) \leq z_{1,2} < \gamma_1/(a(1+\beta))$ then

$$\begin{bmatrix} y_1^*(z_{1,2}) \\ y_2^*(z_{1,2}) \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1-a(1+\beta)z_{1,2}}{2a} \\ 0 \end{bmatrix}, \tag{28}$$

(c) If $\gamma_1/(a(1+\beta)) \leq z_{1,2}$ then

$$\begin{bmatrix} y_1^*(z_{1,2}) \\ y_2^*(z_{1,2}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{29}$$

Without loss of generality, here we assumed that $\gamma_1 \geq \gamma_2$.

The proof of Proposition 1 is given in Appendix C. It helps us obtain *closed-form* expressions for $P_1(\boldsymbol{y}^*(z_{1,2}), z_{1,2})$ and $P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2})$ for any $z_{1,2} \geq 0$ and check conditions (21) and (22) to determine whether the network coding users 1 and 2 can reach a non-zero bargaining solution.

Theorem 4: Assume that parameters $N=2$ and $M=1$.
(a) Among all choices of utility and cost parameters such that

- *Condition 1*: Both (25) and (26) hold, and
- *Condition 2*: Either (21) or (22) holds for $m=1$,

the worst-case efficiency at the subgame perfect equilibrium of Game 2 is $\frac{12}{25}$ and occurs when

$$a = 1 \quad \text{and} \quad \gamma_2 = \frac{\gamma_1}{4}. \tag{30}$$

(b) For *any* bargaining scheme, we have

$$\text{PoA (Game 2, Problem 1)} \leq \frac{12}{25} = 48\%. \tag{31}$$

The proof of Theorem 4 is given in Appendix D. From Condition 2, our focus is on scenarios where any bargaining scheme would lead to $z_{1,2}=0$ and repeated Game 2 is played just like static Game 1. From Condition 1, we further focus on those scenarios where the static Game 1 has poor performance.

2) Network Coding and Routing Sessions ($N>2$ and $M=1$): Next, we consider the case where there is *at least* one routing user in the network together with two network coding users that form one network coding pair.

Proposition 2: Given $z_{1,N} \geq 0$, if conditions (25) and (26) hold, then at Nash equilibrium of the reduced Game 3: (a) If $0 \leq z_{1,N} < (2\gamma_2 - \gamma_1 - q^*(z_{1,N}))/(a(1+\beta))$, then

$$\begin{bmatrix} y_1^*(z_{1,N}) \\ y_2^*(z_{1,N}) \end{bmatrix} = \begin{bmatrix} \frac{2\gamma_1-\gamma_2-a(1+\beta)z_{1,N}-aq^*(z_{1,N})}{3a} \\ \frac{2\gamma_2-\gamma_1-a(1+\beta)z_{1,N}-aq^*(z_{1,N})}{3a} \end{bmatrix}, \tag{32}$$

where $q^*(z_{1,N}) = \sum_{r=2}^{N-1} y_r^*(z_{1,N})$. (b) If $(2\gamma_2 - \gamma_1 - q^*(z_{1,N}))/(a(1+\beta)) \leq z_{1,N} < (\gamma_1 - q^*(z_{1,N}))/(a(1+\beta))$, then

$$\begin{bmatrix} y_1^*(z_{1,N}) \\ y_2^*(z_{1,N}) \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1-a(1+\beta)z_{1,N}-aq^*(z_{1,N})}{2a} \\ 0 \end{bmatrix}. \tag{33}$$

(c) If $(\gamma_1 - q^*(z_{1,N}))(a(1+\beta)) \leq z_{1,N}$, then

$$\begin{bmatrix} y_1^*(z_{1,N}) \\ y_2^*(z_{1,N}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{34}$$

The proof of Proposition 2 is similar to that of Proposition 1. We notice that if $N=2$ (and $q^*(z)=0$), then the expressions in Proposition 2 reduce to the expressions in Proposition 1.

Theorem 5: Assume that parameters $N>2$ and $M=1$.
(a) Among all choices of utility and cost parameters such that

- *Condition 1*: Both (25) and (26) hold, and
- *Condition 2*: Either (21) or (22) holds for $m=1$,

the worst-case efficiency at the subgame perfect equilibrium of Game 2 is $\frac{4}{11}$ and occurs at

$$N \to \infty,\ a=1,\ \gamma_2 = \ldots = \gamma_{N-1} = \frac{3}{4}\gamma_1,\ \gamma_N = \frac{3}{8}\gamma_1.$$

(b) For *any* bargaining scheme, we have

$$\text{PoA (Game 2, Problem 1)} \leq \frac{4}{11} \approx 36\%. \tag{35}$$

The proof of Theorem 5 is given in Appendix E. We can see that the PoA upper bound significantly drops when the network includes both network coding and routing users.

3) Two Network Coding Pairs ($N=4$ and $M=2$): Assume that the network has two parallel network coding sessions. For notational simplicity, we first define $\Phi_{1,4} = a(1+3\beta)z_{1,4} + a(1-2\beta)z_{2,3}$, $\Phi_{2,3} = a(1+3\beta)z_{2,3} + a(1-2\beta)z_{1,4}$, $\Psi_{1,4} = a(1+2\beta)z_{1,4} + a(1-\beta)z_{2,3}$, $\Psi_{2,3} = a(1+2\beta)z_{2,3} + a(1-\beta)z_{1,4}$, $\Lambda_{1,4} = a(1+\beta)z_{1,4} + az_{2,3}$, and $\Lambda_{2,3} = a(1+\beta)z_{2,3} + az_{1,4}$.

Proposition 3: Given $z_{1,4}, z_{2,3} \geq 0$, if (25) and (26) hold, then at Nash equilibrium of the reduced Game 3, we have

(a) If $4\gamma_4 - \gamma_1 - \gamma_2 - \gamma_3 > \Phi_{1,4}$ and $4\gamma_3 - \gamma_1 - \gamma_2 - \gamma_4 > \Phi_{2,3}$, then $y_1^*(z_{1,4}, z_{2,3}) = (4\gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \Phi_{1,4})/(5a)$, $y_2^*(z_{1,4}, z_{2,3}) = (4\gamma_2 - \gamma_1 - \gamma_3 - \gamma_4 - \Phi_{1,4})/(5a)$, $y_3^*(z_{1,4}, z_{2,3}) = (4\gamma_3 - \gamma_1 - \gamma_2 - \gamma_4 - \Phi_{2,3})/(5a)$, and $y_4^*(z_{1,4}, z_{2,3}) = (4\gamma_4 - \gamma_1 - \gamma_2 - \gamma_3 - \Phi_{2,3})/(5a)$.

(b) If $4\gamma_3 - \gamma_1 - \gamma_2 - \gamma_4 \leq \Phi_{2,3} < 3\gamma_2 - \gamma_1 - \gamma_4$ and $3\gamma_4 - \gamma_1 - \gamma_2 > \Psi_{1,4}$, then $y_1^*(z_{1,4}, z_{2,3}) = (3\gamma_1 - \gamma_4 - \gamma_2 - \Psi_{1,4})/(4a)$, $y_2^*(z_{1,4}, z_{2,3}) = (3\gamma_2 - \gamma_1 - \gamma_4 - \Phi_{2,3})/(4a)$, $y_3^*(z_{1,4}, z_{2,3}) = 0$, and $y_4^*(z_{1,4}, z_{2,3}) = (3\gamma_4 - \gamma_1 - \gamma_2 - \Psi_{1,4})/(4a)$.

(c) If $4\gamma_4 - \gamma_1 - \gamma_2 - \gamma_3 \leq \Phi_{1,4} < 3\gamma_1 - \gamma_2 - \gamma_3$ and $3\gamma_3 - \gamma_1 - \gamma_2 > \Psi_{2,3}$, then $y_1^*(z_{1,4}, z_{2,3}) = (3\gamma_1 - \gamma_2 - \gamma_3 - \Phi_{1,4})/(4a)$, $y_2^*(z_{1,4}, z_{2,3}) = (3\gamma_2 - \gamma_1 - \gamma_3 - \Psi_{2,3})/(4a)$, $y_3^*(z_{1,4}, z_{2,3}) = (3\gamma_3 - \gamma_1 - \gamma_2 - \Psi_{2,3})/(4a)$, and $y_4^*(z_{1,4}, z_{2,3}) = 0$.

