Direct Energy Trading of Microgrids in Distribution Energy Market

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Abstract—Recent advancement of distributed renewable generation has motivated microgrids to trade energy directly with one another, as well as with the utility, in order to minimize their operational costs. Energy trading among microgrids, however, confronts challenges such as reaching a fair trading price, maximizing participants’ profit, and satisfying power network constraints. In this paper, we formulate the direct energy trading among multiple microgrids as a generalized Nash bargaining (GNB) problem that involves the distribution network’s operational constraints (e.g., power balance equations, voltage limits). We prove that solving the GNB problem maximizes the social welfare and also fairly distributes the revenue among the microgrids based on their market power. To address the nonconvexity of the GNB problem, we propose a two-phase approach. The first phase involves solving the optimal power flow problem in a distributed fashion using the alternating direction method of multipliers to determine the amount of energy trading. The second phase determines the market clearing price and mutual payments of the microgrids. Simulation results on an IEEE 33-bus system with four microgrids show that the proposed framework substantially reduces total network cost by 37.2%. Our results suggest direct trading need be enforced by regulators to maximize the social welfare.

Keywords: Microgrid, direct energy trading, optimal power flow, distributed optimization, ADMM, generalized Nash bargaining

OMENCLATURE

N Set of buses
E Set of lines
G Graph of distribution network with N and E
M Set of microgrids
M Number of microgrids
t Discrete time slot index
T Set of time slots
V_i(t) Complex voltage of bus i ∈ N in time slot t
v_i(t) |V_i(t)|^2, bus i ∈ N
I_i(j) Complex current from i to j, line (i, j) ∈ E
l_i(j) |I_i(j)|^2, line (i, j) ∈ E
z_{ij} r_{ij} + j\times{l_{ij}}, impedance of line (i, j) ∈ E
r_{ij} Resistance of line (i, j) ∈ E
x_{ij} Reactance of line (i, j) ∈ E
S_{ij}(t) P_{ij}(t) + jQ_{ij}(t), line (i, j) ∈ E
P_{ij}(t) Active power flow from buses i to j
Q_{ij}(t) Reactive power flow from buses i to j
e_i(t) Exporting power from buses i to j
p_i(t) Injected active power into bus i ∈ N
q_i(t) Injected reactive power into bus i ∈ N
e_i(t) Net exporting power from microgrid i
\pi_i Net payment of microgrid i
u_{i}(t) Selling power of microgrid i to utility
s_{i}(t) Selling power of microgrid i to utility
\gamma_i Net importing power of microgrid i
\delta_i Net importing power of microgrid i
\beta_i Market access fee of microgrid i
\beta \sum_{i \in M} \beta_i, (total overhead cost)
C_i Internal cost of microgrid i
\overset{\text{min}}{C_i} (the cost before direct trading)
\delta_i \overset{\text{min}}{C_i}, obtained by solving P2
\delta_i \overset{\text{min}}{C_i}, reduced cost of microgrid i
\gamma_i \delta_i - \pi_i, profit of microgrid i after payment
m Iteration index

I. INTRODUCTION

In the face of climate change and fossil fuel depletion, securing clean and sustainable energy resources is becoming increasingly important for the future generations. Fortunately, recent development of renewable generations (e.g., wind turbine, photovoltaic (PV) panel) has made sustainable energy economically viable. Unlike the conventional large-scale generators, renewable generators are often small-scale, and thus appropriate for serving microgrids. However, the stochastic nature of renewable energy sources and the fluctuations in load demand can cause microgrids to experience intermittent energy shortage or surplus. To this end, direct energy trading among microgrids can be a viable solution to balance energy and lower the operational cost [1].
Direct trading is beneficial to both sellers and buyers compared to the trading with the utility company by reducing the intermediate trading steps. There exist, however, several challenges in designing a direct energy trading mechanism. First, it is difficult to reach an agreement on the trading price, which should not be biased toward either sellers or buyers. Second, it is crucial to determine the power flow from sellers to buyers while satisfying the distribution network constraints. Third, it would be desirable that direct trading can maximize the social welfare (or equivalently, minimize the total cost of energy generation and operation) so that regulators can advocate direct energy trading with legitimate support.

There have been some efforts in the literature to resolve the first challenge associated with trading price [2]–[7]. For example, auction mechanism is applied for direct trading among microgrids in [2]. A coalition of sellers and buyers is considered in [3] to collectively trade energy with the utility company and share the revenue using the Shapley value. The concept of peer-to-peer trading between any pair of microgrids using Nash bargaining solution is proposed in [7]. However, [2]–[7] do not consider the distribution network constraints.

In resolving the second and third challenges associated with physical constraints and optimal operation, several works study the energy management system of microgrids and/or distribution network, but without a market clearing mechanism [8]–[14]. In these works, microgrids are assumed to cooperate to minimize their aggregate cost in a distributed fashion. A number of works have investigated the decentralized analysis of the optimal power flow (OPF) for the energy trading in a distribution network using different techniques such as the predictor corrector proximal multiplier method [8] and the alternative direction method of multipliers (ADMM) [13]–[15]. A comprehensive survey of solving the OPF is given in [16]. Recently, the authors in [13] consider power flow constraints using dedicated DC connections to minimize the total cost, but ignore the market clearing mechanism. The combination of OPF and direct trading is proposed [17], but a heuristic market clearing may not guarantee social welfare.

The aforementioned challenges have not been addressed fully in a unified framework, and thus designing a direct trading mechanism considering physical constraints and social welfare is still an important problem. In particular, we need to address the following questions: 1) how to determine a systematic bargaining process in terms of the quantity and price, which is not biased toward either sellers or buyers, 2) how to perform direct energy trading for networked microgrids even if there is no dedicated distribution line between two microgrids, and 3) how to design a market mechanism which minimizes the total cost considering physical network constraints and power flows in the distribution network.

In this paper, we provide a framework that can address the above challenges. The proposed framework determines the amount of direct energy trading and the corresponding payment among microgrids, considering the operational constraints imposed by the distribution network. We formulate the problem as a generalized Nash bargaining (GNB) problem with a notion of market power [18]. We summarize our key contributions mainly as follows.

- **Direct Trading Framework**: We design a general market mechanism for direct trading among microgrids considering full AC power flow model for the distribution network. Solving the GNB problem can incentivize microgrids to participate in direct trading rather than trading with the utility company. We prove that solving the GNB problem minimizes the total cost, and thereby the proposed framework maximizes the social welfare while each microgrid can maximize its own profit.

- **Distributed Optimization Methods**: We address the non-convexity and obtain an optimal solution of the GNB problem by first solving the OPF and then clearing the market. To solve the OPF problem in a distributed manner, we leverage ADMM to decouple the optimization variables of the microgrids and the distribution network. This enables us to determine the amount of energy trading while concurrently solving the OPF. Then, the market is cleared by using ADMM in a privacy preserving manner. The proposed market mechanism ensures that the profit of each microgrid is proportional to the amount of energy exchange by exploiting the notion of market power.

Simulation results show that the proposed direct trading can reduce the total network cost by 37.2% compared to the case without direct trading. Furthermore, the costs are reduced by 9–42.8%, the revenues are increased by up to 73% depending on microgrids. The power losses are also reduced by 20.6%. Finally, all participating microgrids fairly achieve the same trading profit per kWh.