(d) If $3\gamma_2 - \gamma_1 - \gamma_3 \leq \Psi_{1,4} < 2\gamma_1 - \gamma_2$ and $3\gamma_3 - \gamma_1 - \gamma_2 \leq \Psi_{2,3} < 2\gamma_2 - \gamma_1$, then $y_1^*(z_{1,4}, z_{2,3}) = (2\gamma_1 - \gamma_2 - \Psi_{1,4})/(3a)$, $y_2^*(z_{1,4}, z_{2,3}) = (2\gamma_2 - \gamma_1 - \Psi_{2,3})/(3a)$, $y_3^*(z_{1,4}, z_{2,3}) = y_4^*(z_{1,4}, z_{2,3}) = 0$.

(e) If $2\gamma_2 - \gamma_1 \leq \Lambda_{1,4}$ and $\Phi_{2,3} \geq 3\gamma_2 - \gamma_1 - \gamma_4$, then $y_1^*(z_{1,4}, z_{2,3}) = (2\gamma_1 - \gamma_4 - \Lambda_{1,4})/(3a)$, $y_2^*(z_{1,4}, z_{2,3}) = 0$, $y_3^*(z_{1,4}, z_{2,3}) = 0$, and $y_4^*(z_{1,4}, z_{2,3}) = (2\gamma_4 - \gamma_1 - \Lambda_{1,4})/(3a)$.

(f) If $2\gamma_3 - \gamma_1 \leq \Lambda_{1,4} < \gamma_1$ and $\Phi_{2,3} \geq 2\gamma_2 - \gamma_1$, then $y_1^*(z_{1,4}, z_{2,3}) = (\gamma_1 - \Lambda_{1,4})/(2a)$, $y_2^*(z_{1,4}, z_{2,3}) = 0$, $y_3^*(z_{1,4}, z_{2,3}) = 0$, and $y_4^*(z_{1,4}, z_{2,3}) = 0$.

(g) If $\Phi_{1,4} \geq 3\gamma_1 - \gamma_2 - \gamma_3$ and $\Lambda_{2,3} < 2\gamma_3 - \gamma_2$, then $y_1^*(z_{1,4}, z_{2,3}) = 0$, $y_2^*(z_{1,4}, z_{2,3}) = (2\gamma_2 - \gamma_3 - \Lambda_{2,3})/(3a)$, $y_3^*(z_{1,4}, z_{2,3}) = (2\gamma_3 - \gamma_2 - \Lambda_{2,3})/(3a)$, and $y_4^*(z_{1,4}, z_{2,3}) = 0$.

(h) If $\Psi_{1,4} \geq 2\gamma_1 - \gamma_2$ and $\Lambda_{2,3} \geq \gamma_2$, then $y_1^*(z_{1,4}, z_{2,3}) = 0$, $y_2^*(z_{1,4}, z_{2,3}) = (\gamma_2 - \Lambda_{2,3})/2a$, $y_3^*(z_{1,4}, z_{2,3}) = y_4^*(z_{1,4}, z_{2,3}) = 0$.

(i) If $\Lambda_{1,4} \geq \gamma_1$ and $2\gamma_3 - \gamma_2 \leq \Lambda_{2,3} < \gamma_2$, then $y_1^*(z_{1,4}, z_{2,3}) = 0$, $y_2^*(z_{1,4}, z_{2,3}) = 0$, $y_3^*(z_{1,4}, z_{2,3}) = 0$, and $y_4^*(z_{1,4}, z_{2,3}) = 0$.

The proof of Proposition 3 is similar to Proposition 1.

Theorem 6: Assume that parameters $N = 4$ and $M = 2$. (a) Among all choices of utility and cost parameters such that

- *Condition 1*: Both (25) and (26) hold, and
- *Condition 2*: Either (21) or (22) holds for $m = 1$ or 2,

the worst-case efficiency at the subgame perfect equilibrium of repeated Game 2 is $\frac{4}{9}$ and it occurs at

$$a = 1,\ \gamma_2 \frac{2}{7}\gamma_1,\ \gamma_3 = \frac{5}{7}\gamma_1,\ \gamma_4 = \frac{2}{7}\gamma_1.$$

(b) For *any* bargaining scheme, we have

$$\text{PoA}\,(\text{Game 2, Problem 1}) \leq \frac{4}{9} \approx 44\%. \tag{36}$$

The proof of Theorem 6 is similar to that of Theorems 4 and 5. Theorems 4 - 6 suggest that adding more (network coding or routing) users reduces the PoA upper bounds in Game 2. The PoA upper bounds for other scenarios (e.g., with arbitrary number of network coding and routing users) can be analyzed in a similar way. However, as it is evident from Proposition 3, the computational complexity of the analysis grows drastically as we consider more network coding pairs.

V. Min-Max Bargaining Solution

The idea in the min-max bargaining scheme is to let *each* network coding user m and $N-m+1$, for any $m = 1, \ldots, M$, *individually* make a choice for the coding rate $z_{m,N-m+1}$, and select the bargaining solution such that (14) holds and both users benefit from network coding. Given $\boldsymbol{z}_{-(m,N-m+1)}$, consider the following set for network coding user m:

$$\begin{aligned}\mathcal{Z}_m = \big\{ z_{m,N-m+1} \geq 0 \mid\ & \forall \hat{z}_{m,N-m+1} \in [0, z_{m,N-m+1}],\\ & P_m(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1})\\ & \geq P_m(\boldsymbol{y}^*(\hat{z}_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), \hat{z}_{m,N-m+1})\\ & \geq P_m(\boldsymbol{y}^*(0, \boldsymbol{z}_{-(m,N-m+1)}), 0) \big\}.\end{aligned}$$

User m's payoff is *monotonically increasing* over set $\mathcal{Z}_m$. Any $z_{m,N-m+1} \in \mathcal{Z}_m$ satisfies (14) for user m and is acceptable for user m. Similarly, for user $N-m+1$, we define

$$\begin{aligned}\mathcal{Z}_{N-m+1} = \big\{ z_{m,N-m+1} \geq 0 \mid\ & \forall \hat{z}_{m,N-m+1} \in [0, z_{m,N-m+1}],\\ & P_{N-m+1}(\boldsymbol{y}^*(z_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), z_{m,N-m+1})\\ & \geq P_{N-m+1}(\boldsymbol{y}^*(\hat{z}_{m,N-m+1}, \boldsymbol{z}_{-(m,N-m+1)}), \hat{z}_{m,N-m+1})\\ & \geq P_{N-m+1}(\boldsymbol{y}^*(0, \boldsymbol{z}_{-(m,N-m+1)}), 0) \big\}.\end{aligned}$$

It is clear that the payoffs for both users m and $N-m+1$ are monotonically increasing over the intersection set $\mathcal{Z}_m \cap \mathcal{Z}_{N-m+1}$. Therefore, any choice of $z_{m,N-m+1} \in \mathcal{Z}_m \cap \mathcal{Z}_{N-m+1}$ satisfies (14) for both users m and $N-m+1$ and is a potential bargaining solution. From Theorem 3, we can conclude that the grim-trigger strategy in Definition 8 is a *subgame perfect equilibrium* for Game 2 if we choose any

$$z_{m,N-m+1} \in \mathcal{Z}_m \cap \mathcal{Z}_{N-m+1} \tag{37}$$

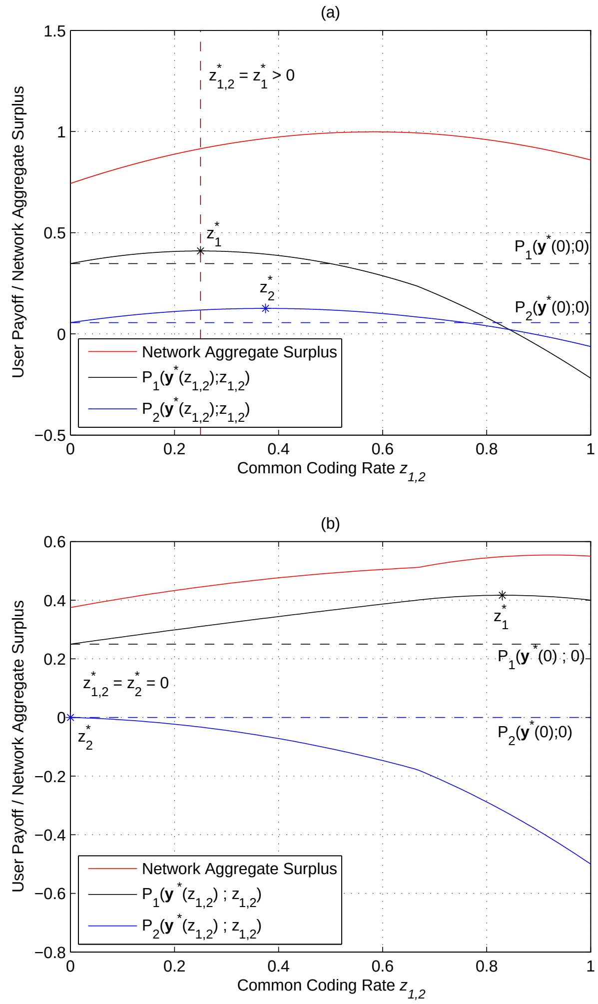

Fig. 2. Calculating the min-max bargaining solution $z_{1,2}^*$ based on the model in (38), where z_1^* and z_2^* are as in (39) and (40), respectively. Dashed lines indicate the payoffs when no network coding is performed. (a) An example with *non-zero* coding rate. (b) An example with *zero* coding rate.

and a discount factor $\delta \geq \delta_{\min}$ for some $\delta_{\min} \in (0, 1]$. Formally, the min-max bargaining solution is calculated as

$$z_{m,N-m+1}^* = \min\{z_m^*, z_{N-m+1}^*\}, \tag{38}$$

where

$$z_m^* = \max_{z_{m,N-m+1} \in \mathcal{Z}_m} z_{m,N-m+1}, \tag{39}$$

and

$$z_{N-m+1}^* = \max_{z_{m,N-m+1} \in \mathcal{Z}_{N-m+1}} z_{m,N-m+1}. \tag{40}$$

Interestingly, z_m^* and z_{N-m+1}^* are the solutions of *selfish* problems (19) and (20) as long as these problems are convex. Otherwise, z_m^* and z_{N-m+1}^* are simply the *smallest* local *maximizers* of problems (19) and (20), respectively.

If $z_m^* < z_{N-m+1}^*$, e.g., as in the example in Fig. 2(a) when $N = 2$ and $M = 1$, then user $N-m+1$ prefers a lower coding rate than user m; however, due to Theorem 2, user $N-m+1$ is worse off by selecting $v_{N-m+1}^k = z_{N-m+1}^k > z_1^*$ at any stage $k \geq 1$. A similar statement is true for user m. Thus, users m and $N-m+1$ can agree on rate $z_{m,N-m+1} = z_{m,N-m+1}^*$, after they *individually* announce z_m^* and z_{N-m+1}^*,

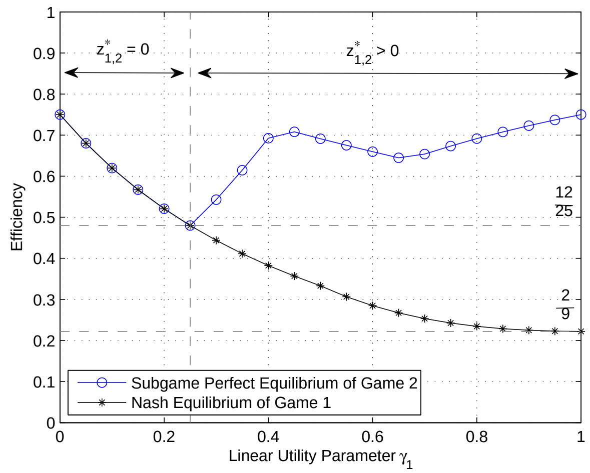

Fig. 3. An example of the impact that changing the utility parameters has on the efficiency of network coding games. Here, $N = 2$ users form $M = 1$ network coding pair. The linear utility parameters are $\gamma_2 = 1$ and $\gamma_1 \in (0, 1]$.

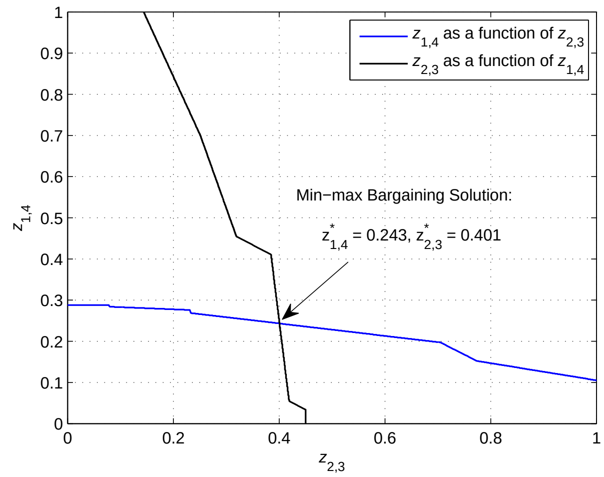

Fig. 4. An example for calculating the min-max bargaining solutions in the presence of two network coding pairs, i.e., when $N = 4$ and $M = 2$.

respectively. Given $z_{m,N-m+1} = z^*_{m,N-m+1}$, for all $m = 1, \ldots, M$, the users can then play the grim-trigger strategy.

The min-max bargaining does not always lead to a non-zero coding rate. If the min-max bargaining solution is zero, then it is only because at least one network coding user is better off if no network coding is performed. This is shown in Fig. 2(b). In this example, we have $N = 2$, $M = 1$, $a = 1$, $b_1 = 0.1$, $b_2 = 0.2$, $\beta = 0.5$, and utility functions are linear with parameters $\gamma_1 = 1$ and $\gamma_2 = 0.2$. We can see that while $z^*_1 = 0.84$ and user 1 is interested in performing network coding with user 2, user 2 has $z^*_2 = 0$ as it is better off not to participate in network coding with user 1. Any other bargaining method similarly results in a zero coding rate. This reveals the main difference between a repeated inter-session network coding and a typical repeated game (e.g., a Prisoner's dilemma): here, the coding rate may still remain zero (i.e., still no cooperation) even if the game is played repeatedly.

Fig. 3 illustrates the improvements in efficiency when the min-max bargaining solution is used for a network with $N = 2$ users that form $M = 1$ network coding pair. Here, we assume that the utility functions are linear with $\gamma_2 = 1$ and γ_1 varying from 0 to 1. We can see that if $0 < \gamma_1 \leq \frac{1}{4}$, i.e., the utility functions of user 1 and user 2 are very different, then user 1 has no interest in participating in network coding and the min-max bargaining solution is $z^*_{1,2} = 0$. The worst-case efficiency occurs when $\gamma_1 = \frac{1}{4}$ as expected based on Theorem 4. As γ_1 increases, the two users have more motivation to agree on a non-zero common coding rate leading to a significant improvement in efficiency in the subgame perfect equilibrium of Game 2 compared to the Nash equilibrium of Game 1. From the results in Fig. 3, for those cases where repeated game cannot do any better than static game, the efficiency of the static game is much better than its worst-case efficiency.

Next, consider the case where $N = 4$ and $M = 2$. We are interested in calculating two min-max bargaining solutions $z^*_{1,4}$ and $z^*_{2,3}$. Since all four users share the bottleneck link, the choice of $z^*_{1,4}$ to be decided by users 1 and 4 also depends on the choice of $z^*_{2,3}$ which is decided by users 2 and 3. Similarly, the choice of $z^*_{2,3}$ to be decided by users 2 and 3 also depends on the choice of $z^*_{1,4}$ which is decided by users 1 and 4. Therefore, min-max bargaining solutions at equilibrium are obtained at the cross section of $z^*_{1,4}$ plotted as a function of $z_{2,3}$ and $z^*_{2,3}$ plotted as a function of $z_{1,4}$. An example is shown in the Fig. 4. In this example, we have $\gamma_1 = 1$, $\gamma_2 = 0.8$, $\gamma_3 = 0.7$, $\gamma_4 = 0.75$, $a = 1$, $b_1 = 0.5$, $b_2 = 0.4$, $b_3 = 0.2$, $b_4 = 0.7$, and $\beta = 0.5$. In this case, we have $z^*_{1,4}$ = 0.243 and $z^*_{2,3} = 0.401$. Efficiency at the sub-game perfect equilibrium of the repeated inter-session network coding game is 0.8443. The efficiency at the Nash equilibrium of the static inter-session network coding game in this scenario is 0.4040.