Our work is in part related with [7], [19] but differs in several aspects. First, the power flow and physical constraints were not considered in [7]. Our work maximizes the social welfare by leveraging the OPF to determine the direct trading payment and the quantity while satisfying the physical constraints (see Proposition 1). Although the authors of [19] addressed bilateral trading considering the OPF and PV, they did not consider energy storage and thus may not be applicable to general multi-period OPF where optimization variables are coupled over time due to energy buffering in the distributed storage. Furthermore, the objective function of the OPF in [19] neglected the power losses, which can be substantial in the distribution network. In addition, in [7], [19], all microgrids achieve the same profits irrespective of the amount of energy trading, e.g., either it is 10 kWh or 1 MWh, which raises a fairness issue. To resolve this issue, we introduce the notion of market power, which ensures that the trading profit per unit energy is equal (see Proposition 2).

The rest of this paper is organized as follows. In Section II we describe the overall system model including the distribution network and the components within each microgrid. We formulate our direct trading with power flow problem in Section III. We develop distributed algorithms for solving the OPF and clearing the market in Section IV. Simulation results are provided in Section V. Conclusion is given in Section VI.

II. SYSTEM MODEL

In this section we present a system model for direct energy trading among microgrids deployed in a distribution network.
Consider a radial distribution network represented by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of branches in the network. We consider a set $\mathcal{M} \subseteq \mathcal{N}$ of $M = |\mathcal{M}|$ microgrids. Let $0 \in \mathcal{N}$ denote the slack bus of $\mathcal{G}$, where the utility is connected as an external power source of the distribution network. The overall structure of direct energy trading among microgrids in a distribution network is shown in Fig. 1. Let $T = \{1, \ldots, T\}$ denote the operational horizon, which is divided into $T$ time slots with equal duration (e.g., one hour) denoted by $\Delta t$.

Assuming a balanced three-phase system, we provide the per-phase analysis. For bus $i \in \mathcal{N}$, let $V_i(t)$ denote the complex voltage in time slot $t \in T$, and let $s_i(t) = p_i(t) + jq_i(t)$ denote the complex power injection into bus $i$, where $i^2 = -1$. For line $(i, j) \in \mathcal{E}$, let $z_{ij} = r_{ij} + jx_{ij}$ denote the line impedance, and $I_{ij}(t)$ denote complex current from bus $i$ to bus $j$ in time slot $t$. For branch $(i, j) \in \mathcal{E}$ and time slot $t \in T$ we have $V_i(t) - V_j(t) = z_{ij}I_{ij}(t)$. Let $V^*_i(t)$ denote the complex conjugate of $I_{ij}(t)$. Then, the complex power flow in line $(i, j) \in \mathcal{E}$ is defined by $S_{ij}(t) = V_i(t)I^*_{ij}(t)$ from which the real power $P_{ij}(t)$ and the reactive power $Q_{ij}(t)$ are determined such that $S_{ij}(t) = P_{ij}(t) + jQ_{ij}(t)$.

The power balance equation for bus $j \in \mathcal{N}$ is given by $s_j(t) = S_{ij}(t) - z_{ij}I^*_{ij}(t)^2 - \sum_{k \neq i;j(k) \in \mathcal{E}} S_{jk}(t)$. Let $l_{ij}(t) = |I_{ij}(t)|^2$ and $v_i(t) = |V_i(t)|^2$. Using the branch flow model in [20], we have the following equations with real variables for all $(i, j) \in \mathcal{E}$ and $t \in T$,

\[ p_{ij}(t) = P_{ij}(t) - r_{ij}l_{ij}(t) - \sum_{k \neq i;j(k) \in \mathcal{E}} P_{jk}(t), \]

\[ q_{ij}(t) = Q_{ij}(t) - x_{ij}l_{ij}(t) - \sum_{k \neq i;j(k) \in \mathcal{E}} Q_{jk}(t), \]

\[ v_i(t) = v_i(t) - 2(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)) + (r_{ij}^2 + x_{ij}^2)l_{ij}(t), \]

\[ l_{ij}(t) = \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)}. \]

We consider the following voltage tolerance constraint:

\[ v^\text{min}_i \leq v_i(t) \leq v^\text{max}_i, \quad i \in \mathcal{N} \setminus \{0\}, \]

where $v^\text{min}_i$ and $v^\text{max}_i$ are the minimum and maximum voltage magnitude of bus $i$, respectively.

### B. Microgrid’s Model

As shown in Fig. 1, the microgrids are interconnected by the distribution network $\mathcal{G}$ through which energy can be traded. For energy trading, only active power can be traded with the utility company or with other microgrids. We assume that each microgrid can be considered as a single bus, but our work can be extended to the case where a microgrid corresponds to multiple buses.

We assume that microgrid $i \in \mathcal{M}$ has its own renewable or fuel-based distributed generator (DG), energy storage, and local loads. The goal of each microgrid is to minimize its total operational cost which includes the cost of purchasing energy from the utility, battery degradation cost, and fuel-based distributed generation operational cost. Similar to [7], [8], one may also include demand response. For the sake of simplicity, we assume fixed loads that can be forecasted with reasonably good accuracy.

1) **Power trading with the utility**: Let $u_{b,i}(t)$ denote the power purchased from the utility company by microgrid $i$ and $\mu_{u,i}(t)$ denote the purchasing price ($/\text{MWh}$) in time slot $t$. Due to the physical or contractual power limit, we have

\[ 0 \leq u_{b,i}(t) \leq u^{\text{max}}_{b,i}, \quad i \in \mathcal{M}, \quad t \in T, \]

where $u^{\text{max}}_{b,i}$ denotes the maximum purchasing power of microgrid $i$. Let $d_i(t)$ denote the load of microgrid $i \in \mathcal{M}$ in time slot $t$. Due to the stochastic nature of the renewable generation, the local generation level of a microgrid can exceed the total local load demands. Then, the microgrid can sell its surplus power to the utility at selling price ($/\text{MWh}$) $\mu_s(t)$ in time slot $t$. The amount of selling power, denoted by $u_{s,i}(t)$, is also subject to the physical or contractual power limit:

\[ 0 \leq u_{s,i}(t) \leq u^{\text{max}}_{s,i}, \quad i \in \mathcal{M}, \quad t \in T, \]

where $u^{\text{max}}_{s,i}$ denotes the maximum selling power of microgrid $i$. Then, the cost of purchasing power by microgrid $i \in \mathcal{M}$ from the utility during time period $T$ is

\[ C_{u,i}(u_i) = \sum_{t \in T} [\mu_u(t)u_{b,i}(t) - \mu_s(t)u_{s,i}(t)]\Delta t, \]

where $u_i = (u_{b,i}(t), u_{s,i}(t), t \in T)$ is a power trading profile of microgrid $i \in \mathcal{M}$ with the utility.