VI. Numerical Results

In this section, we evaluate the min-max bargaining scheme for various choices of parameters N, M, a, $b_1, \ldots, b_N$, and $U_1, \ldots, U_N$. We assume that $\delta = 0.99$. Numerical results for 100 random scenarios are shown in Fig. 5. For the results in Fig. 5(a), we have $N = 2$ and $M = 1$ and the network includes only one network coding pair. For the results in Fig. 5(b), we have $N = 4$ and $M = 2$ and the network includes two parallel network coding pairs. Finally, for the results in Fig. 5(c), N is selected randomly between 5 and 50 and there are always some routing users in the network in addition to $M = 1$ network coding pair. In each scenario, the link cost parameters $a \in (0, 10)$, $b_1 \in (0, 5)$, and $b_2 \in (0, 5)$ are selected randomly. The utility functions are α-fair [35]:

$$U_n(x) = \kappa_n \, (1 - \alpha_n)^{-1} \, x^{1-\alpha_n}, \qquad n \in \mathcal{N}, \tag{41}$$

where $\alpha_n \in [0, 1)$ and $\kappa_n \in (0, 100)$ are selected randomly. We can verify that the utility functions in (41) satisfy Assumption 2. They include the linear case in (25) when $\alpha_n = 0$.

From Fig. 5(a), the subgame perfect equilibrium of Game 2 that is formed when users play the proposed grim-trigger strategy based on the proposed min-max bargaining solution has a higher efficiency than the Nash equilibrium of Game 1 in every scenario. Furthermore, the efficiency of Game 2 is always greater than or equal to 48%, suggesting that $\text{PoA}\,(\text{Game 2}, \text{Problem 1}) \approx 48\%$ for min-max bargaining

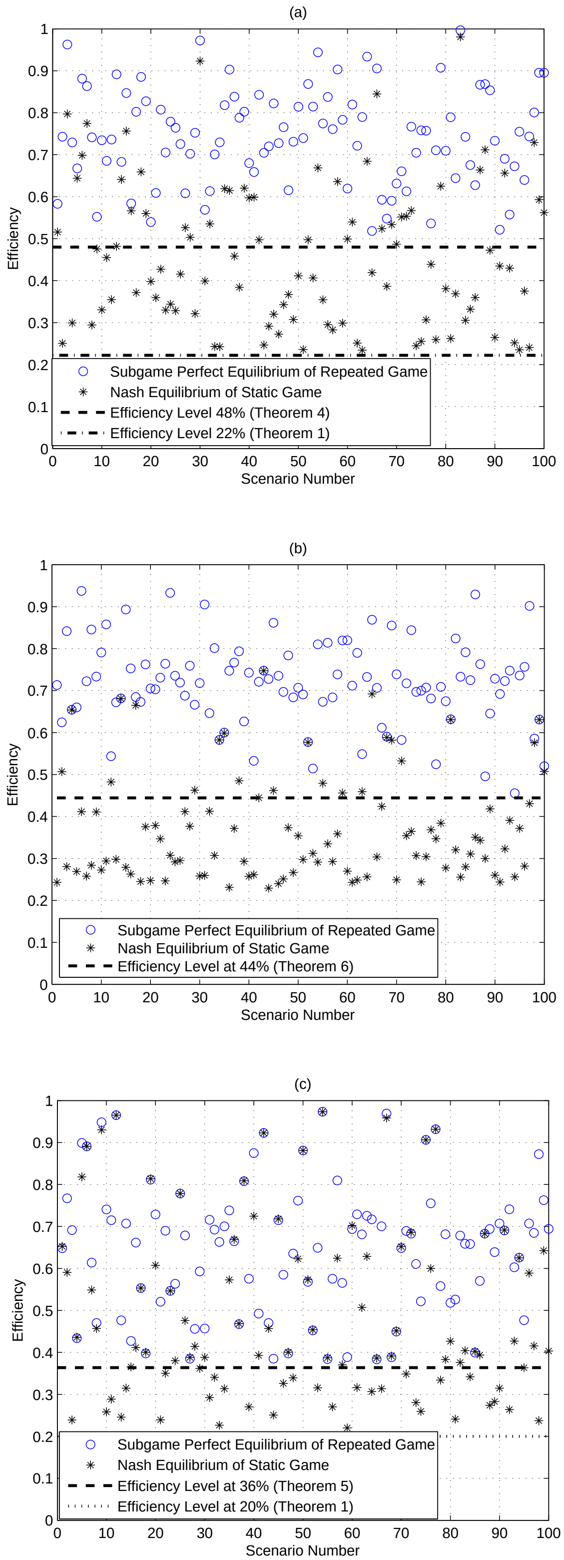

Fig. 5. Efficiency at Nash equilibrium of static Game 1 and subgame perfect equilibrium of repeated Game 2 for 100 random scenarios where the network topology is as in Fig. 1 and the *min-max bargaining solution* is being used. The number of users N and the number of network coding pairs M are chosen as follows: (a) $N = 2$ and $M = 1$. (b) $N = 4$ and $M = 2$. (c) $N > 2$ and $M = 1$, where we randomly choose $N \in (5, 50)$. We can see that the PoA upper bounds in Theorems 4 - 6 are reached when the min-max bargaining solution is used. We also note that in addition to the improvement in the PoA, i.e., the *worst-case* efficiency, the *average* efficiency also increases by 64.2% when the network coding game is played repeatedly rather than only once.

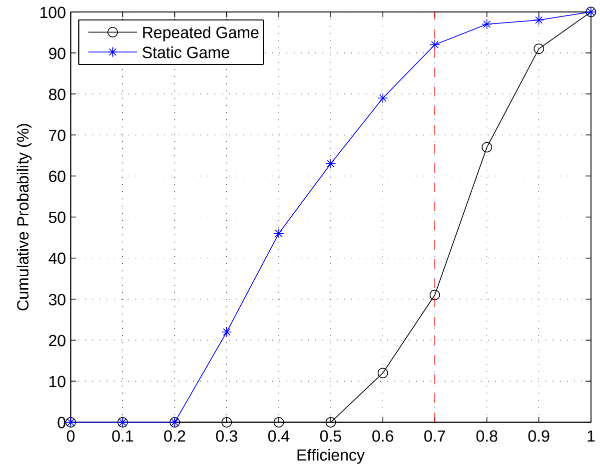

Fig. 6. The cumulative probability of efficiency in static and repeated games as a mean to measure relative performance when $N = 2$ and $M = 1$.

solution. From this, together with the upper bound in Theorem 4, we can conclude that: *the worst-case efficiency of repeated Game 2 when the proposed grim-trigger strategy and min-max bargaining solution are adopted occurs when even the bargaining process cannot help to encourage users to perform network coding*. When the number of network coding pairs increases to $M = 2$, as in Fig. 5(b), the results are similar. Again, efficiency improves when the game is played repeatedly and the users play the proposed grim-trigger strategy based on the proposed min-max bargaining solution. Furthermore, the efficiency of Game 2 is always greater than or equal to 44%, suggesting that $\text{PoA}\,(\text{Game 2}, \text{Problem 1}) \approx 44\%$.

From Fig. 5(c), Game 2 *usually* has a better efficiency than Game 1. We can see that the efficiency of Game 2 is always greater than or equal to 36%. Thus, the PoA of Game 2 achieves the upper bound in Theorem 5. In a few scenarios, e.g., the 1st and the 60th scenarios, the efficiency of static Game 1 is *better* than that of the repeated Game 2. In these scenarios, the bargaining between network coding users leads to a coding rate *higher* than the optimal coding rate of Problem 1, which hurts the utilities of routing users. This interesting observation remains to be further investigated in the future.