2) **Battery operation**: The charging and discharging powers of microgrid $i$ in time slot $t$, denoted by $b_{c,i}(t)$ and $b_{d,i}(t)$, are limited by the capacity of power conditioning system such that

\[ 0 \leq b_{c,i}(t) \leq b^{\text{max}}_{c,i}, \]

\[ 0 \leq b_{d,i}(t) \leq b^{\text{max}}_{d,i}, \]

where $b^{\text{max}}_{c,i}$ and $b^{\text{max}}_{d,i}$ are the maximum of charging power and discharging power of the battery in microgrid $i$, respectively. The stored energy in the battery $E_{b,i}(t)$ changes according to the following equation:

\[ E_{b,i}(t + 1) = E_{b,i}(t) + \left( \eta_{c,i}b_{c,i}(t) - \frac{1}{\eta_{d,i}}b_{d,i}(t) \right) \Delta t, \]

where $\eta_{c,i}$ and $\eta_{d,i}$ are the charging and discharging efficiencies of microgrid $i$. Since battery degradation is known to be
severe at both ends of the state-of-charge (SoC), i.e., either empty or full, \(E_{b,i}(t)\) should be constrained by [21]

\[
\text{SoC}_{i}^\text{min} \leq \frac{E_{b,i}(t)}{E_{b,i}^\text{max}} \leq \text{SoC}_{i}^\text{max},
\]

where \(\text{SoC}_{i}^\text{min}\) and \(\text{SoC}_{i}^\text{max}\) denote the minimum and maximum SoC of the battery and \(E_{b,i}^\text{max}\) denotes the maximum battery capacity in microgrid \(i\).

Although the battery degradation depends on the SoC, the degradation density function of the SoC is almost flat between \(\text{SoC}_{i}^\text{min}\) and \(\text{SoC}_{i}^\text{max}\) [22], [23]. Thus, the battery degradation cost can be computed by the amount of transferred energy:

\[
C_{b,i}(b_i) = c_{b,i} \sum_{t \in \mathcal{T}} [b_{c,i}(t) + b_{d,i}(t)] \Delta t,
\]

where \(b_i = (b_{c,i}(t), b_{d,i}(t), t \in \mathcal{T})\) and \(c_{b,i}\) is the degradation cost coefficient per unit energy.

3) Distributed generation cost: Let \(r_i(t)\) denote renewable generation of microgrid \(i \in \mathcal{M}\) in time slot \(t\). We assume that \(r_i(t)\) can be predicted reasonably well as in [7], [8]. Renewable generation is assumed to have zero marginal cost in the short run [7]. On the other hand, fuel-based generation such as fuel cell, distributed micro turbine or diesel generator has a nonlinear cost function [8], [24]. We use the following quadratic cost function for a fuel-based DG in microgrid \(i\):

\[
C_{g,i}(g_i) = \sum_{t \in \mathcal{T}} (\kappa_{2,i} g_i(t)^2 + \kappa_{1,i} g_i(t) + \kappa_{0,i}) \Delta t,
\]

where \(g_i = (g_i(t), t \in \mathcal{T})\), and the positive coefficients of \(\kappa_{2,i}, \kappa_{1,i}\), and \(\kappa_{0,i}\) depend on the type of DG. The output power of DG in microgrid \(i\) is bounded by

\[
g_i^{\text{min}} \leq g_i(t) \leq g_i^{\text{max}},
\]

where \(g_i^{\text{min}}\) and \(g_i^{\text{max}}\) are the minimum and maximum generation capacities in microgrid \(i\), respectively.

4) Total cost of microgrid: The active power balance equation at microgrid \(i \in \mathcal{M}\) in time slot \(t \in \mathcal{T}\) is

\[
r_i(t) + g_i(t) + u_{b,i}(t) + b_{d,i}(t) = d_i(t) + u_{s,i}(t) + b_{c,i}(t),
\]

where \(d_i(t)\) is the real power demand of microgrid \(i\) in time slot \(t\). Then, the left-hand side corresponds to the power generation and the right-hand side corresponds to the power demand. Then, the internal cost function of microgrid \(i \in \mathcal{M}\) is given by

\[
\bar{C}_i(u_i, b_i, g_i) = C_{u,i}(u_i) + C_{b,i}(b_i) + C_{g,i}(g_i).
\]

5) Microgrid’s local optimization problem: If microgrid \(i \in \mathcal{M}\) does not participate in direct energy trading with other microgrids, it solves the following optimization problem:

\textbf{P0: Microgrid’s Optimization without Direct Trading}

\[
\begin{align*}
\text{minimize} & \quad \bar{C}_i(u_i, b_i, g_i) \\
\text{subject to} & \quad (6), (7), (9a)-11), (14), (15), \\
\text{variables} & \quad \{u_i, b_i, g_i\}.
\end{align*}
\]

Problem \textbf{P0} is a convex problem since the objective function and all constraints are convex. Problem \textbf{P0} is solved by microgrid \(i \in \mathcal{M}\). The optimal value is denoted by \(\bar{C}_i\).

### III. GNB for Direct Energy Trading

In this section, we describe the framework of direct energy trading by exploiting the concept of GNB. Although direct trading can increase the total benefit of the microgrids, it is not clear how to share the increased revenue among the microgrids in a fair manner. In other words, the direct trading price should be impartial to both sellers and buyers so that all participants can agree on it. To resolve direct trading and bargaining, we leverage GNB, which provides a fair Pareto optimal solution that satisfies the four axioms as follows [18];

1) **Individual rationality:** The bargaining solution should increase the benefits\(^1\) of all microgrids participating in direct trading. Otherwise, they would not participate.

2) **Pareto optimality:** At the bargaining solution, one cannot increase the benefit of a microgrid unless it decreases the benefits of some other microgrids.

3) **Independence of irrelevant alternatives:** If the bargaining solution is found on a subset of the feasible set of all possible benefits, then the solution does not change for a feasible set that contains the subset.

4) **Independence of linear transformations:** The bargaining solution is invariant by scaling the benefits and the minimum costs using a linear transformation.

GNB is an optimization problem of maximizing the Nash product, as we will show in (27). The GNB problem differs from the Nash bargaining by removing the axiom of symmetry, and thus can capture the scenario where players have different market powers.

#### A. Problem Formulation Using GNB

When microgrid \(i \in \mathcal{M}\) has energy surplus or deficit, it can trade power through the distribution network \(\mathcal{G}\) as shown in Fig. 1. Let \(e_{ij}(t)\) denote the exporting power from microgrid \(i\) to microgrid \(j\). In a lossless power network, we have \(e_{ij}(t) + e_{ji}(t) = 0\). However, the power losses may not be negligible. Specifically, in the distribution network, we have

\[
e_{ij}(t) + e_{ji}(t) = r_{ij} I_{ij}(t).
\]

Note that the power losses depend on \(I_{ij}(t) = |I_{ij}(t)|^2\), i.e., the solution of OPF. However, before solving the OPF, we do not know the feasibility of direct trading between microgrids \(i\) and \(j\) due to physical constraints. To overcome the complexity to trace \(e_{ij}(t)\) for all tradable \((i, j)\) pairs of microgrids, we focus on the net exporting power \(e_i(t)\) to all other microgrids, which is defined as

\[
e_i(t) = \sum_{j \in \mathcal{M} \setminus \{i\}} e_{ij}(t).
\]

\(^1\)In game theory, the term utility is used. However, in this paper, we reserve the utility to denote an electricity providing company.
or comes from, but only the net export/import power flows. Then, the net power injection into microgrid \( i \) becomes
\[
p_i(t) = u_{b, i}(t) - u_{s, i}(t) - e_i(t), \quad i \in \mathcal{M}, t \in T.
\] (19)

The power balance equation of (15) becomes
\[
r_i(t) + g_i(t) + u_{b, i}(t) + b_{d, i}(t)
= d_i(t) + e_i(t) + u_{s, i}(t) + b_{c, i}(t), \quad i \in \mathcal{M}, t \in T.
\] (20)