The results in Fig. 5 can be evaluated also in terms of *average* performance. When $N = 2$ and $M = 1$, the average efficiency in a Nash equilibrium of static game is 0.45 while the average efficiency in a subgame perfect equilibrium of repeated game when users adopt min-max bargaining solution is 0.75. When $N = 4$ and $M = 2$, the average efficiency in a Nash equilibrium of static game is 0.34 and the average efficiency in a subgame perfect equilibrium of repeated game when users adopt min-max bargaining solution is 0.72. When $N \in (5, 50)$ and $M = 1$, the average efficiency in a Nash equilibrium of static game is 0.48 and the average efficiency in a subgame perfect equilibrium of repeated game when users adopt min-max bargaining solution is 0.66.

The relative performance of static and repeated network coding games can illustrated also in a *probabilistic fashion* as shown in Fig. 6. Here, we assume that $N = 2$ and $M = 1$. At each efficiency level Ω, the cumulative probability is calculated

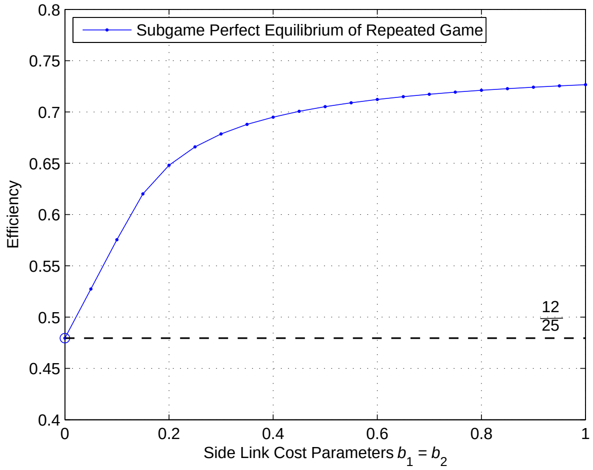

Fig. 7. The impact of the side link cost parameters on efficiency of the repeated network coding game when $N = 2$ and $M = 1$. Here, the values of the side link cost parameters b_1 and b_2 vary from 0 to 1, while a = 1.

as $P\{\text{Efficiency} < \Omega\}$. For example, based on the red dashed vertical line, if the game is played statically, then in 92% of scenarios the efficiency at Nash equilibrium is less than 0.7. In contrast, if the game is played repeatedly based using our proposed grim-trigger strategy based on our proposed min-max bargaining solution, then in only 31% of scenarios the efficiency at the sub-game perfect equilibrium is less than 0.7. Similar comparisons can be made at any other efficiency level.

Last but not least, we note that although utility parameters have a significant impact on the PoA, as suggested by Theorems 4 - 6 and Fig. 3, other parameters also have a major impact on efficiency. For example, Fig. 7 shows the efficiency when the values of the side link cost parameters b_1 and b_2 vary from 0 to 1. Other parameters are fixed to $a = 1$, $\gamma_1 = 1$, $\gamma_2 = 0.25$, and we have $N = 2$ and $M = 1$. As $b_1 \to 0$, and $b_2 \to 0$, efficiency approaches the results in Theorem 4. Therefore, not only the utility parameters but also the cost parameters have an impact on efficiency in the repeated game.

VII. Conclusions and Future Work

This work represents a first step towards understanding *non-cooperative* inter-session network coding in a *repeated* game. Our focus was on a butterfly network with one or two pairs of network coding users and possibly several routing users. We showed that if the network coding game is played repeatedly, then it is possible for network coding users to achieve a mutually desirable positive coding rate via *bargaining*. This is in sharp contrast to static inter-session network coding games, where *no* network coding is performed at the Nash equilibrium. We investigated the *price-of-anarchy* (PoA), i.e., the worst-case efficiency compared to an *optimal* and *cooperative* network design. We showed that for all possible bargaining schemes, the PoA of the repeated network coding game is upper-bounded by 36% (with one network coding pair and several routing users), 44% (with two network coding pairs), and 48% (with one network coding pair). These bounds can be reached by a simple *min-max bargaining*. This indicates a major improvement compared to the 20% and 22% PoA results for static inter-session network coding for the same settings. Numerical results showed that, in addition to the worst-case efficiency, the average efficiency also improves significantly when the network coding game is played repeatedly and the users adopt our proposed min-max bargaining solution.

The results in this paper can be extended in several directions. First, our analysis can be applied to more general network topologies such as those which are superposition of several butterfly networks. Second, efficiency may be improved by using user-specific pricing functions. Third, while we only considered a simple coding approach such as XOR, more general coding schemes may lead to different cooperation behaviors. Finally, non-cooperative network coding models may be studied as games with incomplete information.

Appendix

A. Proof of Theorem 2

Let $\Delta_m^k = z_m^k - z_{N-m+1}^k > 0$. In this case, we have

$$
\begin{aligned}
&P_m\left(\bar{y}_m^k, \bar{z}_m^k, \bar{v}_m^k, \boldsymbol{y}_{-m}^k, \boldsymbol{z}_{-m}^k, \boldsymbol{v}_{-m}^k\right)\\
&\quad= U_m\left(y_m^k + \Delta_m^k + v_{N-m+1}^k\right) - \left(v_m^k - \Delta_m^k\right) p_m\left(v_m^k - \Delta_m^k\right)\\
&\qquad - \left(y_m^k + \Delta_m^k + z_m^k - \Delta_m^k - (1-\beta) z_{N-m+1}^k\right)\\
&\qquad \times p\left(\textstyle\sum_{n=1}^N y_n^k + \Delta_m^k\right.\\
&\qquad\qquad \left. + \textstyle\sum_{r=1, r\neq m}^M \max\{z_r, z_{N-r+1}\} + z_m^k - \Delta_m^k\right)\\
&\quad= U_m\left(y_m^k + \Delta_m^k + v_{N-m+1}^k\right)\\
&\qquad + \Delta_m^k\, p_m\left(v_m^k - \Delta_m^k\right) - v_m^k\, p_m\left(v_m^k - \Delta_m^k\right)\\
&\qquad - \left(y_m^k + z_m^k - (1-\beta) z_{N-m+1}^k\right)\\
&\qquad \times p\left(\textstyle\sum_{n=1}^N y_n^k + \textstyle\sum_{r=1, r\neq m}^M \max\{z_r, z_{N-r+1}\} + z_m^k\right)\\
&\quad> U_m\left(y_m^k + v_{N-m+1}^k\right) - v_m^k\, p_m\left(v_{N-m+1}^k - \Delta_m^k\right)\\
&\qquad - \left(y_m^k + z_m^k - (1-\beta) z_{N-m+1}^k\right)\\
&\qquad \times p\left(\textstyle\sum_{n=1}^N y_n^k + \textstyle\sum_{r=1, r\neq m}^M \max\{z_r, z_{N-r+1}\} + z_m^k\right)\\
&\quad= P_m\left(\boldsymbol{y}^k, \boldsymbol{z}^k, \boldsymbol{v}^k\right),
\end{aligned}
$$

where the inequality is due to $\Delta_m^k\, p_m\left(v_{N-m+1}^k - \Delta_m^k\right) > 0$ and $y_m^k + \Delta_m^k + v_{N-m+1}^k > y_m^k + v_{N-m+1}^k$, and since $U_m(\cdot)$ is increasing. Moreover, we can also show that

$$
\begin{aligned}
&P_{N-m+1}\left(\bar{y}_m^k, \bar{z}_m^k, \bar{v}_m^k, \boldsymbol{y}_{-m}^k, \boldsymbol{z}_{-m}^k, \boldsymbol{v}_{-m}^k\right)\\
&\quad= U_{N-m+1}\left(y_{N-m+1}^k + z_{N-m+1}^k\right)\\
&\qquad - v_{N-m+1}^k p_{N-m+1}(v_{N-m+1}^k) - \left(y_{N-m+1}^k + \beta z_{N-m+1}^k\right)\\
&\qquad \times p\left(\textstyle\sum_{n=1}^N y_n^k\right.\\
&\qquad\qquad \left. + \textstyle\sum_{r=1, r\neq m}^M \max\{z_r, z_{N-r+1}\} + \Delta_m^k + z_m^k - \Delta_m^k\right)\\
&\quad= P_{N-m+1}\left(\boldsymbol{y}^k, \boldsymbol{z}^k, \boldsymbol{v}^k\right).
\end{aligned}
$$