We also have the following constraint
\[
\sum_{i \in \mathcal{M}} e_i(t) = 0, \quad t \in T,
\] (21)

which implies that the sum of all exporting powers should be equal to the sum of all importing powers in time slot \( t \). Let \( e_i = (e_i(t), t \in T) \) denote the trading profile of microgrid \( i \in \mathcal{M} \). To incentivize direct trading, the cost after direct trading should be less than or equal to the cost before direct trading \( \mathcal{C}_i \). To determine the cost after direct trading, we consider two other factors: the distribution network access fee of microgrid \( i \), denoted by \( \beta_i \), and the direct trading payment of microgrid \( i \) denoted by \( \pi_i \). Microgrid \( i \) participates in the direct energy trading only if
\[
\tilde{C}_i(u_i, b_i, g_i) + \beta_i + \pi_i \leq C_i, \quad i \in \mathcal{M}.
\] (22)

We consider a non-profit organization called distribution system operator (DSO), which manages and balances the distribution network. The DSO should compensate power losses in the distribution network by purchasing power from the utility company through the slack bus. Thus, DSO imposes an access fee \( \beta_i \) for microgrid \( i \in \mathcal{M} \) to cover the overhead cost for direct trading. Let \( \beta = \sum_{i \in \mathcal{M}} \beta_i \) denote the total overhead cost. Then, we have
\[
\beta = \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{E}} r_{ij}l_{ij}(t)\xi(t),
\] (23)

where \( \xi(t) \) is a coefficient that accounts for the overhead cost from the power losses and network maintenance. For example, when \( \xi(t) \) is equal to \( \mu_0(t) \), the access fee accounts for the cost from power losses. Note that determining \( \xi(t) \) requires the detailed analysis on the operational cost and is beyond the scope of this paper. When the access fee \( \beta_i \) is imposed in proportion to the amount of direct trading, we have
\[
\beta_i = \frac{\sum_{t \in T} |e_i(t)|}{\sum_{j \in \mathcal{M}} \sum_{t \in T} |e_j(t)|} \beta, \quad i \in \mathcal{M}.
\] (24)

The access fee \( \beta_i \) in proportion to the amount of traded energy is one way of imposing the grid fee, and determining the specific rule is an active ongoing research area, see [25]–[27]. Then, the cost for microgrid \( i \) including the access fee is defined by
\[
C_i(u_i, b_i, g_i) = \tilde{C}_i(u_i, b_i, g_i) + \beta, \quad i \in \mathcal{M}.
\] (25)

Finally, the payment of one microgrid becomes the revenue of the other microgrids, and the sum of payments is zero, i.e.,
\[
\sum_{i \in \mathcal{M}} \pi_i = 0.
\] (26)

Then, the GNBD problem is formulated as follows.

**PI: Generalized Nash Bargaining (GNB) Problem**

\[
\begin{align*}
\text{maximize} & \quad \prod_{i \in \mathcal{M}} \left[ C_i(u_i, b_i, g_i) + \pi_i \right]^\alpha_i \\
\text{subject to} & \quad (1)–(5), (6), (7), (9a)–(11), (14), \\
& \quad (19)–(22), (26),
\end{align*}
\] (27)

variables \( \{e_i, u_i, b_i, g_i, \pi_i, i \in \mathcal{M}, P, Q, v, \ell, s\} \),

where \( P = (P_{ij}(t), (i, j) \in \mathcal{E}, t \in T), Q = (Q_{ij}(t), (i, j) \in \mathcal{E}, t \in T), v = (v_i(t), i \in \mathcal{N} \setminus \{0\}, t \in T), I = (l_{ij}(t), (i, j) \in \mathcal{E}, t \in T), s = (s_i(t), i \in \mathcal{N}, t \in T) \),

the positive parameter \( \alpha_i \) denotes the market power of microgrid \( i \in \mathcal{M} \). We introduce the market power \( \alpha_i \) to reflect the different bargaining power of microgrids in the bargaining process.

For simplicity, we consider the case when constraint (22) is satisfied with strict inequality for all \( i \in \mathcal{M} \), i.e., all microgrids can be better off by participating in direct trading. When some microgrids turn out to have all zero trading profile \( e_i \) after solving the OPF, and thus have no need to trade energy, those microgrids can be simply excluded and will not alter the solution structure.

**B. Analyzing Problem Structure**

Even if we take logarithm of the objective function of \( \textbf{PI} \), it still belongs to nonconvex optimization because (4) is a quadratic equality constraint, i.e., nonconvex constraint. Furthermore, (25) is a nonconvex function because, even though \( \tilde{C}_i(u_i, b_i, g_i) \) is a convex function of \( u_i, b_i, g_i \), the access fee \( \beta_i \) is a nonconvex function of the trading profiles \( e_j, j \in \mathcal{M} \) and all other variables associated with power flows.

Thus, instead of solving \( \textbf{PI} \), we provide Proposition 1 stating that the solution of \( \textbf{PI} \) also minimizes the total cost of the distribution network, which gives us a way to detour in solving \( \textbf{PI} \) in two separate steps: solving the OPF and then determining the market clearing. Then, we will show that this two-phase approach is indeed the solution of \( \textbf{PI} \).

**Proposition 1 (Social Welfare Maximization):** Let \( C^*_i \) denote the optimal value of \( C_i(u_i, b_i, g_i) \) at the solution of \( \textbf{PI} \). Then, the solution of \( \textbf{PI} \) minimizes the total cost of the distribution network, which is given by \( \sum_{i \in \mathcal{M}} C^*_i \).

The proof is given in Appendix A.

**Remark 1:** The implication of Proposition 1 is as follows. The myopic desire of maximizing one’s own benefit by direct trading turns out to be beneficial to the entire society and maximize social welfare. Thus, Proposition 1 underpins the virtue of direct trading from the view point of regulators.

**IV. DISTRIBUTED ALGORITHMS**

In this section, we develop distributed algorithms that solve \( \textbf{PI} \) in two steps. Since the solution of \( \textbf{PI} \) minimizes the total cost as shown in Proposition 1, we first solve the OPF problem in the distribution network. Subsequently, we determine the payments among microgrids based on the solution of the OPF problem. The problem structure and the corresponding decomposition are shown in Fig. 2.
A. OPF and Its Relaxation

The OPF problem is given as follows:

**OPF Problem**

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in \mathcal{M}} \tilde{C}_i(u_i, b_i, g_i) + \sum_{t \in T} \sum_{(i,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t) \\
\text{subject to} & \quad (1) - (5), (6), (7), (9a)-(11), (14), (19)-(21), \\
\text{variables} & \quad \{e_i, u_i, b_i, g_i, i \in \mathcal{M}, \mathcal{P}, \mathcal{Q}, \mathcal{V}, \mathcal{I}, \mathcal{S}\}.
\end{align*}
\]

Note that the cost function is equal to \( \sum_{i \in \mathcal{M}} C_i(u_i, b_i, g_i) = \sum_{i \in \mathcal{M}} \left( \tilde{C}_i(u_i, b_i, g_i) + \beta_i \right) \) according to (23) and (24). The distribution network constraints are given by (1)–(5), the constraints of microgrids are given by (6), (7), (9a)–(11), (14), and the power balancing constraints with trading are given by (19)–(21).

The OPF problem is nonconvex due to the quadratic equality constraint of (4). We apply convex relaxation by replacing (4) with the inequality constraint:

\[
l_{ij}(t) \geq \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)},
\]

which gives us the following relaxed OPF problem.

**OPF-r Problem**

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in \mathcal{M}} \tilde{C}_i(u_i, b_i, g_i) + \sum_{t \in T} \sum_{(i,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t) \\
\text{subject to} & \quad (1) - (3), (5), (28), \\
& \quad (6), (7), (9a)-(11), (14), (19)-(21), \\
\text{variables} & \quad \{e_i, u_i, b_i, g_i, i \in \mathcal{M}, \mathcal{P}, \mathcal{Q}, \mathcal{V}, \mathcal{I}, \mathcal{S}\}.
\end{align*}
\]

The OPF-r is a convex optimization problem, and the relaxation is exact for radial networks as verified in standard IEEE test buses and practical power networks [28], [29]. The sufficient condition for the exactness is when the bus voltage is kept around the nominal value, line impedance is not severe, and the power injection at each bus is not too large. Thus, we assume that convex relaxation is exact, which will be verified by numerical experiments in Section V.

B. Distributed Algorithm for Solving OPF-r

In solving the OPF-r, we leverage ADMM that solves the problems having the following form [30]

\[
\begin{align*}
\text{minimize} & \quad f(x) + g(z) \\
\text{subject to} & \quad Ax + Bz = c, \\
& \quad x \in \mathcal{X}, z \in \mathcal{Z},
\end{align*}
\]

where \( x, z, c \) are vectors, \( A \) and \( B \) are matrices, \( f(x) \) and \( g(z) \) are convex functions, and \( \mathcal{X}, \mathcal{Z} \) are convex sets. The augmented Lagrangian is given by

\[
L(x, z, \lambda) = f(x) + g(z) + \lambda^T (Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|^2,
\]

where \( \rho > 0 \). Then, the distributed updates of the optimization variables \( x, z, \) and the Lagrange multiplier (also called dual variable) \( \lambda \) with an iteration index \( m \) are

\[
\begin{align*}
x^{(m+1)} &= \text{argmin}_{x \in \mathcal{X}} L(x, z^{(m)}, \lambda^{(m)}), \\
z^{(m+1)} &= \text{argmin}_{z \in \mathcal{Z}} L(x^{(m+1)}, z, \lambda^{(m)}), \\
\lambda^{(m+1)} &= \lambda^{(m)} + \rho (Ax^{(m+1)} + Bz^{(m+1)} - c).
\end{align*}
\]

The iteration of two-block ADMM converges to an optimal solution of (29) [30]. In order to exploit ADMM, we reformulate the OPF-r problem as follows.

**P2: OPF-r Problem with ADMM**

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in \mathcal{M}} \tilde{C}_i(u_i, b_i, g_i) + \sum_{t \in T} \sum_{(i,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t) \\
\text{subject to} & \quad \text{MG: (6), (7), (9a)-(11), (14), (20),} \\
& \quad \text{NET: (1)-(3), (5), (19), (21), (28),} \\
& \quad \text{AUX: } e_i = e_i, u_i = u_i, i \in \mathcal{M}, \\
\text{variables} & \quad \{e_i, u_i, b_i, g_i, e_i, u_i, i \in \mathcal{M}, \mathcal{P}, \mathcal{Q}, \mathcal{V}, \mathcal{I}, \mathcal{S}\},
\end{align*}
\]

where the first and the second terms in the objective function correspond to \( f(x) \) and \( g(z) \) in (29), respectively, with \( x = (e_i, u_i, b_i, g_i, i \in \mathcal{M}), z = (e_i, u_i, i \in \mathcal{M}, \mathcal{P}, \mathcal{Q}, \mathcal{V}, \mathcal{I}, \mathcal{S}). \) The constraints of MG, NET, AUX correspond to \( \mathcal{X}, \mathcal{Z} \) and \( Ax + Bz = c, \) respectively. Then, the augmented Lagrangian \( L \) is

\[
L(e_i, u_i, b_i, g_i, e_i, u_i, \lambda_i, i \in \mathcal{M}, \mathcal{P}, \mathcal{Q}, \mathcal{V}, \mathcal{I}, \mathcal{S})
\]

\[
= \sum_{i \in \mathcal{M}} \tilde{C}_i(u_i, b_i, g_i) + \sum_{t \in T} \sum_{(i,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t)
\]

\[
+ \sum_{i \in \mathcal{M}} (\lambda^{x}_{e,i}(e_i - e_i) + \lambda^{x}_{u,i}(u_i - u_i))
\]

\[
+ \frac{\rho}{2} \|e_i - e_i\|^2 + \frac{\rho}{2} \|u_i - u_i\|^2,
\]

where \( \lambda_i = (\lambda^{x}_{e,i}, \lambda^{x}_{u,i}) \) and \( \lambda^{x}_{e,i}, \lambda^{x}_{u,i} \) are dual variable vectors for the constraints of \( e_i = \hat{e}_{i}^{(m)} \) and \( u_i = \hat{u}_{i}^{(m)} \), respectively. Then, using the update rules of (30a)–(30c), we now develop the following MG \( i \) update rule, DSO update rule, and the dual variables update rule.

For MG \( i \) update, at the \( (m+1) \)th iteration, each microgrid \( i \in \mathcal{M} \) solves the following optimization problem with \( \hat{e}_{i} = \hat{e}_{i}^{(m+1)}, \hat{u}_{i} = \hat{u}_{i}^{(m+1)} \), and \( \lambda_i = \lambda_i^{(m)} \).

**MG_i update**

\[
\begin{align*}
\text{minimize} & \quad \tilde{C}_i(u_i, b_i, g_i)
\quad + \lambda^{T}_{e,i}(e_i - e_i) + \lambda^{T}_{u,i}(u_i - u_i)
\quad + \frac{\rho}{2} \|e_i - e_i\|^2 + \frac{\rho}{2} \|u_i - u_i\|^2
\text{subject to} & \quad (6), (7), (9a)-(11), (14), (20),
\text{variables} & \quad \{e_i, u_i, b_i, g_i\}.
\end{align*}
\]
and $u_i = u_{i}^{(m+1)}$, which are used for DSO update below.

**DSO update**

$$\begin{align*}
\text{minimize} & \sum_{i \in \mathcal{T}} \sum_{(s,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t) \\
& + \sum_{m} \left( \lambda_{m}^{T} (\hat{e}_i - e_i) + \lambda_{m}^{T} (\hat{u}_i - u_i) + \frac{\rho}{2} ||\hat{e}_i - e_i||^2 + \frac{\rho}{2} ||\hat{u}_i - u_i||^2 \right) \\
\text{subject to} & (1) - (3), (5), (19), \sum_{i \in \mathcal{M}} e_i = 0, (28), \\
\text{variables} & \{\hat{e}_i, \hat{u}_i, i \in \mathcal{M}, \mathbf{P}, \mathbf{Q}, \mathbf{v}, l, s\}. \\
\end{align*}$$

The DSO update is a convex optimization problem, and the optimal solution can be obtained. Let the solution of DSO update be labeled as $\hat{e}_i^{(m+1)}$ and $\hat{u}_i^{(m+1)}$. Using the solutions of MG $i$ update and DSO update, dual variables are updated as follows.

**Dual variable update**

$$\begin{align*}
\lambda_{e, i}^{(m+1)} &= \lambda_{e, i}^{(m)} + \rho (\hat{e}_i^{(m+1)} - e_i^{(m+1)}), \\
\lambda_{u, i}^{(m+1)} &= \lambda_{u, i}^{(m)} + \rho (\hat{u}_i^{(m+1)} - u_i^{(m+1)}). \\
\end{align*}$$

Then, the iteration of the MG $i$ update, the DSO update, and the dual update converges to an optimal solution. The proposed method using ADMM is scalable in the number of microgrids due to the structure of the distributed algorithm where the microgrids and the DSO solve their own optimization problems in parallel.