Finally, it is easy to verify that for each routing user $n = M+1, \ldots, N-M$, we have

$$
P_n\left(\bar{y}_m^k, \bar{z}_m^k, \bar{v}_m^k, \boldsymbol{y}_{-m}^k, \boldsymbol{z}_{-m}^k, \boldsymbol{v}_{-m}^k\right) = P_n\left(\boldsymbol{y}^k, \boldsymbol{z}^k, \boldsymbol{v}^k\right).
$$

Therefore, user m is better off by switching to new rates $(\bar{y}_m^k, \bar{v}_m^k, \bar{z}_m^k)$, without changing other users payoffs. ■

B. Proof of Theorem 3

We prove that grim-trigger is a subgame perfect equilibrium of Game 2 for user m. Assume that all users cooperate and play Step 1 in Definition 8. In that case, at each stage $k \geq 1$ of Game 2, if user m follows the grim-trigger strategy and sets its rates according to Step 1, it expects a long-term payoff

$$\sum_{t=k}^{\infty} (\delta)^{t-k}\ P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z}) = \frac{P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z})}{1-\delta}. \quad (42)$$

Next, assume that user m can reach the best payoff $\Gamma_m \geq P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z})$ at the current stage of Game 2 if it deviates from Step 1. Then, user m expects long-term payoff

$$\begin{aligned} &P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z}) + \sum_{t=k+1}^{\infty} (\delta)^{t-k}\ P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0}) \\ &= \Gamma_m + \frac{\delta}{1-\delta} P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0}). \end{aligned} \quad (43)$$

Comparing (42) and (43), it is best for user m to cooperate if and only if there exist discount factors $\delta \in (0, 1]$ such that

$$\frac{P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z})}{1-\delta} \geq \Gamma_m + \frac{\delta}{1-\delta} P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0}). \quad (44)$$

After reordering the terms, it is required that

$$\frac{\Gamma_m - P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z})}{\Gamma_m - P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0})} \leq \delta \leq 1. \quad (45)$$

Clearly, the inequality (45) holds for some $\delta \in (0, 1]$ if and only if (14) holds for m. A similar argument is true for any other network coding user, including user $N - m + 1$, for any $m = 1, \ldots, M$. We also note that the proof for routing users $n = M+1, \ldots, N-M$ is evident as the routing users simply play Nash equilibrium for the coding rates given by network coding users. In summary, for the grim-trigger strategy to form a subgame perfect equilibrium, it is required that

$$\delta_{\min} = \max_{m \in \mathcal{M}} \frac{\Gamma_m - P_m(\boldsymbol{y}^*(\boldsymbol{z}), \boldsymbol{z})}{\Gamma_m - P_m(\boldsymbol{y}^*(\boldsymbol{0}), \boldsymbol{0})}.$$

This concludes the proof. ■

C. Proof of Proposition 1

Case I) If $y_1^*(z_{1,2}) > 0$ and $y_2^*(z_{1,2}) > 0$, then

$$\gamma_1 = a\left(y_1^*(z_{1,2}) + y_2^*(z_{1,2}) + (1+\beta) z_{1,2}\right) + a y_1^*(z_{1,2}), \quad (46)$$
$$\gamma_2 = a\left(y_1^*(z_{1,2}) + y_2^*(z_{1,2}) + (1+\beta) z_{1,2}\right) + a y_2^*(z_{1,2}). \quad (47)$$

From (47), we have

$$y_1^*(z_{1,2}) = \frac{\gamma_2 - a(y_2^*(z_{1,2}) + (1+\beta) z_{1,2})}{2a}. \quad (48)$$

Replacing (48) in (46), we have

$$y_1^*(z_{1,2}) = \frac{2\gamma_1 - \gamma_2 - a(1+\beta) z_{1,2}}{3a}, \quad (49)$$

$$y_2^*(z_{1,2}) = \frac{2\gamma_2 - \gamma_1 - a(1+\beta) z_{1,2}}{3a}. \quad (50)$$

From (49) and knowing that $y_1^*(z_{1,2}) > 0$, we have

$$2\gamma_2 - \gamma_1 - a(1+\beta) z_{1,2} > 0 \quad \Rightarrow \quad z_{1,2} < \frac{2\gamma_2 - \gamma_1}{a(1+\beta)}. \quad (51)$$

Similarly, from (50) and knowing that $y_2^*(z_{1,2}) > 0$, we have

$$2\gamma_1 - \gamma_2 - a(1+\beta) z_{1,2} > 0 \quad \Rightarrow \quad z_{1,2} < \frac{2\gamma_1 - \gamma_2}{a(1+\beta)}. \quad (52)$$

Since $\gamma_1 \geq \gamma_2$, inequalities (51) and (52) reduce to

$$0 \leq z_{1,2} < \frac{2\gamma_2 - \gamma_1}{a(1+\beta)}. \quad (53)$$

Thus, the data rates in (49) and (50) hold only if (53) holds.

Case II) If $y_1^*(z_{1,2}) > 0$ and $y_2^*(z_{1,2}) = 0$, then

$$\gamma_1 = a\left(y_1^*(z_{1,2}) + (1+\beta) z_{1,2}\right) + a y_1^*(z_{1,2}), \quad (54)$$
$$\gamma_2 \leq a\left(y_1^*(z_{1,2}) + (1+\beta) z_{1,2}\right). \quad (55)$$

From (54) and after reordering the terms, we have

$$y_1^*(z_{1,2}) = \frac{\gamma_1 - a(1+\beta) z_{1,2}}{2a}. \quad (56)$$

Replacing (56) in (55), we have

$$2\gamma_2 \leq \gamma_1 + a(1+\beta) z_{1,2} \quad \Rightarrow \quad z_{1,2} \geq \frac{2\gamma_2 - \gamma_1}{a(1+\beta)}. \quad (57)$$

Moreover, from (56) and knowing that $y_1^*(z_{1,2}) > 0$, we have

$$\gamma_1 > a(1+\beta) z_{1,2} \quad \Rightarrow \quad z_{1,2} < \frac{\gamma_1}{a(1+\beta)}. \quad (58)$$

Case III) If $y_1^*(z_{1,2}) = 0$ and $y_2^*(z_{1,2}) = 0$, then

$$\gamma_1 \leq a(1+\beta) z_{1,2}, \qquad \gamma_2 \leq a(1+\beta) z_{1,2}. \quad (59)$$

Since $\gamma_1 \geq \gamma_2$, the above leads to $z_{1,2} \geq \gamma_1 / (a(1+\beta))$. ■

D. Proof of Theorem 4

Without loss of generality, we assume that $\gamma_1 \geq \gamma_2$. Given $z_{1,2} \geq 0$, the data rates $y_1^*(z_{1,2})$ and $y_2^*(z_{1,2})$ are obtained from Proposition 1. We consider three cases separately.

Case I) If $\gamma_2 \leq \gamma_1 < 2\gamma_2$ and (25) and (26) hold, then

$$\begin{aligned} &P_2(\boldsymbol{y}^*(0), 0) = \\ &\gamma_2 \left(\frac{2\gamma_2 - \gamma_1}{3a}\right) - a\left(\frac{2\gamma_2 - \gamma_1}{3a}\right)\left(\frac{\gamma_1 + \gamma_2}{3a}\right). \end{aligned} \quad (60)$$

On the other hand, if $0 \leq z_{1,2} < \frac{2\gamma_2 - \gamma_1}{a(1+\beta)}$ and $\beta = \frac{1}{2}$, then

$$\begin{aligned} &P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) = \\ &\gamma_2 \left(\frac{2\gamma_2 - \gamma_1}{3a} + \frac{z_{1,2}}{2}\right) - a\left(\frac{2\gamma_2 - \gamma_1}{3a}\right)\left(\frac{\gamma_1 + \gamma_2}{3a}\right). \end{aligned} \quad (61)$$

From (60) and (61), we have

$$P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) - P_2(\boldsymbol{y}^*(0), 0) = \frac{\gamma_2 z_{1,2}}{2} > 0,$$

for any $z_{1,2}$ such that

$$z_{1,2} \in \left(0, \frac{2\gamma_2 - \gamma_1}{a(1+\beta)}\right). \quad (62)$$

Therefore, (22) does *not* hold. A similar statement is true for (21). Thus, *Condition 1* does not hold if $\gamma_2 \leq \gamma_1 < 2\gamma_2$.