**C. Market Clearing Problem**

Next we address the market clearing problem. Let $C_i^0$ denote the optimal cost of microgrid $i \in \mathcal{M}$ obtained after solving P2. The payment $\pi_i$, $i \in \mathcal{M}$ can be determined using the minimum cost $C_i^0$, $i \in \mathcal{M}$. Note that we use $C_i^0$ instead of $C_i^*$ (which comes from the solutions of P1) because they may not necessarily be the same. After substituting $C_i^0$ into P1, we have the following market clearing problem.

**P3: Market Clearing Problem**

$$\begin{align*}
\text{maximize} & \prod_{i \in \mathcal{M}} \left( \bar{C}_i - (C_i^0 + \pi_i) \right)^{\alpha_i} \\
\text{subject to} & \sum_{i \in \mathcal{M}} \pi_i = 0 \\
\text{variables} & \{\pi_i, i \in \mathcal{M}\}. \\
\end{align*}$$

Although P3 has a closed-form solution of (45) as shown in the Appendix D, it requires $\bar{C}_i - C_i^0$ to be known by the coordinator. In fact, we can solve P3 in a distributed manner to preserve privacy using ADMM. By taking logarithm of (36), we can have the following convex optimization problem:

$$\begin{align*}
\text{minimize} & \sum_{i \in \mathcal{M}} -\alpha_i \log(\delta_i - \pi_i) \\
\text{subject to} & \sum_{i \in \mathcal{M}} \hat{\pi}_i = 0 \\
& \pi_i = \hat{\pi}_i, i \in \mathcal{M} \\
\text{variables} & \{\pi_i, \hat{\pi}_i, i \in \mathcal{M}\}, \\
\end{align*}$$

where $\delta_i = \bar{C}_i - C_i^0$ is the reduced cost of microgrid $i$ by participating in direct energy trading. Let $L_i = -\alpha_i \log(\delta_i - \pi_i) + \lambda_i (\hat{\pi}_i - \pi_i) + \frac{\rho}{2} (\hat{\pi}_i - \pi_i)^2$, and the augmented Lagrangian be $L = \sum_{i \in \mathcal{M}} L_i$. Then, we apply ADMM with $\pi_i$ update, $\hat{\pi}_i$ update and the dual variable $\lambda_i$ update. Note that this optimization is known as exchange ADMM due to the constraint (38), and the ADMM iteration can be further simplified as follows [30]. Microgrid $i \in \mathcal{M}$ solves the following optimization problem:

$$\begin{align*}
\hat{\pi}_i^{(m+1)} &= \arg\min_{\hat{\pi}_i} -\alpha_i \log(\delta_i - \pi_i) - \lambda_i \pi_i \\
& + \frac{\rho}{2} (\hat{\pi}_i - \pi_i)^2. \\
\end{align*}$$

Then, the $\hat{\pi}_i$ update from the coordinator is given by

$$\hat{\pi}_i^{(m+1)} = \pi_i^{(m+1)} - \sum_{i \in \mathcal{M}} \lambda_i^{(m+1)},$$

where $\sum_{i \in \mathcal{M}} \lambda_i^{(m+1)} = \frac{1}{M} \sum_{i \in \mathcal{M}} \pi_i^{(m)}$. Finally, since $\lambda_i, i \in \mathcal{M}$ are all equal in the exchange ADMM [30], only $\lambda$ need be updated,

$$\lambda_i^{(m+1)} = \lambda_i^{(m)} + \frac{\rho}{2} \pi_i^{(m+1)}.$$ 

Now we analyze the property of the optimal solution of P3 by providing Proposition 2.

**Proposition 2 (Fairness):** If the market power $\alpha_i$ is set in proportion to the total traded energy, i.e.,

$$\alpha_i = \frac{\sum_{t \in \mathcal{T}} |e_i^0(t)|}{\sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} |e_i^0(t)|}, \quad i \in \mathcal{M},$$

where $e_i^0(t)$ is an optimal solution of P2, then each microgrid has the equal trading profit per unit energy, denoted by $\Gamma$, after market clearing,

$$\Gamma = \frac{\sum_{i \in \mathcal{M}} \delta_i}{\sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} |e_i^0(t)|}.$$ 

The proof is given in Appendix B.

**Remark 2:** The implication of Proposition 2 is that the trading profits per unit energy for sellers and buyers are all equal, which is fair to both parties, by setting the market power in proportion to the amount of traded energy.

**Remark 3:** Let $\gamma_i = \delta_i - \pi_i$ denote the profit, which is the cost difference before and after direct energy trading. When the market power is the same, i.e., $\alpha_i = 1/\mathcal{M}$, then $\gamma_i = \frac{1}{\mathcal{M}} \sum_{j \in \mathcal{M}} \delta_j$. That is, the profit of each microgrid is all equal irrespective of the total traded energy of microgrid $i$, $\sum_{t \in \mathcal{T}} |e_i^0(t)|$. This is the case of [7] and becomes unfair when the amount of total traded energy is different across microgrids. Consider the following example. Suppose that microgrid $i$ sells 1 MWh while microgrid $j$ sells only 1 kWh at the same time. In the problem formulation from [7], their profits become the same, and this will discourage the deployment of large scale renewables in the microgrids. Thus, the notion of market power should be considered to fairly allocate the profits obtained from direct trading.

To illustrate the implication of Proposition 2, we provide the following corollary as an example.
**Corollary 1:** Suppose each microgrid has either a renewable generator or pure load without energy storage. The distribution network is lossless. Then, the microgrids with renewables always sell power, and the microgrids with loads always purchase power. When these microgrids participate in direct trading, the unit price of trading is always the average of the trading prices with the utility company. Indeed, this is true as formally stated in Proposition 3.

The proof is given in Appendix C.

**D. Equivalence of the Solutions**

In Proposition 1, we showed that the solution of **P1** minimizes the total cost, which allows us to solve the OPF. Thus, the solution of **P1** is the sufficient condition for the solution of **P2**. However, it is not clear whether the reverse is true, i.e., the solutions of **P2** and **P3** maximize **P1**. Indeed, this is true as formally stated in Proposition 3.

**Proposition 3 (Converse):** Suppose that the solution of **P1** exists. Then, the solutions of **P2** and **P3** maximize **P1**. The proof is given in Appendix D.

**E. Discussion on Information Exchange Model**

In this section we describe how to implement direct energy trading using a standard communication protocol. Implementing the ADMM-based distributed algorithm requires iterations between the DSO and microgrids. In doing this, we consider IEC 61850 as discussed in [8], [31]. IEC 61850 is originally designed for communications within substation automation systems, but it can be used for information exchange between the DSO and microgrids as well. In IEC 61850, each DER unit corresponds to a logical device (LD), which is composed of several logical nodes (LNs) [31]. To implement direct energy trading, we use the LN named DER energy and/or ancillary services schedule (DSCH) for each microgrid. The DSO reads or writes an array of the timestamps and values using the IEC 61850 abstract communication service interface (ACSI) (e.g., GetDataValues and SetDataValues). Specifically, four DSCH LNs are used for $e_i^{(m)}$, $u_i^{(m)}$, $\lambda_{e,i}^{(m)}$, $\lambda_{u,i}^{(m)}$ so that the DSO sends the control information to each microgrid $i \in M$ in solving **P2** as shown in Fig. 3. Subsequently, each microgrid $i \in M$ reports its local computation results of $e_i^{(m)}$ and $u_i^{(m)}$ to the DSO using the IEC 61850 report control block (RCB). At convergence, we obtain $e_i^{(m)} = \hat{e}_i^{(m)}$, $u_i^{(m)} = \hat{u}_i^{(m)}$, which are the amounts of direct energy trading and the trading with the utility, respectively. The market clearing process can be implemented using IEC 61850 in a similar manner.

In this section, we provide numerical experiments to demonstrate the virtue of the proposed direct trading technique considering four microgrids interconnected in the IEEE 33-bus test system [20], as shown in Fig. 4. We use the time-of-use (ToU) pricing provided by California Independent System Operator (CAISO) [8], as shown in Fig. 5, which serves as the purchasing price from the utility company in our work. The selling price to the utility is set as half of the purchasing price from the utility company. In Case 1, each microgrid solves **P0**, i.e., schedules its battery and/or DG to minimize the cost function. In Case 2, microgrids trade energy directly (i.e., solve **P1**) in two steps: solving the OPF-1 **P2** and solving the payment problem **P3**. All microgrids have batteries, loads and DGs. Each microgrid has its own load profile and renewable generation profile as shown in Fig. 6; microgrid 1 has PV generation, microgrid 2 has no renewable generation, and microgrids 3 and 4 have wind power generation. Note that the PV generation is during daytime while the wind power...
generations are mostly during nighttime. Table I summarizes key parameters in the simulation.

Fig. 7 shows the power purchased from the utility company before and after direct energy trading. It shows the amount of power purchased from the utility company is substantially reduced for all microgrids, especially, for microgrid 2 that has no renewable generation. Instead, all microgrids directly trade energy as shown in Fig. 8. Microgrid 1 exports power during daytime while PV generation is more than the load, but imports power at night. Microgrid 2 having no renewable generation imports power all the time while microgrids 3 and 4 export power mostly in the morning and at night when they have more wind power generation than the loads.

Fig. 9 shows the battery energy levels of four microgrids, and we observe clear distinction before and after direct trading. For example, after direct trading, microgrid 1 discharges from 16:00 to 20:00 to export power when ToU is high. The batteries of other microgrids also show similar patterns. Interestingly, the energy level trajectories are quite different before direct energy trading but become almost identical after direct energy trading. This is because the batteries are scheduled in a coordinated way to minimize the total cost. Thus, our result suggests the way of harmonizing the distributed batteries in a distribution network using market mechanism. Fig. 10 shows the fuel-based distributed generations, and all DGs operate mainly from 15:00 to 21:00 during which ToU is very high. Interestingly, all DGs run in the same manner. This is due to the fact that all DGs have the same cost functions and their operations are determined by solving the OPF. Finally, Fig. 11 shows the voltage profiles of the microgrid buses and confirms that the solution of OPF satisfies the voltage constraints well. Those buses which do not connect to any microgrids also

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value / Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of time slots per day</td>
<td>24</td>
</tr>
<tr>
<td>Battery size</td>
<td>3 MWh</td>
</tr>
<tr>
<td>Maximum battery power</td>
<td>1 MW</td>
</tr>
<tr>
<td>Battery charging efficiency</td>
<td>0.9</td>
</tr>
<tr>
<td>Battery discharging efficiency</td>
<td>0.9</td>
</tr>
<tr>
<td>Battery degradation cost</td>
<td>$10/MWh</td>
</tr>
<tr>
<td>Maximum power of DG</td>
<td>3 MW</td>
</tr>
<tr>
<td>Maximum SoC</td>
<td>0.9</td>
</tr>
<tr>
<td>Minimum SoC</td>
<td>0.1</td>
</tr>
<tr>
<td>maximum voltage (p.u.)</td>
<td>1.05</td>
</tr>
<tr>
<td>minimum voltage (p.u.)</td>
<td>0.95</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>10</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>61.1</td>
</tr>
</tbody>
</table>

Figure 6. Load and renewable generation profiles of different microgrids.

Figure 7. Power purchased from the utility company.

Figure 8. Direct energy trading (positive for exporting).
satisfy the voltage constraints if the solution of $P_2$ exists. Otherwise, direct energy trading is not feasible.

Next, we present the costs before and after direct energy trading (DET) in Table II. The sum of costs covering all four microgrids is significantly reduced from $2246.74$ (Cost before DET) to $1588.66$ (Cost after DET), i.e., 29.3% of reduction. The power loss cost is also reduced from $284$ to $225$ by 20.6%. The total network cost including the power loss cost in the form of access fee. Finally, we confirm that the exactness condition of (4) is well satisfied as shown in Fig. 13; the left-hand side (LHS) and the right-hand side (RHS) of (4) are exactly same for all cases of the buses in all time slots.

<table>
<thead>
<tr>
<th>Metric</th>
<th>MG1</th>
<th>MG2</th>
<th>MG3</th>
<th>MG4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost before DET</td>
<td>372.37</td>
<td>2175.40</td>
<td>16.69</td>
<td>−317.71</td>
</tr>
<tr>
<td>Cost with OPF</td>
<td>439.42</td>
<td>453.21</td>
<td>386.31</td>
<td>84.15</td>
</tr>
<tr>
<td>Access Fee $\beta_i$</td>
<td>54.65</td>
<td>68.32</td>
<td>23.15</td>
<td>79.45</td>
</tr>
<tr>
<td>GNB Payment $\pi_i$</td>
<td>−281.14</td>
<td>1454.53</td>
<td>−460.30</td>
<td>−713.10</td>
</tr>
<tr>
<td>Cost after DET</td>
<td>212.93</td>
<td>1976.07</td>
<td>−50.84</td>
<td>−549.50</td>
</tr>
<tr>
<td>Profit $\gamma_i$</td>
<td>159.44</td>
<td>199.33</td>
<td>67.53</td>
<td>231.78</td>
</tr>
<tr>
<td>Quantity (MWh)</td>
<td>22.408</td>
<td>28.015</td>
<td>9.491</td>
<td>32.577</td>
</tr>
<tr>
<td>Profit per MWh</td>
<td>7.11</td>
<td>7.11</td>
<td>7.11</td>
<td>7.11</td>
</tr>
<tr>
<td>Market power</td>
<td>0.242</td>
<td>0.303</td>
<td>0.103</td>
<td>0.352</td>
</tr>
<tr>
<td>NBS Payment</td>
<td>−286.22</td>
<td>1489.34</td>
<td>−557.29</td>
<td>−645.83</td>
</tr>
<tr>
<td>Cost with NBS</td>
<td>207.85</td>
<td>2010.88</td>
<td>−147.83</td>
<td>−482.23</td>
</tr>
<tr>
<td>Profit</td>
<td>164.52</td>
<td>164.52</td>
<td>164.52</td>
<td>164.52</td>
</tr>
<tr>
<td>Profit per MWh</td>
<td>7.34</td>
<td>5.87</td>
<td>17.33</td>
<td>5.05</td>
</tr>
<tr>
<td>Market Power</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Fig. 12 shows the convergence of the distributed algorithm that solves $P_2$ using ADMM. The convergence of the total cost is fairly fast, i.e., the number of iterations is about 10 as shown in Fig. 12 (a). We also investigate the convergence of ADMM variables in Fig. 12 (b) by observing $\max_i \{||\hat{e}_i - \hat{e}_i||_\infty, ||u_i - \hat{u}_i||_\infty, i \in M \}$ where $|| \cdot ||_\infty$ is the $l_\infty$ norm. The speed of convergence is exponentially fast after 40 iterations. Finally, we confirm that the exactness condition of (4) is well satisfied as shown in Fig. 13; the left-hand side (LHS) and the right-hand side (RHS) of (4) are exactly same for all cases of the buses in all time slots.
VI. CONCLUSION