Case II) If $2\gamma_2 \leq \gamma_1 \leq 4\gamma_2$ and (25) and (26) hold, then

$$P_2(\boldsymbol{y}^*(0), 0) = 0. \quad (63)$$

On the other hand, if $0 \le z_{1,2} < \frac{\gamma_1}{a(1+\beta)}$ and $\beta = \frac{1}{2}$, then

$$P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) = \gamma_2 z_{1,2} - \frac{a z_{1,2}}{2}\left(\frac{\gamma_1}{2a} + \frac{z_{1,2}}{4}\right). \quad (64)$$

Furthermore, we have

$$\lim_{z_{1,2}\to 0} \frac{d\, P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2})}{d\, z_{1,2}} = \lim_{z_{1,2}\to 0} \gamma_2 - \frac{\gamma_1}{4} - \frac{a z_{1,2}}{4} = \gamma_2 - \frac{\gamma_1}{4} > 0.$$

Therefore, (22) does *not* hold. A similar statement is true for (21). Thus, *Condition 1* does not hold if $2\gamma_2 \le \gamma_1 < 4\gamma_2$.

Case III) If $4\gamma_2 \le \gamma_1$ and (25) and (26) hold, then

$$P_2(\boldsymbol{y}^*(0), 0) = 0. \quad (65)$$

If $0 \le z_{1,2} < \frac{\gamma_1}{a(1+\beta)}$ and $\beta = \frac{1}{2}$, then (64) holds and we have

$$\begin{aligned} &P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) - P_2(\boldsymbol{y}^*(0), 0) \\ &\quad = z_{1,2}\left(\gamma_2 - \tfrac{\gamma_1}{4}\right) - \tfrac{a z_{1,2}^2}{4} < 0, \end{aligned} \qquad \forall z_{1,2} \in \left(0, \tfrac{\gamma_1}{a(1+\beta)}\right), \quad (66)$$

where the inequality is due to $4\gamma_2 \le \gamma_1$. On the other hand, if $\frac{\gamma_1}{a(1+\beta)} \le z_{1,2}$ and $\beta = \frac{1}{2}$, then

$$P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) = \gamma_2 z_{1,2} - \frac{a z_{1,2}^2}{2}. \quad (67)$$

Therefore,

$$\begin{aligned} &P_2(\boldsymbol{y}^*(z_{1,2}), z_{1,2}) - P_2(\boldsymbol{y}^*(0), 0) \\ &\quad = z_{1,2}\left(\gamma_2 - \tfrac{a z_{1,2}}{2}\right) < 0, \end{aligned} \qquad \forall z_{1,2} \ge \tfrac{\gamma_1}{a(1+\beta)}, \quad (68)$$

where the inequality is due to

$$\gamma_2 - \frac{a z_{1,2}}{2} < \gamma_2 - \frac{a}{2}\left(\frac{\gamma_1}{a(1+\frac{1}{2})}\right) = \gamma_2 - \frac{\gamma_1}{3} < 0. \quad (69)$$

From (66) and (69), inequality (22) holds if and only if

$$0 < \gamma_2 \le \frac{\gamma_1}{4}. \quad (70)$$

In that case, $z_{1,2} = 0$, $y_1^*(0) = \frac{\gamma_1}{2a}$, and $y_2^*(0) = 0$. Thus,

$$\mathbb{S}\left(\boldsymbol{y}^*(z_{1,2}), z_{1,2}\right) = \gamma_1\left(\frac{\gamma_1}{2a}\right) - \frac{a}{2}\left(\frac{\gamma_1}{2a}\right)^2 = \frac{3{\gamma_1}^2}{8a}. \quad (71)$$

On the other hand, we can show that

$$\mathbb{S}(\boldsymbol{y}^S, \boldsymbol{z}^S, \boldsymbol{v}^S) = \frac{(\gamma_1 + \gamma_2)^2}{2a}. \quad (72)$$

Therefore, the worst-case efficiency of Game 2 is obtained by solving the following optimization problem

$$\begin{aligned} \underset{\gamma_1, \gamma_2, a}{\textbf{minimize}} \quad & \frac{\frac{3{\gamma_1}^2}{8a}}{\frac{(\gamma_1+\gamma_2)^2}{2a}} \\ \textbf{subject to} \quad & 0 < \gamma_2 \le \frac{\gamma_1}{4}. \end{aligned} \quad (73)$$

We can see that the worst-case efficiency does not depend on the value of shared-link cost parameter a. It only depends on the *relative* value of utility parameters γ_1 and γ_2, c.f. 73. The objective function in (73) is *decreasing* in γ_2. Thus, the minimum occurs when $\gamma_2 = \frac{\gamma_1}{4}$. Thus, the efficiency becomes

$$\frac{\frac{3{\gamma_1}^2}{8a}}{\frac{(\gamma_1 + \frac{\gamma_1}{4})^2}{2a}} = \frac{\frac{3}{8}}{\frac{25}{32}} = \frac{12}{25}. \quad (74)$$

This concludes the proof. ■

E. Proof of Theorem 5

Let $\gamma_{\max} = \max_{n\in\mathcal{N}} \gamma_n$ and assume that $\gamma_1 \ge \gamma_N$.

From [6, Theorem 10], no network coding is desired at optimal resource allocation if $\gamma_1 + \gamma_N < \gamma_{\max}$. Therefore, we focus only on the case when $\gamma_1 + \gamma_N \ge \gamma_{\max}$. We have

$$\mathbb{S}(\boldsymbol{y}^S, \boldsymbol{z}^S, \boldsymbol{v}^S) = \frac{(\gamma_1 + \gamma_N)^2}{2a}. \quad (75)$$

Moreover, we can verify that

$$P_N(\boldsymbol{y}^*(z_{1,N}), z_{1,N}) < P_N(\boldsymbol{y}^*(0), 0), \quad \forall z_{1,N} > 0, \quad (76)$$

if and only if

$$0 < \gamma_N \le \frac{\gamma_1}{4} + \frac{a q^*(0)}{4}. \quad (77)$$

We notice that if $q^*(0) = 0$ then condition (77) reduces to (70) and the results will be as in Theorem 4. Therefore, we only focus on the case when $q^*(z_{1,N}) > 0$. Next, we can verify that the worst-case efficiency occurs when

$$N \to \infty, \qquad \gamma_2 = \ldots = \gamma_{N-1}. \quad (78)$$

The proof is similar to that of [6, Theorem 11(b)] and [26, Theorem 3]. On the other hand, we can show that for each routing user $n = 2, \ldots, N-1$, we have

$$\begin{aligned} \gamma_n = {} & a(q^*(z_{1,N}) + y_1^*(z_{1,N}) + y_N^*(z_{1,N}) + z_{1,N}) \\ & + a y_n^*(z_{1,N}). \end{aligned} \quad (79)$$

Replacing (78) in (79), we have

$$\gamma_2 = a(q^*(z_{1,N}) + y_1^*(z_{1,N}) + y_N^*(z_{1,N}) + z_{1,N}). \quad (80)$$

We notice that if (77) holds, then $z_{1,N} = 0$ and (80) becomes

$$\gamma_2 = a(q^*(0) + y_1^*(0) + y_N^*(0)). \quad (81)$$

Next, we consider three cases separately:

Case I) If $y_1^*(0) > 0$ and $y_N^*(0) > 0$, then

$$\gamma_1 = a\left(q^*(0) + y_1^*(0) + y_N^*(0)\right) + a y_1^*(0), \quad (82)$$

and

$$\gamma_N = a\left(q^*(0) + y_1^*(0) + y_N^*(0)\right) + a y_N^*(0). \quad (83)$$

From Proposition 2, we have

$$y_N^*(0) = \frac{2\gamma_2 - \gamma_1 - a q^*(0)}{3a}. \quad (84)$$

However, replacing (77) in (84), we have $y_N^*(0) \le 0$. This contradicts the initial assumption that $y_N^*(0) > 0$. Thus, (76) does *not* occur. Thus, *Condition 2* does not hold in this case.

Case II) If $y_1^*(0) > 0$ and $y_N^*(0) = 0$, then

$$\begin{aligned} &\gamma_1 = a\left(q^*(0) + y_1^*(0)\right) + a y_1^*(0), \\ &\gamma_N \le a\left(q^*(0) + y_1^*(0)\right). \end{aligned} \quad (85)$$

From Proposition 2, we have

$$y_1^*(0) = \frac{\gamma_1 - a q^*(0)}{2a}. \quad (86)$$

On the other hand, from (81) and the fact that $y_N^*(0) = 0$,

$$a q^*(0) = \gamma_2 - a y_1^*(0). \quad (87)$$

By replacing (87) in (86) and after reordering the terms,

$$y_1^*(0) = \frac{\gamma_1 - \gamma_2}{a}. \tag{88}$$

From (87) and (88), we can further show that $aq^*(0) = 2\gamma_2 - \gamma_1$. Replacing this in (77), inequality in (76) holds if and only if

$$0 < \gamma_N \leq \frac{\gamma_1}{4} + \frac{2\gamma_2 - \gamma_1}{4} = \frac{\gamma_2}{2}. \tag{89}$$

Therefore, in this case, we have

$$\begin{aligned} &\mathbb{S}(\boldsymbol{y}^*(z_{1,N}=0), z_{1,N}=0) = \gamma_1 \left(\frac{\gamma_1 - \gamma_2}{a}\right) + \gamma_N \times 0 \\ &\quad + \gamma_2 \left(\frac{2\gamma_2 - \gamma_1}{a}\right) - \frac{a}{2}\left(\frac{\gamma_1 - \gamma_2}{a} + \frac{2\gamma_2 - \gamma_1}{a}\right)^2 \\ &= \frac{\gamma_1^2 - 2\gamma_1\gamma_2 + \frac{3}{2}\gamma_2^2}{a}. \end{aligned} \tag{90}$$

From (75) and (90), the worst-case efficiency of Game 2 is obtained by solving the following optimization problem

$$\begin{aligned} \underset{\gamma_1,\gamma_2,\gamma_N}{\textbf{minimize}} \quad & \frac{\gamma_1^2 - 2\gamma_1\gamma_2 + \frac{3}{2}\gamma_2^2}{0.5\left(\gamma_1 + \gamma_N\right)^2} \\ \textbf{subject to} \quad & \gamma_N \leq \frac{\gamma_2}{2}, \\ & \gamma_1 + \gamma_N \geq \gamma_{\max}, \\ & \gamma_N \leq \gamma_1, \\ & 0 < \gamma_1, \gamma_2, \gamma_N \leq \gamma_{\max}. \end{aligned} \tag{91}$$

From the 1st, 3rd, and 4th constraints in (91), we have

$$\gamma_2 \leq \gamma_{\max} \leq \gamma_1 + \gamma_N \leq 2\gamma_1 \quad \Rightarrow \quad \gamma_1 \geq \frac{\gamma_2}{2}. \tag{92}$$

Thus, we can remove constraints $\gamma_N \leq \gamma_1$ and $\gamma_N \leq \gamma_{\max}$. The optimization problem (91) reduces to

$$\begin{aligned} \underset{\gamma_1,\gamma_2,\gamma_N}{\textbf{minimize}} \quad & \frac{\gamma_1^2 - 2\gamma_1\gamma_2 + \frac{3}{2}\gamma_2^2}{0.5\left(\gamma_1 + \gamma_N\right)^2} \\ \textbf{subject to} \quad & \gamma_N \leq \frac{\gamma_2}{2}, \\ & \gamma_1 + \gamma_N \geq \gamma_{\max}, \\ & 0 < \gamma_1, \gamma_2 \leq \gamma_{\max}. \end{aligned} \tag{93}$$

The objective function in (93) is decreasing in γ_N. Thus, the worst-case efficiency occurs at upper bound of γ_N, i.e., when we have $\gamma_N = \frac{\gamma_2}{2}$. Therefore, problem (93) further reduces to

$$\begin{aligned} \underset{\gamma_1,\gamma_2}{\textbf{minimize}} \quad & \frac{\gamma_1^2 - 2\gamma_1\gamma_2 + \frac{3}{2}\gamma_2^2}{0.5\left(\gamma_1 + \frac{\gamma_2}{2}\right)^2} \\ \textbf{subject to} \quad & \gamma_1 + \frac{\gamma_2}{2} \geq \gamma_{\max}, \\ & 0 < \gamma_1, \gamma_2 \leq \gamma_{\max}. \end{aligned} \tag{94}$$

Problem (94) is not a convex minimization problem with respect to variables γ_1 and γ_2. However, we can still solve problem (94) as follows. We first assume that γ_2 is fixed. By solving the Karush-Kuhn-Tucker (KKT) conditions of problem (94) with respect to γ_1, we can identify three KKT points:

$$\gamma_1 = \gamma_{\max}, \tag{95}$$

$$\gamma_1 = \gamma_{\max} - \tfrac{\gamma_2}{2}, \tag{96}$$

$$\gamma_1 = \tfrac{4}{3}\gamma_2. \tag{97}$$

The global minimizer choice of γ_1 is among the above three KKT conditions. We start by replacing (95) in problem (94). After reordering the terms, problem (94) becomes

$$\begin{aligned} \underset{\gamma_2}{\textbf{minimize}} \quad & \frac{\gamma_{\max}^2 - 2\gamma_{\max}\gamma_2 + \frac{3}{2}\gamma_2^2}{0.5\left(\gamma_{\max} + \frac{\gamma_2}{2}\right)^2} \\ \textbf{subject to} \quad & 0 < \gamma_2 \leq \gamma_{\max}. \end{aligned} \tag{98}$$

Problem (98) is convex with respect to γ_2. By taking derivatives, we can show that the minimum occurs when $\gamma_2 = \frac{3}{4}\gamma_{\max}$. Replacing this in the objective function in (98), the worst-case efficiency in this case becomes

$$\frac{2\,\gamma_{\max}^2\left(1 - 2 \times \frac{3}{4} + \frac{3}{2} \times \left(\frac{3}{4}\right)^2\right)}{\gamma_{\max}^2\left(1 + \frac{3}{8}\right)^2} = \frac{4}{11}. \tag{99}$$

Next, we replace (96) in problem (94). The minimum occurs when $\gamma_2 = \frac{6}{11}\gamma_{\max}$ and the worst-case efficiency is $\frac{4}{11}$. Finally, we replace (97) in (94). The worst-case efficiency is obtained as in (99) which occurs when $\gamma_2 = \frac{3}{4}\gamma_{\max}$.

Case III) If $y_1^*(0) = 0$ and $y_N^*(0) = 0$, then

$$\gamma_N \leq \gamma_1 \leq aq^*(0). \tag{100}$$

Furthermore, from (81), we have $\gamma_2 = aq^*(0)$. Thus, in this case, $\gamma_N \leq \gamma_1 \leq \gamma_2$. By following similar steps as in Case II, we can verify that the worst-case efficiency in this case is $\frac{4}{9}$.

Combining the results in Cases I, II, and III, the worst-case efficiency when *Condition 1* and *Condition 2* hold becomes

$$\min\left\{\frac{4}{11}, \frac{4}{9}\right\} = \frac{4}{11}, \tag{101}$$

which occurs when (36) holds. Before concluding this proof, We note that although the above $\frac{4}{11}$ bound also appears in [6, Theorem 8], Theorem 5 in this paper is very different from [6, Theorem 8]. Here, the key concern is to find scenarios where no non-zero network coding rate (and thus no solution of any bargaining approach) can benefit one of the two users involved in network coding. However, this concept was not studied in [6, Theorem 8] or any other part of Section IV in [6]. ■

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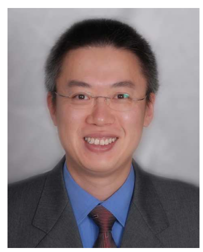

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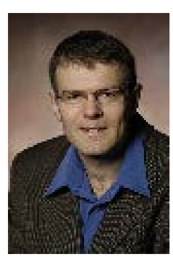

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