In this paper, we investigated direct energy trading among microgrids considering both the economical and technical aspects of the distribution power market and network constraints. We formulated direct energy trading as a nonconvex generalized Nash bargaining problem and showed that the problem can be solved by decomposing it into two phases: solving the OPF and solving the payment. In both cases, we leveraged ADMM to decouple the optimization variables of the DSO and microgrids, and to preserve the privacy of microgrids. The proposed DSO-based market mechanism is efficient in maximizing the social welfare and minimizing the network loss, and also fair by guaranteeing the equal trading profit per unit energy among microgrids. Simulation results demonstrated that direct energy trading reduces the total cost including the costs of all microgrids and network loss by 37.2% compared to the case without direct trading. Resolving the stochastic uncertainties of loads and generations remains as future work.

APPENDIX

A. Proof of Proposition 1

For notational simplicity, we omit the variables \((u_i, b_i, g_i)\) in \(C_i\). We prove by contradiction. Let \(\{C_i^*, \pi_i^*, i \in \mathcal{M}\}\) be obtained from the solution of P1. Suppose that \(\sum_{i \in \mathcal{M}} C_i^*\) does not minimize \(\sum_{i \in \mathcal{M}} C_i\). Then, there exists \(C_i'\) such that \(\sum_{i \in \mathcal{M}} C_i' < \sum_{i \in \mathcal{M}} C_i^*\).

Let \(\Delta C_i = C_i' - C_i^*\). Then, we have
\[
\sum_{i \in \mathcal{M}} \Delta C_i < 0. \tag{44}
\]

We consider another cost \(C_i\) and the payment \(\pi_i\) such that \(C_i = C_i^* + \Delta C_i\) for \(i = 1, \ldots, M\) and \(\pi_i = \pi_i^* - \Delta C_i\) for \(i = 1, \ldots, M - 1\), and \(\pi_M = \pi_M^* - \Delta C_M + \epsilon\). Then, plugging in \(C_i\) and \(\pi_i\) gives us
\[
\prod_{i=1}^{M} \left[ C_i - (C_i + \pi_i) \right]^{\alpha_i}
\]
\[
= \prod_{i=1}^{M} \left[ C_i - (C_i^* + \Delta C_i + \pi_i^* - \Delta C_i) \right]^{\alpha_i}
\times \left[ C_M - (C_M^* + \Delta C_M + \pi_M^* - \Delta C_M + \epsilon) \right]^{\alpha_M}
\]
\[
= \prod_{i=1}^{M} \left[ C_i - (C_i^* + \pi_i^*) \right]^{\alpha_i}
\times \left[ C_M - (C_M^* + \pi_M^* + \epsilon) \right]^{\alpha_M}.
\]

From (26) and (44), we have \(\epsilon = \sum_{i \in \mathcal{M}} \Delta C_i < 0\). Thus
\[
\prod_{i=1}^{M} \left[ C_i - (C_i + \pi_i) \right]^{\alpha_i} > \prod_{i=1}^{M} \left[ C_i - (C_i^* + \pi_i^*) \right]^{\alpha_i}.
\]

This contradicts that \(C_i^*\) and \(\pi_i^*\) maximize P1, and it completes the proof.

B. Proof of Proposition 2

By taking log and negating the objective function, we have the following minimization problem

\[
\begin{align*}
\text{minimize} & \quad - \sum_{i \in \mathcal{M}} \alpha_i \log(\delta_i - \pi_i) \\
\text{subject to} & \quad \sum_{i \in \mathcal{M}} \pi_i = 0 \\
\text{variables} & \quad \{\pi_i, i \in \mathcal{M}\}.
\end{align*}
\]

Then, the Lagrangian is given by
\[
L = \sum_{i \in \mathcal{M}} \left( -\alpha_i \log(\delta_i - \pi_i) + \lambda \pi_i \right).
\]

From \(\frac{\partial L}{\partial \pi_i} = 0\), we have \(\pi_i = \delta_i + \frac{\alpha_i}{\lambda}\). From \(\sum_{i \in \mathcal{M}} \pi_i = 0\) and \(\sum_{i \in \mathcal{M}} \alpha_i = 1\), we have \(\frac{1}{\lambda} = -\sum_{i \in \mathcal{M}} \delta_i\). Thus, the payment of microgrid \(i\) is simply given by
\[
\pi_i = \delta_i - \alpha_i \sum_{j \in \mathcal{M}} \delta_j. \tag{45}
\]

Since the profit of microgrid \(i\) is defined as the reduced cost after the payment, we have
\[
\gamma_i = \delta_i - \pi_i = \alpha_i \sum_{j \in \mathcal{M}} \delta_j. \tag{46}
\]

Thus, the total profit \(\sum_{i \in \mathcal{M}} \delta_i\) of direct trading is allocated to each microgrid \(i\) based on its market power \(\alpha_i\). Then,
When direct trading is not applied, a seller\(i\) selling microgrids and a set of buying microgrids, respectively.\[\text{Proof of Corollary 1}\]

Since there is no energy storage, we only need to consider a specific time slot \(t \in T\). Let \(M_s\) and \(M_b\) denote a set of selling microgrids and a set of buying microgrids, respectively. When direct trading is not applied, a seller \(i \in M_s\) injects all renewable generation \(r_i(t)\) into the grid at the unit price of \(\mu_s(t)\). Meanwhile, a buyer \(j \in M_b\) serves its local load \(d_j(t)\) by purchasing power at the unit price of \(\mu_b(t)\). Now, we consider how the price of direct trading is set. The selling microgrids with renewable generation export all their generations, i.e., \(e_i(t) = r_i(t) > 0\), and the buying microgrids import all their required loads, i.e., \(e_j(t) = -d_j(t) < 0\).

For \(i \in M_s\), the cost before trading is \(C_i = \mu_s r_i(t)\), and the cost after trading is \(C_i = 0\) because there is no trading with the utility. Thus, we have \(\delta_i = -\mu_s r_i(t)\). Similarly, for \(j \in M_b\), we have \(C_j = -\mu_b d_j(t)\), \(C_j = 0\), and \(\delta_j = \mu_b d_j(t)\). Then, the total profit is \(\sum_{i \in M_s} \delta_i = \sum_{i \in M_s} -\mu_s r_i(t) + \sum_{j \in M_b} \mu_b d_j(t)\). Since the total selling and buying amounts are the same, \(\sum_{i \in M_s} r_i(t) = \sum_{j \in M_b} d_j(t)\) assuming there is no loss. The total profit is given by

\[
\sum_{i \in M_s} \delta_i = \frac{\mu_s(t) - \mu_b(t)}{2} \sum_{i \in M_s} |e_i(t)|. \tag{48}
\]

Next, we compute the unit selling price. From (43), (45) and (48), we have \(\pi^i_s = -\frac{\mu_s(t) + \mu_b(t)}{2}\), where the negative sign means selling. Similarly, for \(j \in M_b\), the unit buying price is given by \(\pi^j_b = \frac{\mu_s(t) - \mu_b(t)}{2}\).

**References**


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